Lorentz-invariant Bohmian Mechanics

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A derivation of the Bohm model, and some general comments about it, are given. A modification of the model which is formally local and Lorentz-invariant is introduced, and its properties studied for a simple experiment.

1. **The Bohm model as a simple realistic quantum theory.**

Non-relativistic quantum mechanics, as it is presented in almost all text-books, is a theory which is either incorrect or incomplete, even within the domain where non-relativistic approximations are adequate. It apparently gives the correct predictions for the outcome of measurements, but nowhere within it does it contain those outcomes, or allow any description of the processes whereby they are produced. By far the simplest method of curing this problem is that introduced by David Bohm in 1952. Here quantum theory is correct, but it is incomplete. The complete theory has, in addition to the wavefunction, trajectories for individual particles exactly as in classical mechanics. Indeed, as we discuss in the next section, it is possible to regard the Bohm model as being an extension of classical mechanics.

We shall first consider the simple logical steps which allow us to derive the Bohm model directly from the rules of orthodox quantum theory. The starting point here is to note that all observations are in reality observations of *position*. We deduce the results of measurements of other quantities by an observation of position (consider for example the measurement of a spin projection by a Stern-Gerlach device). Exactly *why* this is so is an interesting question itself (Squires, 1990). Next we recall that quantum theory gives statistical predictions. Thus we require a model in which objects at all times have positions, and which gives the correct statistical distribution of these positions.

To see what this means we suppose that we have N particles with positions $\mathbf{x}_i(t)$, where t is the time.

We can represent these by the vector $\mathbf{X(t)}$ in the $3N$ dimensional configuration space. If we denote the probability distribution of the positions at time t by $\rho(\mathbf{X}, t)$, then the condition that $\rho(\mathbf{X}, t + dt)$ gives the probability distribution at $t + dt$ is clearly

$$\rho(\mathbf{X}, t) d^{3N}\mathbf{X} = \rho(\mathbf{X} + \dot{\mathbf{X}} dt, t + dt) d^{3N}\mathbf{X}(t + dt), \tag{1.1}$$

which leads to the standard continuity equation

$$\nabla.(\rho \dot{\mathbf{X}}) \equiv \sum_i \nabla_i.(\rho \dot{\mathbf{x}}_\mathbf{i}) = -\frac{\partial \rho}{\partial t}, \tag{1.2}$$

where $\dot{\mathbf{X}}(\mathbf{X}, t)$ is the vector field giving the velocity of a particle at $\mathbf{X}$.

The rhs of eq.(1.2) can be evaluated if we write the density in terms of a wavefunction evolving according to the Schrödinger equation:

$$\rho = \Psi^* \Psi, \tag{1.3}$$

with

$$i\hbar \dot{\Psi} = \left(-\frac{\hbar^2}{2} \sum_i \frac{1}{m_i} \frac{\partial^2}{\partial x_i^2} + V \right) \Psi. \tag{1.4}$$

A simple calculation yields

$$\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2} \sum_i \frac{1}{m_i} \nabla_i . \left[\Psi^* \nabla_i \Psi - \Psi \nabla_i \Psi^* \right] \tag{1.5}$$

$$= -\sum_i \nabla_i . \frac{1}{m_i} \Re \left(\Psi^* \mathbf{p}_i \Psi \right), \tag{1.6}$$

where $\mathbf{p}_i$ is the momentum operator for the i^{th} particle,

$$\mathbf{p}_i = -i\hbar \nabla_i. \tag{1.7}$$

Comparing eq. (1.6) with (1.2) we see that

$$\dot{\mathbf{x}}_i = \frac{1}{m_i} \Re \left(\frac{\mathbf{p}_i \Psi}{\Psi} \right) + (\frac{1}{\rho}) \mathbf{c}_i, \tag{1.8}$$

where the $\mathbf{c}_i$ are arbitrary vectors which satisfy

$$\sum_i \nabla_i . \mathbf{c}_i = 0. \tag{1.9}$$

To obtain the Bohm expression we take the simplest form, i.e., zero, for the $\mathbf{c}_i$. This can be justified essentially on the grounds of simplicity, together with the fact that any arbitrary vectors $\mathbf{c}$, not dependent on Ψ, would break rotational invariance, and would give the very unphysical result that the velocity would go to infinity as the density went to zero. It is also the obvious choice if we use the standard form for the probability current, which is similarly underdetermined.

Before proceeding we note, however, that there is a simple case where the neglect of the final term is rather less "natural". This is when Ψ, and hence ρ, is independent of time. Then the rhs of eq. 1.2 is zero, which would suggest zero velocity as the natural solution. The fact that the Bohm model need not give zero velocity in such a situation may be significant in quantum cosmology (Valentini, 1992, Vink, 1992: Squires, 1992, 1994). Here, according to the Wheeler-deWitt equation, the wavefunction of the universe (which is the only wavefunction that actually *exists*!) is independent of time. This is a consequence of the fact that the theory must be invariant under reparameterisation of time. For any real solution of this equation, the straightforward generalisation of Bohmian mechanics to quantum cosmology predicts zero velocities, i.e., a universe in which nothing ever moves. Presumably this is not a good prediction! There is of course an analogous prediction in the microscopic world where for example the model predicts that an electron in the ground state of a hydrogen atom does not move. In this case, however, the result is not a problem because we know that predictions for the results of *measurements* of the electron velocity, which will be related to positions of certain probes, will be correct. There is no similar escape in the cosmological case - a stationary universe is simply a stationary universe! Thus it is essential to select a (non-trivially) complex solution of the Wheeler-deWitt equation, and to use the fact that such a wavefunction can give non-zero velocities, even if the wavefunction itself is constant.

Eqs. (1.4) and (1.8), with the $\mathbf{c}_i$ equal to zero, completely define the theory. Provided that in any experiment the initial distribution of positions agrees with that given by the quantum rule at the initial time then they will do so at the end of the experiment, which then guarantees that the model will always agree exactly with the predictions of quantum theory. Note, especially, that the model automatically avoids the hidden-variable "no-go" theorems (Bell, 1966, Hardy, 1995, Clifton and Pagonis, 1995); in other words, the form of eq.(1.8) ensures that there is the necessary contextuality of measurements (see remarks below eq. 3.2).

To find an analogue of Newton's second law of motion we put

$$\ddot{\mathbf{x}}_i = \frac{\partial \dot{\mathbf{x}}_i}{\partial t} + \sum_j \frac{d\mathbf{x}_j}{dt}.\nabla_j \dot{\mathbf{x}}_i \tag{1.10}$$

$$= \frac{1}{m_i}\Re\left[-i\hbar\frac{\partial}{\partial t}\left(\frac{\nabla_i\Psi}{\Psi}\right) + \sum_j \dot{\mathbf{x}}_j.\nabla_j\left(\frac{-i\hbar\nabla_i\Psi}{\Psi}\right)\right], \tag{1.11}$$

where we have used eqs. (1.7) and (1.8). After a little rearrangement this becomes

$$m_i\ddot{\mathbf{x}}_i = -\nabla_i\left[V + Q\right], \tag{1.12}$$

which is Newton's second law with the potential, $V+Q$, given by

$$V+Q=\Re\left(\frac{H\Psi}{\Psi}\right)-\left(\frac{1}{2}\right)\sum_j m_j\dot{\mathbf{x}}_j^2, \tag{1.13}$$

where H is the hamiltonian. We can separate out the "quantum potential", Q, by using eq. 1.4. This gives

$$Q=\Re\sum_j \frac{1}{2m_j}\left(\frac{\mathbf{p}_j^2\Psi}{\Psi}-m_j^2\dot{\mathbf{x}}_j^2\right), \tag{1.14}$$

This equation reveals some interesting features of the model. For example, if we replace the operator $\mathbf{p}$ by its classical value $m\dot{\mathbf{x}}$ the quantum potential becomes zero, so in this sense the Bohm equation for the velocity may be said to contain Netwon's second law. Nevertheless, although we expect that the quantum potential should be a small quantum correction, it clearly exactly cancels (up to an irrelevant constant) the "classical" potential in the case when the state is an energy eigenstate with a constant phase, as occurs in particular for a lowest energy bound state.

It is important to note, however, that this step of introducing the potential is not necessary. Unlike Newtonian mechanics, Bohmian mechanics gives an equation for the velocities, not the accelerations. The initial conditions for a Bohmian universe are not positions and velocites, but positions (together of course with the wavefunction). Even the positions are not free, but have to satisfy the constraint that, at some initial time, they will give probability distributions consistent with the Born rule (see Dürr, Goldstein and Zhangi, 1992, for a detailed discussion).

2. Comparison with classical mechanics.

Before proceeding it is of interest to see how the Bohm model relates to other models. In my opinion the model looks much more convincing if we emphasise its similarity to *classical* mechanics rather than to (orthodox) *quantum* mechanics. The ontology of the model is that of classical mechanics; it has real particles, which at all times have positions. The law describing how the particles move has the same form as in classical mechanics; the trajectories are defined by Newton's second law of motion.

On the other hand, there is none of the indeterminism, or special role of observations, which are characteristic of quantum theory.

There are, of course, differences between Bohmian mechanics and Newtonian mechanics, although to some extent these can be regarded as additions to the latter rather than changes. One such difference is that, as we have noted, the second order equation of motion can be integrated to give a first order equation, without any need for additional boundary conditions. To appreciate the significance of this point we might imagine a world in which time is discrete. Then it is clear that in Newtonian mechanics, the equations plus conditions at one time do not completely determine the state at future times. In the Bohm model however they do.

The other difference is that there is an additional "quantum" force. Of course classical physics is easily able to cope with new forces, but this particular addition is not as innocent as it sounds. The quantum force is unlike all the other forces in nature because it is not derivable from the positions of the particles. We can compare it with, for example, gravity. The force of gravity acting on a given particle is a unique function of the positions of the other particles, which act as sources in the Poisson equation. This of course is only true if we neglect the "complimentary function" (i.e. solution of the Laplace equation), a procedure which we normally justify by imposing some sort of boundary condition at infinity. The quantum potential, however, is derived from the Schrödinger equation in which there are no sources, so the analogue of the neglected term is here everything. Also the actual quantum force is independent of the *magnitude* of the field from which it derived (see eq. 1.14).

These differences strongly suggest that there is some underlying theory, which is not quantum theory, and not classical mechanics, but which combines (and explains?) certain aspects of both. It is one of the great merits of the Bohm model that, in addition to giving a proper, respectable, explanation of all quantum phenomena, it encourages speculation about such an underlying theory, and even about possible theories which give results different to those of standard quantum theory. An example is discussed below.

3. Relativistic invariance and the Bohm model.

The Bohm model exposes the non-locality which has long been recognised as one of the significant features of the difference between quantum mechanics and classical physics. Bell's theorem demonstrated that this non-locality is not peculiar to the particular form of realistic model used by Bohm. Any "completion" of quantum theory which is consistent with all its predictions must be non-local. Since the experimental tests (e.g., Aspect, et al., 1981, 1982a,b) seem to agree with quantum theory, most physicists have come to accept this non-locality in some form or other. Although the discussions are normally carried out in a non-relativistic context, it is clear that agreement with the predictions of quantum theory strongly suggests that a realistic model should be non-Lorentz invariant, in particular should require that there exists a preferred frame (Hardy, 1992, Hardy and Squires, 1992).

The need for such a frame is of course evident in the Bohm model because the expression for the velocity of one particle (eq. 1.8) requires knowledge of the position of all the others *at the same time*, which clearly is not a frame-independent concept. However, the fact that the model so clearly reveals the non-locality means that it also shows how it might be removed, at the cost of course of a failure to agree at all times with the predictions of quantum theory. The idea is suggested by the analogy with a classical potential given in section 2. Any classical potential, e.g., the electrostatic potential, is also defined in the configuration space of the particles, and requires simultaneous positions. However, we know how this is dealt with in a proper relativistic treatment: we use the "retarded" potential in which the positions are determined on the backward light cone.

It is possible to do something similar in the Bohm model (Squires, 1993, Mackman and Squires, 1995) and to replace eq. 1.8 by

$$\dot{\mathbf{x}}_i(t_i) = \frac{1}{m_i}\Re\left(\frac{\mathbf{p}_i\Psi(\mathbf{x}_1(t_1), \mathbf{x}_2(t_2)....)}{\Psi(\mathbf{x}_1(t_1), \mathbf{x}_2(t_2)....)}\right), \tag{3.1}$$

where

$$t_k = t_i - \frac{|\mathbf{x}_i(t_i) - \mathbf{x}_k(t_k)|}{c}. \tag{3.2}$$

In equation (3.1) we have ignored the explicit time dependence of the wavefunction, and it is not clear how we should treat this. Part of the problem is that we are working within the framework of non-relativistic quantum theory. At the fundamental level, we could perhaps take refuge in the fact noted above that the actual wavefunction of the universe is constant. Further work is needed here but for the moment we shall ignore the problem. In the example discussed below an unambiguous procedure suggests itself.

It is clear that eq. (3.1) goes some way towards removing the obvious non-locality from the Bohm model, and it is important to study the nature of its inevitable disagreement with quantum theory predictions. In principle this is possible because the model allows explicit calculations.

Consider, for example, an experiment in one space dimension in which a photon is emitted from an origin in the form of two wave-packets, of equal size, one travelling in the positive direction and one in the negative. We suppose that there are photon detectors at positions l and $-l$, and that the purpose of the experiment is to determine in which direction the photon is observed to travel, i.e., which detector actually "sees" the photon. The difficulty with this type of discussion is how we model a real photon detector. One simple possibility (see Squires, 1993, and Squires and Mackman, 1994) is to take the detectors to be free particles, initially in zero-momentum gaussian wave-packets, which receive a momentum p when they detect a photon of momentum p. Thus, just after the photon is emitted, the wavefunction of the system is given by

$$|\Psi>= 2^{-\frac{1}{2}}[\phi_L(y) + \phi_R(y)]\psi_L(x_L)\psi_R(x_R), \tag{3.3}$$

where $\phi_{L,R}(y)$ is the part of the photon wavefunction moving to the L, R respectively, and $\psi_{L,R}$ are the two detector states given by

$$\psi_{L,R}(x_{L,R}) = (\frac{a}{\pi})^{\frac{1}{4}} \exp[-\frac{1}{2}a(x_{L,R} \pm l)^2]. \tag{3.4}$$

At a later time, after the photon wave-packets have interacted with the detectors, the wavefunction has the form:

$$|\Psi >= 2^{-\frac{1}{2}}[\phi_L(y)\psi_L^{-p}(x_L)\psi_R(x_R) + \phi_R(y)\psi_L(x_L)\psi_R^p(x_R)], \tag{3.5}$$

where the $\psi^{\pm p}$ now represent moving wavepackets, e.g.,

$$\psi^{-p}(x_L) = (\frac{a}{\pi})^{\frac{1}{4}} \exp[i(x_L p + \frac{p^2 t}{2m}) - \frac{1}{2}a(x_L + l + \frac{pt}{m})^2]. \tag{3.6}$$

Note that in the last expression we have neglected the quantum evolution of the free detector state.

Now we recall that in a spin measurement, as for example in the experiments of Aspect et al., the actual outcome, i.e., the value of the spin that is observed, is determined in the Bohm model by the value(s) of the, so-called, hidden variable(s) in the detector. To study something analogous to this we suppose that there are no photon trajectories (this is the case in at least some versions of the Bohm model). Then, as we shall see below, the measurement outcome, which we refer to as the position of the photon, again depends upon the values of the hidden variables of the detectors, i.e., the positions of the particles.

First, it is necessary to modify the standard Bohm formula for particle velocities by integrating over the positions of those particles without trajectories (Bell, 1981, Squires and Mackman, 1994). Thus, in general, eq. (1.8) must be replaced by

$$\dot{\mathbf{x}}_i = \Re\left(\frac{\int d^3\mathbf{y}\,\Psi^*\mathbf{p}_i\Psi}{m_i\int d^3\mathbf{y}\,\Psi^*\Psi}\right), \tag{3.7}$$

In our experiment this leads to the result

$$m\dot{x}_L = \Re\left(\frac{|\psi_R|^2\psi_L^{-p*}p_L^{op}\psi_L^{-p} + |\psi_R^p|^2\psi_L^*p_L^{op}\psi_L}{|\psi_R|^2|\psi_L^{-p}|^2 + |\psi_R^p|^2|\psi_L|^2}\right) \tag{3.8}$$

and a similiar equation for $\dot{x}_R$, where m is the mass of the detector particle. Here we have assumed that there is no overlap between the right and left moving photon wave-packets. This will be approximattely true for any reasonable definition of the space-time surface implicit in eq. (3.7).

We first use the non-retarded, and hence non-local, Bohm model. Then, inserting the previous wavefunctions into Eq. (3.8), we find

$$\dot{u} = (1 + \exp[2t(v-u)])^{-1}, \tag{3.9}$$

and

$$\dot{v} = (1 + \exp[2t(u-v)])^{-1}, \tag{3.10}$$

where we have simplified the notation by using units in which $a = \frac{p}{m} = 1$ and by defining

$$u = x_R - l \tag{3.11}$$

and

$$v = -(x_L + l). \tag{3.12}$$

Adding (3.9) and (3.10) we obtain $\dot{u} + \dot{v} = 1$, hence

$$u + v = t + u_0 + v_0, \tag{3.13}$$

where u_0 and v_0 are the values of u and v at the time when the interaction occurs, taken to be $t = 0$. If we substitute (3.13) into (3.9) and (3.10) we find

$$\dot{u} = \left(1 + \exp[-2t(2u - t - u_0 - v_0)]\right)^{-1}, \tag{3.14}$$

and

$$\dot{v} = \left(1 + \exp[-2t(2v - t - u_0 - v_0)]\right)^{-1}. \tag{3.15}$$

We compare these results with what we obtain with only one detector, say the one at the left. Then the Bohm equation would give

$$\dot{v} = \left(1 + \exp[-t(2v - t)]\right)^{-1}. \tag{3.16}$$

Clearly the small t behaviour is $v \simeq \frac{1}{2}t + v_0$, leading to

$$\dot{v} \simeq \left(1 + \exp[-2tv_0]\right)^{-1}, \tag{3.17}$$

for small t. It follows that this detector will record the photon (in the sense that the detector particle will continue to move, with velocity approaching 1), if $v_0 > 0$, but not if $v_0 < 0$. Incidently we can here see the fact that an initial distribution agreeing with quantum theory will give the correct quantum theoretic outcome: such an initial distribution will have the v_0 equally distributed between positive and negative values, leading to half the photons being detected in the left detector, as required.

If we treat eq. (3.15) in a similar way we find

$$\dot{v} \simeq \left(1 + \exp[-2t(v_0 - u_0)]\right)^{-1}. \tag{3.18}$$

Hence in this case the condition that the left detector records (fails to record) the photon is that $v_0 - u_0$ is positive (negative). Clearly the opposite is true for the right detector, so (as required) one, and only one, detector will see the photon. Again an initial distribution agreeing with quantum theory will have $v_0 - u_0$

positive and negative with equal frequency, so giving the expected ouput results. We note also that this example reveals the contextuality of this version of the Bohm model: comparison of (3.17) with (3.18) shows that the result emerging from the right detector, say, is affected by the presence of the left detector.

The differential equations (3.14) and (3.15) can in fact be solved directly to give, for example,

$$\int_{\frac{-(u_0-v_0)}{2}}^{\frac{(u_0-v_0)}{2}} dy e^{-2y^2} = \int_{t-(u-\frac{(u_0+v_0)}{2})}^{u-\frac{(u_0+v_0)}{2}} dy e^{-2y^2}. \tag{3.19}$$

Clearly, if $(u_0 - v_0)$ is positive, then $(u - t)$ must remain constant as t becomes large; on the other hand if it is negative, then u becomes constant and small for large t. This confirms that we will obtain the expected measurement outcomes.

We must now consider what happens in this experiment if we use the retarded Bohm model. Since we have well-localised wave-packets we can solve the problem noted below eq. 3.1 by evaluating the wave-packet from eq. 3.6 at the appropriate retarded time as well as retarded position. We define T, the time for light to travel from one detector to the other, according to

$$T = \frac{2l}{c}. \tag{3.20}$$

Then, for $t < T$, the two detectors will behave as if the other one was not present. Hence, for $t < T$,

$$\dot{u}(t) = (1 + \exp[-t(2u(t) - t)])^{-1} \tag{3.21}$$

and

$$\dot{v}(t) = (1 + \exp[-t(2v(t) - t)])^{-1}, \tag{3.22}$$

where, for reasons which will be immediately evident, we have explicitly written the time arguments.

Thus, up to $t = T$, the detectors behave independently and record the presence, or otherwise, of the photon strictly according to their own initial position. Clearly this means that in some cases "wrong" results are occuring, i.e., both, or neither, detector is seeing the photon. However, at $t = T$, the situation changes because information about the presence of the other detector becomes available. Thus, for $t > T$,

$$\dot{u}(t) = (1 + \exp[-2u(t)t + 2v(t-T).(t-T) + T.(2t-T)])^{-1} \tag{3.23}$$

and

$$\dot{v}(t) = (1 + \exp[-2v(t)t + 2u(t-T).(t-T) + T.(2t-T)])^{-1}. \tag{3.24}$$

Note that, as expected, these equations agree with eqs. (3.9) and (3.10) if T is put equal to zero.

I have not been able to solve these equations analytically but it is clear that they give the expected results (Squires, 1993). In particular, if T is sufficiently small, and $|v_0 - u_0|$ sufficiently large, then again, one, and only one, detector will record the photon. There are, however, circumstances in which both, or neither, detector will see the photon., This would correspond to a "wrong" result, in the sense that the predictions of orthodox quantum theory would be violated.

More precisely, the condition that there will be a significant number of "wrong" results (zero or two photons), is that

$$T \geq |v_0 - u_0|_{typical}. \tag{3.25}$$

With the units restored this means

$$\frac{l}{c} \geq \frac{m}{pa^{\frac{1}{2}}}. \tag{3.26}$$

If we now assume that the detector acquires all the initial momentum of the detected photon then $p = \frac{\hbar}{\lambda}$ so the condition for wrong results becomes

$$\frac{l\hbar}{mcd\lambda} \geq 1, \tag{3.27}$$

where $d \sim a^{-1/2}$ is the spatial spread of the initial wavefunction of the detector particle. In fact numerical solutions of eqs. (3.23) and (3.24) show that when

$$\frac{l\hbar}{mcd\lambda} = 1, \tag{3.28}$$

then about one in ten events give wrong results (H.Movahhedian, private communication).

In a typical experiment the separation l is only a few metres, so if for m we take the mass of a macroscopic pointer, it is clear that the condition in eq. (3.27) is not satisfied. On the other hand if we suppose the detection comes about by the photon being absorbed by an electron, then for an optical photon, the LHS of (3.27) is of the order of $\frac{10^{-5}}{d}$, with d measured in metres. If the electron is initially confined to within an atomic distance then clearly this is much greater than 1, so there will be many events in which both, or neither, electron records the photon. Of course, we would certainly not directly observe a single electron, and it would be essential here to use a device which was genuinely responsive to the electron trajectory (this is not always a trivial issue - see, for example, Englert, et al., 1992 and Dewdney, et al., 1993).

The tentative conclusion of this analysis is that it is unlikely that deviations from the quantum theory results, which would arise in our retarded model, would have been seen in any experiments that have been performed. Nevertheless a better analysis of the actual experiments is required, and such an analysis could well reveal the possibility of realistic tests for retarded effects in future, carefully designed experiments. Such experiments would need the largest possible values of l, detectors where the effective "mass" of the detector is as small as possible, and of course efficient detectors.

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