FEW-BODY DESCRIPTIONS OF THE πNN SYSTEM IN THREE AND FOUR DIMENSIONS [1]

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Abstract

We summarise the recent theoretical progress in few-body descriptions of the πNN system. Previous descriptions, both three- and four-dimensional, are shown to possess serious theoretical defficiencies. We illustrate how three-dimensional approaches suffer from renormalisation problems, and how four-dimensional descriptions contain both overcounting and undercounting of diagrams. We then show how such theoretical problems have been recently overcome, leading to new practical few-body equations for the πNN system.

1. Introduction

In the absence of three-body forces, Faddeev equations provide the theoretically exact way to describe three-body systems. It is this fact that has enabled models of the three-nucleon system where the effect of the missing three-body forces can be accurately studied. It may at first seem that the system consisting of one pion and two nucleons can be similarly described in an accurate way with Faddeev equations. This, however, is not the case. The problem, of course, is that a pion can get absorbed by one nucleon and then emitted by the other nucleon, thus making the standard three-body description inappropriate. Indeed any number of pions can get created and absorbed by nucleons, and it is clear that field theory must be used to describe the πNN system, and not the standard three-body theory of quantum mechanics. The problem of formulating a few-body description for the πNN system, that is the analogue of the Faddeev description for three nucleons, has by now a long history.[1] Yet until very recently, both three- and four-dimensional formulations have had serious theoretical inconsistencies. Here we would like to summarise these inconsistencies, and to describe our recently developed few-body descriptions that appear to overcome all these theoretical problems. The new descriptions give rise to πNN scattering equation that can be written in various equivalent forms. Here we present our equations in a form that is most easily compared with previous works, while at the same time being especially convenient for numerical solution.

2. Three-dimensional formulation

Quantum field theory requires the use of four dimensions to assure manifestly covariant descriptions. However, solving four-dimensional equations presents an enormous numerical task, and consequently three-dimensional formulations of the πNN system are most desirable and indeed have been the most numerous.

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For the last fifteen years or so, the most sophisticated few-body description of the πNN system has been the "unitary $NN-\pi NN$ model".[2]-[5] This is a field-theoretic model based on time-ordered perturbation theory, it takes into account pion absorpion, and has the desirable properties of two- and three-body unitarity. The essential feature of the model is that it describes all the processes $\pi d \rightarrow \pi d$, $\pi d \rightarrow \pi NN$, $NN \rightarrow \pi NN$, $NN \rightarrow \pi d$, and $NN \rightarrow NN$ all within the one set of coupled equations.

The derivation of the unitary $NN-\pi NN$ equations is based upon truncating Hilbert space to states of at most one pion. In practise, this means retaining all diagrams contributing to subsystem πN and NN potentials, but neglecting all other diagrams having two or more pions in an intermediate state. Many calculations have been performed with the $NN-\pi NN$ model.[6]-[11] In general, one can say that the model can account for an extensive amount of data, albeit only in a qualitative way.

2.1 The renormalization problem

Despite the modest successes of the $NN-\pi NN$ model, it has become clear that the model itself has a serious theoretical inconsistency.[12] The origin of the problem lies in the truncation of Hilbert space used to derive the $NN-\pi NN$ equations. This truncation has serious consequences for the renormalization of both the two-nucleon propagator and the πNN vertex. In Fig. 1(a) we show the πN nucleon pole diagram where the intermediate state nucleon is dressed by one-pion loops; however, the initial and final state nucleons do not include dressing since two-pion states are neglected in the truncation. Since close to the nucleon pole the dressed one-nucleon propagator is of the form $g(E) \sim Z/(E-m)$, where Z is the residue at the pole, Fig. 1(a) illustrates how each πNN vertex $f(E)$ gets effectively renormalized by a factor of $Z^{1/2}$. Thus $f_{\pi NN} = Z^{1/2} f(m)$ is essentially the πNN coupling constant, and this fact is used to fix the strength parameter in the form factor $f(E)$. With all other parameters of $f(E)$ fixed to reproduce experimental πN phase shifts, this form factor then enters the unitary $NN-\pi NN$ equations as an input. As illustrated in Fig. 1(b), when the NN one pion exchange (OPE) amplitude is calculated in the unitary $NN-\pi NN$ model, the initial and final nucleons are dressed by pions and consequently each external nucleon obtains a renormalization factor of $\tilde{Z}^{1/2}$. The first renormalization problem is the fact that $\tilde{Z} \neq Z$. This arises because two nucleons cannot be dressed at the same time in the truncated Hilbert space; thus, each nucleon in a two-nucleon state cannot obtain its full dressing. This, however, may not be such a serious problem since, in practice, the difference between Z and $\tilde{Z}$ turns out to be quite small. The serious problem, instead, is the size of the effective πNN coupling constant in the $NN-\pi NN$ equations. Taking $Z \approx \tilde{Z}$, Fig. 1(b) illustrates that each vertex gets renormalized by a factor of Z, so that the effective πNN coupling constant here becomes $Zf(m)$; this is a factor $Z^{1/2}$ times the physical coupling constant. With Z being typically between 0.6 and 0.8, we come to the disturbing conclusion that the effective πNN coupling constant in the $NN-\pi NN$ equations is smaller than the one used in constructing the input. This observation helps explain why one typically obtains

Figure 1: Allowed dressing in the unitary $NN-\pi NN$ model, with associated Z renormalization factors. (a) πN nucleon pole graph, (b) NN OPE graph.

much too small $pp \to \pi^+ d$ cross sections using this model.[7]-[10].

These important observations about the renormalization problem in the unitary $NN-\pi NN$ model were already made in 1985 by Sauer *et al.*,[13] yet they seem to have gone largely unnoticed. Perhaps this is partly because one could find less "fatal" reasons for the low cross sections; for example, it was legitimately argued that off-shell effects and the lack of a "backward-going pion" in the $NN \to N\Delta$ amplitude can lead to the underestimation of $pp \to \pi^+ d$ cross sections[10]. However, with the advent of calculations where the nucleon and Δ are treated on an equal footing in the $NN-\pi NN$ model (the so-called $BB-\pi BB$ equations[12]), the effective πNN coupling constant is lowered by yet a further factor of $Z^{1/2}$ in the most important $NN \to N\Delta$ amplitude, and it has become very apparent that the renormalization problem is indeed fatal to this type of approach to the πNN system.

It may seem that one can fix the renormalization problem "by hand" by strategically including extra $Z^{1/2}$ factors in either πNN propagators or πNN form factors, or both. But it soon becomes apparent that there is no easy way of doing this without destroying the three-body unitarity of the equations.

2.2 The πNN convolution equations

Here we describe a new formulation of the πNN problem where unitary equations are obtained without having to truncate the Hilbert space to some maximum number of pions. Consequently, all possible dressings of one-particle propagators and vertices are retained in our model. The essential technique that enables the calculation of all such dressings is a novel use of convolution integrals. In this way we overcome the renormalization problems discussed above.

As an explicit derivation of the new πNN equations can be found in Ref.[14], here we prefer to simply state the final equations, and to describe their essential features. The new πNN equations can be expressed in many different forms, all of which are equivalent. The form we shall choose here is the one that most closely resembles the unitary $NN-\pi NN$ equations as given by Afnan and Blankleider[5] (AB). Choosing this AB form has two essential advantages: firstly, we are able to directly compare the differences between our πNN convolution equations and the unitary $NN-\pi NN$ equations, and secondly, this form is ideal for numerical solution, especially since advantage may be taken of existing codes for the unitary $NN-\pi NN$ equations where essentially the AB form has been used.

The πNN convolution equations may be expressed as a set of coupled equations for the reactions $NN \to NN$, $NN \to \pi d$, $\pi d \to NN$, and $\pi d \to \pi d$ using the following (4×4) matrix form:

$$\begin{pmatrix} T_{NN} & \underline{\bar{T}}_N \\ \underline{T}_N & \underline{T} \end{pmatrix} = \begin{pmatrix} V_{NN} & \underline{\bar{F}} \\ \underline{F} & G_0^{-1}\underline{I} \end{pmatrix} \left\{ I + \begin{pmatrix} G_{NN} & \underline{0} \\ \underline{0} & G_0 \underline{w}^0 G_0 \end{pmatrix} \begin{pmatrix} T_{NN} & \underline{\bar{T}}_N \\ \underline{T}_N & \underline{T} \end{pmatrix} \right\}. \qquad (1)$$

Before explaining the symbols in this equation, let us define what we mean by a product of two symbols. For any two quantities B and A, describing processes $m \to k$ and $k \to n$, respectively, we define the product symbol AB to mean the the integral

$$AB \equiv \int d\mathbf{p}_1'' \dots d\mathbf{p}_k''\, A(\mathbf{p}_1' \dots \mathbf{p}_n', \mathbf{p}_1'' \dots \mathbf{p}_k''; E)\, B(\mathbf{p}_1'' \dots \mathbf{p}_k'', \mathbf{p}_1 \dots \mathbf{p}_m; E) \qquad (2)$$

where $\mathbf{p}_i$ is the three-momentum of particle i and E is the total energy. Although, momentum conserving δ-functions are assumed to be contained in both A and B, it is easy to see that

Figure 2: The vertices F_1 and $\bar{F}_1$. The dark circles represent all possible intermediate states. Vertices F_2 and $\bar{F}_2$ are obtained by interchanging 1 and 2.

such δ-functions can be factored out without affecting the symbolic equations. In Eq. (1) the unknown quantities are T_{NN}, together with $T_{\alpha N}$, $T_{N\beta}$, and $T_{\alpha\beta}$ which are elements of the matrices $\underline{T}_N$, $\underline{\bar{T}}_N$, and $\underline{T}$, respectively (here indices α and β take on values 1, 2, and 3). The physical amplitudes for $NN \rightarrow NN$, $NN \rightarrow \pi d$, $\pi d \rightarrow NN$, and $\pi d \rightarrow \pi d$, are then given by

$$X_{NN} = T_{NN} \quad ; \quad X_{Nd} = T_{N3}\Psi_d \quad ; \quad X_{dN} = \bar{\Psi}_d T_{3N} \quad ; \quad X_{dd} = \bar{\Psi}_d T_{33}\Psi_d, \tag{3}$$

respectively, where Ψ_d is the deuteron wave function in the presence of a spectator pion. On the r.h.s. of Eq. (1) G_0 is the fully dressed πNN propagator, G_{NN} is the fully dressed NN propagator, $\underline{\mathcal{I}}$ is a 3×3 matrix whose elements are $\bar{\delta}_{\alpha\beta} = 1 - \delta_{\alpha\beta}$, and V_{NN} is the dressed one-pion exchange potential given by

$$V_{NN} = \sum_{i,j=1}^{2} \bar{F}_i \bar{\delta}_{ij} G_0 F_j \tag{4}$$

where F_i and $\bar{F}_i$ are fully dressed πNN vertices in the two-nucleon sector as illustrated in Fig. 2. Finally we have the matrices

$$\underline{F} = \left(\sum_{j=1}^{2} \bar{\delta}_{\alpha j} F_j\right) \quad ; \quad \underline{\bar{F}} = \left(\sum_{i=1}^{2} \bar{F}_i \bar{\delta}_{i\beta}\right) \quad ; \quad \underline{w}^0 = \begin{pmatrix} w_1^0 & w_4^0 & 0 \\ w_5^0 & w_2^0 & 0 \\ 0 & 0 & w_3^0 \end{pmatrix} \tag{5}$$

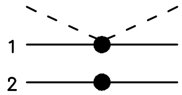
where the w_α^0 ($\alpha = 1 \ldots 5$) are the disconnected NN-irreducible amplitudes for $\pi NN \rightarrow \pi NN$, to be discussed shortly.

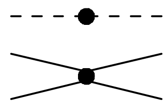
By form, Eq. (1) is very similar to the unitary $NN - \pi NN$ equations as given in Eq. (59) of AB. However, the essential feature of Eq. (1) that distinguishes it from the $NN - \pi NN$ equations, is that all input quantities in Eq. (1) are fully dressed. In this way the renormalisation problems of the $NN - \pi NN$ equations have been overcome. However this would only be a formal solution to the renormalisation problem if it were not for the fact that all the necessary dressings can be calculated exactly using convolution integrals. That this is so follows from Ref.[15] where we showed that any disconnected Green function is equal to the convolution of all its disconnected parts; thus for example, the dressed two-nucleon propagator G_{NN} is expressed in terms of the dressed one-nucleon propagators g_1 and g_2 as

$$G_{NN}(E) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} dz\, g_1(E-z) g_2(z) \tag{6}$$

where, for the sake of simplicity, we have set the momenta of the nucleons to zero. To further save on notation, we introduce the shorthand $G_{NN} = g_1 \otimes g_2$ to mean the convolution integral of Eq. (6). Giving labels 1 and 2 to the two nucleons, and label 3 to the pion, in the same way we have that the fully dressed πNN propagator G_0 is given by the double convolution

$$G_0 = g_1 \otimes g_2 \otimes g_3. \tag{7}$$

$w_1 =$ 

$w_3 =$ 

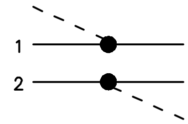
$w_4 =$ 

Figure 3: The amplitudes w_α. The dark circles represent all possible intermediate states. Amplitudes w_2 and w_5 are obtained by interchanging the two nucleons in w_1 and w_4 respectively.

To see how the amplitudes w_α^0 are calculated, we first define the amplitudes w_α to be the disconnected $\pi NN \to \pi NN$ amplitudes, illustrated in Fig. 3, each corresponding to a different type of disconnectedness, and containing *all* possible contributing diagrams. It is just because the w_α contain all possible contributions that one can express them through convolution integrals as

$$\tilde{w}_1 = \tilde{t}_1 \otimes g_2 \quad ; \quad \tilde{w}_2 = \tilde{t}_2 \otimes g_1 \quad ; \quad \tilde{w}_3 = \tilde{t}_3 \otimes g_3 \quad ; \quad \tilde{w}_4 = \tilde{f}_1 \otimes \tilde{\bar{f}}_2 \quad ; \quad \tilde{w}_5 = \tilde{f}_2 \otimes \tilde{\bar{f}}_1 \tag{8}$$

where the "tilde" denotes a Green function quantity consisting of the corresponding amplitude with additional initial and final-state propagators; thus, for example, $\tilde{w}_\alpha = G_0 w_\alpha G_0$, and $\tilde{t}_1 = g_{\pi N_1} t_1 g_{\pi N_1}$ where t_1 is the t-matrix and $g_{\pi N_1}$ the dressed propagator for scattering of a pion off nucleon 1. As we have shown in Ref.[15], the convolution integrals effectively sum over all the relative time orderings of one subamplitude of a disconnected diagram with respect to another.

Once the w_α are calculated, we may then write them as

$$w_\alpha = w_\alpha^0 + w_\alpha^P \tag{9}$$

where w_α^P is the part of w_α that is two-nucleon reducible, while w_i^0 is two-nucleon irreducible. Since we consider all possible contributions, it is clear that

$$w_i^P = F_i G_{NN} \bar{F}_i \quad ; \quad w_3^P = 0 \quad ; \quad w_4^P = F_1 G_{NN} \bar{F}_2 \quad ; \quad w_5^P = F_2 G_{NN} \bar{F}_1. \tag{10}$$

In this way, all the essential input to Eq. (1) has been specified.

We may finally note a second major difference between Eq. (1) and the unitary $NN-\pi NN$ equations. The input matrix $\underline{w}^0$ in Eq. (1) has off-diagonal elements, while the corresponding matrix for the $NN-\pi NN$ equations is diagonal. Recalling that the amplitudes of $\underline{w}^0$ are two-nucleon irreducible, we can see from Fig. 3, that the off-diagonal elements $\underline{w}_4^0$ and $\underline{w}_5^0$ correspond to what has been called the Jennings terms. As pointed out by Jennings[16], these terms may be important for the understanding of πd scattering. In our case, the Jennings terms are also fully dressed, and form an essential part of the convolution equations. Indeed, since our NN propagator G_{NN} is fully dressed, it contains two-pion states coming from intermediate Jennings-like terms. It is then necessary to retain $\underline{w}_4^0$ and $\underline{w}_5^0$ in the convolution equations because they combine with G_{NN} in just the right way to guarantee three-body unitarity.

We recall, that the only approximation made in deriving the convolution equations of Eq. (1) is the neglect of all connected πNN-irreducible diagrams for the $\pi NN \to \pi NN$ process[14]. Yet it is very easy to include some types of connected contributions. One such contribution would involve intermediate state potentials $V_{NN}^{(1)}$ that are πNN-irreducible. Then Eq. (1) would be modified simply by replacing V_{NN} with $V_{NN}+V_{NN}^{(1)}$. This observation suggests that a

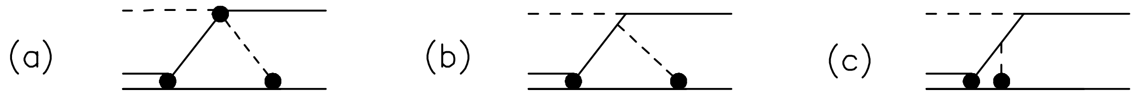


Figure 4: Example of overcounting in $NN \to \pi d$. (a) The $NN \to \pi d$ Feynman diagram where dark circles represent all possible contributions. (b) One of the contributions included in (a). (c) Another way of drawing diagram (b) showing how overcounting arises.

way to include heavy meson exchange into our NN potential would be as a phenomenological model for $V^{(1)}_{NN}$.

3. Four-dimensional formulation

Although three-dimensional equations may be easier to solve than those in four-dimensions, there are important reasons why the formulation of four-dimensional equations is necessary. Firstly, such equations are based on relativistic quantum field theory, and retain the fundamental property of off-shell covariance. Secondly, having the correct four-dimensional equations, one can then do a three-dimensional reduction using one of the well-known reduction schemes. We may also add, that with the ever increasing power of computers, the numerical solution of four-dimensional equations becomes ever more feasible.

The first attempts to formulate few-body equations using relativistic quantum field theory were made already in the early 1960's.[17]-[19] Both such general formulations and ones more specific to the πNN system have been pursued until the present time.[20]-[23] Yet as in the three-dimensional case, all these attempts have had theoretical inconsistencies. In particular, all previous attempts have contained either overcounting or undercounting of Feynman diagrams.

3.1 Overcounting and undercounting problems

Perhaps the easiest way to illustrate the overcounting problem in the πNN system is with an example. Consider the "triangle" diagram of Fig. 4(a) for the process $NN \to \pi d$, where the dark circles represent the full $\pi N \to \pi N$ amplitude, the dressed πNN vertex, and the dressed deuteron vertex. If one were to calculate this diagram in four dimensions, as is, using covariant forms for the off-shell πN t-matrix, πNN vertex, and the deuteron vertex, then one would have the mistake of overcounting of diagrams. This is illustrated in Fig. 4(b) where we consider just the crossed-pion graph contribution to the input πN t-matrix. As these are Feynman graphs, there is no meaning associated with the slope of the lines, and one could just as well have drawn Fig. 4(c). However Fig. 4(c) clearly illustrates that this contribution corresponds to the dressing of the already fully dressed deuteron vertex.

This type of overcounting arises in four-dimensional approaches whenever one tries to formulate multiple-scattering graphs in terms of fully dressed vertices and full amplitudes for all subprocesses. In once-off cases, like that of Fig. 4(a), one can easily fix the overcounting problem by making a necessary subtraction (here one would subtract the graph of Fig. 4(b) from the calculation of Fig. 4(a)). However, the way to solve the overcounting problem for the case of coupled integral equations is highly non-trivial as an infinite number of overcounted contributions are involved.

In a similar way, let us illustrate how undercounting arises in the covariant πNN problem. As in the three-dimensional formulation, one neglects three-body forces also in the four-dimensional case. Only in this way can one obtain few-body equations where (in the c.m.)

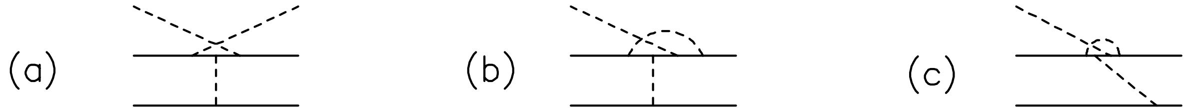


Figure 5: Example of undercounting in $NN \to \pi NN$. (a) A $\pi NN \to \pi NN$ graph that has usually been neglected since it corresponds to a three-body force. (b) The coupling of the graph in (a) to the NN channel. (c) Another way of drawing diagram (b) reveals a two-body process.

no more than two independent momenta are involved. However, one does need to be very careful about neglecting three-body forces in the four-dimensional theory. Consider, for example the Feynman diagram of Fig. 5(a). This is a graph for the process $\pi NN \to \pi NN$ that is both connected and πNN-irreducible. It therefore corresponds precisely to what is meant by a three-body force. However, neglecting this contribution from a few-body theory of the πNN system would be a bad mistake. This is illustrated in Fig. 5(b) where we allow the graph of Fig. 5(a) to couple to the NN channel. Again no meaning can be attached to the slope of the propagator lines, and we can equally well draw this diagram as in Fig. 5(c). This, however, reveals that the three-body force of the $\pi NN \to \pi NN$ process has now become a two-body rescattering contribution in the $NN \to \pi NN$ process. Thus neglecting the three-body force of Fig. 5(a) would lead to an undercounting of important two-body contributions.

3.2 Four-dimensional πNN equations

In a recent paper, we have solved both the overcounting and undercounting problems in the formulation of few-body equations in field theory[24]. The few-body equations for the πNN system then follow as a particular case. The method used to derive the equations involves the classification of Feynman diagrams according to their irreducibility. The overcounting problem is handled by a procedure where, in formally identical cases like that of Figs. 4(b) and (c), one of the two right-most vertices is "pulled out" further to the right. The undercounting of diagrams is handled simply by retaining all three-body forces until the end of the derivation where the ones that did not lead to two-body interactions are safely neglected. It is gratifying that Phillips and Afnan[25] have recently confirmed our equations using a modified version of Taylor's original classification of diagram scheme.[17, 26]

As before, the four-dimensional equations for the πNN system can be written in any number of equivalent forms, and for reasons already outlined, we choose here the form closest to Eq. (59) of AB.

With three-body forces neglected as described, one might still find it useful to retain, as in the three-dimensional case, the $NN \to NN$ πNN-irreducible potential $V_{NN}^{(1)}$, as well as the simultaneously NN- and πNN-irreducible connected $NN \to \pi NN$ amplitide $F_c^{(1)}$. However, let us at first consider the simplest case where these contributions are neglected (they are in fact completely absent in the usual case of a $\phi\bar{\psi}\psi$ interaction).

In this case the four-dimensional πNN equations can be written as Eq. (1), but with the following modifications: (1) The product of two quantities A and B is now defined as in Eq. (2), but with all momenta and integrations being four-dimensional, (2) All convolutions of Green functions are replaced by usual products, (3) the matrix $\bar{w}^0$ is now diagonal, and (4) the following replacements are made,

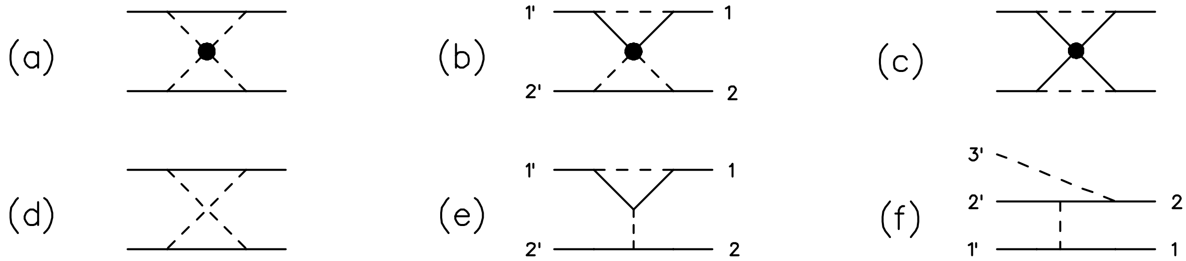


Figure 6: The subtraction terms in the four-dimensional πNN equation: (a) $W_{\pi\pi}$, (b) W_1, (c) W_{NN}, (d) X, (e) Y_1, and (f) B_1. The dark circles represent the following two-body amplitudes: (a) full $\pi\pi$ t-matrix, (b) one-nucleon irreducible πN t-matrix, and (c) full NN t-matrix minus the NN one-pion-exchange potential. Amplitudes W_2, Y_2, and B_2 are obtained by exchanging the 1 and 2 labels.

$$F_i \to F_i - \tfrac{1}{2}B \quad ; \quad \bar{F}_i \to \bar{F}_i - \tfrac{1}{2}\bar{B} \quad ; \quad V_{NN} \to V_{NN} - \Delta \tag{11}$$

where the terms B and Δ are subtraction terms that exactly compensate all the overcounting due to the use of full off-shell amplitudes and fully dressed vertices in the coupled scattering equations. To specify these subtraction terms, we express them as

$$\Delta = W_{\pi\pi} + W_1 + W_2 + W_{NN} + Y_1 + Y_2 \qquad ; \qquad B = B_1 + B_2 \tag{12}$$

and illustrate each of these terms in Fig. 6.

In the more general case where $V_{NN}^{(1)}$ and $F_c^{(1)}$ are retained, it turns out that only the subtraction terms of Eq. (12) need be modified. In particular, we only need to do the replacements $\Delta \to \Delta - V_{NN}^{(1)}$ and $B \to B - F^{(1)}$.

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References

[1] H. Garcilazo and T. Mizutani, *πNN Systems*, (World Scientific, Singapore, 1990).

[2] Y. Avishai, T. Mizutani, *Nucl. Phys.* **A326** (1979) 352 ; **A338** (1980) 377 ; **A352** (1981) 399; *Phys. Rev.* C **27** (1983) 312.

[3] A. S. Rinat, *Nucl. Phys.* **A287** (1977) 399.

[4] A. W. Thomas and A. S. Rinat, *Phys. Rev.* C **20** (1979) 216.

[5] I. R. Afnan and B. Blankleider, *Phys. Rev.* C **22** (1980) 1638.

[6] B. Blankleider and I. R. Afnan, *Phys. Rev.* **24** (1981) 1572.

[7] A. S. Rinat and Y. Starkand, *Nucl. Phys.* **A397** (1983) 381; A. S. Rinat and R. S. Bhalerao, Weizmann Institute of Science Report, WIS-82/55 Nov-Ph, 1982.

[8] G. H. Lamot, J. L. Perrot, C. Fayard, and T. Mizutani, *Phys. Rev.* C **35** (1987) 239.

[9] F. Sammarruca and T. Mizutani, *Phys. Rev.* C **41** (1990) 2286.

[10] C. Fayard, G. H. Lamot, T. Mizutani and B. Saghai, *Phys. Rev.* C **46** (1992) 118.

[11] T. Mizutani, C. Fayard, G.H. Lamot, and B. Saghai, *Phys. Rev.* C **47** (1993) 56.

[12] B. Blankleider, *Nucl. Phys.* **A543** (1992) 163c.

[13] P. U. Sauer, M. Sawicki and S. Furui, *Prog. Theor. Phys.* **74** (1985) 1290.

[14] A. N. Kvinikhidze and Blankleider, *Phys. Lett.* **B307** (1993) 7.

[15] A. N. Kvinikhidze and Blankleider, *Phys. Rev.* C **48** (1993) 25.

[16] B. K. Jennings, *Phys. Lett.* **B205** (1988) 187; B. K. Jennings and A. S. Rinat, *Nucl. Phys.* **A485** (1988) 421.

[17] J. G. Taylor, *Nuovo Cimento Suppl.* **1** (1963) 857; *Phys. Rev.* **150** (1966) 1321.

[18] M. M. Broido, *Rep. Prog. Phys.* **32** (1969) 493.

[19] A. Tucciarone, *Nuovo Cimento* **41** (1966) 204.

[20] Y. Avishai and T. Mizutani, Phys. Rev. C **27** (1983) 312.

[21] I. R. Afnan and B. Blankleider, *Phys. Rev.* **32** (1985) 2006.

[22] M. Araki and I. R. Afnan, *Phys. Rev.* C **38** (1988) 213.

[23] H. Haberzettl, *Phys. Rev.* C **47** (1993) 1237; *ibid.* **49** (1994) 2142.

[24] A. N. Kvinikhidze and B. Blankleider, *Nucl. Phys.* **A574** (1994) 788.

[25] D. R. Phillips and I. R. Afnan, e-preprint nucl-th/9502040, Los Alamos Electronic Preprint Archive, http://xxx.lanl.gov.

[26] D. R. Phillips and I. R. Afnan, *Ann. Phys.* **240** (1995) 266.