

Two observers calculate the trace anomaly

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Abstract

We adapt a calculation due to Massacand and Schmid [1] to the coordinate independent definition of time and vacuum given by Capri and Roy [2] in order to compute the trace anomaly for a massless scalar field in a curved spacetime in 1+1 dimensions. The computation only requires normal ordering and yields the well-known result $\frac{R}{24\pi}$ in a straightforward manner.

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I. INTRODUCTION

One of the more interesting results of the study of quantum field theory in curved space-time is the fact that the expectation value of the trace of the stress tensor of a conformally coupled field does not vanish. It has an anomaly. This trace or conformal anomaly, as it is known, was first noticed by Capper and Duff [3] using a dimensional regularization scheme. Since then many other regularization procedures have been used and when used correctly lead to same result [4]. Unfortunately, as anyone who has ever calculated this trace anomaly knows, the computations required are rather lengthy and certainly less than illuminating. On the other hand, if one has a particle interpretation the problem can be handled more simply. This fact was first exploited by Massacand and Schmid [1]. In this paper we adapt their method to a computation in 1+1 dimensions using only the following two inputs.

- 1) The frame components of the stress tensor at a given point are, for two frames based at this point, related by a Lorentz transformation.
- 2) The vacuum expectation value of the energy momentum density (relative to a given frame) should vanish. Thus, the vacuum can have pressure, but no energy or momentum.

In general there would remain the vexing question, “Which vacuum?” The answer we propose is to use the coordinate independent definition of Capri and Roy. In section II we give a brief review of this construction of the vacuum and apply the result to a calculation of the vacuum expectation value of the trace of the stress tensor in section III. Our conclusions are set out in section IV.

II. COORDINATE INDEPENDENT DEFINITION OF TIME AND VACUUM

In a globally hyperbolic spacetime one can choose a foliation based solely on geodesics. Thus, given a timelike (unit) vector $N_\mu(P_0)$ at the point P_0 one establishes a frame (zweibein) at P_0 with components:

$$e^{\mu\hat{0}} = N^\mu(P_0)$$

$$e^{\mu\hat{1}} = p^\mu(P_0) \quad (2.1)$$

where $p^\mu(P_0)$ is a unit vector orthogonal to $N^\mu(P_0)$ at P_0 . The spacelike hypersurface (line) consisting of the geodesic through P_0 with tangent vector $p^\mu(P_0)$ defines the surface $t = 0$. The “time” t corresponding to an arbitrary point P is the distance along a geodesic $P_1 - P$ which intersects the line $t = 0$ orthogonally at some point P_1 . The geodesic distance $P_0 - P_1$ along the line $t = 0$ yields the space coordinate x . These geodesic normal coordinates prove to be very useful since in these coordinates the metric becomes

$$ds^2 = dt^2 - \alpha^2(t, x)dx^2 \quad (2.2)$$

where

$$\begin{aligned} \alpha(0, 0) &= 1 \\ \frac{\partial\alpha}{\partial t}\Big|_{t=0} &= \frac{\partial\alpha}{dx}\Big|_{t=0} = 0 = \frac{\partial^2\alpha}{\partial t\partial x}\Big|_{P_0} = \frac{\partial^2\alpha}{\partial x^2}\Big|_{P_0} \end{aligned} \quad (2.3)$$

Also,

$$\frac{2}{\alpha} \frac{\partial^2\alpha}{\partial t^2} = R \quad (2.4)$$

where R is the curvature scalar.

The field equations in these coordinates, for a massless scalar field read:

$$\begin{aligned} \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu) \phi &= 0 \\ \frac{\partial^2\phi}{\partial t^2} + \frac{\dot{\alpha}}{\alpha} \frac{\partial\phi}{\partial t} + \frac{\alpha'}{\alpha^3} \frac{\partial\phi}{\partial x} - \frac{1}{\alpha^2} \frac{\partial^2\phi}{\partial x^2} &= 0. \end{aligned} \quad (2.5)$$

Here,

$$\dot{\alpha} = \frac{\partial\alpha}{\partial t} \quad \text{and} \quad \alpha' = \frac{\partial\alpha}{\partial x}. \quad (2.6)$$

The positive frequency modes ϕ of this field are obtained by solving these field equations with the two initial conditions

$$1) \quad \phi_{p,\epsilon}(0, x) = \frac{1}{\sqrt{4\pi p}} \exp(ip\epsilon x) \quad p > 0. \quad (2.7)$$

Here we have

$$\begin{aligned}\epsilon = +1 & \text{ corresponds to right travelling waves} \\ \epsilon = -1 & \text{ corresponds to left travelling waves.}\end{aligned}\tag{2.8}$$

and

$$2) \quad i \frac{\partial \phi_{p,\epsilon}}{\partial t} \Big|_{t=0} = p \phi_{p,\epsilon} \Big|_{t=0}.\tag{2.9}$$

A useful Ansatz to implement these initial conditions is:

$$\phi_{p,\epsilon}(t, x) = \frac{1}{\sqrt{4\pi p}} \exp(-ipf_\epsilon(t, x))\tag{2.10}$$

where f_ϵ is real. Equation (2.5) then yields that

$$\frac{\partial f_\epsilon}{\partial t} = \frac{\epsilon}{\alpha} \frac{\partial f_\epsilon}{\partial x}\tag{2.11}$$

The initial conditions become

$$f_\epsilon(0, x) = \epsilon x\tag{2.12}$$

and near $t = 0$

$$f_\epsilon(t, x) \approx t + \epsilon x\tag{2.13}$$

The quantized field is now given by

$$\Psi(t, x) = \sum_{\epsilon=\pm 1} \int_0^\infty d(\epsilon p) \left(\phi_{p,\epsilon}(t, x) a_{p,\epsilon} + \phi_{p,\epsilon}^*(t, x) a_{p,\epsilon}^\dagger \right)\tag{2.14}$$

with the vacuum defined by

$$a_{p,\epsilon} |0\rangle = 0.\tag{2.15}$$

These modes have been normalized such that

$$\begin{aligned}
(\phi_{p,\epsilon}, \phi_{q,\epsilon}) &= i\epsilon \int_{-\infty}^{\infty} dx \sqrt{g} \left(\phi_{p,\epsilon}^*(t, x) \overleftrightarrow{\partial}_t \phi_{q,\epsilon}(t, x) \right) \\
&= \frac{p+q}{4\pi\sqrt{pq}} \int_{-\infty}^{\infty} dx \epsilon \alpha \frac{\partial f_\epsilon}{\partial t} \exp(i(p-q)f_\epsilon(t, x)) \\
&= \frac{p+q}{4\pi\sqrt{pq}} \int_{-\infty}^{\infty} dx \frac{\partial f_\epsilon}{\partial x} \exp(i(p-q)f_\epsilon(t, x)) \\
&= \frac{p+q}{4\pi\sqrt{pq}} 2\pi \delta(p-q) \\
&= \delta(p-q)
\end{aligned} \tag{2.16}$$

III. THE TRACE ANOMALY

We begin with two ‘‘observers’’ with tangents to their world lines given by

$$N^\mu(P_0) = (1, 0) \quad \text{and} \quad \bar{N}^\mu(P_0) = (\cosh(\chi), \frac{\sinh(\chi)}{\bar{\alpha}}) \tag{3.1}$$

The corresponding frames are:

$$e^{\mu\hat{0}} = (1, 0) \quad e^{\mu\hat{1}} = (0, -\frac{1}{\alpha}) \tag{3.2}$$

$$\bar{e}^{\mu\hat{0}} = (\cosh(\chi), \frac{\sinh(\chi)}{\bar{\alpha}}) \quad \bar{e}^{\mu\hat{1}} = (\sinh(\chi), -\frac{\cosh(\chi)}{\bar{\alpha}}) \tag{3.3}$$

Corresponding to this the metric has the two forms

$$ds^2 = dt^2 - \alpha^2(t, x) dx^2 = d\bar{t}^2 - \bar{\alpha}^2(\bar{t}, \bar{x}) d\bar{x}^2 \tag{3.4}$$

We can solve for the positive frequency modes in the barred as well as in the unbarred coordinates to obtain the corresponding quantized fields $\bar{\Psi}(\bar{t}, \bar{x})$ and $\Psi(t, x)$. Their respective sets of annihilation and creation operators are $(\bar{a}_{p,\epsilon}, \bar{a}_{p,\epsilon}^\dagger)$ and $(a_{p,\epsilon}, a_{p,\epsilon}^\dagger)$.

At P_0 , the point with coordinates $(0, 0)$ in both coordinate systems, the two fields coincide, as do their first time derivatives. Corresponding to these two fields we have their respective Fock space vacuums $|\bar{0}\rangle$, $|0\rangle$ defined by

$$\bar{a}_{p,\epsilon} |\bar{0}\rangle = 0 \quad , \quad a_{p,\epsilon} |0\rangle = 0 \tag{3.5}$$

Any bilinear expression in the field operators which, for physical reasons, should have vanishing vacuum expectation value is defined by normal ordering with respect to its own vacuum.

Thus since we expect the vacuum to be the state of zero energy and momentum density we require that

$$\langle \bar{0} | : \bar{T}^{\hat{0}\hat{\mu}} : | \bar{0} \rangle = 0 \quad (3.6)$$

and

$$\langle 0 | : T^{\hat{0}\hat{\mu}} : | 0 \rangle = 0, \quad (3.7)$$

where,

$$\begin{aligned} T^{\hat{\alpha}\hat{\beta}} &= e^{\mu\hat{\alpha}} e^{\nu\hat{\beta}} T_{\mu\nu} \\ \bar{T}^{\hat{\alpha}\hat{\beta}} &= \bar{e}^{\mu\hat{\alpha}} \bar{e}^{\nu\hat{\beta}} \bar{T}_{\mu\nu}. \end{aligned} \quad (3.8)$$

Furthermore, since the barred and unbarred frames $\bar{e}^{\mu\hat{\alpha}}$, $e^{\mu\hat{\alpha}}$ are related by a Lorentz transformation

$$\Lambda_{\hat{\alpha}}^{\hat{\beta}} = \begin{pmatrix} \cosh(\chi) & \sinh(\chi) \\ \sinh(\chi) & \cosh(\chi) \end{pmatrix} \quad (3.9)$$

we have that at P_0

$$: T^{\hat{\alpha}\hat{\beta}} : |_{P_0} = \Lambda_{\hat{\gamma}}^{\hat{\alpha}} \Lambda_{\hat{\delta}}^{\hat{\beta}} : \bar{T}^{\hat{\gamma}\hat{\delta}} : |_{P_0} \quad (3.10)$$

so that in particular

$$: T^{\hat{0}\hat{0}} : |_{P_0} = \cosh^2(\chi) : \bar{T}^{\hat{0}\hat{0}} : |_{P_0} + 2 \cosh(\chi) \sinh(\chi) : \bar{T}^{\hat{0}\hat{1}} : |_{P_0} + \sinh^2(\chi) : \bar{T}^{\hat{1}\hat{1}} : |_{P_0}. \quad (3.11)$$

Taking the vacuum expectation value with respect to the barred vacuum of this equation, and using (3.7) we have

$$\langle \bar{0} | : T^{\hat{0}\hat{0}} : |_{P_0} | \bar{0} \rangle = \sinh^2(\chi) \langle \bar{0} | : \bar{T}^{\hat{1}\hat{1}} : |_{P_0} | \bar{0} \rangle \quad (3.12)$$

Since $\langle \bar{0} | : \bar{T}^{\hat{0}\hat{0}} : | \bar{0} \rangle = 0$ we find that the vacuum expectation value of the trace is:

$$\langle \bar{0} | \eta_{\hat{\alpha}\hat{\beta}} : \bar{T}^{\hat{\alpha}\hat{\beta}} : |_{P_0} | \bar{0} \rangle = - \langle \bar{0} | : \bar{T}^{\hat{1}\hat{1}} : |_{P_0} | \bar{0} \rangle = - \frac{1}{\sinh^2(\chi)} \langle \bar{0} | : T^{\hat{0}\hat{0}} : |_{P_0} | \bar{0} \rangle \quad (3.13)$$

To evaluate this expression we have to take the term $:T^{\hat{0}\hat{0}}:|_{P_0}$ which has been normal ordered with respect to the vacuum $|0\rangle$, rewrite it in terms of the operators $(\bar{a}_{p,\epsilon}, \bar{a}_{p,\epsilon}^\dagger)$ and commute the terms so that the resulting expression is normal ordered with respect to the vacuum $|\bar{0}\rangle$. To do this we write out the term $:T^{\hat{0}\hat{0}}:|_{P_0}$ explicitly. A simplification due to the use of equation (2.11) occurs so that only time derivatives of the field operators appear. Also since at P_0 the fields Ψ and $\bar{\Psi}$ as well as $\partial_t\Psi$ and $\partial_t\bar{\Psi}$ coincide we may write

$$:T^{\hat{0}\hat{0}}:|_{P_0} =: \frac{\partial\Psi}{\partial t} \frac{\partial\Psi}{\partial t} :|_{P_0} =: \frac{\partial\bar{\Psi}}{\partial t} \frac{\partial\Psi}{\partial t} :|_{P_0} = \sum_{\epsilon=\pm 1} \int d(\epsilon p) \left[\frac{\partial\bar{\Psi}}{\partial t} \frac{\partial\phi_{p,\epsilon}}{\partial t} a_{p,\epsilon} + \frac{\partial\phi_{p,\epsilon}^*}{\partial t} a_{p,\epsilon}^\dagger \frac{\partial\bar{\Psi}}{\partial t} \right] |_{P_0}. \quad (3.14)$$

To simplify the notation we drop the $|_{P_0}$, but keep in mind that these equations only apply at the point P_0 . Also we only evaluate this expression for a fixed ϵ . Thus,

$$:T_\epsilon^{\hat{0}\hat{0}}:= \int_0^\infty d(\epsilon p) \int_0^\infty d(\epsilon q) \left[\left(\frac{\partial\bar{\phi}_{q,\epsilon}}{\partial t} \bar{a}_{q,\epsilon} + \frac{\partial\bar{\phi}_{q,\epsilon}^*}{\partial t} \bar{a}_{q,\epsilon}^\dagger \right) a_{p,\epsilon} \frac{\partial\phi_{p,\epsilon}}{\partial t} + \frac{\partial\phi_{p,\epsilon}^*}{\partial t} a_{p,\epsilon}^\dagger \left(\frac{\partial\bar{\phi}_{q,\epsilon}}{\partial t} \bar{a}_{q,\epsilon} + \frac{\partial\bar{\phi}_{q,\epsilon}^*}{\partial t} \bar{a}_{q,\epsilon}^\dagger \right) \right] \quad (3.15)$$

The operators $(a_{p,\epsilon}, a_{p,\epsilon}^\dagger)$ are related to the barred operators $(\bar{a}_{k,\epsilon}, \bar{a}_{k,\epsilon}^\dagger)$ by a Bogolubov transformation

$$a_{k,\epsilon} = \int d(\epsilon q) (\alpha_{k,q} \bar{a}_{q,\epsilon} + \beta_{k,q}^* \bar{a}_{q,\epsilon}^\dagger) \quad (3.16)$$

where

$$\begin{aligned} \alpha_{k,q} &= (\phi_{k,\epsilon}, \bar{\phi}_{q,\epsilon}) \\ \beta_{k,q} &= (\phi_{k,\epsilon}^*, \bar{\phi}_{q,\epsilon}) \end{aligned} \quad (3.17)$$

In our evaluation of the vacuum expectation value, the only term of interest is the c-number term that results from the commutator

$$\bar{a}_{q,\epsilon'} \bar{a}_{k,\epsilon}^\dagger = \bar{a}_{k,\epsilon}^\dagger \bar{a}_{q,\epsilon'} + \delta_{\epsilon,\epsilon'} \delta(\vec{k} - \vec{q}) \quad (3.18)$$

Thus, we get

$$\langle \bar{0} | : T_\epsilon^{\hat{0}\hat{0}} : |_{P_0} | \bar{0} \rangle = \int d(\epsilon p) d(\epsilon q) \left[\frac{\partial \bar{\phi}_{q,\epsilon}}{\partial \bar{t}} \frac{\partial \phi_{p,\epsilon}}{\partial t} \beta_{p,q}^* + c.c. \right] \quad (3.19)$$

These terms are evaluated by replacing β by its expression (3.17) and interchanging the order of integration to first do the momentum integrals. In doing so the only regularization required is to define an integral of the form

$$\int_0^\infty dx x \exp(ixp) \quad (3.20)$$

This is accomplished by replacing p by $p + i\delta$. No further regularizations are needed. Further details of such a calculation are in the paper by Massacand and Schmid [1] and yield a Schwarz derivative. The final result is:

$$\langle \bar{0} | : T_\epsilon^{\hat{0}\hat{0}} : |_{P_0} | \bar{0} \rangle = -\frac{1}{24\pi} \frac{\frac{\partial^3 \bar{f}}{\partial x^3} |_{P_0}}{\frac{\partial \bar{f}}{\partial x} |_{P_0}} \quad (3.21)$$

So we only have to evaluate these terms. Now,

$$\frac{\partial \bar{f}}{\partial x} = \frac{\partial \bar{f}}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial x} + \frac{\partial \bar{f}}{\partial \bar{t}} \frac{\partial \bar{t}}{\partial x} \quad (3.22)$$

and as initial conditions at P_0 we have

$$\frac{\partial \bar{x}}{\partial x} |_{P_0} = \cosh(\chi) \quad , \quad \frac{\partial \bar{t}}{\partial x} |_{P_0} = \sinh(\chi) \quad (3.23)$$

Furthermore, we also have that

$$\bar{f}_\epsilon(0, \bar{x}) = \epsilon \bar{x} \quad , \quad \frac{\partial \bar{f}}{\partial \bar{t}} |_{P_0} = 1 \quad , \quad \frac{\partial \bar{f}}{\partial \bar{x}} |_{P_0} = \epsilon \bar{\alpha} \frac{\partial \bar{f}}{\partial \bar{t}} |_{P_0} = \epsilon \quad (3.24)$$

since $\bar{\alpha} |_{P_0} = 1$. Also, as we stated earlier,

$$\frac{\partial \bar{\alpha}}{\partial \bar{t}} |_{P_0} = \frac{\partial \bar{\alpha}}{\partial \bar{x}} |_{P_0} = 0 \quad (3.25)$$

$$\frac{\partial^2 \bar{\alpha}}{\partial \bar{t} \partial \bar{x}} |_{P_0} = \frac{\partial^2 \bar{\alpha}}{\partial \bar{x}^2} |_{P_0} = 0 \quad (3.26)$$

and

$$\frac{\partial^2 \bar{\alpha}}{\partial \bar{t}^2} |_{P_0} = \frac{R}{2}. \quad (3.27)$$

By repeatedly using the barred version of equation (2.11) , namely

$$\frac{\partial \bar{f}}{\partial \bar{x}} = \epsilon \bar{\alpha} \frac{\partial \bar{f}}{\partial \bar{t}} \quad (3.28)$$

as well as (3.22), (3.23) and (3.25) we find:

$$\frac{\partial^2 \bar{f}}{\partial \bar{t}^2} \Big|_{P_0} = \frac{\partial^2 \bar{f}}{\partial \bar{t} \partial \bar{x}} \Big|_{P_0} = \frac{\partial^2 \bar{f}}{\partial \bar{x}^2} \Big|_{P_0} = 0 \quad (3.29)$$

as well as

$$\frac{\partial^3 \bar{f}}{\partial \bar{x}^3} \Big|_{P_0} = \epsilon \frac{\partial^3 \bar{f}}{\partial \bar{t}^3} \Big|_{P_0} + \epsilon \frac{\partial^2 \bar{\alpha}}{\partial \bar{t}^2} \frac{\partial \bar{f}}{\partial \bar{t}} \Big|_{P_0} = 0 \quad (3.30)$$

Thus we arrive at the result that

$$\frac{\partial^3 \bar{f}}{\partial \bar{t}^3} \Big|_{P_0} = -\frac{\partial^2 \bar{\alpha}}{\partial \bar{t}^2} \Big|_{P_0} = -\frac{R}{2} \Big|_{P_0} \quad (3.31)$$

This result now allows us to obtain that

$$\begin{aligned} \frac{\partial^3 \bar{f}}{\partial x^3} \Big|_{P_0} &= \left[\frac{\partial^3 \bar{t}}{\partial x^3} + \epsilon \bar{\alpha} \frac{\partial^3 \bar{x}}{\partial x^3} + \epsilon \frac{\partial^2 \bar{\alpha}}{\partial \bar{t}^2} \frac{\partial \bar{x}}{\partial x} \left(\frac{\partial \bar{t}}{\partial x} \right)^2 \right] \frac{\partial \bar{f}}{\partial \bar{t}} \Big|_{P_0} + \left(\frac{\partial \bar{t}}{\partial x} + \epsilon \frac{\partial \bar{x}}{\partial x} \right) \frac{\partial^3 \bar{f}}{\partial \bar{t}^3} \left(\frac{\partial \bar{t}}{\partial x} \right)^2 \Big|_{P_0} \\ &= \frac{R}{2} \sinh^2(\chi) \epsilon (\cosh(\chi) - \exp(\epsilon \chi)) + \frac{\partial^3 \bar{t}}{\partial x^3} + \epsilon \frac{\partial^3 \bar{x}}{\partial x^3} \end{aligned} \quad (3.32)$$

To evaluate the last two terms in this expression we use the fact that (t, x) as well as (\bar{t}, \bar{x}) satisfy the geodesic equations, but have different initial data on the spacelike geodesic that passes through P_0 . These initial data are:

$$\frac{dx}{ds} \Big|_{P_0} = 1 \quad , \quad \frac{dt}{ds} \Big|_{P_0} = 0 \quad (3.33)$$

$$\frac{d\bar{x}}{ds} \Big|_{P_0} = \cosh(\chi) \quad , \quad \frac{d\bar{t}}{ds} \Big|_{P_0} = \sinh(\chi) \quad (3.34)$$

The geodesic equations read:

$$\begin{aligned} \frac{d^2 t}{ds^2} &= -\alpha \frac{\partial \alpha}{\partial t} \left(\frac{dx}{ds} \right)^2, \\ \frac{d^2 x}{ds^2} &= -\frac{2}{\alpha} \frac{\partial \alpha}{\partial t} \frac{dx}{ds} \frac{dt}{ds} - \frac{1}{\alpha} \frac{\partial \alpha}{\partial x} \left(\frac{dx}{ds} \right)^2 \end{aligned} \quad (3.35)$$

$$\begin{aligned} \frac{d^2 \bar{t}}{ds^2} &= -\bar{\alpha} \frac{\partial \bar{\alpha}}{\partial \bar{t}} \left(\frac{d\bar{x}}{ds} \right)^2, \\ \frac{d^2 \bar{x}}{ds^2} &= -\frac{2}{\bar{\alpha}} \frac{\partial \bar{\alpha}}{\partial \bar{t}} \frac{d\bar{x}}{ds} \frac{d\bar{t}}{ds} - \frac{1}{\bar{\alpha}} \frac{\partial \bar{\alpha}}{\partial \bar{x}} \left(\frac{d\bar{x}}{ds} \right)^2 \end{aligned} \quad (3.36)$$

By differentiating these equations as well as using (3.23) we find that

$$\frac{\partial^2 \bar{t}}{\partial x^2} \Big|_{P_0} = \frac{\partial^2 \bar{x}}{\partial x^2} \Big|_{P_0} = 0 \quad (3.37)$$

and

$$\frac{\partial^3 \bar{t}}{\partial x^3} \Big|_{P_0} = -\frac{\partial^2 \bar{\alpha}}{\partial \bar{t}^2} \Big|_{P_0} \sinh(\chi) \cosh^2(\chi) = -\frac{R}{2} \sinh(\chi) \cosh^2(\chi) \quad (3.38)$$

$$\frac{\partial^2 \bar{x}}{\partial x^3} \Big|_{P_0} = -2 \frac{\partial^2 \bar{\alpha}}{\partial \bar{t}^2} \Big|_{P_0} \cosh(\chi) \sinh^2(\chi) = -R \cosh(\chi) \sinh^2(\chi). \quad (3.39)$$

Combining these results we obtain that

$$\langle \bar{0} \rangle : T_\epsilon^{\hat{0}\hat{0}} : |_{P_0} [\bar{0}] \rangle = -\frac{R}{48\pi} \epsilon \exp(\epsilon\chi) \sinh(\chi) \quad (3.40)$$

Adding the results for both values of ϵ we obtain

$$\langle \bar{0} \rangle : T^{\hat{0}\hat{0}} : |_{P_0} [\bar{0}] \rangle = -\frac{R}{24\pi} \sinh^2(\chi) \quad (3.41)$$

Inserting this into equation (3.12) we finally obtain the vacuum expectation value of the trace of the stress-energy tensor, namely $\frac{R}{24\pi}$.

IV. CONCLUSION

For the case of a conformally coupled massless scalar field in 1+1 dimensions it is much simpler to evaluate the trace anomaly using a particle picture than to avoid this. The only regularization required is very simple, but it must be this very simple regularization that suffices to break the conformal symmetry and thus give a non-zero result for the vacuum expectation value of the trace.

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