Does the quark cluster model predict any isospin two dibaryon resonance?

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Abstract

We analyze the possible existence of a resonance in the $J^P = 0^-$ channel with isospin two by means of nucleon-Δ interactions based on the constituent quark model. We solve the bound state and the scattering problem using two different potentials, a local and a non-local one. The non-local potential results to be the more attractive, although not enough to generate the experimentally predicted resonance.

The existence of bound states of negative pions and neutrons (pineuts) was predicted theoretically years ago [1]. A system of neutrons and negative pions gives rise to a structure similar to an ordinary nucleus, where the protons have been replaced by negative pions. Since these systems can only decay through weak interactions, they should be stable and have lifetimes comparable to that of the pion. Then, a question arises immediately as to if one could observe even the simple of these possible pineuts, a bound state of a pion and two neutrons, just like a nucleon-Δ bound state.

This problem has been studied from the theoretical point of view by means of different methods [2,3] and with different conclusions, but never definitively excluding the possibility of a resonance. Mainly due to this controversial situation, a lot of experiments were done to find evidences of such a resonance. After some signatures in experiments with poor resolutions [4], an experiment with high intensity proton beams almost excluded the existence of these structures [5].

The situation has been recently renewed from the experimental point of view. In Ref. [6], a $J^P = 0^-$ resonance has been proposed to explain a sharp peak seen on the pionic double charge exchange cross section in several nuclei from ^{14}C to ^{48}Ca. The narrow width of these peaks suggested that the resonance must have isospin even, otherwise decay into nucleon-nucleon (NN) would be allowed, causing a much large width. Besides, based on QCD string models, they assumed that the resonance has isospin zero. However, in Refs. [3,7] was pointed out that the narrow width of this structure could be related with the vicinity of the nucleon-Δ ($N\Delta$) threshold and therefore the resonance most likely must have isospin two (the $N\Delta$ system cannot coupled to isospin zero).

Our aim in this paper is to present the predictions of a quark model based potential about the possibility of a $N\Delta$ resonance with isospin two. Previous calculations based on the three-body formalism of the πNN system predict a large attraction in the 0^- channel [2], the same proposed in Ref. [6]. Therefore, the 0^- channel is the ideal candidate to posses a resonance and we will concentrate on it. Moreover, we will see that in case the resonance exists there is a strong correlation between its mass and its width due to the proximity of the $N\Delta$ threshold.

We have derived a $N\Delta$ interaction using the same two-center quark-cluster model of Ref. [9]. Quarks acquire a dynamical mass as a consequence of the chiral symmetry breaking. To restore this symmetry one has at least to introduce the exchange of a pseudoscalar (pion) and a scalar (sigma) boson between quarks. Besides, a perturbative contribution is obtained from the non-relativistic reduction of the one-gluon exchange diagram in QCD.

Therefore, the ingredients of the quark-quark interaction are the confining potential (CON), the one-gluon exchange (OGE), the one-pion exchange (OPE) and the one-sigma exchange (OSE). The explicit form of these interactions is given by (see Ref. [9] for details),

$$V_{CON}(\vec{r}_{ij}) = -a_c\, \vec{\lambda}_i \cdot \vec{\lambda}_j\, r_{ij}^2\,, \tag{1}$$

$$V_{OGE}(\vec{r}_{ij}) = \frac{1}{4}\,\alpha_s\, \vec{\lambda}_i \cdot \vec{\lambda}_j \left\{ \frac{1}{r_{ij}} - \frac{\pi}{m_q^2}\left[1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right]\delta(\vec{r}_{ij}) - \frac{3}{4m_q^2\, r_{ij}^3}\, S_{ij} \right\}, \tag{2}$$

$$\begin{aligned} V_{OPE}(\vec{r}_{ij}) = \frac{1}{3}\,\alpha_{ch}\frac{\Lambda^2}{\Lambda^2 - m_\pi^2}\, m_\pi \Bigg\{ &\left[Y(m_\pi\, r_{ij}) - \frac{\Lambda^3}{m_\pi^3}\, Y(\Lambda\, r_{ij})\right] \vec{\sigma}_i \cdot \vec{\sigma}_j + \\ &\left[H(m_\pi\, r_{ij}) - \frac{\Lambda^3}{m_\pi^3}\, H(\Lambda\, r_{ij})\right] S_{ij} \Bigg\}\, \vec{\tau}_i \cdot \vec{\tau}_j\,, \end{aligned} \tag{3}$$

$$V_{OSE}(\vec{r}_{ij}) = -\alpha_{ch}\, \frac{4\, m_q^2}{m_\pi^2}\frac{\Lambda^2}{\Lambda^2 - m_\sigma^2}\, m_\sigma \left[Y(m_\sigma\, r_{ij}) - \frac{\Lambda}{m_\sigma}\, Y(\Lambda\, r_{ij})\right]. \tag{4}$$

The main advantage of this model comes from the fact that it works with a single qq-meson vertex. Therefore, its parameters (coupling constants, cut-off masses,...) are independent of the baryon to which the quarks are coupled, the difference among them being generated by SU(2) scaling. This makes the generalization of the NN interaction to any other non-strange baryonic system straightforward, and in particular to the $N\Delta$ system.

Once the quark-quark interaction is chosen, an effective nucleon-Δ potential can be obtained as the expectation value of the energy of the six-quark system minus the self-energies of the two clusters, which can be computed as the energy of the six-quark system when the two quark clusters do not interact:

$$V_{N\Delta(L\,S\,T)\to N\Delta(L'\,S'\,T)}(R, R') = \xi_{L\,S\,T}^{L'\,S'\,T}(R, R') \;-\; \xi_{L\,S\,T}^{L'\,S'\,T}(\infty, \infty)\,, \tag{5}$$

where,

$$\xi^{L'\,S'\,T}_{L\,S\,T}(R,R') \;=\; \frac{\left\langle \Psi^{L'\,S'\,T}_{N\Delta}(\vec{R}') \mid \sum_{i<j=1}^{6} V_{qq}(\vec{r}_{ij}) \mid \Psi^{L\,S\,T}_{N\Delta}(\vec{R}) \right\rangle}{\sqrt{\left\langle \Psi^{L'\,S'\,T}_{N\Delta}(\vec{R}') \mid \Psi^{L'\,S'\,T}_{N\Delta}(\vec{R}') \right\rangle}\sqrt{\left\langle \Psi^{L\,S\,T}_{N\Delta}(\vec{R}) \mid \Psi^{L\,S\,T}_{N\Delta}(\vec{R}) \right\rangle}} \,. \tag{6}$$

The parameters of the model are those of Ref. [9]. As local potential we will assume $R = R'$.

In order to determine the nature (attractive or repulsive) of the 0^- $N\Delta$ channel, we will first calculate the Fredholm determinant of that channel as a function of energy assuming a stable delta and nonrelativistic kinematics. That means, we will use the Lippmann-Schwinger equation

$$T_{ij}(q,q_0) = V_{ij}(q,q_0) + \sum_k \int_0^\infty q'^2 dq' V_{ik}(q,q')G_0(E,q')T_{kj}(q',q_0)\,, \tag{7}$$

where the two-body propagator is

$$G_0(E,q) = \frac{1}{E - q^2/2\eta + i\epsilon}\,, \tag{8}$$

with reduced mass

$$\eta = \frac{m_N m_\Delta}{m_N + m_\Delta}\,. \tag{9}$$

The energy and on-shell momentum are related as

$$E = q_0^2/2\eta\,, \tag{10}$$

and we will restrict ourselves to the region $E \leq 0$.

If we replace the integration in Eq. (7) by a numerical quadrature, the integral equations take the form

$$T_{ij}(q_n,q_0) = V_{ij}(q_n,q_0) + \sum_k \sum_m w_m q_m^2 V_{ik}(q_n,q_m)G_0(E,q_m)T_{kj}(q_m,q_0)\,, \tag{11}$$

where q_m and w_m are the abscissas and weights of the quadrature (we use a 40-point Gauss quadrature). Eq. (11) gives rise to the set of inhomogeneous linear equations

$$\sum_k \sum_m M^{ik}_{nm}(E)T_{kj}(q_m,q_0) = V_{ij}(q_n,q_0)\,, \tag{12}$$

with

$$M^{ik}_{nm}(E) = \delta_{ik}\delta_{nm} - w_m q_m^2 V_{ik}(q_n, q_m) G_0(E, q_m)\,. \tag{13}$$

If a bound state exists at an energy E_B, the determinant of the matrix $M^{ik}_{nm}(E_B)$ (the Fredholm determinant) must vanish, i.e.,

$$\left| M^{ik}_{nm}(E_B) \right| = 0. \tag{14}$$

Even if there is no bound state, the Fredholm determinant is a very useful tool to determine the nature of a given channel. If the Fredholm determinant is larger than one that means that channel is repulsive. If the Fredholm determinant is less than one that means the channel is attractive. Finally, if the Fredholm determinant passes through zero that means there is a bound state at that energy.

In Figure 1 we compare the Fredholm determinant generated by the local and non-local quark model based potentials. The non-locality of the interaction generates additional attraction, enough to produce a resonance (it goes through zero).

To determine the exact location of the resonance, we calculate Argand diagrams between a stable and an unstable particle using the formalism of Ref. [8]. In this case, however, we will use relativistic kinematics and will include the width of the delta. That means, instead of the propagator (8) we will use [8]

$$G_0(S, q) = \frac{2m_\Delta}{s - m_\Delta^2 + i m_\Delta \Gamma_\Delta(s, q)}\,, \tag{15}$$

where S is the invariant mass squared of the system, while s is the invariant mass squared of the πN subsystem (those are the decay products of the Δ) and is given by

$$s = S + m_N^2 - 2\sqrt{S(m_N^2 + q^2)}\,. \tag{16}$$

The width of the Δ is taken to be [8]

$$\Gamma_\Delta(s, q) = \frac{2}{3}\, 0.35\, p_0^3 \frac{\sqrt{m_N^2 + q^2}}{m_\pi^2 \sqrt{s}}\,, \tag{17}$$

where p_0 is the pion-nucleon relative momentum given by

$$p_0 = \left(\frac{[s - (m_N + m_\pi)^2][s - (m_N - m_\pi)^2]}{4s} \right)^{1/2} . \tag{18}$$

We show in Figure 2 the phase shifts for the 0^- channel. As it can be seen from this figure, the attraction is only strong enough to produce a resonance with the non-local potential (it reaches 90 degrees). This resonance lies at 2145.6 MeV and has a width of 148.12 MeV for the mass of the sigma predicted by chiral symmetry requirements $m_\sigma \sim 675$ MeV.

The proposed resonance has a mass of 2065 MeV and a very small width of 0.51 MeV [6]. It is therefore very interesting to investigate whether the nucleon-Δ system exhibit the features of this resonance, and particularly such a tiny width.

In order to do this, we have artificially varied the mass of the σ meson with both potential models, such as to increase the amount of attraction. We show in Table I the mass and width of the resonance and the corresponding mass of the sigma meson necessary to generate it. The width of the resonance drops dramatically when its mass approaches the πNN threshold (2017 MeV). This result can be understood from simple angular momentum barrier considerations. If we call q and L to the relative momentum and relative orbital angular momentum between a nucleon and the π-nucleon pair, respectively, then since $L = 1$ the width of the resonance will be proportional to $q^{2L+1} = q^3$, so that it will drop very fast as one approaches the πNN threshold since there $q \to 0$.

In both local and non local potential models, when the mass of the sigma is taken to reproduce the predicted mass of the resonance (2065 MeV) the width is very narrow, which is in very good agreement with the predictions extracted by Bilger and Clement [6]. Therefore, the sharp peak seen in the double charge exchange reactions could be justified as a nucleon-Δ resonance in the isospin 2 channel, without resorting to other more exotic processes.

As a summary, we have studied the nucleon-Δ system in the 0^- channel with isospin two, within the quark cluster model of the baryon-baryon interaction. We have used a local and a non-local potential. We found that the non-local effects generate additional attraction, although not enough to reproduce the resonance predicted in Ref. [6]. However, due to the proximity of the nucleon-Δ threshold, if we force the resonance mass to reach the experimental predicted value of 2065 MeV, then its width is very narrow, in very good agreement with the width extracted in Ref. [6].

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TABLES

TABLE I. Mass and width of the 0^- resonance with the corresponding mass of the sigma meson using the local and non-local nucleon-Δ potentials based on the constituent quark model.

Potential model	$m_\sigma (MeV)$	$M_{Res} (MeV)$	$\Gamma_{Res} (MeV)$
Local	234.0	2064.4	0.6
Non local	422.0	2064.5	1.6