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Chiral symmetry breaking in strongly coupled QED?

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Abstract

The coupled system of renormalized Dyson-Schwinger equations for the electron self-energy and the photon propagator are supplied with the tree level vertex as Ansatz for the renormalized three point function. The system is investigated numerically. In the case of a massive electron, the theory is “weakly renormalizable”, i.e. cutoff independent for values of the cutoff below an upper limit. In this regime of cutoff independence, the quenched approximation yields good results for the electron self-energy. In the chiral limit, a logarithmic cutoff dependence of the electron self-energy is found. The question, whether a regime of cutoff independence with a spontaneously broken chiral symmetry exists in strongly coupled QED, remains open.

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1 Introduction

The occurrence of spontaneous chiral symmetry breaking in strongly coupled QED is one of the most challenging issues of non-perturbative quantum field theory nowadays. In his pioneering work, Miranski reported a regime of cutoff independence (CI-regime)¹ corresponding to a phase of QED with spontaneous broken chiral symmetry [1]. His results are based on a study of the Dyson-Schwinger equation for the electron self-energy, where electron loop corrections to the photon propagator are neglected (quenched approximation). In the quenched approximation, no renormalization of the electric charge is required, and spontaneous symmetry breakdown occurs, if the bare charge exceeds a critical value [1, 2, 3]. Subsequently, it was argued by Bardeen, Leung and Love that a Nambu-Jona-Lasinio type interaction [4], must be included in order to render the renormalization of the Dyson-Schwinger equation consistent with the known renormalization properties of the theory [2]. The results of the quenched approximation are caught into question, since vacuum polarization effects might enforce the renormalized coupling to vanish in the the infinite cutoff limit [5] (triviality). First investigations of this effect were done by Kondo by parametrizing the effect of the vacuum polarization [6]. Further insight in the issue of triviality was gained by the important work of Rakow [7]. He numerically solved the coupled set of (bare) Dyson-Schwinger equations for the electron self-energy and the photon propagator, and therefore included vacuum polarization effects self-consistently. He finds a second order chiral phase transition and zero renormalized charge at the critical point [7]. This seems to rule out the quenched approximation, since the quenched approximation predicts an interacting infinite cutoff limit.

Parallel to the studies of the truncated Dyson-Schwinger equations, extensive lattice investigations were performed in order to clarify the triviality problem of QED [8, 9, 10]. Lattice simulations are not bounded by any approximation, but suffer from a small correlation length compared with the intrinsic fermionic energy scale due to the finite lattice sizes. In order to overcome the difficulty of a small correlation length, one fits the lattice data to two Ansätze of the equation of state (the bare electron mass as function of the fermionic condensate). One of these Ansätze favors an interacting infinite cutoff limit [9], whereas the other Ansatz is compatible with triviality [10]. The present status is that the lattice data are not conclusive enough in order to distinguish between the two equations of state [8].

In this paper, we further develop the results of [7] and study the coupled set of renormalized Dyson-Schwinger equations for the electron self-energy and the photon propagator. The tower of Dyson-Schwinger equations is truncated by using the tree level electron-photon-vertex as Ansatz for the renormalized three point func-

¹We do not use the notion of a “scaling limit” in order to avoid confusion with the term in solid state physics, where it corresponds to the case of exact scaling (zero masses).

tion. In order to introduce our notation and to make the renormalization procedure transparent, we first derive the renormalized Dyson-Schwinger equations (section 2). All Ansätze are made for renormalized quantities. This will later turn out to be crucial in order to compare the full results with those of the quenched approximation. We first concentrate on the case of a massive electron (section 3). A cutoff independence is found for sufficiently small values of the cutoff. The theory is called to be *weakly renormalizable*. In contrast, the quenched approximation always predicts a CI-regime with no upper limit on the cutoff. Although the quenched approximation is incapable to predict the correct CI-behavior, it provides good results for physical quantities in the CI-regime (subsection 3.2). We then focus onto the chiral limit (section 4). A logarithmic cutoff dependence is seen as well in the electron self-energy as in the vacuum polarization. The problem of the existence of a CI-regime characterized by a spontaneous breakdown of chiral symmetry is still unsolved.

2 Renormalized Dyson-Schwinger equations

The generating functional for *bare* Green's functions of QED in Euclidean space is given by the functional integral

$$W[j_\mu^B, \eta^B, \bar{\eta}^B](e_B, m_B) = \int \mathcal{D}A_\mu \mathcal{D}q \mathcal{D}\bar{q} \exp \left\{ - \int d^4x \left[L_0(x) - \bar{\eta}^B(x)q(x) - \bar{q}(x)\eta^B(x) - j_\mu^B(x)A_\mu(x) \right] \right\}, \quad (1)$$

$$L_0(x) = \frac{1}{4e_B^2} F_{\mu\nu}[A](x)F_{\mu\nu}[A](x) + \bar{q}(x)(i\rlap{\not{\partial}} + im_B)q(x) + \bar{q}(x)A(x)q(x), \quad (2)$$

where $F_{\mu\nu}[A](x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the field strength tensor, and e_B, m_B represent the bare electric charge and the bare electron mass respectively. At zero external sources, the generating functional $W[0, 0, 0](m^B, e^B)$ is invariant under U(1) gauge transformations of the integration variables, i.e.

$$q(x) \rightarrow \exp\{i\alpha(x)\}q(x), \quad A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\alpha(x). \quad (3)$$

Bare Green's functions are divergent implying the theory must be regularized and renormalized in order to make sense. Renormalization is performed by absorbing the divergences into the renormalization constants $Z_{2,3,e,m}$, which relate the bare sources and the bare parameters to the renormalized ones, i.e.

$$\begin{aligned} \eta^B(x) &= Z_2^{-1/2} \eta(x), \quad \bar{\eta}^B(x) = Z_2^{-1/2} \bar{\eta}(x), \quad j_\mu^B(x) = Z_3^{-1/2} Z_e^{-1} j_\mu(x), \\ e_B &= Z_e e_R, \quad m_B = Z_m m_R. \end{aligned} \quad (4)$$

The generating functional for *renormalized* Green's functions is obtained from $W[j_\mu^B, \eta^B, \bar{\eta}^B](e_B, m_B)$ by replacing the bare sources and bare parameters by the renormalized sources and the renormalized parameters respectively, i.e.

$$W_R[j_\mu, \eta, \bar{\eta}](e_R, m_R) = W[Z_3^{-1/2} Z_e^{-1} j_\mu, Z_2^{-1/2} \eta, Z_2^{-1/2} \bar{\eta}](Z_e e_R, Z_m m_R). \quad (5)$$

The renormalized Green's functions are obtained by taking functional derivatives of W_R (5) with respect to the corresponding external sources. For example, the renormalized electron propagator S_R and the renormalized photon propagator $D_{\mu\nu}^R$ are given by

$$S_R(x, y) = \frac{\delta \ln W_R[j_\mu, \eta, \bar{\eta}]}{\delta \eta(x) \delta \bar{\eta}(y)} \Big|_{j, \eta, \bar{\eta}=0}, \quad D_{\mu\nu}^R(x, y) = \frac{\delta \ln W_R[j_\mu, \eta, \bar{\eta}]}{\delta j_\mu(x) \delta j_\nu(y)} \Big|_{j, \eta, \bar{\eta}=0}. \quad (6)$$

After a change of the integration variables, the generating functional (5) can be cast into

$$W_R[j_\mu, \eta, \bar{\eta}](e_R, m_R) = \int \mathcal{D}A_\mu \mathcal{D}q \mathcal{D}\bar{q} \exp \left\{ - \int d^4x \left[L(x) - \bar{\eta}(x)q(x) - \bar{q}(x)\eta(x) - j_\mu(x)A_\mu(x) \right] \right\}, \quad (7)$$

$$L(x) = \frac{Z_3}{4e_R^2} F_{\mu\nu}[A](x)F_{\mu\nu}[A](x) + Z_2 \bar{q}(x)i\cancel{\partial}q(x) + Z_0 im_R \bar{q}(x)q(x) \quad (8)$$

$$+ Z_1 \bar{q}(x)\cancel{A}(x)q(x), \quad (9)$$

where we have introduced

$$Z_1 = Z_2 Z_3^{1/2} Z_e, \quad Z_0 = Z_2 Z_m. \quad (10)$$

The generating functional Γ for renormalized one-particle irreducible functions (in the following vertex functions) is obtained from W_R (5) by a Legendre transformation, i.e.

$$\Gamma[\mathcal{A}_\mu, \psi, \bar{\psi}] = \ln W_R[j_\mu, \eta, \bar{\eta}] - \int d^4x \left[j_\mu(x)\mathcal{A}_\mu(x) + \bar{\eta}(x)\psi(x) + \bar{\psi}(x)\eta(x) \right], \quad (11)$$

where the external sources are implicitly related to the fields \mathcal{A} , ψ , $\bar{\psi}$ by

$$\mathcal{A}_\mu(x) = \frac{\delta \ln W_R[j_\mu, \eta, \bar{\eta}]}{\delta j_\mu(x)}, \quad \psi(x) = \frac{\delta \ln W_R[j_\mu, \eta, \bar{\eta}]}{\delta \bar{\eta}(x)}, \quad \bar{\psi}(x) = \frac{\delta \ln W_R[j_\mu, \eta, \bar{\eta}]}{\delta \eta(x)}. \quad (12)$$

Below we will use the renormalized electron photon vertex, which is defined by

$$\Lambda_\mu(x, y, z) = \frac{\delta \Gamma[\mathcal{A}_\mu, \psi, \bar{\psi}]}{\delta \mathcal{A}_\mu(x) \delta \psi(y) \delta \bar{\psi}(z)} \Big|_{\mathcal{A}_\mu, \psi, \bar{\psi}=0}. \quad (13)$$

Exploiting the fact that the functional integral $W_R[j_\mu, \eta, \bar{\eta}]$ (7) is not changed by a shift of the integration variables A_μ, q, \bar{q} generates the *renormalized* Dyson-Schwinger equations [11]. In particular, one finds

$$(S_R(x, y))^{-1} = [Z_2 i\not{\partial} + Z_0 im_R] \delta(x - y) - Z_1 \int d^4z d^4w \gamma_\mu D_{\mu\nu}^R(x, z) S_R(x, w) \Lambda_\nu(z, w, y), \quad (14)$$

$$\delta_{\mu\nu} \delta(x - y) = \frac{Z_3}{e_R^2} \left(-\partial^2 \delta_{\mu\alpha} + \partial_\mu \partial_\alpha \right) D_{\alpha\nu}^R(x, y) + Z_1 \int d^4z d^4w d^4v \text{tr} \{ \gamma_\mu S_R(x, z) \Lambda_\alpha(v, z, w) S_R(w, x) \} D_{\alpha\nu}^R(v, y). \quad (15)$$

Throughout this paper, we will truncate the tower of Dyson-Schwinger equations by making an Ansatz for the renormalized vertex function Λ_ν . In this case, the renormalized electron propagator and the renormalized photon propagator can be obtained by solving the coupled system (14,15).

In order to investigate the consistency of the Ansatz, one studies the Ward-Takahashi identities [11], which provide relations among Green's functions induced by gauge invariance. One first realizes that $Z_1 = Z_2$ must hold in the renormalized Lagrangian (8) in order to preserve the invariance under the transformation (3). Combining (10) with (4), one finds that the electric charge is renormalized by the photon wave function renormalization constant, i.e. $e_R = Z_3^{1/2} e_B$. If the bare charge e_B acquires a finite value in the limit of large cutoff (as suggested by the quenched approximation), and if further this limit enforces Z_3 to vanish, then the theory is only consistent with a zero renormalized electric charge. This scenario is referred to as triviality of QED in the literature.

Exploring the fact that the generating functional W_R (7) is invariant under small gauge rotations (3), one obtains

$$\partial_\mu^{(x)} \Lambda_\mu(x, y, z) = iS_R^{-1}(y, x) \delta(x - z) - iS_R^{-1}(x, z) \delta(x - y). \quad (16)$$

Once the coupled system (14,15) was solved for a particular choice of the renormalized vertex Λ_μ , one inserts the solution for S_R into (16) in order to check the accuracy of the Ansatz for the vertex function Λ_μ .

In the following, we will work in Landau gauge and set $Z_1 = Z_2$ as imposed by gauge invariance. We will study the Ansatz

$$\Lambda_\mu(z, x, y) = \gamma_\mu \delta(z - x) \delta(z - y) \quad (17)$$

for the renormalized vertex function, which is the tree level electron-photon vertex. Recently, the full one-loop QED vertex was obtained [12]. The results might provide the structure for a more general ansatz than (17). Note that the choice of the

renormalized vertex is the only freedom we have. There is no further possibility to argue in favor of a four fermion interaction first introduced in [2]. Note also that a four fermion interaction arises and can be naturally incorporated in renormalized Dyson-Schwinger equations in the dual formulation of QED [13].

The coupled Dyson-Schwinger equations are then solved by parametrizing the renormalized electron propagator and the renormalized photon propagator in momentum space by

$$\tilde{S}_R(p) = \frac{1}{F(p^2)\not{p} + i\Sigma(p^2)}, \quad \tilde{D}_{\mu\nu}^R(p) = \frac{4\pi D_R(p^2)}{p^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right). \quad (18)$$

Up to the occurrence of the renormalization constants $Z_{1,3}$, the derivation yields the same results as reported in [7]. The first Dyson-Schwinger equation is straightforwardly reduced to two equations to determine $F(p^2)$ and $\Sigma(p^2)$. Using a sharp $O(4)$ -invariant cutoff Λ to regularize the momentum integration, these equations are

$$\Sigma(p^2) = m_0 + Z_1 \frac{3}{2\pi^2} \int_0^{\Lambda^2} dk^2 k^2 \int_0^\pi d\theta \sin^2 \theta \frac{\Sigma(k^2)}{s(k^2)} \frac{D_R(q^2)}{q^2}, \quad (19)$$

$$F(p^2) = Z_1 \left[1 + \frac{1}{\pi^2 p} \times \int_0^\Lambda dk k^4 \int_0^\pi d\theta \sin^2 \theta \frac{F(k^2)}{s(k^2)} \frac{D_R(q^2)}{q^4} \{3q^2 \cos \theta - 2kp \sin^2 \theta\} \right], \quad (20)$$

where we have introduced $m_0 = Z_m m_R$, $s(k^2) = F(k^2)k^2 + \Sigma(k^2)$ and $q^2 = k^2 + p^2 - 2kp \cos \theta$. The equation for the photon propagator (15) needs more thoughts. The vacuum polarization $\Pi_{\mu\nu}(p)$ due to the electron loop is transverse, i.e. $p_\mu \Pi_{\mu\nu}(p) = 0$, if the regularization prescription does not violate gauge invariance. Unfortunately, the sharp $O(4)$ -invariant cutoff prescription spoils gauge invariance implying that the vacuum polarization $\Pi_{\mu\nu}(p)$ acquires spurious terms proportional to $\delta_{\mu\nu}$, which are, in addition, quadratic divergent. On the other hand, regularization schemes consistent with gauge invariance are very time consuming when solving the coupled equations (14,15) numerically. In order to circumvent this problem, one starts calculating $\Pi_{\mu\nu}(p)$ using a regularization scheme which respects gauge invariance (e.g. Schwinger proper time, Pauli-Villars regularization). The result for $\Pi_{\mu\nu}(p)$ is the transverse projector times a function depending on p^2 . In this function, one first introduces a second regularization which corresponds to the $O(4)$ -invariant cutoff scheme, and then removes the regulator of the scheme which is compatible with gauge invariance. The final result allows a fast numerical treatment and is consistent with the transversality of the vacuum polarization $\Pi_{\mu\nu}(p)$. In the limit of a large cutoff, all regularization schemes which respect the symmetries of the model are supposed to yield the same result for physical quantities. In agreement with the

result stated in [7], this lengthy procedure yields

$$\frac{1}{D_R(p^2)} = \frac{Z_3}{\alpha_R} - Z_1 \frac{2}{3\pi^2} \times \int_0^{\Lambda^2} dk^2 k^2 \int_0^\pi d\theta \sin^2 \theta \frac{F(q_+^2) F(q_-^2)}{s(q_+^2) s(q_-^2)} \left\{ \frac{k^2}{p^2} (8 \cos^2 \theta - 2) - 3/2 \right\}, \quad (21)$$

where $\alpha_R = e_R^2/4\pi$ and $q_\pm = (k \pm p/2)^2$.

3 QED with massive electrons

In order to calculate the renormalization constants $Z_{1,3}$ and m_0 as function of the cutoff, we impose renormalization conditions. In order to fix Z_1 , we demand that the inverse renormalized electron propagator has the canonical kinetic term, i.e. $F(0) = 1$. Our conventions imply that the residue of the renormalized photon propagator $D_R(0)$ can be identified with α_R . Providing a value for α_R determines the constant Z_3 . Finally, we set the scale of the electron self energy $\Sigma(p^2)$ by the constraint $\Sigma(0) = \Sigma_0$, which yields a condition to calculate m_0 . Although m_0 might tend to zero for infinite cutoff, the renormalized (current) mass m_R is non-zero. The behavior of m_0 in the quenched approximation might serve as an illustrative example [2, 3], i.e.

$$m_0 = \left(\frac{\mu}{\Lambda} \right)^w m_R(\mu), \quad \text{with} \quad w = 1 - \sqrt{1 - \frac{3e^2}{4\pi^2}}, \quad (22)$$

where μ is an arbitrary renormalization point. Since m_0 will turn out to be different from zero for any fixed value of the cutoff, the renormalized (current) mass m_R is also non-zero implying that the above set of renormalization conditions corresponds to the case of a massive electron. The chiral symmetry is explicitly broken.

3.1 Numerical results

The set of coupled equations (19,20,21) for the functions $F(p^2)$, $\Sigma(p^2)$, $D_R(p^2)$ was numerically solved by iteration. The integrals were done by Simpson's integration with a step-size control. A CI-regime is found for sufficiently small cutoff. A representative result is shown in figure 1 for $\alpha_R = D_R(0) = 0.35$. The function $F(p^2)$ is weakly p -dependent. The maximum deviation from one occurs for $p^2 = \Lambda^2$ and is 0.04776 for $\ln \Lambda^2/\Sigma_0^2 = 16$. The self-energy roughly stays constant for $\ln p^2/\Sigma_0^2 \lesssim 4$ and smoothly decays afterwards. The vacuum polarization also stays constant up to approximately the same momentum and then decays according to

$$\frac{1}{D_R(p^2)} = c_1(\alpha_R) - c_2(\alpha_R) \ln \Lambda^2/p^2, \quad \text{for} \quad (\ln p^2/\Sigma_0^2 > 4), \quad (23)$$

where $c_1(0.35) \approx 2.97$ and where $c_2 \approx 1/\pi^2$ for a wide range of renormalized couplings. The renormalization constants are given in the table below.

$\ln \Lambda^2 / \Sigma_0^2$	Z_1	Z_3	m_0 / Σ_0
7	1.0097	0.75537	0.49437
10	1.0183	0.64295	0.33425
13	1.0305	0.52927	0.20643
16	1.0497	0.41307	0.10936

The renormalization group flow is shown in figure 2 for $\alpha_R = 0.35$. The vertex renormalization constant Z_1 (which is identical to the wave function renormalization constant Z_2 of the electron) stays close to one. The photon wave function renormalization constant almost decays logarithmically. The behavior of the bare mass m_0 corresponds to a powerlaw decay with logarithmic corrections.

In order to check the accuracy of the Ansatz (17) for the vertex, we study the Ward identity (16) in momentum space and at zero photon momentum, i.e.

$$\tilde{\Lambda}_\mu(0, p) = \frac{\partial}{\partial p_\mu} S^{-1}(p). \quad (24)$$

Using the parametrization (18) of the electron propagator and neglecting derivatives of the functions $F(p^2)$ and $\Sigma(p^2)$, the Ward identity (24) becomes

$$\tilde{\Lambda}_\mu(0, p) = F(p^2) \gamma_\mu. \quad (25)$$

The deviation of $F(p^2)$ from unity measures the violation of the ward identity due to the Ansatz (17) for the vertex. The numerical result shows that this violation is small, since the deviation of $F(p^2)$ from unity is always beyond 5% for the values of the cutoff used in table 1.

Increasing the cutoff for fixed renormalized coupling α_R , one observes a critical upper limit Λ_C of the cutoff. Beyond this critical value, no solution of the set of equations (18,19,20) was found. This result can be anticipated from the asymptotic behavior of the vacuum polarization (23). For a fixed (positive) value of the renormalized coupling $\alpha_R = D_R(0)$, equation (23) cannot be satisfied for an arbitrarily large cutoff Λ . Figure 3 shows the relation between the maximal possible renormalized coupling and the critical cutoff. The results indicate that in the limit $\Lambda \rightarrow \infty$ a solution of the coupled Dyson-Schwinger equations only exists for $\alpha_R \rightarrow 0$. This phenomenon is referred to as triviality of massive QED. Note, however, that for values of the cutoff smaller than the critical value the electron self-energy is cutoff independent. The theory is called weakly renormalizable. For moderate values of the renormalized coupling, the theory allows for a large value of the critical cutoff and therefore for a cutoff many times bigger than the typical energy scale set by the mass of the

electron. This implies that weakly renormalizable QED is perfectly compatible with the observations in nature. The lack of a cutoff independence at high momentum does no harm, since QED in the real world is embedded in the Weinberg-Salam model at high energies.

3.2 The quenched approximation

Neglecting fermion loop effects in the photon propagator is called quenched approximation. In our formulation, this corresponds to replacing (21) by

$$\frac{1}{D_R(p^2)} = \frac{Z_3}{\alpha_R}. \quad (26)$$

The vacuum polarization $D_R(p^2)$ is momentum independent, and from the renormalization conditions (see beginning of section 3) one immediately obtains $Z_3 = 1$. For a constant vacuum polarization D_R , the angle integral in (20) can be done explicitly and yields zero. This implies that $F(p^2) = Z_1$ is also constant, and taking into account the renormalization conditions, we have $Z_1 = 1$. The only remaining non-trivial equation is the integral equation for the electron self-energy (19). For standard QED (massless photon), this integral equation can be transformed into a non-linear differential equation with appropriate boundary conditions [1, 2, 6, 3]. In the table below, we compare the renormalization constants of the full and the quenched approach for $\alpha_R = 0.35$ and a cutoff in the CI-region $\ln \Lambda^2/\Sigma_0^2 = 16$.

	Z_1	Z_3	m_0/Σ_0
quenched	1	1	0.23172
full	1.0497	0.41307	0.10936

Large deviations are found. Note, however, that renormalization constants are not physical observables, and the quenched approximation might improve when physical quantities are studied. Figure 4 shows the electron self-energy in the quenched approximation in comparison with the full result. One finds that the quenched approximation yields good results for the electron self-energy at least for small momentum p . This behavior can be understood by a scaling argument. Due to cutoff independence, the momentum dependence of the electron-self-energy for a small momentum p and for cutoff slightly below the upper critical value is also obtained in a calculation with a cutoff far beyond the critical cutoff and momentum p comparable to this cutoff (see figure 1). At small cutoff, however, the bare electron mass m_0 is large (see table 1), and the electron loop contributing to the inverse vacuum polarization in (21) is negligible, hence the quenched approximation is good.

If the cutoff exceeds the upper critical limit, a solution of the full equations ceases to exist, whereas the quenched approximation still predicts a solution. This implies that polarization effects are important to address the CI-behavior of the model.

We conclude that within the quenched approximation one cannot decide whether a CI-regime exists or not. If a CI-regime exists, the quenched approximation may be a good approximation for physical observables, although non-observables (e.g. renormalization constants) might turn out to be completely different.

4 CI-violation in chiral symmetric QED

In the following, we will study the chiral limit $m_R = 0$ (implying $m_0 \equiv 0$) of QED, which requires a new set of renormalization conditions. Again we demand that the residue of the electron propagator is one, i.e. $F(0) = 1$, which fixes Z_1 . We are interested in a phase with spontaneously broken chiral symmetry. We therefore insist on $\Sigma(0) = \Sigma_0$, which must now be accomplished by a choice of Z_3 since m_0 is now identical zero. All renormalization constants are fixed. The renormalized coupling $\alpha_R = D_R(0)$ will be self-consistently calculated.

We numerically solved the coupled equations (19,20,21) for this set of renormalization conditions. The result for the electron self-energy $\Sigma(p^2)$ and the vacuum polarization $D_R(p^2)$ is shown in figure 5, which should be compared with figure 1. We find a logarithmic dependence on the cutoff. Although the qualitative behavior of $1/D_R(p^2)$ is qualitatively the same as in the case of massive electron, the plateau at small p^2 does not stabilize, but continuously increases with increasing cutoff. One might think that this CI-violation in the self-energy is due to the change of the plateau of $D_R(p^2)$ when the cutoff is varied. One might therefore be tempted to parametrize the vacuum polarization by

$$\frac{1}{D_R(p^2)} = c_3(\alpha_R) - c_4(\alpha_R) \ln \Lambda^2/p^2, \quad \text{for} \quad \Lambda^2 \rightarrow \infty \quad (27)$$

and to search for a CI-regime by solving the reduced set of equations (19,20). However, it turns out that the CI-violation in the electron self-energy qualitatively remains the same.

Two possible explanations of this cutoff dependence are immediate: the theory is trivial in a sense that the limit $\Lambda \rightarrow \infty$ is only compatible with $\alpha_R = 0$. In this case, the interaction is not strong enough to spontaneously break chiral symmetry, and one cannot keep the renormalization condition $\Sigma(0) = \Sigma_0 \neq 0$. A second possible explanation is that in the chiral limit the solutions are sensitive to the infrared behavior of the integrals in (19,20,21). The momentum independent Ansatz for the renormalized vertex (17) might be too crude for the chiral limit and induces the logarithmic CI-violation. An improved Ansatz for the vertex function might provide a CI-regime with a spontaneous broken symmetry. None of these two explanations can be favored without further studies beyond the Ansatz (17) for the vertex studied here.

5 Conclusions

We numerically studied the coupled set of renormalized Dyson-Schwinger equations for the electron and the photon propagator using a tree level electron-photon-vertex as an Ansatz for the renormalized vertex function. We imposed the constraint $Z_2 = Z_1$ between the electron wave function renormalization constant Z_2 and the vertex renormalization constant Z_1 as required by gauge invariance. The violation of the Ward identity is smaller than 5%.

In the case of a massive electron (explicitly broken chiral symmetry) and fixed renormalized coupling α_R , a regime of cutoff independence (CI-regime) was found, if the cutoff is below an upper critical value. The theory is weakly renormalizable. For larger values of the cutoff than the critical value, no solution of the Dyson-Schwinger equations exists. The results indicate that the infinite cutoff limit is only compatible with a zero renormalized charge. In the CI-regime, the quenched approximation yields reasonable results for the electron self-energy, although the values of the renormalization constants completely differ from the full result.

In the case of chiral symmetry (zero renormalized mass of the electron) a logarithmic dependence on the cutoff was found. The question, whether this cutoff dependence is due to the crude Ansatz for the renormalized vertex or induced by triviality, cannot be answered at the present stage of investigations. The question, whether a CI-regime with spontaneously broken chiral symmetry exists in strongly coupled QED, cannot be answered from the viewpoint of the truncated Dyson-Schwinger equations supplied with tree level vertex.

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Figure captions

Figure 1: CI-behavior: the electron self-energy $\Sigma(p^2)$ and the vacuum polarization D_R as function of the momentum p^2 for several values of the cutoff Λ in units of $\Sigma_0 = \Sigma(0)$.

Figure 2: The renormalization group flow for $\alpha_R = 0.35$; $Z_1(\Lambda)$ short dashed line, $Z_3(\Lambda)$ long dashed line, $m_0(\Lambda)/\Sigma_0$ solid line.

Figure 3: The maximal possible renormalized coupling α_R as function of the critical cutoff Λ_c (left). The ratio Z_3/α_R at the critical cutoff (right).

Figure 4: The electron self-energy $\Sigma(p^2)$ in the quenched approximation compared with the full result for $\alpha_R = 0.35$ and $\Lambda^2 = 8.9 \cdot 10^6 \Sigma_0$.

Figure 5: CI-violations: the electron self-energy $\Sigma(p^2)$ and the vacuum polarization D_R as function of the momentum p^2 for several values of the cutoff Λ in units of $\Sigma_0 = \Sigma(0)$ in the chiral limit $m_R = 0$.