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Electrostatic quadrupoles

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Abstract

The equations of motion of the charged particle under the action of electric forces in the simple Electrostatic Quadrupole (ESQ) and in the Helical Electrostatic Quadrupole (HESQ) are solved. The HESQ electric field is realized by the four pole tips forming concentric helices of pitch β. The transformation matrices for ESQ and HESQ are found as the basic elements for designing more complex optical systems.

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1 Introduction

Commonly, a system of magnetic lenses as, for example, solenoids and magnetic quadrupoles, is used for focusing and transport of particle beams. On the other hand, a system composed of electrostatic lenses forming a section of linac has been applied only in some cases. For example, a helical electrostatic quadrupole (HESQ) was used for transport and matching of an H^- beam to a RFQ [1]. Note that the beam must be azimuthally symmetric and highly convergent to be matched to the RFQ acceptance.

A similar system for focusing low energy and high current negative Cu^- and Au^- ion beams was developed at the National Laboratory for High Energy Physics (KEK) [2]. Reasons for such a choice exposed in [3] were the following:

1. Electrostatic focusing is more effective at lower particle velocities than magnetic focusing because of the velocity term in the force equation.

2. Difficulties concerning the beam emittance growth caused by large space charge forces in the beam are easily surmountable in the case of electrostatic focusing.

3. Electrostatic focusing is very flexible.

Because of the high voltage required for electrostatic focusing the problem of discharge breakdown arises and spherical as well as chromatic aberrations take place if Einzel lenses or electrostatic quadrupoles are utilized. The helical electrostatic quadrupole provides a more suitable system for the transport and focusing of the beam with low velocities. The focusing forces are continuously spread in space thus reducing the possibility of breakdown and also maintaining the beam size during the transport. This property of a helical electrostatic quadrupole influences favourably the aberrations. The helical electrostatic quadrupole represents a first-order focusing optical system with high focusing power.

The aim of this work is to calculate transformation matrices for a simple electrostatic quadrupole and a helical electrostatic quadrupole. They are the basis for designing much more complex optical systems for transport and focusing of heavy ion beams, which should become a topic of further theoretical studies.

2 Simple electrostatic quadrupole

Let us consider the motion of a charged particle in the electrostatic field given by the potential:

$$\Phi = \frac{G}{2}(x^2 - y^2), \quad (1)$$

where

$$G=V\frac{}{a^2} \quad (2)$$

(V - d.c.voltage, a - distance of the vanes from the axis, x, y are the coordinates in the plane perpendicular to the optical axis z). The potential,

Φ, is the solution of the Laplace equation with boundary conditions: for $x = a, y = 0$ is $\Phi = V/2$ and

for $x = 0, y = a$ is $\Phi = -V/2$.

This represents electrostatic quadrupole field corresponding to the fixed geometry of the electrodes with an alternating potential.

From (1) x and y components of the electric field intensity are:

$$\mathrm{E}_x = -Gx$$

(3)

$$\mathrm{E}_y = +Gy$$

(4)

Let us first treat the case of the projection of the particle trajectory in the xz plane. Then, using (3), we find the equation of the motion to be

$\mathrm{md}^2x\frac{}{dt^2 = -eGx}$

(5)

where m and e are mass and charge of the particle.

If $G > 0$, the solution of eq.(5) is

x=A$\cos\left(\frac{eG}{m}\right)^{\frac{1}{2}}t + B\sin\left(\frac{eG}{m}\right)^{\frac{1}{2}}t$

(6)

with constants A, B, which are determined by the initial conditions:

$$x(t=0) = x_0, \qquad dx/dt(t=0) = \dot{x}_0.$$

We now set

dx$_{\overline{dt=\frac{dx}{dz}\frac{dz}{dt}=x'\dot{z}=x'v}}$

(7)

and we obtain the projection of the particle trajectory and derivative in xz plane:

x=x$_0 \cos Kz + \frac{x_0'}{K}\sin Kz$

(8)

x' = -Kx$_o$ $\sin Kz + x_0' \cos Kz$.

(9)

with $K = [(eG)/(mv^2)]^{1/2}$.

In matrix notation the equations may be written as:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos Kz & \frac{1}{K}\sin Kz \\ -K\sin Kz & \cos Kz \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

Evidently, if the sign of the gradient G is reversed we have:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cosh Kz & \frac{1}{K}\sinh Kz \\ K\sinh Kz & \cosh Kz \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

It follows from eqs.(3),(4) that if the beam is focused in xz plane for $G > 0$ then the beam is defocused in the yz plane and vice versa. Corresponding transformation matrices of the trajectory projection in the yz plane will be:
for $G > 0$

$$\mathbf{T}_D = \begin{pmatrix} \cosh Kz & \frac{1}{K}\sinh Kz \\ K\sinh Kz & \cosh Kz \end{pmatrix}$$

and for negative G

$$\mathbf{T}_F = \begin{pmatrix} \cos Kz & \frac{1}{K}\sin Kz \\ -K\sin Kz & \cos Kz \end{pmatrix}$$

If we let the particle pass through the two successive field regions with $G > 0$ and $G < 0$ we find that such system is highly astigmatic. The focal points in the xz and yz planes are at very different locations.
The behaviour of the electrostatic quadrupole system is similar to the magnetic quadrupole but the action of forces is different. It follows directly from mathematically equal forms of the equations of motion (within an approximation of the first order). An analogous treatment of the magnetic quadrupole leads to the same form of the transformation matrices $\mathbf{T_F}$ and $\mathbf{T_D}$ with the constant $K = \sqrt{k}$,
where
k is the magnetic quadrupole strength $k = eg/p$ and g is the field gradient
[4].

3 Helical Electrostatic Quadrupole

The Helical Electrostatic Quadrupole provides a stronger first-order focusing
and it is also stronger than the alternating gradient focusing [3].
Electric focusing of this kind is a spatially continuous focusing.
It is realized by a structure of four vanes with an alternating voltage bias $\pm V/2$. The vanes form a helix with
the pitch β, which represents a free parameter of the focusing structure.
β is defined as an angular rate per unit length along the axis.
The helical quadrupole field can be described by the potential:

$$\Phi = const\, I_2(\beta r)\, \cos(2\vartheta - \beta s) \qquad (10)$$

($I_2(\beta r)$ is the modified Bessel function of the second order)
which satisfies the Laplace equation fulfilling the boundary conditions:

$$\Phi = \frac{V}{2} \quad \text{if} \quad 2\vartheta - \beta s = 0 \qquad (11)$$

$$\Phi = -\frac{V}{2} \quad \text{if} \quad 2\vartheta - \beta s = \frac{\pi}{2} \qquad (12)$$

For small value of the argument the Bessel function $I_2(\beta r)$

can be expanded in a power series and the expression (10) is reduced to

$$\Phi = \frac{1}{2}\, G\, r^2\, cos(2\vartheta - \beta s) \tag{13}$$

if we restrict ourselves to the lowest order term r^2. Using (13), the components of the quadrupole field are

$$d\Phi \overline{dx = -G[x\cos(\beta s) + y\sin(\beta s)]} \tag{14}$$

$$d\Phi \overline{dy = -G[x\sin(\beta s) - y\cos(\beta s)]} \tag{15}$$

$$d\Phi \overline{ds = G\beta[(x^2 - y^2)\sin(\beta s) - 2xy\cos(\beta s)]}$$

(16)

They are written in Cartesian coordinates having s direction along the optical axis.

The equations of motion for charged particle in electrostatic field can be derived from the principle of the least action or simply from Newton's law. In the system of coordinates (x, y, s) they have the form

$$x" = e\frac{}{\dot{s}p(E_x - x'E_s)} \tag{17}$$

$$y" = e\frac{}{\dot{s}p(E_y - y'E_s)} \tag{18}$$

where $p = mv$, $\dot{s} = ds/dt$ and x', y' are the derivatives with respect to s variable.

Considering the particle moving near the axis, the approximative expression (13) can be used to describe the quadrupole field.

Then the equation for transverse motion in the continuously rotated quadrupole system will be:

$$x'' = -Kx\cos z - Ky\sin z \tag{19}$$

$$y'' = -Kx\sin z + Ky\cos z \tag{20}$$

where $z = \beta s$ and $K = (eG)/(\beta^2 \dot{s} p)$.

The task is to find the transformation matrix. That means we shall have to express the transverse amplitude and angle of an arbitrary trajectory at any point of the optical system as a function of the optical conditions at the beginning of the system. It is seen that there is a coupling between the trajectory projections into perpendicular planes xz and yz. Thus, the transformation matrix will have 4×4 dimensions.

To solve the system of equations (19), (20) , we define a new function

$$W = x + iy \ . \tag{21}$$

Differentiating (21) twice and using
eq.(19),(20) gives

W"=-K$e^{iz}(x-iy)$
(22)

Calculating the third and fourth derivate of W from (22) and their combinations
yield final differential equation

W""- 2iW"'- W"-K$^2W=0$
(23)

In such a way we obtained the differential equation of the fourth order with
constant coefficients, which is easily solvable. Its solution is:

W=expiz$\frac{}{2[W_1 exp(ipz)+W_2 exp(-ipz)+W_3 exp(iqz)+W_4 exp(-iqz)]}$
(24)

where $p = \frac{1}{2}\sqrt{1+4K}$ and $q = \frac{1}{2}\sqrt{1-4K}$.

Taking into account the conditions upon x, y the arbitrary complex constants in eq.(24) can be specified. To do it we divide the function $W(x+iy)$ into real and imaginary parts in the complex plane x, iy:

W $=$ exp(iz$_{\overline{2)[(a+i\bar{a})}}$

exp(ipz)+(b+i$\bar{b}$)$exp(-ipz)+$

$+$(c+i$\bar{c}$)$exp(iqz)] + (d+i\bar{d})exp(-iqz)]$.

(25)

Performing the algebraic operations and after some rearrangement we obtain

the system of four equations for x, y, x', y':

x $=$ A$\cos\frac{z}{2}\cos pz - \bar{A}\sin\frac{z}{2}\sin pz-$

B$\sin\frac{z}{2}\cos pz - \bar{B}\cos\frac{z}{2}\sin pz -$

C$\sin\frac{z}{2}\cos qz - \bar{C}\cos\frac{z}{2}\sin qz+$

D$\cos\frac{z}{2}\cos qz - \bar{D}\sin\frac{z}{2}\sin qz$

(26)

x' = -Ap$\cos\frac{z}{2}\sin pz - \frac{1}{2}A\sin\frac{z}{2}\cos pz$
-$\bar{A}p\sin\frac{z}{2}\cos pz - \frac{1}{2}\bar{A}\cos\frac{z}{2}\sin pz$
+Bp$\sin\frac{z}{2}\sin pz - \frac{1}{2}B\cos\frac{z}{2}\cos pz$
-$\bar{B}p\cos\frac{z}{2}\cos pz + \frac{1}{2}\bar{B}\sin\frac{z}{2}\sin pz$
+Cq$\sin\frac{z}{2}\sin qz - \frac{1}{2}C\cos\frac{z}{2}\cos qz$
-$\bar{C}q\cos\frac{z}{2}\cos qz + \frac{1}{2}\bar{C}\sin\frac{z}{2}\sin qz$
-Dq$\cos\frac{z}{2}\sin qz - \frac{1}{2}D\sin\frac{z}{2}\cos qz$
-$\bar{D}q\sin\frac{z}{2}\cos qz - \frac{1}{2}\bar{D}\cos\frac{z}{2}\sin qz$
(27)

y = A$\sin\frac{z}{2}\cos pz + \bar{A}\cos\frac{z}{2}\sin pz +$
B$\cos\frac{z}{2}\cos pz - \bar{B}\sin\frac{z}{2}\sin pz +$
C$\cos\frac{z}{2}\cos qz - \bar{C}\sin\frac{z}{2}\sin qz -$
D$\sin\frac{z}{2}\cos qz - \bar{D}\cos\frac{z}{2}\sin qz$
(28)

y' = -Ap$\sin\frac{z}{2}\sin pz + \frac{1}{2}A\cos\frac{z}{2}\cos pz$
+$\bar{A}p\cos\frac{z}{2}\cos pz + \frac{1}{2}\bar{A}\sin\frac{z}{2}\sin pz$
-Bp$\cos\frac{z}{2}\sin pz - \frac{1}{2}B\sin\frac{z}{2}\cos pz$
-$\bar{B}p\sin\frac{z}{2}\cos pz - \frac{1}{2}\bar{B}\cos\frac{z}{2}\sin pz$
-Cq$\cos\frac{z}{2}\sin qz - \frac{1}{2}C\sin\frac{z}{2}\cos qz$

-$\bar{C}q \sin \frac{z}{2} \cos qz - \frac{1}{2}\bar{C} \cos \frac{z}{2} \sin qz$

-Dqsin $\frac{z}{2} \sin qz - \frac{1}{2}D \cos \frac{z}{2} \cos qz$

$+\bar{D}q \cos \frac{z}{2} \cos qz - \frac{1}{2}\bar{D} \sin \frac{z}{2} \sin qz$

(29)

with eight unknown real constants:

A = a+b, B $= \bar{a} + \bar{b}, \bar{A} = a - b, \bar{B} = \bar{a} - \bar{b}$

(30)

D = c+d, C $= \bar{c} + \bar{d}, \bar{D} = c - d, \bar{C} = \bar{c} - \bar{d}.$

(31)

For determination of the constants we have four initial conditions:

x(z=0) = $x_0, y(z = 0) = y_0, x'(x = 0) = x'_0, y'(z = 0) = y'_0.$

(32)

The second and third derivatives of x and y provide the other four conditions:

$$x''_0 = Kx_0, \qquad x_0''' = Kx_0' + Ky_0, \tag{33}$$

$$y''_0 = -Ky_0', \qquad y_0''' = Kx_0 - Ky_0'. \tag{34}$$

Here zero subscript indicates the initial values at $z = 0$. Summarizing we get for the calculation of the constants (30) and (31) the system of eight equations:

$x_0 = A + D(35)$

$y_0 = B + C(36)$

$x'_0 = -\frac{1}{2}(C + B) - \bar{B}p + \bar{C}q(37)$

$y'_0 = \frac{1}{2}(A + D) + \bar{A}p + \bar{D}q(38)$

$0 = (Ap + \bar{A})p + (Dq + \bar{D} + x_0 q)q(39)$

$0 = (Bp + \bar{B} + y_0 p)p + (Cq + \bar{C})q(40)$

$0 = Ky_0 + Kx_0' + \frac{1}{8}y_0 + \bar{B}p^3 + \bar{C}q^3 + \frac{3}{2}(Bp^2 + Cq^2) + \frac{3}{4}(\bar{B}p + \bar{C}q)(41)$

$0 =$ -K$x_0 + Ky_0' + \frac{1}{8}x_0 + \bar{A}p^3 + \bar{D}q^3 + \frac{3}{2}(Ap^2 + Dq^2) + \frac{3}{4}(\bar{A}p + \bar{D}q)$
(42)

from which it follows:

A $=$ $x_0 - \frac{y_0'}{2K}$
B $=$ x'$_0 \overline{2K\qquad}$
C $=$ $y_0 - \frac{x_0'}{2K}$
D $=$ y'$_0 \overline{2K(43)}$
$\bar{A} = -\frac{A}{2p}$
$\bar{B} = -2pB$
$\bar{C} = -\frac{C}{2q}$
$\bar{D} = -2qD.$
(44)

After an amount of elementary but tedious algebra we find the following transformation matrix:

$$\mathbf{T_1} =$$

$$\left(\begin{array}{cc} \frac{\sin pz}{2p}\sin\frac{z}{2} + \cos pz\cos\frac{z}{2} & (\cos qz - \cos pz)\sin\frac{z}{2} + \left(-\frac{\sin qz}{2q} - 2p\sin pz\right)\cos\frac{z}{2} \\ -\frac{\sin pz}{2p}\cos\frac{z}{2} & \cos pz\cos\frac{z}{2} + \frac{\sin qz}{2q}\sin\frac{z}{2} \\ \cos pz\sin\frac{z}{2} - \frac{\sin pz}{2p}\cos\frac{z}{2} & (\cos pz - \cos qz)\cos\frac{z}{2} + \left(2p\sin pz - \frac{\sin qz}{2q}\right)\sin\frac{z}{2} \\ -\frac{\sin pz}{2p}\sin\frac{z}{2} & \cos pz\sin\frac{z}{2} - \frac{\sin qz}{2q}\cos\frac{z}{2} \end{array} \right.$$

$$\begin{pmatrix} -\cos qz \sin\frac{z}{2} + \frac{\sin qz}{2q}\cos\frac{z}{2} & (\cos qz - \cos pz)\cos\frac{z}{2} + \left(\frac{-\sin pz}{2p} + 2q\sin qz\right)\sin\frac{z}{2} \\ -\frac{\sin qz}{2q}\sin\frac{z}{2} & -\cos qz\sin\frac{z}{2} + \frac{\sin pz}{2p}\cos\frac{z}{2} \\ \cos qz\cos\frac{z}{2} + \frac{\sin qz}{2q}\sin\frac{z}{2} & (\cos qz - \cos pz)\sin\frac{z}{2} + \left(2q\sin qz - \frac{\sin pz}{2p}\right)\cos\frac{z}{2} \\ \frac{\sin qz}{2q}\cos\frac{z}{2} & \cos qz\cos\frac{z}{2} + \frac{\sin pz}{2p}\sin\frac{z}{2} \end{pmatrix}$$

Let us consider K reverse. It is equivalent to keeping K positive, but changing the signs in eqs.(19),(20).

They take the form:

x"= +Kx cosz + Ky sinz

(45)

y"= +Kx sinz - Ky cosz

(46)

(K is again considered to be positive)

from which it follows for the second and third derivatives of x and y at $z = 0$:

x"$_0 = -Kx_0, \qquad x_0''' = -Kx_0' - Ky_0,$

(47)

y"$_0 = Ky_0', \qquad y_0''' = -Kx_0 + Ky_0'.$

(48)

Then the equations for
the calculation of the constants (30) are:

x$_0 = A + D$(49)

y$_0 = B + C$(50)

x'$_0 = -\frac{1}{2}(C + B) - \bar{B}p - \bar{C}q$(51)

y'$_0 = \frac{1}{2}(A + D) + \bar{A}p + \bar{D}q$(52)

$0 =$ (Ap $+ \bar{A} + x_0p)p + (Dq + \bar{D})q$(53)

$0 =$ (Bp $+ \bar{B})p + (Cq + \bar{C} + y_0q)q$(54)

$0 =$ -Ky$_0 - Kx_0' + \frac{1}{8}y_0 + \bar{B}p^3 + \bar{C}q^3 + \frac{3}{2}(Bp^2 + Cq^2) + \frac{3}{4}(\bar{B}p + \bar{C}q)$(55)

$0 =$ Kx$_0 - Ky_0' + \frac{1}{8}x_0 + \bar{A}p^3 + \bar{D}q^3 + \frac{3}{2}(Ap^2 + Dq^2) + \frac{3}{4}(\bar{A}p + \bar{D}q)$

(56)

The solution is:

$A = -\, y'_0 \frac{\qquad}{2K}$
$B = y_0 + \frac{x'_0}{2K}$
$C = -\, x'_0 \frac{\qquad}{2K}$
$D = x_0 + \frac{y'_0}{2K}$ (57)
$\bar{A} = -2pA$
$\bar{B} = -\frac{B}{2p}$
$\bar{C} = -2qC$
$\bar{D} = -\frac{D}{2q}$.
(58)

Using these constants we obtain a new transformation matrix:

$$\mathbf{T_2} =$$

$$\left(\begin{array}{cc} \frac{\sin qz}{2q} \sin \frac{z}{2} + \cos qz \cos \frac{z}{2} & (\cos qz - \cos pz) \sin \frac{z}{2} + \left(\frac{\sin pz}{2p} - 2q \sin qz \right) \cos \frac{z}{2} \\ \frac{\sin qz}{2q} \cos \frac{z}{2} & \cos qz \cos \frac{z}{2} + \frac{\sin pz}{2p} \sin \frac{z}{2} \\ \cos qz \sin \frac{z}{2} - \frac{\sin qz}{2q} \cos \frac{z}{2} & (\cos pz - \cos qz) \cos \frac{z}{2} + \left(-2q \sin qz + \frac{\sin pz}{2p} \right) \sin \frac{z}{2} \\ \frac{\sin qz}{2q} \sin \frac{z}{2} & \cos qz \sin \frac{z}{2} - \frac{\sin pz}{2p} \cos \frac{z}{2} \end{array} \right.$$

$$\left.\begin{array}{cc} -\cos pz \sin \frac{z}{2} + \frac{\sin pz}{2p} \cos \frac{z}{2} & (\cos qz - \cos pz) \cos \frac{z}{2} + \left(\frac{\sin qz}{2q} - 2p \sin pz\right) \sin \frac{z}{2} \\ \frac{\sin pz}{2p} \sin \frac{z}{2} & -\cos pz \sin \frac{z}{2} + \frac{\sin qz}{2q} \cos \frac{z}{2} \\ \cos pz \cos \frac{z}{2} + \frac{\sin pz}{2p} \sin \frac{z}{2} & (\cos qz - \cos pz) \sin \frac{z}{2} + \left(2p \sin pz - \frac{\sin qz}{2q}\right) \cos \frac{z}{2} \\ -\frac{\sin pz}{2p} \cos \frac{z}{2} & \cos pz \cos \frac{z}{2} + \frac{\sin qz}{2q} \sin \frac{z}{2} \end{array}\right)$$

Now we introduce the rotation matrix $\mathbf{R}$ for rotation of the coordinate system x, y by an angle α :

$$\mathbf{R} = \begin{pmatrix} \mathbf{U} \cos \alpha & -\mathbf{U} \sin \alpha \\ \mathbf{U} \sin \alpha & \mathbf{U} \cos \alpha \end{pmatrix}$$

where

$$\mathbf{U} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

It is possible to demonstrate fairly simply that the change of sign of K in the equations of motion is related to a rotation by an angle $\pi/2$.
It follows from the relation:

$$\mathbf{R}\mathbf{T_2}\mathbf{R}^{\mathrm{T}} = \mathbf{T_1}$$

(where $\mathbf{R}^{\mathrm{T}}$ denotes the transposed matrix $\mathbf{R}$)
with the angle $\pi/2$ substituted for α in the matrix

R.

Performing the calculation of both the matrices $\mathbf{T_1}$,$\mathbf{T_2}$

we supposed K to be positive and less than $1/4$. In this case the characteristic equation has imaginary roots and the matrices contain only the functions *sin* and *cos* of the arguments pz and qz in contrast to the behaviour of the simple electrostatic quadrupole. In case of $K > 1/4$ the functions *sin* and *cos* in the matrices $\mathbf{T_1}$,$\mathbf{T_2}$

are replaced by *sinh* and *cosh* and the system is defocusing.

4 Conclusion

It is seen from the foregoing results that the ESQ transformation matrices have the same forms as magnetic quadrupole matrices (see, for example, [4]). Consequently,

the same mathematical formalism can be applied to designing a more complex

beam transport lines. As an example, we can cite the work [5], in which a suitable mathematical formalism is briefly described and applied to

configurations consisting of several rotated permanent magnetic quadrupoles. Clearly, making conclusions for any configurations constructed from ESQ disks and drift spaces one must keep in mind a different action of electric and magnetic forces on the moving particle.

The general features of the continuously rotated magnetic quadrupole system

for transport and focusing of high current beams were analyzed in [6]. If we extended this analysis to HESQ we should obtain similar results. HESQ exhibits the same features as rotated magnetic quadrupole and the similar conclusions about its transport and focusing properties as in [6] can be made if we account for the fact that the electrostatic focusing is more effective in case of small particle velocities than the magnetic one.

In this work we analysed the trajectory of a charged particle moving in an electrostatic field in the helical quadrupole geometry. The result is the transformation matrix which should serve for design of more complicated transport lines of the high current beams of heavy ions. It is supposed that the TRANSPORT code will be used for this purpose. The resulting tranformation matrix is too complex to calculate the transport parameters of the configuration consisting of HESQ analytically.

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