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Observational Tests of One-Bubble Open Inflationary Cosmological Models

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Abstract

Motivated by recent studies of the one-bubble inflationary scenario, simple open cold dark matter models are tested for consistency with cosmological observations. The initial perturbation spectrum is derived by solving for the evolution of fluctuations in an open inflationary stage. A likelihood analysis is performed for the Cosmic Microwave Background anisotropies using the two-year COBE DMR data and considering models based on both the Bunch-Davies and conformal vacua. Having normalized the perturbation spectrum to fit the COBE data, we reconsider the validity of the open model from the view point of cosmic structure formation. Open models may be severely constrained by the COBE likelihood analysis. In particular, small values of Ω_0 are ruled out in the Bunch-Davies case: we find that $\Omega_0 \geq 0.34$ at 95% confidence for this model.

Subject headings: cosmology: theory — cosmic microwave background — large scale structure

1 Introduction

The recent discovery of anisotropy in the cosmic microwave background (CMB) lends support to the hypothesis that structure in the universe formed via gravitational instability from small density perturbations. Furthermore, the observed anisotropy allows us to characterize the initial perturbations statistically with enough precision to draw important cosmological conclusions. In particular, the COBE detection (Smoot et al. 1992, Bennett et al. 1994) shows that density perturbations existed on scales larger than the horizon at the epoch of recombination. Furthermore, the data are consistent with the hypothesis that the perturbations are Gaussian distributed and can be used to place constraints on the power spectrum of the initial density perturbations. CMB anisotropy is already one of the most important pieces of cosmological data, and in coming years its importance will only increase.

In recent years, many cosmologists have favored theories in which the density parameter Ω_0 is equal to one. In particular, this value for Ω_0 is predicted by the simplest versions of the popular inflationary scenario. However, many dynamical measurements indicate a low-density universe (Peebles 1993, Ratra & Peebles 1994a). It is well known that the standard COBE-normalized Cold Dark Matter (CDM) scenario, in which universe is flat and contains only baryons and CDM particles, and in which the initial density perturbations are of the Harrison-Zel'dovich type, predicts fluctuations that are too large in amplitude on scales of galaxy clusters and below, although a number of slight variants on this model can be devised that fit the data better (*e.g.*, Bunn, Scott, & White 1995; White et al. 1995).

There are two classes of low-density cosmological model that can be motivated by the inflationary universe scenario. One is the Λ -model. In this model the matter density is small, so $\Omega_0 < 1$; however, the cosmological constant contributes to the total mean density required to make the universe flat: $\Omega_0 + \Omega_\Lambda = 1$. The usual inflationary scenarios predict that the universe is flat, so that $\Omega_0 + \Omega_\Lambda$ must be extremely close to unity (Kashlinsky, Tkachev, & Frieman 1994). The cosmological constant is equal to the vacuum energy density of the Universe. Although we have no reason to be certain that it must be zero, the values that are of interest to cosmologists (*i.e.*, the values that make Ω_Λ of order unity) are unnaturally small from the point of view of particle physics (Weinberg 1989), and so these Λ -models are often regarded as unappealing.

The other class of low-density inflationary scenario is the open model, in which the universe has negative spatial curvature. Recently, the possibility of realizing an open universe has been discussed in the context of inflation theory (Bucher, Goldhaber, & Turok 1994; Yamamoto, Sasaki, & Tanaka 1995; Linde 1995; Linde & Mezhlumian 1995). The essential idea is based on the semiclassical picture of a bubble nucleation, which is described by a bounce solution (Coleman & De Luccia 1980). One bubble nucleation process can be regarded as the creation of a homogeneous and isotropic spacetime with negative spatial curvature inside the bubble due to the $O(4)$ -symmetry of the bounce solution.

It is of great interest to ask whether an open universe created in this one-bubble inflationary scenario is observationally acceptable or not. Open CDM models have been investigated by many authors (Lyth & Stewart 1990; Ratra & Peebles 1994ab; Sugiyama & Silk 1994; Kamionkowski et al. 1994; Górski et al. 1995; Liddle et al. 1995). Their investigations are based on the simple assumption that the quantum state of a scalar field is in the conformal vacuum state at the inflationary stage; however, this is unlikely to be the prediction of the one-bubble inflationary

scenario. It has been pointed out that if we take the Bunch-Davies vacuum state as the state of scalar field, the CMB anisotropy in a low Ω_0 universe appears quite different due to the super-curvature mode (Yamamoto, Sasaki, & Tanaka 1995).

Following the usual inflationary picture in which the quantum fluctuation of a scalar field generates the density perturbation, we must investigate the quantum state of fields inside the bubble. Attempts have been made to study this problem by developing a field-theoretical formalism based on a multidimensional tunneling wave function (Tanaka, Sasaki, & Yamamoto 1994; Sasaki et al. 1994; Tanaka & Sasaki 1994; Yamamoto, Tanaka, & Sasaki 1995; Hamazaki et al. 1995). Bucher et al. have also considered this problem (Bucher, Goldhaber & Turok 1994; Bucher & Turok 1995). However, this problem requires further investigation.

In this paper, we consider the simple case in which the quantum state of a scalar field is in the Bunch-Davies vacuum state, and compare the predictions of this model with several cosmological observations. This situation is physically definite and clear, and as long as the bubble nucleation occurs in the de Sitter inflationary background (where the initial inflationary period has lasted sufficiently long), the field can be approximated by the Bunch-Davies vacuum state, provided that the effect of the bubble nucleation process is negligible.

In section 2, we first consider the initial spectrum by solving for the evolution of cosmological perturbations in the open inflationary stage. In section 3 we use this initial power spectrum to calculate various cosmological quantities and compare these predictions with observations. We also compare these results with those of a previous analysis of the open inflationary model based on the conformal vacuum state, and a Λ -model with Harrison-Zel'dovich spectrum. Section 4 is devoted to a discussion of our results. We will work in units where $c = 1$ and $\hbar = 1$.

2 Initial Conditions

In this section, we consider the evolution of cosmological perturbations in an open inflationary stage and derive the initial spectrum of perturbations. Ratra & Peebles (1994b) have investigated the evolution of cosmological perturbations in an open inflationary universe with gauge fixed. Bucher, Goldhaber & Turok (1994) have also investigated cosmological perturbations in an inflationary stage in the gauge-invariant formalism. In the first half of this section, we follow the work by Bucher, Goldhaber & Turok (1994).

In an open universe the line element can be written as

$$ds^2 = a^2(\eta) \left[-d\eta^2 + d\chi^2 + \sinh^2 \chi d\Omega_{(2)}^2 \right], \quad (1)$$

where η is conformal time. We consider a scalar field ϕ with potential $V(\phi)$, which drives inflation. Writing the scalar field as a homogeneous part and a small inhomogeneous part, i.e., $\phi = \phi_0 + \delta\phi$, the equations for the homogeneous part are

$$\phi_0'' + 2\mathcal{H}\phi_0' + a^2 \frac{\partial V}{\partial \phi} = 0, \quad (2)$$

and

$$\mathcal{H}^2 - 1 = \frac{8\pi G}{3} \left[\frac{\phi_0'^2}{2} + a^2 V \right], \quad (3)$$

where $\mathcal{H} := a'/a$ and the prime denotes differentiation with respect to η .

The small fluctuation $\delta\phi$ gives rise to a metric perturbation. As we are interested in a scalar perturbation in a scalar field dominated universe, the metric perturbation can be written,

$$ds^2 = a^2(\eta) \left[-(1 + 2\Phi)d\eta^2 + (1 - 2\Phi) \left(d\chi^2 + \sinh^2 \chi d\Omega_{(2)}^2 \right) \right]. \quad (4)$$

Φ corresponds to the curvature perturbation or gravitational potential. Then we get the evolution equations for the perturbation (Mukhanov, Feldman, & Brandenberger 1992),

$$\Phi'' + 2 \left(\mathcal{H} - \frac{\phi_0''}{\phi_0'} \right) \Phi' + \left(-\mathbf{L}^2 + 4 + 2\mathcal{H}' - 2\mathcal{H} \frac{\phi_0''}{\phi_0'} \right) \Phi = 0, \quad (5)$$

$$\delta\phi = \frac{1}{4\pi G \phi_0'} (\Phi' + \mathcal{H}\Phi), \quad (6)$$

where

$$\mathbf{L}^2 := \frac{1}{\sinh^2 \chi} \frac{\partial}{\partial \chi} \left(\sinh^2 \chi \frac{\partial}{\partial \chi} \right) + \frac{1}{\sinh^2 \chi} \mathbf{L}_\Omega^2, \quad (7)$$

and \mathbf{L}_Ω^2 is the Laplacian on the unit sphere.

Now let us consider an inflationary stage of the universe inside a bubble. To solve the above equations analytically, we assume that the potential is nearly flat, and use the approximation, $V \simeq V_0 + V'\phi$, where $V' = \text{const}$. We also assume that the background spacetime is approximated by de Sitter spacetime, that is, $a(t) = -1/H \sinh \eta$.

Then the field equation for the homogeneous part is

$$\phi_0'' - 2 \coth \eta \phi_0' = \frac{-V'}{H^2 \sinh^2 \eta}, \quad (8)$$

with $H^2 = 8\pi G V_0/3$.

Eq. (8) can be integrated, giving

$$\phi_0' = \frac{-V' - \cosh^3 \eta + 3 \cosh \eta \sinh^2 \eta + 2 \sinh^3 \eta}{H^2 3 \sinh \eta}, \quad (9)$$

and the perturbation equation (5) reduces to,

$$\Phi'' - \frac{6(1 - e^{2\eta})}{3 - e^{2\eta}} \Phi' + \left(-\mathbf{L}^2 + 4 - \frac{4(3 + e^{2\eta})}{3 - e^{2\eta}} \right) \Phi = 0. \quad (10)$$

To solve these equations, we need the initial values. To determine the initial values, we must investigate the problem of what the quantum state is inside the bubble. As mentioned before, this is a very important problem, which demands further investigation. We here consider the case in which the scalar field is in the Bunch-Davies vacuum state. This is the case provided that the effect of bubble nucleation process is small and negligible. But we should keep in mind that this point needs examination in the various models of one-bubble inflation scenario, taking into consideration the effect of bubble nucleation.

Recently, quantum field theory in de Sitter space-time associated with the open chart has been investigated (Sasaki, Tanaka, & Yamamoto 1995). According to this analysis, a quantized scalar field with mass $m^2 \ll H^2$ in the Bunch-Davies Vacuum state is described in the second quantized manner as,

$$\delta\phi = \int_0^\infty dp \sum_{\sigma, l, m} \chi_{p, \sigma}(\eta) Y_{plm}(\chi, \Omega_{(2)}) \hat{a}_{p\sigma lm} + \sum_{lm} v_{(*)lm}(t, \chi, \Omega_{(2)}) \hat{a}_{(*)lm} + \text{h.c.}, \quad (11)$$

where

$$\chi_{\sigma, p}(\eta) = \frac{-1}{\sqrt{8p(p^2 + 1) \sinh \pi p}} \left[e^{\pi p/2} (ip + \coth \eta) e^{-ip\eta} + \sigma e^{-\pi p/2} (ip - \coth \eta) e^{ip\eta} \right] \frac{1}{a(\eta)}, \quad (12)$$

$$\begin{aligned} v_{(*)lm}(t, \chi, \Omega_{(2)}) &= \frac{H}{2} \sqrt{\Gamma(l+2)\Gamma(l)} \frac{P_{1/2}^{-l-1/2}(\cosh \chi)}{\sqrt{\sinh \chi}} Y_{lm}(\Omega_{(2)}) \\ &=: \frac{H}{2} W_{(*)l}(\chi) Y_{lm}(\Omega_{(2)}), \quad (l > 0), \end{aligned} \quad (13)$$

σ takes on the values ± 1 , \hat{a} is the annihilation operator, and $\Gamma(z)$ is the gamma function. The orthonormal harmonics on a three-dimensional unit hyperboloid $Y_{plm}(\chi, \Omega)$ are

$$Y_{plm}(\chi, \Omega_{(2)}) = \left| \frac{p\Gamma(ip + l + 1)}{\Gamma(ip + 1)} \right| \frac{P_{ip-1/2}^{-l-1/2}(\cosh \chi)}{\sqrt{\sinh \chi}} Y_{lm}(\Omega_{(2)}), \quad (14)$$

with normalization

$$\int_0^\infty d\chi \int d\Omega_{(2)} \sinh^2 \chi Y_{p_1 l_1 m_1}(\chi, \Omega_{(2)}) \overline{Y_{p_2 l_2 m_2}(\chi, \Omega_{(2)})} = \delta(p_1 - p_2) \delta_{l_1 l_2} \delta_{m_1 m_2}. \quad (15)$$

In the above expression, the usual harmonics behave as $Y_{plm} \propto e^{-\chi}$ at scales larger than the curvature scale, $\chi \gg 1$, although $v_{(*)lm}$ is constant for $\chi \gg 1$. Thus $v_{(*)lm}$ represents a fluctuation larger than the curvature scale, so we call this mode a super-curvature mode. The necessity of super-curvature modes for a complete description of a random field in an open universe has also been discussed by Lyth & Woszczyna (1995).

Next, let us consider the curvature perturbation, which can be written in the mode-expanded form

$$\Phi = \int_0^\infty dp \sum_{\sigma, l, m} \Phi_{p, \sigma}(\eta) Y_{plm}(\chi, \Omega_{(2)}) + \sum_{l, m} \Phi_{(*)}(\eta) W_{(*)l}(\chi) Y_{lm}(\Omega_{(2)}). \quad (16)$$

For the continuous mode (p, l, m) , Eq.(5) reduces to

$$\Phi_p'' - \frac{6(1 - e^{2\eta})}{3 - e^{2\eta}} \Phi_p' + \left(p^2 + 5 - \frac{4(3 + e^{2\eta})}{3 - e^{2\eta}} \right) \Phi_p = 0. \quad (17)$$

The solution that behaves like $\tilde{\Phi}_p \rightarrow e^{(1-ip)\eta}$ as $\eta \rightarrow -\infty$ is (Bucher & Turok 1995)

$$\tilde{\Phi}_p = e^{(1-ip)\eta} \left(1 + \frac{1+ip}{1-ip} \frac{e^{2\eta}}{3} \right). \quad (18)$$

The equation for the super-curvature mode is

$$\Phi_{(*)}'' - \frac{6(1-e^{2\eta})}{3-e^{2\eta}} \Phi_{(*)}' + \left(4 - \frac{4(3+e^{2\eta})}{3-e^{2\eta}} \right) \Phi_{(*)} = 0, \quad (19)$$

and we find a solution

$$\tilde{\Phi}_{(*)} = e^{2\eta}. \quad (20)$$

To determine the amplitude of Φ , we use Eq.(6). From the behavior at $\eta \rightarrow -\infty$, we find for the continuous mode

$$\Phi_{p,\sigma}(\eta) = \frac{2\pi G V'}{H} \frac{1}{\sqrt{8p(p^2+1)} \sinh \pi p} \left\{ e^{\pi p/2} \frac{1-ip}{2-ip} \tilde{\Phi}_p + \sigma e^{-\pi p/2} \frac{1+ip}{2+ip} \tilde{\Phi}_{-p} \right\}, \quad (21)$$

and for the super-curvature mode

$$\Phi_{(*)}(\eta) = \frac{2\pi G V'}{H} \frac{1}{3} \tilde{\Phi}_{(*)}(\eta). \quad (22)$$

We therefore have the following spectrum at the end of inflation, by taking the limit $\eta \rightarrow 0$,

$$\begin{aligned} \Phi_p(0)^2 &:= \lim_{\eta \rightarrow 0} \sum_{\sigma=\pm 1} \Phi_{p,\sigma}(\eta)^2 \\ &= \left(\frac{2\pi G V'}{H} \right)^2 \frac{\coth \pi p}{2p(p^2+1)} \frac{p^2+1}{p^2+4} \lim_{\eta \rightarrow 0} |\tilde{\Phi}_p(\eta)|^2 \\ &= \left(\frac{4\pi G V'}{3H} \right)^2 \frac{\coth \pi p}{2p(p^2+1)}, \end{aligned} \quad (23)$$

and

$$\begin{aligned} \Phi_{(*)}(0)^2 &= \lim_{\eta \rightarrow 0} \Phi_{(*)}(\eta)^2 \\ &= \left(\frac{4\pi G V'}{3H} \right)^2 \frac{1}{4}. \end{aligned} \quad (24)$$

For comparison, we also investigate the case when the scalar field is assumed to be in the conformal vacuum state (Lyth & Stewart 1990, Ratra & Peebles 1994b). In this case, the scalar field is written

$$\delta\phi = \int_0^\infty dp \sum_{l,m} \chi_p(\eta) Y_{plm}(\chi, \Omega_{(2)}) \hat{b}_{plm} + \text{h.c.}, \quad (25)$$

where

$$\chi_p(\eta) = \frac{(ip + \coth \eta) e^{-ip\eta}}{\sqrt{2p(p^2+1)}} \frac{1}{a(\eta)}. \quad (26)$$

After a similar analysis, we get the spectrum of curvature perturbations at the end of inflation,

$$\lim_{\eta \rightarrow 0} \Phi_p(\eta)^2 = \left(\frac{4\pi G V'}{3H} \right)^2 \frac{1}{2p(p^2 + 1)}. \quad (27)$$

The conformal vacuum case differs from the Bunch-Davies vacuum case in two ways, the factor $\coth \pi p$ and the super-curvature mode.

Lyth & Stewart (1990) have investigated perturbations in an open inflationary universe, and have given a relation to relate the curvature perturbation and the scalar field perturbation $\mathcal{R} \simeq -(H/\dot{\phi})\delta\phi$, though a paper justifying this relation has never been published. But the above investigation shows the correctness of their result on all scales except for the small difference of the former coefficient.

3 Observational Confrontations

Now we start testing the predictions of the open universe in the context of CDM cosmology with the initial conditions obtained above. The matter-dominated open universe has the line element (1) with $a(\eta) = \cosh \eta - 1$. In this section we use $\eta (> 0)$ as the conformal time in the matter-dominated universe.

(1) CMB Anisotropies

Let us first consider the CMB temperature fluctuation. Having obtained an initial perturbation spectrum, we can compute the temperature fluctuations in the gauge-invariant formalism (Sugiyama & Gouda 1992). As usual, we write the temperature autocorrelation in the form,

$$C(\alpha) = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos \alpha). \quad (28)$$

Figure 1(a) shows the power spectrum of temperature fluctuations, $l(l+1)C_l \times 10^{10}/2\pi$, for various values of Ω_0 with initial conditions associated with the Bunch-Davies vacuum state. We have taken $\Omega_B h^2 = 0.0125$ and Hubble parameter $h = 0.75, 0.70, 0.65, 0.65, 0.60$, for $\Omega_0 = 0.1, 0.2, 0.3, 0.4, 0.5$, respectively, to take the age problem into consideration. Figure 1(b) shows the same quantities with the conformal vacuum state (Kamionkowski et al. 1994; Górski et al. 1995). For reference, we show the corresponding results for a Λ -model with a Harrison Zel'dovich spectrum in Figure 1(c) (Sugiyama 1995). Here the parameter $\Omega_B h^2 = 0.0128$ and $h = 0.8$. We also show the results of several CMB experiments, taken from the paper by Scott, Silk & White (1995). Open models may have trouble fitting the data near the ‘‘Doppler peak’’ on degree scales, although assessing the significance of this problem will require very careful investigation (Ratra et al. 1995).

The differences between Fig.1(a) and Fig.1(b) at low multipoles come almost entirely from the contribution of the super-curvature mode (Yamamoto, Sasaki, & Tanaka 1995),

$$C_{(*)l} = \left(\frac{2\pi G V'}{3H} \right)^2 \left\{ \frac{1}{3} f(\eta_{LS}) W_{(*)l}(\eta_0 - \eta_{LS}) + 2 \int_{\eta_{LS}}^{\eta_0} d\eta' \frac{df(\eta')}{d\eta'} W_{(*)l}(\eta_0 - \eta') \right\}^2, \quad (29)$$

where η_{LS} and η_0 are the recombination time and the present time, respectively, $W_{(*)l}$ is defined in Eq.(13), and $f(\eta)$ is the decay factor of the curvature perturbation,

$$f(\eta) = 5 \frac{\sinh^2 \eta - 3\eta \sinh \eta + 4 \cosh \eta - 4}{(\cosh \eta - 1)^3}. \quad (30)$$

The most accurate and reliable CMB anisotropy data at the present time come from the COBE DMR experiment. In addition to providing us with accurate estimates of the fluctuation amplitude, the data from this experiment can be used to constrain the shape of the power spectrum. We have used the two-year COBE data (Bennett et al. 1994) to place constraints on open inflationary models, following a procedure based on the Karhunen-Loève transform (Bunn, Scott, & White 1994; Bunn & Sugiyama 1995; White & Bunn 1995; Bunn 1995ab). This procedure gives results that are generally consistent with the spherical-harmonic technique devised by Górski (1994). We will now describe this procedure.

In inflationary cosmological models, the CMB anisotropy is a realization of a Gaussian random field. If we expand the anisotropy in spherical harmonics,

$$\Delta T(\hat{\mathbf{r}}) = \sum_{l=2}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\hat{\mathbf{r}}), \quad (31)$$

then each coefficient a_{lm} is an independent Gaussian random variable of zero mean. Furthermore, the variance of a_{lm} is simply C_l . With this information, we can in principle compute the probability density $p(\vec{d} | C_l)$ of getting the actual COBE data \vec{d} given a power spectrum C_l : since each data point is a linear combination of Gaussian random variables, the probability distribution \vec{d} is simply a multivariate Gaussian,

$$p(\vec{d} | C_l) \propto \exp\left(-\frac{1}{2} \vec{d}^T M^{-1} \vec{d}\right), \quad (32)$$

where the covariance matrix M can be written in terms of the power spectrum and the noise covariance matrix.

Let us restrict our attention to a few-parameter family of possible power spectra. We will denote the parameters generically by \vec{q} . In this paper, for example, we will consider power spectra that are parameterized by two parameters, the density parameter Ω_0 and the power spectrum normalization $Q \equiv \sqrt{5C_2/4\pi}$, and so \vec{q} will be a two-dimensional vector. The probability density in equation (32) is then simply the probability density $p(\vec{d} | \vec{q})$ of the data \vec{d} given the parameters \vec{q} . If we adopt a Bayesian view of statistics, we can convert this into a probability density for the parameters given the data:

$$p(\vec{q} | \vec{d}) \propto p(\vec{d} | \vec{q})p(\vec{q}), \quad (33)$$

where $p(\vec{q})$ is the prior probability density we choose to adopt. $p(\vec{d} | \vec{q})$ is generally denoted $L(\vec{q})$ and called the likelihood.

The choice of prior distribution is a notoriously troublesome issue. In practice, one generally chooses a prior that is a smooth, slowly-varying function of the parameters. In this paper, we will adopt a prior that is uniform in Ω_0 and one that is uniform in $\ln Q$. (This prior is approximately equivalent to one that is uniform in the power spectrum normalization C_{10} near

the “pivot point.” It differs slightly from one that is uniform in Q , although not enough to affect our results significantly.)

Unfortunately, in order to compute the probability density $p(\vec{d}|\vec{q})$, and hence the likelihood L , one must invert a matrix of dimension equal to the number of data points. For the COBE DMR data, this number is of order 4000. Such exact likelihoods have been computed for a small class of models (Tegmark & Bunn 1995); however, this is quite a time-consuming procedure. The Karhunen-Loève transform allows us to “compress” the data from 4000 numbers to only 400 in a way that throws away very little of the actual cosmological signal. The likelihoods estimated from the transformed data approximate the true likelihoods well, and are much more efficient to compute. For details on how the Karhunen-Loève transform is performed, see White & Bunn (1995) and Bunn (1995ab).

Once we know L , it is quite easy to place constraints on the parameters \vec{q} . Since L is a probability distribution for \vec{q} , it should be normalized so that

$$\int L(\vec{q}) d\vec{q} = 1. \quad (34)$$

Now suppose that we choose some subset R of possible parameter values. Then if

$$\int_R L(\vec{q}) d\vec{q} = c \quad (35)$$

then we can say that \vec{q} lies in the region R with probability c . If we want to find a 95% confidence interval, we simply find a region R such that $c = 0.95$. One frequently chooses R to be the region enclosed by a contour of constant likelihood.

If one of the parameters is deemed to be uninteresting, the standard practice is to “marginalize” over it. For example, if we are interested in constraining Ω_0 but not Q , then we replace $L(\Omega_0, Q)$ by

$$L_{\text{marg}}(\Omega_0) = \int L(\Omega_0, Q) dQ. \quad (36)$$

This is a natural thing to do: if L is the joint probability density for Ω_0 and Q , then L_{marg} is the probability density for Ω_0 alone.

Figure 2 shows the contours of the likelihood $L(\Omega_0, Q)$ for open models associated with the Bunch-Davies vacuum state. In computing these likelihoods, we use a linear combination of the 53 and 90 GHz two-year COBE maps, with weights chosen to minimize the noise. We use the ecliptic-projected maps; maps that were made in Galactic coordinates, and therefore have different pixelization, give normalizations that are generally lower by a few percent (Stompor, Górski & Banday 1995; Bunn 1995a). The choice of pixelization appears to affect primarily the overall normalization of models; likelihood ratios of models with power spectra of different shapes are less affected (Bunn 1995ab).

Figure 3 shows the marginal likelihoods for Ω_0 for both the Bunch-Davies and the conformal vacuum open models. In the Bunch-Davies case, we find that $\Omega_0 > 0.34$ at 95% confidence and $\Omega_0 > 0.15$ at 99% confidence. We also show the confidence levels for various Ω_0 in Table 1. For the conformal vacuum models, the likelihood is bimodal, and so the allowed regions are not connected. If we take a cut at small Ω_0 and only consider the region $\Omega_0 \geq 0.03$, we can state that at 95% confidence either $\Omega_0 < 0.085$ or $\Omega_0 > 0.36$, and 99% confidence, either $\Omega_0 < 0.14$

or $\Omega_0 > 0.23$. There are difficulties associated with the interpretation of the likelihoods in this case (Górski et al. 1995).

Of course, the likelihood $L(\Omega_0, Q)$ provides us with accurate normalizations in addition to shape constraints. For any particular value of Ω_0 , we find the value of Q that maximizes the likelihood and use this value as the power spectrum normalization. The normalizations determined in this way have typical one-sigma fractional uncertainties of approximately 7.5%. The maximum-likelihood normalizations computed in this way are listed in the second column of Table 2(a) for the Bunch-Davies vacuum case. For comparison, we have also computed the conformal vacuum case; these normalizations are given in the second column of Table 2(b).

(2) Linear density power spectrum

We next consider the matter inhomogeneities using the COBE normalization as described above. As the density perturbation Δ is related to the curvature perturbation Φ by the gravitational Poisson equation (Kodama & Sasaki 1984),

$$(p^2 + 4)\Phi_p(\eta) = 4\pi G\rho(\eta)a^2(\eta)\Delta_p(\eta), \quad (37)$$

in linear perturbation theory, we can write the power spectrum of the matter perturbation in an open universe from Eq.(23),

$$\begin{aligned} a_0^3 P(k) \left(:= a_0^3 \Delta_p^2 \right) &= \left(\frac{2(1 - \Omega_0)}{3\Omega_0} \right)^2 (p^2 + 4)^2 a_0^3 \Phi_p^2(0) f^2(\eta_0) T(k)^2 \\ &=: \mathcal{A}(p^2 + 4)^2 \frac{\coth \pi p}{p(p^2 + 1)} T(k)^2, \end{aligned} \quad (38)$$

where $p = a_0 k$, $a_0 = 1/H_0 \sqrt{1 - \Omega_0}$, and $H_0 = 100 \text{ hkm/s/Mpc}$. The super-curvature mode does not contribute on small scales. The model based on the conformal vacuum state leads to the same form but without the factor $\coth \pi p$. In the CDM cosmology, the following transfer function is useful (Bardeen et al. 1986, Sugiyama 1995),

$$T(k) = \frac{\log(1 + 2.34q)}{2.34q} \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4 \right]^{-1/4}, \quad (39)$$

with

$$q = \left(\frac{2.726}{2.7} \right)^2 \frac{k}{\Omega_0 h \exp(-\Omega_B - \sqrt{2h}\Omega_B/\Omega_0)} \text{ hMpc}^{-1}. \quad (40)$$

This transfer function is for a flat universe; however, since we are interested in small-scale perturbations, the curvature of the universe can be neglected and the transfer function above is acceptable.

The COBE DMR normalization determines the amplitude of the fluctuation. We give the numerical value of \mathcal{A} determined from the likelihood normalization in the second column in Table 2(a). The second column in Table 2(b) gives the value for the conformal vacuum case, in which the power spectrum is obtained by Eq.(38) without the factor $\coth \pi p$.

We show in Figure 4(a) the density power spectrum $a_0^3 P(k)$, for various Ω_0 , with initial conditions based on the Bunch-Davies vacuum state. The Hubble parameter is the same as that

in Fig.1(a). The points are from Peacock & Dodds (1994). Figure 4(b) shows the density power spectra for Λ -models with Harrison-Zel'dovich power spectrum, in which the Hubble parameter is same as that in Fig.1(c).

Given the density perturbation spectrum $P(k)$, we are able to calculate $\sigma_8^2 = (\delta M/M)_{8h^{-1}\text{Mpc}}^2$, the variance of the mass fluctuation in a sphere of a radius $R = 8h^{-1}\text{Mpc}$,

$$\sigma^2(R) = \frac{1}{2\pi^2} \int k^2 dk P(k) W^2(kR), \quad (41)$$

where the top-hat window function W is defined by $W(x) = 3(\sin x - x \cos x)/x^3$. In Tables 2(a) and 2(b), we give σ_8 for various Ω_0 and h in the open model associated with Bunch-Davies vacuum and the conformal vacuum respectively. We also show values for the Λ model in Table 2(c) (Sugiyama 1995; Stompor et al. 1995).

The difference between the Bunch-Davies case and the conformal vacuum case is very small. The difference is 10 percent at $\Omega_0 = 0.05$, but is a few percent even at $\Omega_0 = 0.1$. This is because the COBE normalization based on a likelihood analysis gives more weight to the behavior of the power spectrum at $l \simeq 10$, and less weight at lower multipoles (White & Bunn 1995). When we take a $\sigma(10^\circ)$ normalization, there is a 10 percent difference between the two cases at $\Omega_0 = 0.1$. The values obtained are consistent with those in Górski et al. (1995).

A precise comparison between predictions and observations of the matter power spectrum is difficult. One of the primary problems is that we do not know whether the galaxy distribution is an unbiased tracer of the mass distribution. However, if we make the reasonable assumptions that the galaxies are not anti-biased and are not extremely strongly biased (say $b \equiv \sigma_8^{-1} \lesssim 2.5$), then these calculations suggest that $0.3 \lesssim \Omega_0 \lesssim 0.5$. Note that the values of Ω_0 preferred by the COBE likelihood analysis tend to be higher than this range; however, one might be inclined to argue that a model with $\Omega_0 \simeq 0.4 - 0.5$ passes both tests.

(3) Large-scale bulk velocity

Next, we consider large-scale bulk velocities, which are given by the following expression,

$$v_R^2 = \frac{H_0^2 a_0^2}{2\pi^2} \Omega_0^{1.2} \int dk P(k) W(kR)^2 \exp(-k^2 R_s^2), \quad (42)$$

where $W(kR)$ is the window function, and $R_s = 12h^{-1}\text{Mpc}$ is the Gaussian smoothing length for comparison with the observational data. In Table 3, we have summarized the computation of v_R^2 with $R = 40h^{-1}\text{Mpc}$ for various Ω_0 for open models with the initial conditions associated with the Bunch-Davies vacuum state. These results are consistent with those of Górski et al. (1995).

We can compare this results with the recent data from the POTENT analysis (Dekel 1994; Liddle et al. 1995): $v_{R=40h^{-1}\text{Mpc}} = 373 \pm 50\text{km/s}$. Large values of Ω_0 clearly provide a better fit to the velocities. It appears difficult to reconcile models with $\Omega_0 \lesssim 0.3$ with these data; however, it is difficult to make precise statistical statements based on these observations. As is discussed by Liddle et al. (1995), this measurement of the bulk velocity contains additional uncertainty due to cosmic variance. In addition, it is quite difficult to assess the uncertainties and potential

biases in the POTENT analysis, and one should therefore be reluctant to draw firm conclusions on the basis of such a comparison.

(4) epoch of galaxy formation

Liddle et al. (1995) have performed a detailed investigation of abundances of galaxy clusters and damped Lyman-alpha systems in open CDM models using Press-Schechter theory. In this paper we will rely on a simple and rough estimate of the epoch of galaxy formation, following the work of Gottlöber, MÜchet, & Starobinsky (1994) and Peter, Polarski, & Starobinsky (1994). According to Press-Schechter theory, the fraction of the matter in the universe which is in gravitationally bound objects above a given mass M_R at a redshift z has the form

$$F(> M_R) = \operatorname{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M_R, z)}\right), \quad (43)$$

where

$$\sigma(M_R, z) = \sigma(R) \frac{1}{1+z} \frac{f(\eta_0)}{f(\eta(z))}, \quad (44)$$

where $M_R = (4/3)\pi R^3 \rho$, and $f(\eta)$ is the decay factor of the curvature perturbation.

The choice of δ_c depends on the collapse model. The spherical collapse of a top-hat perturbation gives $\delta_c = 1.69$, although non-spherical collapse models suggest other values. Here let us consider the range ($1.33 < \delta_c < 2$) (Gottlöber et al. 1994). Observations suggest that many galaxies seem to have formed at $z = 1$, then, assuming $F(> 10^{12}M_\odot) \gtrsim 0.1$ at $z = 1$, we have $\sigma(M_R = 10^{12}M_\odot, z = 1) \gtrsim 2 \pm 0.4$.

Figure 5(a) shows a contour plot of $\sigma(M_R = 10^{12}M_\odot, z = 1)$ in the $\Omega_0 - h$ plane for the open model. Figure 5(b) is the same but for the Λ -model with a Harrison Zel'dovich spectrum. If we take the age problem into consideration, it indicates a lower bound $\Omega_0 \gtrsim 0.4$ for the open model. Note that the above estimate is very rough, although a similar constraint has been obtained from the exact estimation of cluster abundances (Liddle et al. 1995). The bound is weaker in the Λ -model than in the open model.

4 Discussion

A low-density universe is well motivated from several dynamical observations of galaxies and clusters. The simplest such low-density models are those in which the universe is open. In the context of inflation theory, however, we need a special idea such as the one-bubble inflationary scenario in order to produce an open universe. In this paper, motivated by the one-bubble inflationary universe scenario, we have examined the cosmological predictions based on the assumption that the scalar field is initially in the Bunch-Davies vacuum state. The initial perturbation spectrum has been derived by considering the evolution of perturbations in an open inflationary stage. Then the CMB anisotropies and the matter inhomogeneities have been examined.

As the first test, we have performed a likelihood analysis for the CMB anisotropies by using the COBE DMR data. Interestingly, the COBE likelihood analysis gives severe constraints on

the model. Models with $\Omega_0 \leq 0.4$, $\Omega_0 \leq 0.5$ are excluded at confidence levels of 92%, 83%, respectively. In a previous analysis associated with the conformal vacuum state (Górski et al. 1995), the likelihood function has another steep peak below $\Omega_0 \lesssim 0.15$. This complicates the statistical interpretation of the results (Górski et al. 1995). In the case of the Bunch-Davies vacuum state, no such peak appears in the range of Ω_0 we are interested in, and so the likelihood analysis gives clear results. The COBE likelihood analysis is therefore a powerful probe of these open models.

We have used the the COBE DMR maximum-likelihood normalization to predict the amplitude of matter fluctuations. According to this normalization method, there is little difference between the predictions of the Bunch-Davies vacuum and conformal vacuum cases. Even for the case $\Omega_0 = 0.1$, the discrepancy of σ_8 is a few percent. We obtain results that are similar to previous analyses: the power spectrum of the mass fluctuation fits the observations of galaxies and clusters for $0.3 \lesssim \Omega_0 \lesssim 0.5$. The required bias is unacceptably high for $\Omega_0 \lesssim 0.1$, while high values of Ω_0 demand anti-biasing. For example, $\Omega_0 \gtrsim 0.6$ needs anti-biasing when $h = 0.65$. On the other hand, the Λ -models with Harrison Zel'dovich spectrum have higher amplitude compared with open models. The Λ -model needs anti-biasing for $\Omega_0 \gtrsim 0.4$ even when $h = 0.65$. The Λ -model therefore needs low h or a tilted spectrum (Ostriker & Steinhardt 1995). For the range $0.3 \lesssim \Omega_0 \lesssim 0.5$, open models give unacceptably small bulk velocities compared with the POTENT analysis. However, given the present quality of the velocity data and the problem of cosmic variance, one might be reluctant to draw strong conclusions from this fact. The rough estimation associated with the galaxy formation gives lower bound of Ω_0 consistent with the value discussed above.

It is very interesting that the COBE likelihood analysis has given the most severe constraint on this open model. The COBE likelihood analysis strongly prefers a high value of Ω_0 . The peak value is around $0.7 \lesssim \Omega_0 \lesssim 0.8$, and we can state that $\Omega_0 \geq 0.5$ with 83% confidence in this model. Considering both the COBE analysis and the matter inhomogeneity, we are led to prefer a value of $\Omega_0 \simeq 0.5$ if the one-bubble inflationary scenario is correct. Such a model is consistent with the Press-Schechter analysis of the epoch of galaxy formation and is marginally consistent with the bulk velocity data. As the CMB data continue to improve, particularly on degree scales, we should be able to test this model.

It is premature to rule out low Ω_0 inflationary models on the basis of this investigation at present, because we do not include the effect of bubble nucleation in the calculation of initial density power spectrum. Previous analysis indicates that the bubble nucleation effect in general excites fluctuations, and amplifies the perturbations on scales larger than curvature scale (Yamamoto, Tanaka, & Sasaki 1995; Hamazaki et al. 1995). One might therefore expect low-density models to fit the COBE data even more poorly once this effect is taken into account; however, since the calculation has not been done, we cannot be certain. In particular, the status of the super-curvature mode is still quite uncertain.

Various modifications of the open model may also be viable. We must investigate the effect of gravity waves in an open inflationary universe. One might also consider the effect of tilting the primordial power spectrum; however, in order to improve the fit to the data one would probably need to tilt the power spectrum to have increased power on small scales, and such “blue” power spectra are not naturally produced by inflation. Such a model is probably too contrived to be plausible.

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Table 1
 Confidence levels for Bunch-Davies open model

$\Omega_0(>)$	confidence level (%)
0.1	99.4
0.2	98.4
0.3	96.2
0.4	91.8
0.5	83.2

Table 2(a)
 Amplitude of density perturbation for Bunch-Davies open model

Ω_0	\mathcal{A} ($h^{-1} \text{Mpc}$) ³	σ_8		
		h=0.5	h=0.65	h=0.8
0.05	$1.96(Q/27.9\mu\text{K})^2 \times 10^2$	0.0041	0.011	0.021
0.1	$2.42(Q/28.8\mu\text{K})^2 \times 10^2$	0.032	0.063	0.099
0.2	$2.83(Q/27.8\mu\text{K})^2 \times 10^2$	0.15	0.25	0.35
0.3	$2.94(Q/25.8\mu\text{K})^2 \times 10^2$	0.31	0.48	0.66
0.4	$2.90(Q/23.4\mu\text{K})^2 \times 10^2$	0.49	0.74	1.00
0.5	$2.72(Q/21.1\mu\text{K})^2 \times 10^2$	0.69	1.01	1.33
0.6	$2.43(Q/19.3\mu\text{K})^2 \times 10^2$	0.89	1.26	1.63
0.7	$2.03(Q/18.3\mu\text{K})^2 \times 10^2$	1.05	1.45	1.90
0.8	$1.54(Q/18.3\mu\text{K})^2 \times 10^2$	1.20	1.66	2.10
0.9	$1.00(Q/18.9\mu\text{K})^2 \times 10^2$	1.31	1.80	2.25

Table 2(b)

Amplitude of density perturbation for conformal vacuum open model

Ω_0	\mathcal{A} ($h^{-1}\text{Mpc}$) ³	σ_8		
		h=0.5	h=0.65	h=0.8
0.05	$2.45(Q/18.7\mu\text{K})^2 \times 10^2$	0.0045	0.012	0.023
0.1	$2.54(Q/23.1\mu\text{K})^2 \times 10^2$	0.032	0.064	0.10
0.2	$2.89(Q/26.5\mu\text{K})^2 \times 10^2$	0.15	0.25	0.36
0.3	$3.01(Q/25.9\mu\text{K})^2 \times 10^2$	0.31	0.49	0.67
0.4	$2.96(Q/23.5\mu\text{K})^2 \times 10^2$	0.50	0.75	1.01
0.5	$2.77(Q/20.6\mu\text{K})^2 \times 10^2$	0.69	1.02	1.34
0.6	$2.46(Q/18.3\mu\text{K})^2 \times 10^2$	0.88	1.27	1.64
0.7	$2.04(Q/17.2\mu\text{K})^2 \times 10^2$	1.06	1.50	1.90
0.8	$1.55(Q/17.5\mu\text{K})^2 \times 10^2$	1.20	1.67	2.10
0.9	$1.00(Q/18.7\mu\text{K})^2 \times 10^2$	1.31	1.80	2.25

Table 2(c)

Amplitude of density perturbation for Harrison-Zel'dovich Λ -model

Ω_0	σ_8		
	h=0.5	h=0.65	h=0.8
0.1	0.15	0.29	0.46
0.2	0.41	0.68	0.98
0.3	0.65	1.0	1.4
0.4	0.85	1.3	1.7
0.6	1.2	1.7	2.2

Table 3

Large scale bulk velocity for Bunch-Davies open model

Ω_0	$v_{R=40h^{-1}\text{Mpc}}$ (km/s)		
	h=0.5	h=0.65	h=0.8
0.05	6.3	8.6	11
0.1	21	29	37
0.2	71	90	106
0.3	130	160	180
0.4	200	230	260
0.5	260	300	330
0.6	320	360	390
0.7	370	410	450
0.8	410	450	490
0.9	440	480	510

Figure Captions

Figure 1. Power spectra of the CMB temperature anisotropy $l(l+1)C_l \times 10^{10}/2\pi$ for (a) the Bunch-Davies vacuum open model with $\Omega_0 = 0.1, 0.2, 0.3, 0.4, 0.5$. The data were provided by N. Sugiyama. These theoretical curves are normalized by the COBE likelihood. The curves are in descending order of Ω_0 as one moves down at $l = 50$. The results of several CMB experiments are also shown, taken from the paper by Scott, Silk, & White (1995). To compare with degree-scale observations, careful investigations are required (Ratra et al. 1995).

Figure 1(b). CMB power spectra for the conformal vacuum open model (Górski et al. 1995). The curves and points are as in Figure 1(a).

Figure 1(c). Power spectrum of the CMB temperature anisotropy $l(l+1)C_l \times 10^{10}/2\pi$ for the Harrison-Zel'dovich Λ -model with $\Omega_0 = 0.1, 0.2, 0.3, 0.4, 1.0$. (Sugiyama 1995)

Figure 2. Contour plot of the likelihood function $L(\Omega_0, Q)$ for the Bunch-Davies open model. The contour range is from $L = 0.25$ to $L = 1.5$, where the likelihoods are scaled so that $L = 1$ corresponds to a flat Harrison-Zel'dovich spectrum with maximum-likelihood normalization.

Figure 3. The marginal likelihood $L_{\text{marg}}(\Omega_0)$ as a function of Ω_0 for both the Bunch-Davies open model (solid line) and the conformal vacuum open model (dashed line).

Figure 4(a). Power spectrum of density perturbation $a_0^3 P(k)$ for the open model with $\Omega_0 = 0.1, 0.2, 0.3, 0.4, 0.5$, for $h = 0.75, 0.70, 0.65, 0.65, 0.60$, respectively. We have taken $\Omega_B h^2 = 0.0125$. The points are from Peacock & Dodds (1994).

Figure 4(b). Same figure as Fig.4(a) but for the Harrison-Zel'dovich Λ -model with $\Omega_0 = 0.1, 0.2, 0.3, 0.4, 1.0$. We have taken $\Omega_B h^2 = 0.0128$ and $h = 0.8$.

Figure 5(a). Contours of $\sigma(M_R = 10^{12} M_\odot, z = 1)$, in the $(\Omega_0 - h)$ plane for the Bunch-Davies open models. The contour range is from $\sigma = 1.0$ to $\sigma = 6.0$. The dashed lines are $\sigma = 1.6$ and 2.4 .

Figure 5(b). Same figure as Fig.5(a) but for Harrison-Zel'dovich Λ -models.