

Inverse Neutrinoless Double Beta Decay Revisited

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Abstract

We critically reexamine the prospects for the observation of the $\Delta L = 2$ lepton-number-violating process $e^-e^- \rightarrow W^-W^-$ using the e^-e^- option of a high-energy e^+e^- collider (NLC). We find that, except in the most contrived scenarios, constraints from neutrinoless double beta decay render the process unobservable at an NLC of $\sqrt{s} < 2$ TeV. Other $\Delta L = 2$ processes such as $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$, $e^-\gamma \rightarrow \nu_e\ell^-\ell^-W^+$, $e^-e^- \rightarrow \nu_e\nu_e\ell^-\ell^-$ ($\ell = \mu, \tau$), and $e^-\gamma \rightarrow e^+W^-W^-$, which use various options of the NLC, require a \sqrt{s} of at least 4 TeV for observability.

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1 Introduction

One of the most intriguing puzzles in modern particle physics is whether the neutrino has a mass. In fact, it is doubly interesting since, if the neutrino is massive, one will want to know whether it has a Dirac or a Majorana mass. If the neutrino has a Majorana mass, then it will contribute to $\Delta L = 2$ lepton-number-violating processes such as neutrinoless double beta decay ($\beta\beta_{0\nu}$). The key subprocess in $\beta\beta_{0\nu}$ is $W^-W^- \rightarrow e^-e^-$, mediated by a Majorana ν_e .

One possible future collider which is being vigorously investigated at the moment is a high-energy linear e^+e^- collider, known generically as the Next Linear Collider (NLC). With such a collider, it is possible to replace the positron by another electron and look at e^-e^- collisions. If the electron neutrino has a Majorana mass, it may be possible to observe the process $e^-e^- \rightarrow W^-W^-$. This is essentially the inverse of neutrinoless double beta decay.

In fact, this is not a new idea. The process $e^-e^- \rightarrow W^-W^-$ has been looked at several times, by different authors, over the last decade or so [1]-[6]. In the most recent analysis, the authors of Ref. [6] found that this process could be observable at an NLC of $\sqrt{s} = 500$ GeV or 1 TeV. One of the purposes of the present paper is to reexamine this analysis. Once the constraints from $\beta\beta_{0\nu}$ are taken into account, we find that, in fact, except for extremely contrived scenarios, the cross section for $e^-e^- \rightarrow W^-W^-$ is simply too small for it to be seen at a 500 GeV or 1 TeV NLC. An NLC of at least $\sqrt{s} = 2$ TeV will be necessary in order to have a hope of observing this process.

The limits from $\beta\beta_{0\nu}$ apply only to ν_e . Should the ν_μ have a Majorana mass, it will contribute to the processes $\mu^-\mu^- \rightarrow W^-W^-$ and its inverse (and similarly for the ν_τ), with no constraints from low-energy processes. However, unless a $\mu^-\mu^-$ collider is built [7], such lepton-number-violating processes cannot take place directly. Fortunately, there are other possibilities at the NLC. It is possible to backscatter laser light off one or both of the beams, creating an $e\gamma$ or $\gamma\gamma$ collider [8]. $\mu^-\mu^- \rightarrow W^-W^-$ can then be observed as a subprocess in one of the various modes of the NLC. For example, the observation of $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$ would be evidence for a Majorana ν_μ . This is the second purpose of the paper – to investigate the possibilities for the detection of $\Delta L = 2$ lepton-number violation in the muon or tau sectors at the NLC. We will see that an NLC with a centre-of-mass energy of at least 4 TeV is necessary.

The paper is organized as follows. In the following section we discuss the process $e^-e^- \rightarrow W^-W^-$, paying careful attention to the constraints from unitarity and $\beta\beta_{0\nu}$. In

Sec. 3 we elaborate on the possibilities for detecting $\Delta L = 2$ lepton-number violation in the muon or tau sectors. Sec. 4 contains a discussion of the prospects for detecting a Majorana ν_e if no e^-e^- collider is ever built. We conclude in Sec. 5.

2 $e^-e^- \rightarrow W^-W^-$

2.1 Neutrino Mixing

Suppose that the ν_e mixes with other neutrinos. For the moment, we leave the number of new neutrinos unspecified, as well as their transformation properties under $SU(2)_L$. (The ν_e could even mix with ν_μ and/or ν_τ , although this will lead to flavour-changing neutral currents, which are extremely stringently constrained.) Once the mass matrix is diagonalized, ν_e can be expressed in terms of the mass eigenstates N_i :

$$\nu_e = \sum_i U_{ei} N_i, \quad (2.1)$$

where the mixing matrix U is unitary. Phenomenologically, we have observed two things. First, the ν_e does not mix much with other neutrinos [9]:

$$\sum_{i \neq 1} |U_{ei}|^2 < 6.6 \times 10^{-3} \quad (90\% \text{ c.l.}) . \quad (2.2)$$

This limit is essentially independent of the $SU(2)_L$ transformation properties of the neutrino(s) with which the ν_e mixes. Also, the limit is quite conservative – it allows for the possibility that the other charged fermions also mix with new, exotic charged particles [10]. If one assumes that the only new particles are neutrinos, then the above limit improves somewhat to 5.0×10^{-3} . Thus, the ν_e is mainly N_1 . Second, from muon decay, we know that the N_1 is very light: $M_1 < 7$ eV [11].

2.2 Cross Section for $e^-e^- \rightarrow W^-W^-$

Assuming that the N_i are Majorana neutrinos, they will contribute to the process $e^-e^- \rightarrow W^-W^-$ through the diagrams of Fig. 1. (If right-handed W 's exist, they can also be produced, either singly or in pairs, through similar diagrams. In this paper we consider only ordinary W 's in the final state – the production of W_R 's is discussed in Refs. [1, 4, 5].) Neglecting the electron mass, the differential cross section for unpolarized electrons is

$$\begin{aligned}
\frac{d\sigma}{d\cos\theta} &= \frac{g^4}{512\pi s} \left(1 - \frac{4M_w^2}{s}\right)^{1/2} \sum_{ij} M_i M_j (U_{ei})^2 (U_{ej})^2 \\
&\times \left[\frac{1}{(t - M_i^2)(t - M_j^2)} \left(\frac{(s - 2M_w^2)(t - M_w^2)^2}{2M_w^4} + \frac{(t - M_w^2)(u - M_w^2)}{M_w^2} + \frac{s}{2} \right) \right. \\
&\quad + u \leftrightarrow t + \left. \left(\frac{1}{(t - M_i^2)(u - M_j^2)} + \frac{1}{(t - M_j^2)(u - M_i^2)} \right) \right. \\
&\quad \times \left. \left(\frac{(tu - M_w^4)(s - 2M_w^2)}{2M_w^4} - \frac{(t - M_w^2)(u - M_w^2)}{M_w^2} + \frac{3s}{2} \right) \right]. \quad (2.3)
\end{aligned}$$

Although this expression is rather complicated, it simplifies considerably in the limit that $s \gg M_w^2$, which is a reasonable approximation for the NLC. In this case, the terms in the square brackets which are proportional to $1/M_w^4$ dominate – they are larger than the terms proportional to $1/M_w^2$ by a factor $\sim s/M_w^2$. Keeping only the dominant terms, the differential cross section then becomes simply

$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{1024\pi M_w^4} \left(\sum_i M_i (U_{ei})^2 \left[\frac{t}{(t - M_i^2)} + \frac{u}{(u - M_i^2)} \right] \right)^2. \quad (2.4)$$

(Although this is an excellent approximation to the differential cross section, we nevertheless use the full expression (Eq. 2.3) when presenting numerical results.)

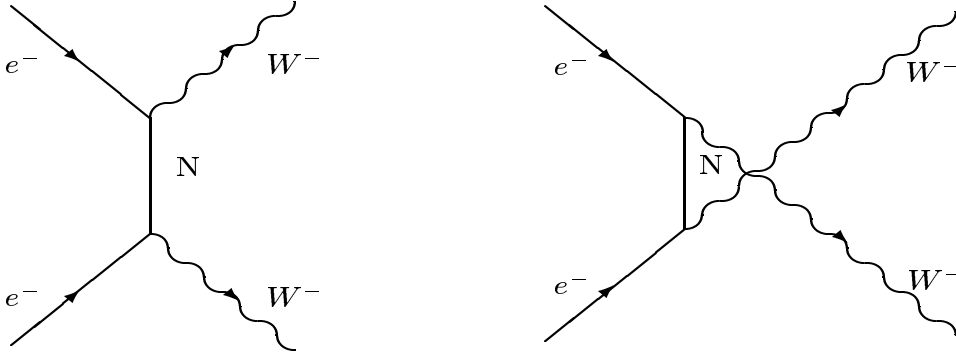


Figure 1: *Diagrams contributing to $e^-e^- \rightarrow W^-W^-$.*

Note that Eq. 2.4 is precisely what is obtained if one calculates the cross section for the process $e^-e^- \rightarrow W_{long}^- W_{long}^-$, where W_{long}^- is the longitudinal component of the W^- , as we have verified by explicit calculation. We thus confirm the observation of Ref. [6] that the production of longitudinal W 's dominates the process $e^-e^- \rightarrow W^-W^-$ if $s \gg M_w^2$. The full helicity amplitudes are given in the Appendix.

There are two limiting cases of Eq. 2.4 which will be useful in what follows. First, if $s \gg M_i^2$, the cross section becomes

$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{256\pi M_W^4} \left(\sum_i M_i (U_{ei})^2 \right)^2. \quad (2.5)$$

Second, in the other limit, $M_i^2 \gg s$, we get

$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{1024\pi M_W^4} s^2 \left(\sum_i \frac{(U_{ei})^2}{M_i} \right)^2. \quad (2.6)$$

Note that, in this limit, the cross section grows like s^2 , as was observed in Ref. [6].

2.3 Unitarity Considerations

From Eq. 2.5, we see that, in the high-energy limit ($s \rightarrow \infty$), the cross section tends towards a constant:

$$\sigma_{s \rightarrow \infty} = \frac{g^4}{128\pi M_W^4} \left(\sum_i M_i (U_{ei})^2 \right)^2. \quad (2.7)$$

In this particular case this indicates a violation of unitarity, since the amplitude (which is a pure s-wave) grows as \sqrt{s} .

There are basically two ways in which this unitarity violation can be cured. The first through the inclusion of a Higgs triplet. If the neutrinos with which the ν_e mixes are $SU(2)_L$ doublets, then they can acquire Majorana masses by giving the Higgs triplet a vacuum expectation value (v.e.v.). This Higgs triplet includes a doubly-charged Higgs, H^{--} . In this case unitarity is restored through the inclusion of a diagram in which the H^{--} is exchanged in the s-channel.

However, this type of solution has been virtually eliminated phenomenologically. The v.e.v. of the Higgs triplet breaks lepton number spontaneously, producing a Majoron. But light Majorons would contribute significantly in Z decays, and have been ruled out by the precision LEP data. Such models are therefore untenable. There are ways to evade the LEP bounds – for instance, one can add a Higgs singlet and allow the triplet to mix with the singlet [12]. However, in addition to being somewhat artificial, this solution does not explain the large range of neutrinos masses. If all neutrinos are $SU(2)_L$ doublets, then all their masses would be Majorana, and would come from the v.e.v. of the Higgs triplet. Precision measurements on the Z peak constrain such a v.e.v. to be at most a few percent of that of the standard Higgs doublet [13]. It would therefore require an extremely large Yukawa coupling to produce a neutrino mass in the TeV range. Such large Yukawa

couplings typically lead to other problems, such as the breakdown of perturbation theory, etc. In addition, there is no natural explanation why some neutrino masses should be in the eV range, while others are in the TeV range. Not even the charged fermions of the standard model cover such a large range in mass. For all of the above reasons, we discard the Higgs triplet as a solution to the unitarity problem in $e^-e^- \rightarrow W^-W^-$.

In the absence of Higgs triplets, the only way to restore unitarity is to require that the neutrinos' masses and mixing angles satisfy

$$\sum_i (U_{ei})^2 M_i = 0 . \quad (2.8)$$

Although this relation may appear arbitrary at first sight, it is in fact automatically satisfied. It is straightforward to show that

$$\sum_i (U_{ei})^2 M_i = M_{ee}^* , \quad (2.9)$$

where M_{ee} is the Majorana mass of the ν_e . However, because there are no Higgs triplets, this mass is equal to zero, so that Eq. 2.8 holds.

As an explicit example, consider the famous seesaw mechanism: one adds a right-handed neutrino N_R to the spectrum. This neutrino acquires a large Majorana mass M through the v.e.v. of a Higgs singlet, and the combination $\bar{N}_R \nu_{eL} + h.c.$ obtains a Dirac mass m once the ordinary Higgs doublet gets a v.e.v. The mass matrix looks like

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}. \quad (2.10)$$

The two mass eigenstates are N_1 and N_2 , with masses $-m^2/M$ and M , respectively (the minus sign in front of M_1 can be removed by a γ_5 rotation). For m of the order of the electron mass and M about 1 TeV, one obtains a mass of about 1 eV for the lightest neutrino. (Thus, in such models, the large range of neutrino masses is explained in a natural way, unlike the Higgs triplet models.) The ν_e is a linear combination of these two physical neutrinos:

$$\nu_e = \cos \theta N_1 + \sin \theta N_2 , \quad \sin \theta = \frac{m}{M} . \quad (2.11)$$

It is clear that, with these masses and mixing angles, the relation in Eq. 2.8 is automatically satisfied.

The downside of this particular solution is that the mixing of the ν_e with the N is tiny: for $m \sim m_e$ and $M \sim 1$ TeV, $\sin \theta \sim 10^{-6}$! This would make the cross section for $e^-e^- \rightarrow W^-W^-$ invisible, since $\sigma(e^-e^- \rightarrow W^-W^-) \propto \sin^4 \theta$, and is typical of what

happens in left-right symmetric models [1]. However, if the ν_e mixes with more neutrinos, it is, in principle, possible to satisfy Eq. 2.8 without having such small values of U_{ei} . (This is the assumed solution in Ref. [3].) This is perhaps a bit artificial, and probably requires some fine-tuning, but it is possible. If this is how unitarity restoration comes about, then Eq. 2.3 contains all the contributions to $e^-e^- \rightarrow W^-W^-$.

Although it is interesting to understand how unitarity is restored in different models, the above discussion demonstrates that the cross section for $e^-e^- \rightarrow W^-W^-$ is essentially unconstrained by such considerations – the U_{ei} and M_i can take any values consistent with the phenomenological limits in Sec. 2.1. This is not the case when the experimental limits on neutrinoless double beta decay are taken into account, which we do in the next subsection.

2.4 Limits from $\beta\beta_{0\nu}$

As mentioned in the introduction, $e^-e^- \rightarrow W^-W^-$ is essentially the inverse of neutrinoless double beta decay. We might therefore expect that the limits on the latter process could constrain the former.

If some of the neutrinos have masses $M_i \ll 1$ GeV, then, for these neutrinos, the quantity which contributes to $\beta\beta_{0\nu}$ is

$$\langle m_\nu \rangle = \sum_i' (U_{ei})^2 M_i, \quad (2.12)$$

where the sum is over the light neutrinos. (For simplicity, we have ignored factors corresponding to complications from the nuclear matrix elements – their inclusion does not change our conclusions. For more details we refer the reader to Ref. [14].) The experimental limit on $\langle m_\nu \rangle$ is [14]

$$\langle m_\nu \rangle \lesssim 1 \text{ eV}. \quad (2.13)$$

As for the neutrinos which are heavy, $M_i \gg 1$ GeV, they can still mediate $\beta\beta_{0\nu}$ decay. In this case the relevant quantity is

$$\langle m_\nu^{-1} \rangle_H = \sum_i'' (U_{ei})^2 \frac{1}{M_i}, \quad (2.14)$$

where the sum is over the heavy neutrinos. Now the experimental limit on $\beta\beta_{0\nu}$ implies the following:

$$\langle q^2 \rangle \sum_i'' (U_{ei})^2 \frac{1}{M_i} \lesssim 1 \text{ eV}, \quad (2.15)$$

where q is an average nuclear momentum transfer. If one takes q to be roughly about 100 MeV, one obtains the right order-of-magnitude constraint. However, a more careful calculation, including all the nuclear effects, gives [15]

$$\sum_i'' (U_{ei})^2 \frac{1}{M_i} < 5 \times 10^{-5} \text{ TeV}^{-1} . \quad (2.16)$$

Assuming no cancellations, this implies a generic lower bound on the mass of the heavy neutrino:

$$M_i > 2 \times 10^4 (U_{ei})^2 \text{ TeV} . \quad (2.17)$$

For $(U_{ei})^2 \sim 5 \times 10^{-3}$, this gives $M_i > 100 \text{ TeV}$!

However, it is possible to evade this order-of-magnitude bound if one allows cancellations among the various terms. This can come about in one of two ways: either (i) all the heavy neutrino masses are roughly equal, or (ii) they are different.

- If all masses are equal, then we obtain

$$M > 2 \times 10^4 \sum_i'' (U_{ei})^2 \text{ TeV} . \quad (2.18)$$

In this case, if $\sum_i'' (U_{ei})^2$ is small, M will be as well. Note that, since the mixing angles may be complex, it is possible that each of the individual $(U_{ei})^2$'s is large (up to the constraint of Eq. 2.2), but that their sum is small.

- In the second scenario involving quite different neutrino masses, there can again be cancellations among different terms. This requires either that the heavier neutrinos have larger mixings with the ν_e than the lighter ones, or that there be a large number of heavy neutrinos. For example, just to give a feel for the numbers, the contribution of a 1 TeV neutrino with a mixing of $U^2 = 5 \times 10^{-3}$ can be cancelled by (a) a 100 GeV neutrino with a mixing $U^2 = -5 \times 10^{-4}$ or (b) ten 10 TeV neutrinos with mixings of $U^2 = -5 \times 10^{-3}$. There are many other possibilities, of course, but these illustrate roughly what is required for cancellation.

We will return to these when discussing $e^- e^- \rightarrow W^- W^-$.

2.5 $e^- e^- \rightarrow W^- W^-$ at the NLC

The constraints from $\beta\beta_{0\nu}$ give us one of two conditions, depending on whether the new neutrinos are very light (Eq. 2.13: $M \ll 1 \text{ GeV}$) or very heavy (Eq. 2.16: $M \gg 1$

GeV), relative to the energy scale of neutrinoless double beta decay. For the case of light neutrinos, we can use Eq. 2.5 to calculate the cross section for $e^-e^- \rightarrow W^-W^-$, which is independent of \sqrt{s} . It is minuscule:

$$\sigma(e^-e^- \rightarrow W^-W^-) = 1.3 \times 10^{-17} \text{ fb} . \quad (2.19)$$

Such a signal is clearly unobservable at any future collider.

If no cancellations are allowed in Eq. 2.16, then $\beta\beta_{0\nu}$ constrains the neutrinos to be very massive (Eq. 2.17). For NLC's with centre-of-mass energies of order 1 TeV, we have $M_i \gg \sqrt{s}$, and Eq. 2.6 can be used to calculate the cross section for $e^-e^- \rightarrow W^-W^-$. Using the limit in Eq. 2.16, we find that, at a $\sqrt{s} = 1$ TeV NLC,

$$\sigma(e^-e^- \rightarrow W^-W^-) < 2.5 \times 10^{-3} \text{ fb} . \quad (2.20)$$

The hoped-for luminosity at a $\sqrt{s} = 1$ TeV NLC is 80 fb^{-1} . Clearly the process $e^-e^- \rightarrow W^-W^-$ is unobservable at such a collider. (Since the cross section grows like s^2 , the 500 GeV NLC fares even worse.)

However, this does not cover all the possibilities. As discussed in the previous subsection, the constraint from Eq. 2.16 can be evaded if one allows cancellations among the various contributions. Thus we must also consider neutrino masses considerably lighter than 100 TeV. Nevertheless, as we discuss below, even for such masses the process $e^-e^- \rightarrow W^-W^-$ is still unobservable at the NLC, except in the most contrived, fine-tuned models.

We consider again the two scenarios for evading the constraint from Eq. 2.16: (i) roughly equal heavy neutrino masses, and (ii) different heavy neutrino masses.

- In the scenario where all the neutrino masses are roughly equal, there is an upper limit on the mixing as a function of the neutrino mass. From Eq. 2.18 we have

$$\sum_i \text{"}(U_{ei})^2 < 5 \times 10^{-5} \left(\frac{M}{1 \text{ TeV}} \right) . \quad (2.21)$$

(Of course, even for super-heavy neutrinos, the mixing cannot be larger than the phenomenological limit of Eq. 2.2.) It is therefore possible to have neutrino masses lighter than 100 TeV, but only at the expense of smaller mixing angles. This is the key point. Even though the lighter neutrino masses soften, and even remove, the $1/M^2$ suppression of Eq. 2.6, the smaller mixing angles render the process $e^-e^- \rightarrow W^-W^-$ unobservable. For M in the range 500 GeV to 10 TeV, the cross section

for $e^-e^- \rightarrow W^-W^-$ is in the range $O(10^{-4})$ - $O(10^{-3})$ fb . In fact, the largest cross section occurs for heavier neutrinos, $M \gtrsim 10$ TeV, where the mixing angles are the largest. In this case, we simply reproduce the cross section of Eq. 2.20, which is, as we stated previously, too small to be observed.

- In the second scenario the cancellations occur between terms involving neutrinos of quite different masses. This in itself is quite contrived – it requires a fair amount of fine tuning, since the masses and mixing angles have to be carefully adjusted to have such a cancellation. However, one has to go even further to obtain an observable cross section for $e^-e^- \rightarrow W^-W^-$.

One important observation is that a neutrino of mass $M < \sqrt{s}$ which has a significant mixing with the ν_e would be first observed directly at the NLC in the process $e^+e^- \rightarrow \nu_e N_l$ [16]. The decay products of the N_l would indicate that it is a Majorana neutrino. And since such a neutrino would by itself violate the constraint from $\beta\beta_{0\nu}$ (Eq. 2.16), one could deduce the presence of additional, heavier Majorana neutrinos. Thus, if one has to add light ($M < \sqrt{s}$) neutrinos in order to evade the constraints from Eq. 2.16 and make the cross section for $e^-e^- \rightarrow W^-W^-$ observable, then the measurement of the process $e^-e^- \rightarrow W^-W^-$ is not even necessary – the neutrinos will be observed, or their presence inferred, before $e^-e^- \rightarrow W^-W^-$ is ever measured. A rather amusing situation.

Suppose there were one heavy neutrino of mass $M \sim 1$ TeV, with a mixing $U^2 = 5 \times 10^{-3}$. In this case the cross section for $e^-e^- \rightarrow W^-W^-$ at a 1 TeV NLC is $\sigma \sim 10$ fb , which is easily observable. However, as we have argued previously, if this is the only heavy neutrino, this set of parameters is ruled out by the constraints from $\beta\beta_{0\nu}$. But if we add other heavy neutrinos whose contributions conspire to evade the constraint from $\beta\beta_{0\nu}$ (Eq. 2.16), a neutrino with such a mass and mixing could conceivably be allowed. One possibility is to add a lighter neutrino N_l , say with mass $M = 100$ GeV and a mixing $U^2 = -5 \times 10^{-4}$. However, as we have discussed above, such a light neutrino would be first observed directly. Another possibility is to add ten neutrinos of mass $M = 10$ TeV and mixing $U^2 = -5 \times 10^{-3}$. This possibility is clearly exceedingly baroque.

As a final example, if there were one heavy neutrino of $M \sim 1$ TeV, with a mixing $U^2 = 5 \times 10^{-4}$, then the constraint from $\beta\beta_{0\nu}$ could be evaded through the addition of a single heavier neutrino of $M = 10$ TeV and mixing $U^2 = -5 \times 10^{-3}$. In this case, the cross section for $e^-e^- \rightarrow W^-W^-$ is $\sigma \simeq 0.04$ fb , which might be just observable. Still, in addition to requiring the fine-tuned cancellation of two terms,

this scenario requires the heavier neutrino to have a *larger* mixing angle than the lighter neutrino. This is rather unnatural, and is not what happens in the quark sector.

Of course, there are many other ways of arranging the neutrino masses and mixings in order to evade the low-energy constraint from $\beta\beta_{0\nu}$, and to produce an observable cross section from $e^-e^- \rightarrow W^-W^-$. However, the above examples give a flavour of what is necessary – one must construct extremely contrived models in order to do this.

From here on, we assume that there are no fine-tuned cancellations, and that the constraint in Eq. 2.17 holds for all neutrinos. Furthermore, when we present our results for the cross section for $e^-e^- \rightarrow W^-W^-$ (and the other processes in the subsequent sections), we assume that it is dominated by the exchange of a single neutrino. (Of course, additional, heavier neutrinos must be present to satisfy the bound from unitarity.) Even if one assumes that more than one neutrino contributes to $e^-e^- \rightarrow W^-W^-$, this will not change the cross section significantly, since the mixing angles of all the neutrinos must be correspondingly reduced in order to satisfy the constraint in Eq. 2.16.

In Fig. 2 we present the discovery limit for $e^-e^- \rightarrow W^-W^-$ at the NLC for several centre-of-mass energies as a function of M_i and $(U_{ei})^2$. We demand 10 events for discovery, and assume unpolarized e^- beams and a luminosity of $80(\sqrt{s}/(1 \text{ TeV}))^2 \text{ fb}^{-1}$. We present the discovery curves for $\sqrt{s} = 500 \text{ GeV}, 1 \text{ TeV}, 2 \text{ TeV}, 4 \text{ TeV}$ and 10 TeV . We also superimpose the phenomenological limit on $(U_{ei})^2$, as well as the constraint from $\beta\beta_{0\nu}$. Note that we have not included efficiencies for the detection of the W 's, nor have we included any backgrounds. Our discovery limits are therefore quite conservative.

As is clear from this figure, for $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV , the values of M_i and $(U_{ei})^2$ which produce an observable $e^-e^- \rightarrow W^-W^-$ cross section are already ruled out by neutrinoless double beta decay. For $\sqrt{s} = 2 \text{ TeV}$, the discovery limit and the limit from $\beta\beta_{0\nu}$ are roughly equal. Note, however, that if polarized e^- beams are used, the 2 TeV NLC opens a very small region of parameter space, and hence does slightly better than $\beta\beta_{0\nu}$. On the other hand, by the time such a collider is built the $\beta\beta_{0\nu}$ limits will probably have become more stringent, so the prospects for a 2 TeV NLC to improve upon neutrinoless double beta decay are marginal at best. Finally, for 4 TeV and 10 TeV NLC's, there exists a sizeable region of M_i - $(U_{ei})^2$ parameter space, not ruled out by $\beta\beta_{0\nu}$, which produces an observable signal for $e^-e^- \rightarrow W^-W^-$.

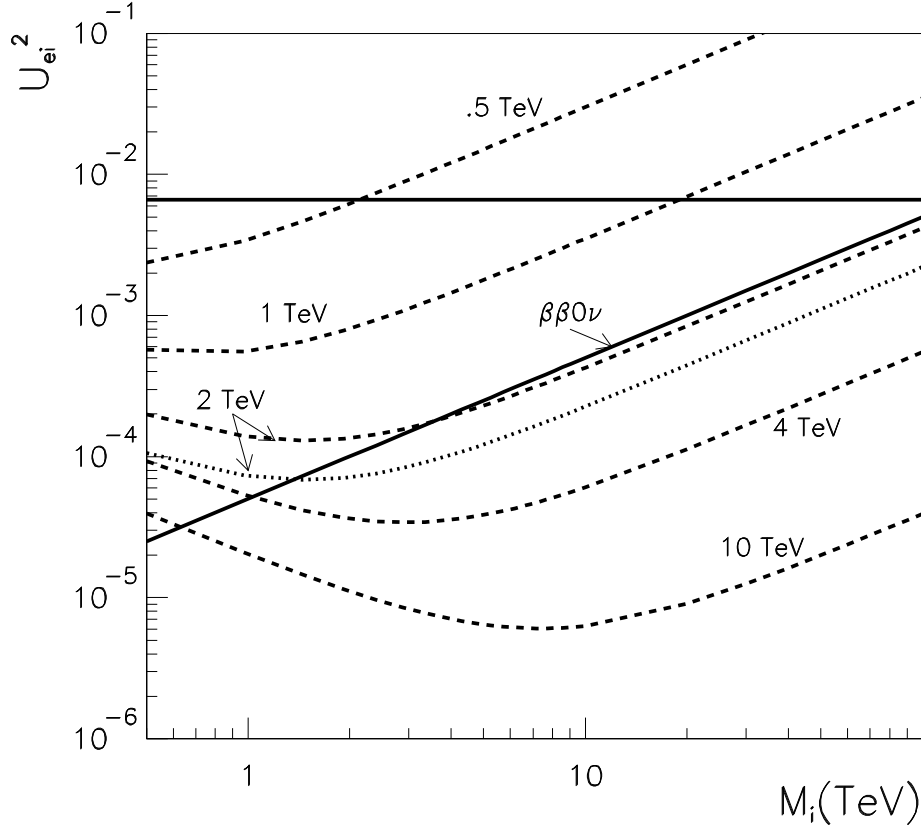


Figure 2: *Discovery limit for $e^-e^- \rightarrow W^-W^-$ at the NLC as a function of M_i and $(U_{ei})^2$ for $\sqrt{s} = 500 \text{ GeV}$, 1 TeV , 2 TeV , 4 TeV and 10 TeV (dashed lines). We assume unpolarized e^- beams and a luminosity of $80(\sqrt{s}/(1 \text{ TeV}))^2 \text{ fb}^{-1}$. For $\sqrt{s} = 2 \text{ TeV}$, the limit assuming polarized e^- beams is also shown (dotted line). In all cases, the parameter space above the line corresponds to observable events. We also superimpose the experimental limit from $\beta\beta_{0\nu}$ (diagonal solid line), as well as the limit on $(U_{ei})^2$ (horizontal solid line). Here, the parameter space above the line is ruled out.*

3 Other $\Delta L = 2$ Processes at the NLC

In the last section, we saw that the constraints from neutrinoless double beta decay are so stringent that an NLC of at least $\sqrt{s} = 2$ TeV is required to be able to observe the process $e^-e^- \rightarrow W^-W^-$. However, $\beta\beta_{0\nu}$ constrains only the ν_e – it says nothing about the ν_μ or the ν_τ . It therefore seems reasonable to ask about the possibilities for observing other $\Delta L = 2$ processes at the NLC, specifically those involving a Majorana ν_μ or ν_τ . We address this issue in this section.

If the ν_ℓ ($\ell = \mu, \tau$) is Majorana, it will mediate processes such as $\ell^-\ell^- \rightarrow W^-W^-$. This is exactly like the ν_e , except that there are no constraints from $\beta\beta_{0\nu}$. On the other hand, there is a major disadvantage – the NLC involves e^+/e^- beams, not ℓ^+/ℓ^- . Thus, $\ell^-\ell^- \rightarrow W^-W^-$ cannot be observed directly as a $2 \rightarrow 2$ process at the NLC, unlike $e^-e^- \rightarrow W^-W^-$. However, it does appear as a subprocess in a number of $2 \rightarrow 4$ processes involving the various modes of the NLC. Specifically, if the ν_ℓ is Majorana, it will mediate $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$, $e^-\gamma \rightarrow \nu_\ell\ell^-\ell^-W^+$ and $e^-e^- \rightarrow \nu_\ell\nu_\ell\ell^-\ell^-$. (This is similar to the analysis of Ref. [3], where the process $pp \rightarrow (jet)_1(jet)_2e^+e^+$ was considered.) We discuss these possibilities in turn in the subsections which follow. In principle, the e^+e^- option of the NLC can also be used: $e^+e^- \rightarrow e^+\nu_\ell\ell^-\ell^-W^+$. However, since this is a $2 \rightarrow 5$ process, it will be smaller than the others, so we do not consider it further.

3.1 Neutrino Masses and Mixing

The limits on the masses of the ν_μ and ν_τ are [11]

$$\begin{aligned} m_{\nu_\mu} &< 0.27 \text{ MeV}, \\ m_{\nu_\tau} &< 31 \text{ MeV}. \end{aligned} \tag{3.22}$$

Suppose that the masses of the neutrinos are given by their upper limits. If ν_μ and ν_τ are Majorana, but do not mix with heavy neutrinos, then the cross section for $\ell^-\ell^- \rightarrow W^-W^-$ is still unobservable – from Eq. 2.19 it is at most $O(10^{-3})$ fb. Thus, in order to observe $\Delta L = 2$ processes involving the ν_μ or ν_τ , these neutrinos must mix with heavy Majorana neutrinos, just as was the case for the ν_e .

The limits on the mixing of the ν_μ and ν_τ are [9]

$$\begin{aligned} \sum_{i \neq 1} |U_{\mu i}|^2 &< 6.0 \times 10^{-3} \quad (90\% \text{ c.l.}), \\ \sum_{i \neq 1} |U_{\tau i}|^2 &< 1.8 \times 10^{-2} \quad (90\% \text{ c.l.}). \end{aligned} \tag{3.23}$$

As with the ν_e , these conservative limits are for the case where the other fermions also mix with new particles. If one assumes that only the neutrinos mix, then the limits improve to 1.8×10^{-3} and 9.6×10^{-3} for $\sum_{i \neq 1} |U_{\mu i}|^2$ and $\sum_{i \neq 1} |U_{\tau i}|^2$, respectively. In our analyses, we will use the conservative limits above.

3.2 $\gamma\gamma \rightarrow \ell^+ \ell^+ W^- W^-$

A large number of Feynman diagrams contribute to $\gamma\gamma \rightarrow \ell^+ \ell^+ W^- W^-$. However, it can be argued that a single one dominates. First, the diagrams can be separated into two categories: “fusion” and “bremmstrahlung.” In the fusion diagrams, each photon splits into a real and a quasi-real (*i.e.* almost on-shell) particle. The two quasi-real particles then interact, creating an internal $2 \rightarrow 2$ process. In bremmstrahlung diagrams, the two photons interact in a $2 \rightarrow 2$ process, followed by the radiation of particles from one of the final lines. The fusion diagrams are clearly much larger than the bremmstrahlung diagrams, since they involve the propagators of almost on-shell particles.

There are 3 fusion diagrams, involving the internal $2 \rightarrow 2$ subprocesses $\ell^- \ell^- \rightarrow W^- W^-$, $\ell^- W^+ \rightarrow \ell^+ W^-$ and $W^+ W^+ \rightarrow \ell^+ \ell^+$. We remind the reader that it is primarily W_{long} , the longitudinal component of the W , which is involved in the subprocesses. In order to compare the sizes of these 3 fusion diagrams, it is not necessary to calculate the entire $2 \rightarrow 4$ process – one can simply convolute the internal $2 \rightarrow 2$ process with the structure functions of the ℓ and/or W_{long} in the photon. Thus, a comparison of the luminosity spectrum for $\ell\ell$, ℓW_{long} and $W_{long} W_{long}$ in $\gamma\gamma$ will suffice to tell us which, if any, of the 3 fusion diagrams dominates. In Fig. 3 we show the luminosity for $\ell = \mu$ and W_{long} as a function of the energy fraction ($\tau = \hat{s}/s_{\gamma\gamma}$) of the photons carried by the quasi-real particles, μ or W_{long} . The luminosity is defined as

$$\frac{dL}{d\tau} = N \int dx f_{i/\gamma}(x, Q^2) f_{j/\gamma}(\tau/x, Q^2) , \quad (3.24)$$

where $N = 1$ ($N = 2$) if $i = j$ ($i \neq j$) and $i, j = \mu$ or W_{long} . Q^2 is a typical scale for the subprocess. Here we take $Q^2 = s_{\gamma\gamma}/4$. The structure functions for the leptons are taken from Ref. [17] and those for the longitudinal W were given in Ref. [18][†]. It is clear that there is very little W_{long} in the photon, since over most of the energy range, and especially in the high-energy region which gives the main contribution to the process under study, $W_{long} W_{long} \ll \ell W_{long} \ll \ell\ell$. Thus, the dominant diagram is the one in which the two

[†]The structure function describing the W_{long} content in the photon consists of two parts – one where the spectator W is transverse and the other where it is longitudinal. It has been found that the former is much larger [18] and shows scaling behaviour. Our numbers are based only on this component.

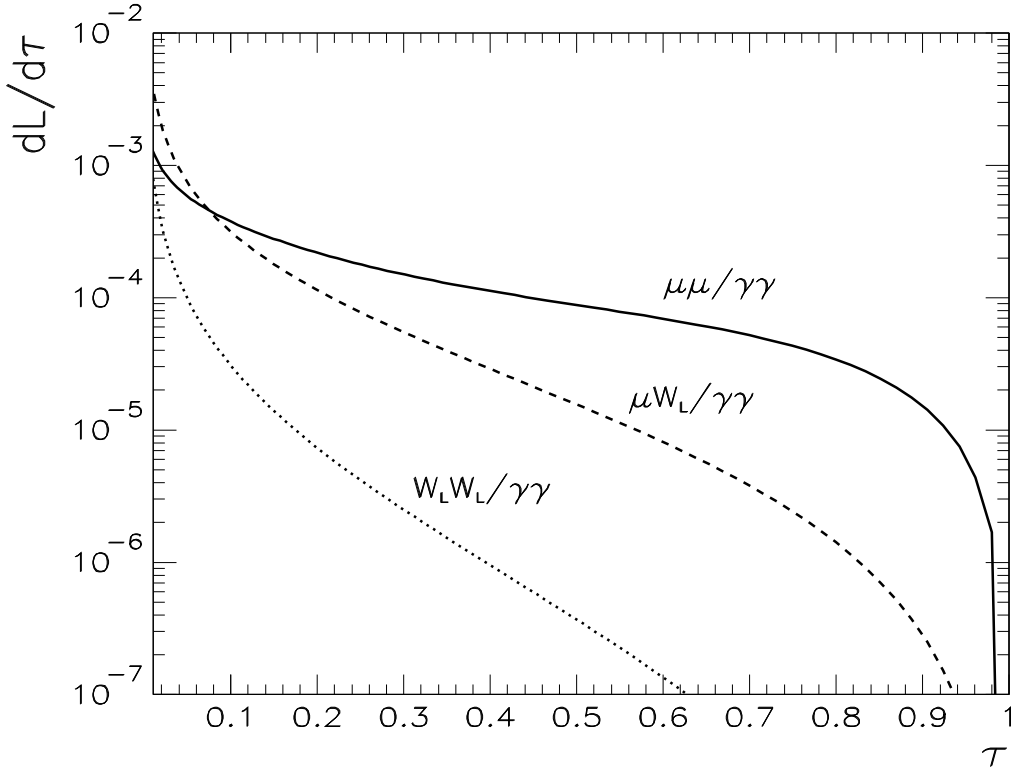


Figure 3: *The luminosity spectra for $\mu\mu$, μW_{long} and $W_{\text{long}}W_{\text{long}}$ in $\gamma\gamma$.*

quasi-real particles are ℓ and the internal $2 \rightarrow 2$ subprocess is $\ell^-\ell^- \rightarrow W^-W^-$. This is shown in Fig. 4.

In Fig. 5 we present the cross section for the process $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$ as a function of the neutrino mass M_i for three centre-of-mass energies: 2 TeV, 4 TeV and 10 TeV. We take $(U_{\mu i})^2 = 6.0 \times 10^{-3}$. Note that, in all cases, if $M_i < \sqrt{s}$, the new neutrino is far more likely to be first discovered via single production in $e^+e^- \rightarrow \nu_\mu N_i$ than in $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$ (see the discussion in Sec. 2.5). Thus, although we present the cross section for a large range of neutrino masses, we really should consider only $M_i > \sqrt{s}$. Assuming a luminosity of $80(\sqrt{s}/(1 \text{ TeV}))^2 \text{ fb}^{-1}$, we see that this process is unobservable at $\sqrt{s} = 2 \text{ TeV}$, regardless of the neutrino mass. And a signal of 10 events can be observed at $\sqrt{s} = 4 \text{ TeV}$ only for $M_i \lesssim 3 \text{ TeV}$. One has to go to higher energies to be able

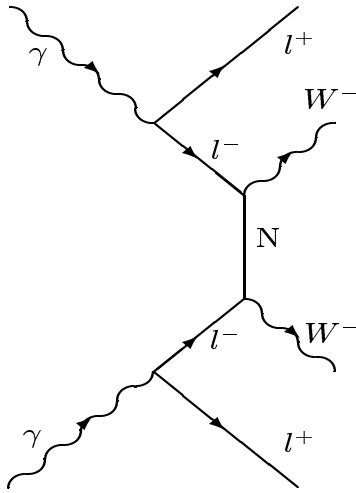


Figure 4: *The dominant diagram in $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$.*

to observe $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$ for $M_i > \sqrt{s}$: for example, for $\sqrt{s} = 10$ TeV, the process is observable for $M_i \lesssim 90$ TeV. Of course, for the higher-energy NLC's, the luminosity assumed is considerable – the reality could be quite different. But if the luminosity scales as we have assumed, and if the neutrino mixing is as large as we have taken it to be, the $\Delta L = 2$ process $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$ can be observed at an NLC with a centre-of-mass energy above 4 TeV.

We must also stress again that we have not considered here any backgrounds and have only looked for processes with a few event signals. A more careful analysis would include backgrounds from standard processes without lepton number violation, such as $\gamma\gamma \rightarrow \mu^+\mu^-W^+W^-$. In addition, we have not folded in the photon energy spectrum due to the backscattering of laser light off the e^+/e^- beams. Since the backscattered photons are not monochromatic, the inclusion of this spectrum would somewhat reduce the cross sections in our figures.

There are certain numerical differences for the process $\gamma\gamma \rightarrow \tau^+\tau^+W^-W^-$. Although the mixing can be three times as large (see Eq. 3.23), there is also a suppression $(\ln[s/4m_\tau^2]/\ln[s/4m_\mu^2])^2$ due to the larger τ mass. Putting the factors together, we estimate that the cross section for $\gamma\gamma \rightarrow \tau^+\tau^+W^-W^-$ can be roughly 4 times larger than that for $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$. However, when one folds in the much smaller efficiencies for detecting τ 's, not to speak of the increased backgrounds, it is more promising to search for $\Delta L = 2$ lepton number violation through $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$.

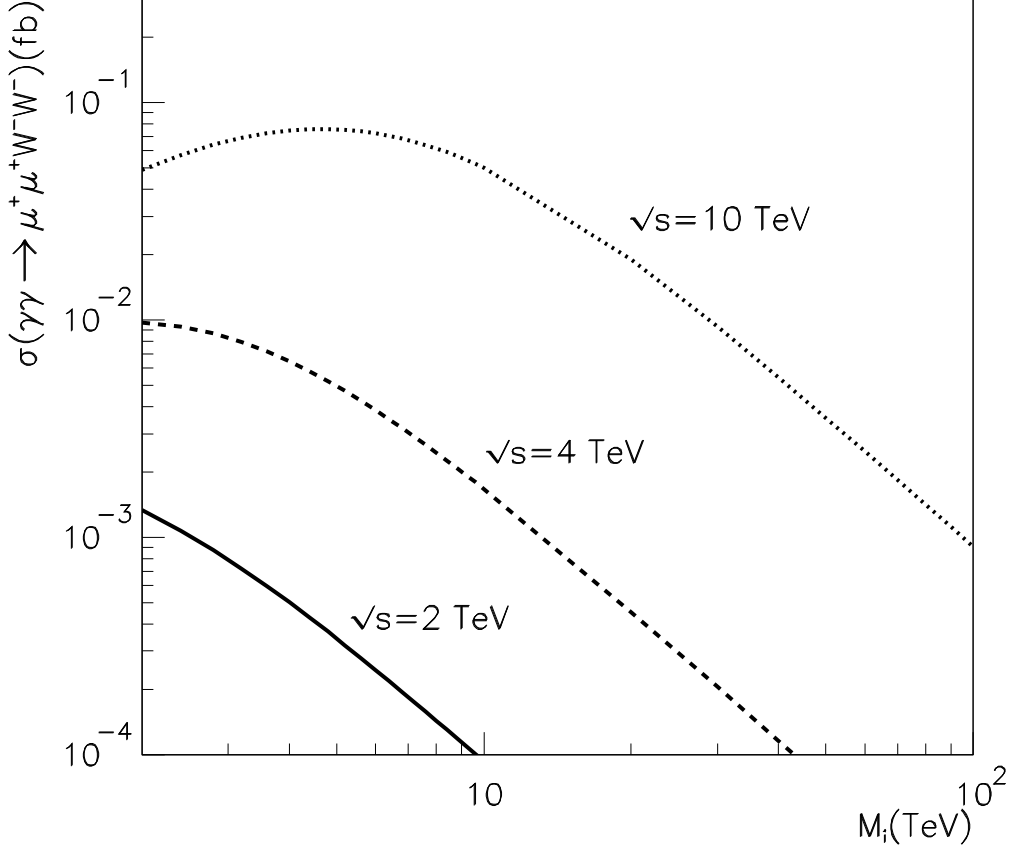


Figure 5: Cross section for $\gamma\gamma \rightarrow \mu^+\mu^+W^-W^-$ at the NLC assuming $(U_{\mu i})^2 = 6.0 \times 10^{-3}$ for $\sqrt{s} = 2$ TeV (solid line), 4 TeV (dashed line) and 10 TeV (dotted line).

3.3 $e^-\gamma \rightarrow \nu_e \ell^-\ell^-W^+$

The process $e^-\gamma \rightarrow \nu_e \ell^-\ell^-W^+$ also involves a large number of Feynman diagrams. However, just as was the case for $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$, there is a single diagram which dominates, shown in Fig. 6. (The argument leading to this is essentially the same as for $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$.) On the other hand, in contrast to $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$, note that this diagram involves an internal W_{long} . Just like the photon, there is relatively little W_{long} in the electron (the dominant term in the two sets of structure functions is the same up to a factor of $4\sin^2\theta_w \approx 1$ [18]). We therefore expect the cross section for $e^-\gamma \rightarrow \nu_e \ell^-\ell^-W^+$ to be suppressed relative to that for $\gamma\gamma \rightarrow \ell^+\ell^+W^-W^-$.

This is indeed the case. In Fig. 7 we present the cross section for $e^-\gamma \rightarrow \nu_e \mu^-\mu^-W^+$

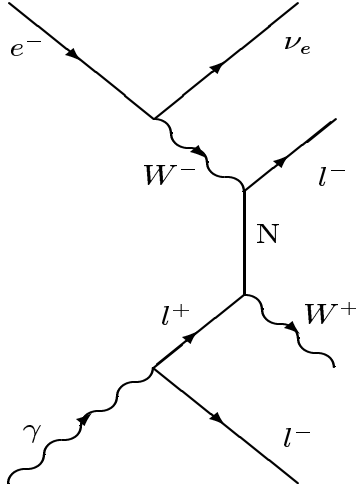


Figure 6: *The dominant diagram in $e^- \gamma \rightarrow \nu_e \ell^- \ell^- W^+$.*

as a function of the neutrino mass M_i for three centre-of-mass energies: 2 TeV, 4 TeV and 10 TeV. We again take $(U_{\mu i})^2 = 6.0 \times 10^{-3}$. It is clear that the situation is worse than for $\gamma\gamma \rightarrow \mu^+ \mu^+ W^- W^-$. Again assuming a luminosity of $80(\sqrt{s}/(1 \text{ TeV}))^2 \text{ fb}^{-1}$, we see that at $\sqrt{s} = 4 \text{ TeV}$, this process is unobservable even for $M_i < \sqrt{s}$. And at $\sqrt{s} = 10 \text{ TeV}$, the process is observable, but the reach is reduced compared to $\gamma\gamma \rightarrow \mu^+ \mu^+ W^- W^-$ – a signal of 10 events requires $M_i \lesssim 40 \text{ TeV}$.

As far as $e^- \gamma \rightarrow \nu_e \tau^- \tau^- W^+$ is concerned, although the cross section may be somewhat larger than that of $e^- \gamma \rightarrow \nu_e \mu^- \mu^- W^+$, the process suffers from the same problems as $\gamma\gamma \rightarrow \tau^+ \tau^+ W^- W^-$: larger backgrounds and worse detection efficiencies. When all is folded together, $e^- \gamma \rightarrow \nu_e \tau^- \tau^- W^+$ is probably worse than $e^- \gamma \rightarrow \nu_e \mu^- \mu^- W^+$.

3.4 $e^- e^- \rightarrow \nu_e \nu_e \ell^- \ell^-$

Should the $e^- e^-$ option of the NLC be available, not only can it be used to search for a Majorana ν_e via $e^- e^- \rightarrow W^- W^-$, as discussed in Sec. 2, but a Majorana ν_ℓ ($\ell = \mu, \tau$) can in principle be detected through the process $e^- e^- \rightarrow \nu_e \nu_e \ell^- \ell^-$. The diagram is shown in Fig. 8. However, note that this diagram involves two internal W_{long} 's. Therefore the cross section for this process will be suppressed relative to that for $e^- \gamma \rightarrow \nu_e \ell^- \ell^- W^+$, and very suppressed relative to $\gamma\gamma \rightarrow \ell^+ \ell^+ W^- W^-$. It is clear, therefore, that $e^- e^- \rightarrow \nu_e \nu_e \ell^- \ell^-$ is far from the optimal way to search for $\Delta L = 2$ processes involving a Majorana ν_μ or ν_τ , and we do not consider it further.

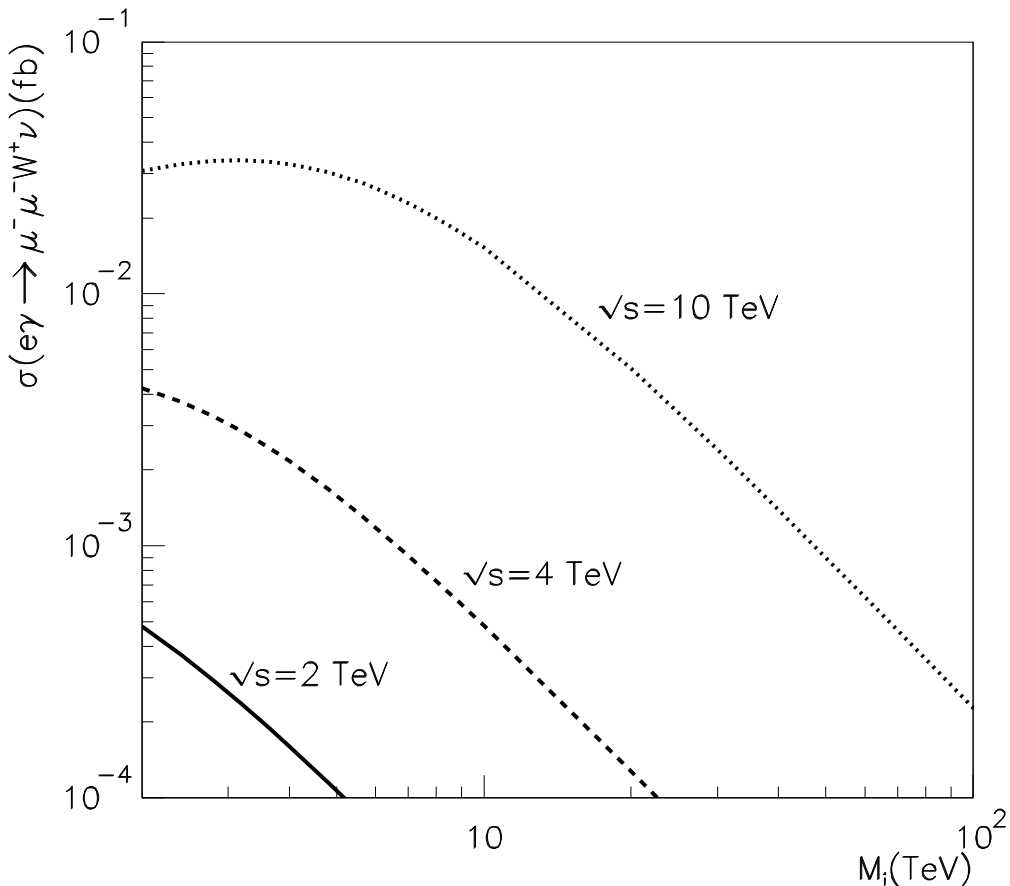


Figure 7: Cross section for $e^- \gamma \rightarrow \nu_e \mu^- \mu^- W^+$ at the NLC assuming $(U_{\mu i})^2 = 6.0 \times 10^{-3}$ for $\sqrt{s} = 2$ TeV (solid line), 4 TeV (dashed line) and 10 TeV (dotted line).

4 Detecting a Majorana ν_e Without an $e^- e^-$ Collider

For various reasons, it is conceivable that, even if an NLC is built, the $e^- e^-$ option may never be used. In the absence of an $e^- e^-$ collider, what are the prospects for detecting a Majorana ν_e through $\Delta L = 2$ processes similar to those discussed to this point? The only real possibility is the $2 \rightarrow 3$ process $e^- \gamma \rightarrow e^+ W^- W^-$.[‡] The dominant diagram for this process is shown in Fig. 9.

In Fig. 10 we present the discovery limit for $e^- \gamma \rightarrow e^+ W^- W^-$ at the NLC for $\sqrt{s} = 4$

[‡]The $\Delta L = 2$ process $\gamma \gamma \rightarrow e^+ e^+ W^- W^-$ could also occur, with a cross-section slightly larger than the one presented for $\gamma \gamma \rightarrow \mu^+ \mu^+ W^- W^-$ due to the larger electron content of the photon. However, with the constraint from $\beta \beta_{0\nu}$, a few events are expected only for very heavy neutrinos.

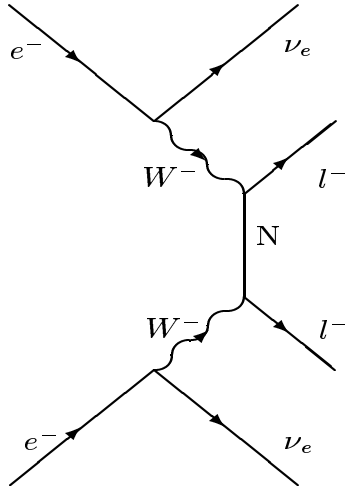


Figure 8: *The dominant diagram in $e^-e^- \rightarrow \nu_e\nu_e\ell^-\ell^-$.*

TeV and 10 TeV as a function of M_i and $(U_{ei})^2$. We consider two scenarios. In the optimistic (conservative) scenario, the e^- is polarized (unpolarized), and we demand 10 (25) events for discovery. As is clear from the figure, for $\sqrt{s} = 4$ TeV, even in the optimistic scenario the values of M_i and $(U_{ei})^2$ which produce an observable $e^-\gamma \rightarrow e^+W^-W^-$ cross section are already ruled out by neutrinoless double beta decay. However, for $\sqrt{s} = 10$ TeV, there exists a sizeable allowed region of $M_i-(U_{ei})^2$ parameter space which produces an observable signal for $e^-\gamma \rightarrow e^+W^-W^-$. Therefore, even if the e^-e^- option of the NLC is never used, it will be possible to detect a Majorana ν_e . However, one must go to extremely high energies and luminosities.

5 Conclusions

We have critically reexamined the prospects for the observation of $e^-e^- \rightarrow W^-W^-$ at a high-energy e^-e^- collider. This process is essentially the inverse of neutrinoless double beta decay ($\beta\beta_{0\nu}$). Once the constraints from $\beta\beta_{0\nu}$ are taken into account, we have found that $e^-e^- \rightarrow W^-W^-$ is unobservable at an NLC of $\sqrt{s} < 2$ TeV. It is possible to evade the constraints, but this requires models which are extremely contrived and fine-tuned. A $\sqrt{s} = 2$ TeV NLC essentially reproduces the limits from $\beta\beta_{0\nu}$, and for $\sqrt{s} > 2$ TeV, there is a sizeable region of parameter space, not ruled out by $\beta\beta_{0\nu}$, which produces an observable signal for $e^-e^- \rightarrow W^-W^-$.

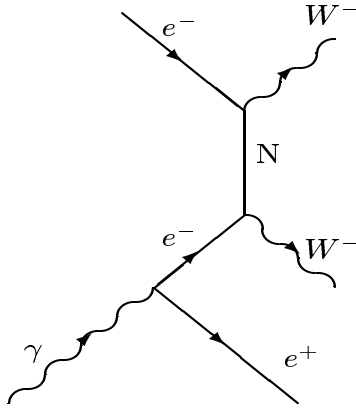


Figure 9: *The dominant diagram in $e^- \gamma \rightarrow e^+ W^- W^-$.*

The constraints from $\beta\beta_{0\nu}$ apply only to Majorana neutrinos which mix with the ν_e . $\Delta L = 2$ processes in the μ or τ sectors are unconstrained by $\beta\beta_{0\nu}$. We have therefore also considered other $\Delta L = 2$ processes at the NLC, involving μ - and τ -lepton-number violation. The process $\gamma\gamma \rightarrow \ell^+ \ell^+ W^- W^-$ ($\ell = \mu, \tau$) can be observed for $\sqrt{s} > 4$ TeV, while the observation of $e^- \gamma \rightarrow \nu_e \ell^- \ell^- W^+$ requires $\sqrt{s} \sim 10$ TeV.

Finally, we have examined the possibilities for the observation of $\Delta L = 2$ e -lepton-number violation in the absence of an $e^- e^-$ collider. The most promising process is $e^- \gamma \rightarrow e^+ W^- W^-$. Taking into account the constraints from $\beta\beta_{0\nu}$, we have found that its observation requires $\sqrt{s} \sim 10$ TeV.

Note added: While writing up this paper, we received Ref. [19], which also discusses $e^- e^- \rightarrow W^- W^-$. These authors arrive at the conclusion that this process is observable at a 1 TeV NLC. However, like Ref. [6], they have not included the constraints from $\beta\beta_{0\nu}$.

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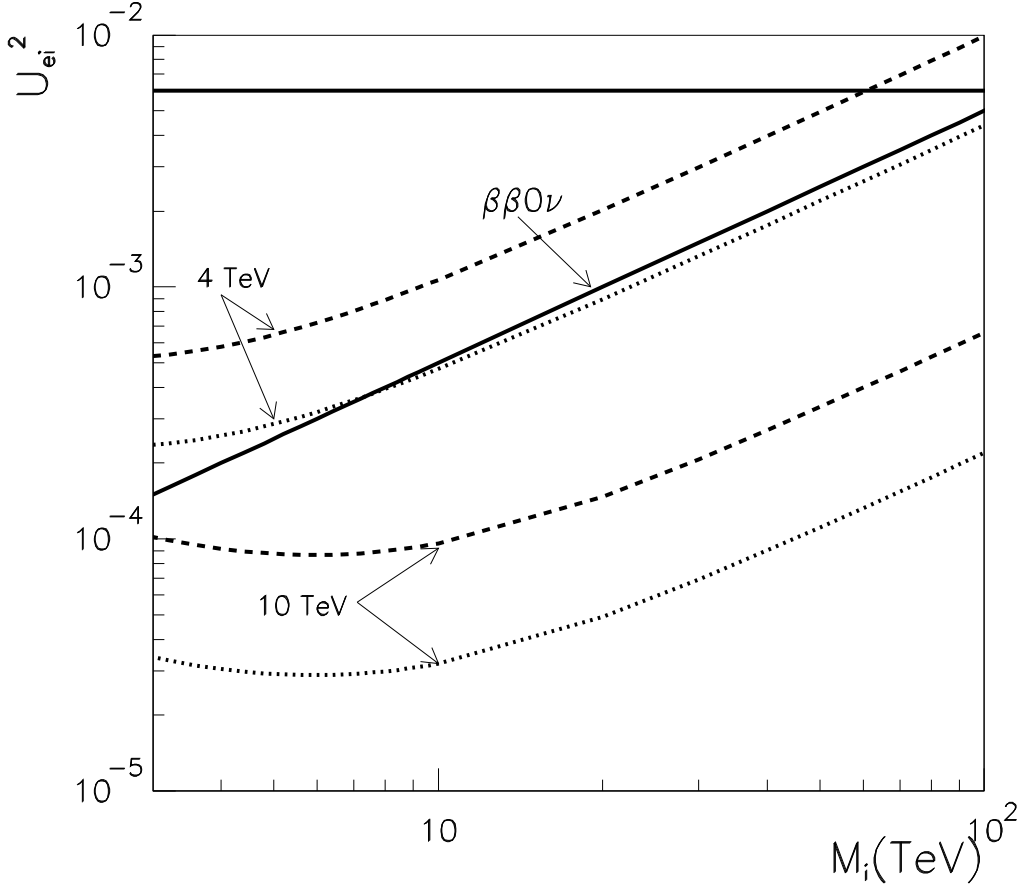


Figure 10: Discovery limit for $e^- \gamma \rightarrow e^+ W^- W^-$ at the NLC as a function of M_i and $(U_{ei})^2$. We assume a luminosity of $80(\sqrt{s}/(1 \text{ TeV}))^2 \text{ fb}^{-1}$. The dash-dot (dotted) line corresponds to an unpolarized (polarized) e^- beam, and we require 25 (10) events for discovery. In all cases, the parameter space above the line corresponds to observable events. We also superimpose the experimental limit from $\beta\beta_{0\nu}$ (diagonal solid line), as well as the limit on $(U_{ei})^2$ (horizontal solid line). Here, the parameter space above the line is ruled out.

6 Appendix

The helicity amplitudes for $e^-e^- \rightarrow W^-W^-$ can be written in a very compact way. The only non-vanishing helicity amplitudes are those involving left-handed electrons, *i.e.* we have $2\lambda_e = 2\lambda'_e = -1$ where λ_e is the helicity of the electron. The helicities of the W^- are denoted by h_i , with $h_i = 0, \tau$, $\tau = \pm 1$ and 0 is the longitudinal contribution. $\beta = \sqrt{1 - 4M_W^2/s}$ will denote the velocity of the W in the centre-of-mass system.

The amplitude is given by

$$\mathcal{M}_{h_1, h_2}^{\lambda, \lambda'} = \frac{\delta_{2\lambda, -1} \delta_{2\lambda', -1}}{2\sqrt{s}} \left(\frac{g}{\sqrt{2}} \right)^2 \sum_j M_j U_{ej}^2 A_{h_1, h_2}^{(j)}, \quad (6.25)$$

where

$$\begin{aligned} A_{00}^{(j)} &= -\frac{2s}{M_W^2} \left(\frac{t}{t - M_j^2} + \frac{u}{u - M_j^2} \right) \\ A_{\tau, \tau'}^{(j)} &= 2\delta_{\tau, \tau'} \left\{ \frac{s}{t - M_j^2} \left(1 - \frac{\tau t - u}{\beta s} \right) + t \leftrightarrow u \right\} \\ A_{0, \tau}^{(j)} &= \sqrt{\frac{s}{M_W^2}} \left(\frac{u - t}{\beta} \sqrt{\frac{2}{ut - M_W^4}} \right) \frac{s^2}{(t - M_j^2)(u - M_j^2)} \\ &\quad \left\{ \frac{ut - M_W^4}{s^2} - \frac{\tau}{\beta} \left(\frac{(t + M_W^2)(u + M_W^2)}{s^2} + \frac{M_W^2}{s} \frac{(t - u)^2}{s^2} \right) \right\}. \end{aligned} \quad (6.26)$$

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