RESOLUTION OF THE STRONG CP AND U(1) PROBLEMS*

H. Banerjee
S.N.Bose National Centre For Basic Sciences
DB-17, Sector-I, Salt Lake, Calcutta-700064, INDIA

Abstract

Definition of the determinant of Euclidean Dirac operator in the nontrivial sector of gauge fields suffers from an inherent ambiguity. The popular Osterwalder-Schrader (OS) recipe for the conjugate Dirac field leads to the option of a vanishing determinant. We propose a novel representation for the conjugate field which depends linearly on the Dirac field and yields a nonvanishing determinant in the nontrivial sector. Physics, it appears, chooses this second option becuase the novel representation leads to a satisfactory resolution of two outstanding problems, the strong CP and U(1) problems, attributed to instanton effects.

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1 Introduction

The strong CP and U(1) problems are two outstanding problems in the QCD sector of the standard model. The former consists in the gross disagreement of theoretical estimates of electric dipole moment of neutron (NEDM), which are invariably larger, by nearly nine to ten orders of magnitude, than the experimental upper limit [1]. The U(1) problem epitomises the difficulties in formulating a theoretically consistent framework to interpret the mass of the flavour singlet pseudoscalar meson $\eta^{'}$ [2], which, unlike the other Goldstone bosons, is very heavy.

The genesis of both the problems is the anomaly for space-time indepedent 'global' chiral rotation of fermi (quark) fields

$$q(x) \rightarrow e^{i\alpha\gamma_5} q(x), \quad \bar{q}(x) \rightarrow \bar{q}(x) e^{i\alpha\gamma_5} \tag{1}$$

In perturbation theory there is no trace of this 'global' anomaly [3]. A space-time indepedent chiral phase in fermion mass drops out from all amplitudes diagram by diagram if the interactions are chirally invariant. This is not in conflict with the ABJ anomaly in the four divergence of the axial vector current which arises from the triangle diagram in perturbation theory. The 'global' anomaly is just the space-time integral of the ABJ anomaly, which, in path integral approach, corresponds to space-time dependent 'local' chiral transformations

$$q(x) \rightarrow e^{i\alpha(x)\gamma_5} q(x), \qquad \bar{q}(x) \rightarrow \bar{q}(x) e^{i\alpha(x)\gamma_5} \tag{2}$$

In perturbation theory the integrand is nontrivial but the integral vanishes.

The sine qua non of a nonvanishing integral and hence of 'global' chiral anomaly is the zero modes of Euclidean Dirac operator which live in compactified space-time and are inaccessible in perturbative framework. The carrier of the virus of fermion zero modes in popular path integral approach is identified to be the Osterwalder-Schrader (OS) [4] recipe for the conjugate Dirac field in Euclidean metric, viz., $\bar{q}(x)$ is indepedent of $q(x)$. In relativistic metric the relation $\bar{q}(x) = (\gamma_o q(x))^+$ relates the Dirac field to its conjugate. The OS recipe, therefore, requires that the degrees of freedom are doubled in Euclidean metric. This is neither natural nor necessary.

We propose a prescription which is just the opposite of the OS recipe. To be precise, we require $\bar{q}(x)$ to be antilinear in $q(x)$ and the relationship to obey reciprocity [5]. The representation of $\bar{q}(x)$ is unique if it is required that $\bar{q}(x)$ has the correct chiral properties, i.e., obeys eq.(1). The novel representation reproduces the correct two-point correlation function, and hence, by antisymmetry, all the 2n-point fermion correlation functions of perturbation theory [5].This assures that the novel representation is consistent with all the standard axioms of Euclidean field theory [6]. The point of interest is that the fermion zero modes are evaded and the novel representation leads to a formulation of QCD which is free from 'global' chiral anomaly. This solves the strong CP and the U(1) problems [5].

It is remarkable that the novel representation yields, in path integral approach, a nonvanishing determinent of the Dirac operator $\not{D}$ in the nontrivial sector $\nu \neq 0$ of gauge fields

$$\nu \equiv \frac{g^2}{16\pi^2} \int tr F_{\mu\nu} \tilde{F}_{\mu\nu} d^4x \tag{3}$$

whereas, the OS recipe gives a vanishing result. This reflects an inherent ambiguity in the definition of the determinant of Dirac operator in non-trivial sector. The ambiguity becomes transparent in Weyl space where one has the option to write Dirac determinant either as

$$det \not{D} = det(-DD^+) or \;\; det(-D^+D) \tag{4}$$

in terms of Weyl operators D, D^+ defined through

$$\begin{aligned} \not{D} &\equiv \gamma_\mu(i\partial_\mu - gA_\mu) \\ &= \begin{pmatrix} 0 & D \\ D^+ & 0 \end{pmatrix} \end{aligned} \tag{5}$$

In nontrivial sector $\nu \neq 0$ one of the options in (4) vanishes while the other does not. This follows from the index theorem

$$\begin{aligned} \nu &= dimker(DD^+) - dimker(D^+D) \\ &= n_+ - n_- \end{aligned} \tag{6}$$

and the theorem that there are no 'wrong chirality' solutions of Dirac operator [7], i.e., for $\nu \geq 0$ ($\nu \leq 0$), the number of normalisable negative (positive) chirality solution $n_-(n_+)$ is zero. The novel representation chooses the non-vanishing option for the Dirac determinant while the OS recipe leads to the vanishing option in nontrivial sector.

Resolution of the strong CP and the U(1) problems, and hence, it appears, physics chooses the option of the novel representation.

2 Global Chiral Anomaly

In Euclidean metric the QCD action is given by

$$S_{QCD} = S_G + S_F + \theta_{ew}\Delta^{'}S + \gamma_{QCD}\Delta S \tag{7}$$

where S_F is the fermionic piece

$$S_F \equiv \int \bar{q}(x)(\not{D} - iM)q(x)d^4x \tag{8}$$

and S_G is the gluon action. The (light) quarks have three flavours. For convenience, we assume the mass matrix M to be diagonal in flavour space and suppress the flavour indices.

The two terms ΔS and $\Delta^{'}S$ are potential sources of CP violation. While the θ_{QCD} term is attributed to the topological structure of the QCD vacuum

$$\Delta S \equiv \frac{g^2}{16\pi^2}\int trF_{\mu\nu}\bar{F}_{\mu\nu}d^4x \tag{9}$$

the chiral phase θ_{ew} in quark mass

$$\Delta^{'}S \equiv \int \bar{q}(x)M\gamma_5 q(x)d^4x \tag{10}$$

arises from Higgs interactions in the electroweak sector. In compactified space-time ΔS assumes integral values ν.

The degrees of freedom of $q(x)$ are the Grassmann gennators which appear as coefficients in the expansion of $q(x)$ in a complete set of basis functions.For convenience, the orthonormal set of eigenfunctions of the Dirac operator are chosen as basis functions,

$$q(x) = \sum_r (a_r + a_{-r}\gamma_5)\varphi_r(x) + \sum_i a_{oi}\varphi_{oi}(x) \tag{11}$$

where the normalised eigenfunctions obey the equations,

$$\begin{aligned} \not{D}\varphi_r(x) = \lambda_r\varphi_r(x) \quad &, \qquad \not{D}\gamma_5\varphi_r(x) = -\lambda_r\gamma_5\varphi_r(x) \\ \not{D}\varphi_{oi}(x) = 0 \quad &, \qquad \gamma_5\varphi_{oi}(x) = \epsilon_i\varphi_{oi} \end{aligned} \tag{12}$$

The zero eigenmodes φ_{oi} have definite chiralities, positive ($\epsilon_i = 1$) or negative ($\epsilon_i = -1$). The OS recipe is implemented by choosing an independent set of Grassmann generators for $\bar{q}(x)$

$$\bar{q}(x) = \sum (b_r + b_{-r}\gamma_5)\varphi_r(x) + \sum b_{oi}\varphi_{oi}(x) \tag{13}$$

The Jacobian of the measure in the fermionic partition function Z_F

$$Z_F \equiv \int d\mu \, exp[-S_F] \tag{14}$$

has the form

$$lnJ(\alpha) = -2i\alpha \int A(x) d^4x \tag{15}$$

for 'global' chiral rotation (1). The integrand A(x) is identified as the 'local' chiral anomaly.

The measure corresponding to OS recipe for $\bar{q}(x)$ is given by

$$d\mu^I = \Pi_r da_r db_r \Pi da_{oi} db_{oi}$$

The 'local' chiral anomaly for this measure was obtained by Fujikawa [8],

$$\begin{aligned} A^I(x) &= 2\sum \varphi_r^+(x)\gamma_5\varphi_r(x) + \sum \epsilon_i \varphi_{oi}^+(x)\varphi_{oi}(x) \\ &= \frac{g^2}{16\pi^2} tr F_{\mu\nu}(x)\bar{F}_{\mu\nu}(x) \end{aligned} \tag{16}$$

Nonzero eigenmodes drop out because of orthogonality of $\varphi_r(x)$ and $\gamma_5\varphi_r(x)$ and only zero modes of $A^I(x)$ survive in the integral (15) for the Jacobian

$$\begin{aligned} lnJ^I(\alpha) &= -2i\alpha(n_+ - n_-) \\ &= -2i\alpha \frac{g^2}{16\pi^2} \int tr F_{\mu\nu}\bar{F}_{\mu\nu} d^4x \end{aligned} \tag{17}$$

This means that under 'global' chiral rotation QCD action changes according to the formula

$$S_{QCD} \to S^I_{QCD}(\alpha) = S_G + S_F + (\theta_{ew} + 2\alpha)\Delta^{'} S + (\theta_{QCD} - 6\alpha)\Delta S \tag{18}$$

In effective Lagrangians for chiral models there is no scope for a nontrivial Jacobian. The global chiral U(1) anomaly (17) in underlying QCD can, therefore, be reproduced in effective Lagrangians through an 'anomaly term' which breaks chiral symmetry explicitly. A popular representation of the anomaly term is [9]

$$\Delta S^I_{eff} = -m^2_{\eta'} f^2_\pi \int [tr ln(\frac{U}{U^+}) - \theta_{QCD}]^2 d^4x \tag{19}$$

where f_π and $m_\eta^{'}$ are respectively the pion decay constant and the mass of the flavour singlet Goldstone boson. The meson matrix U transforms as $U \to Ue^{i\alpha}$ under global chiral rotation (1). The problems with this 'anomaly term' are (a) its chiral variation depends explicitly on θ_{QCD}, and (b) its second order variation does not vanish. Neither of these properties hold in the underlying QCD. This is the crux of the controversy between 't Hooft and Crewther [2], and the reason why the popular resolution (19) of the U(1) problem is regarded as unsatisfactory.

The transformation law (18) shows that neither θ_{ew} nor θ_{QCD} can be physical. Only the chirally invariant combination $\bar{\theta} \equiv (\theta_{QCD} + 3\theta_{ew})$ can appear in CP violating effects. Theoretical estimates in various chiral models suggests [1] NEDM in the range

$$d_n \approx \bar{\theta} \times 10^{-15\pm1} e.cm$$

Experimental upper limit $|d_n| < 10^{-25} e.cm$, therefore, puts a stringent constraint $\bar{\theta} < 10^{-10}$. This is the crux of the strong CP problems. The strong CP problem is, therefore, a serious problem of fine tuning. Two parameters θ_{QCD} and $3\theta_{ew}$ which are of completely different origins in the standard model must be so fine tuned as to cancel each other completely.

3 Novel representation of Euclidean Dirac fermion

We start from the ansatz which is just the opposite of the OS recipe, i.e., we assume that in Euclidean metric the conjugate field $\chi(x) \equiv [\bar{q}(x)]^+$ is linear in $q(x)$. This means that the Grassmann generators defining the degrees of freedom of $\chi(x)$ are a subset of the generators $\{a_r, a_{oi}\}$ appearingin $q(x)$. The resulting representation of $\chi(x)$ is unique, modulo an overall sign, if one requires that, (a) chiral charge of $\chi(x)$ is opposite to that of $q(x)$, i.e., if $q(x) \to e^{i\alpha\gamma_5} q(x)$ then $\chi(x) \to e^{-i\alpha\gamma_5} q(x)$, and (b) the linear relation obeys reciprocity,

$$\chi(x) = \sum_r [a_r - a_{-r}\gamma_5]\varphi_r(x) \tag{20}$$

The crucial point to note is that $\chi(x)$ cannot contain the zero mode generators a_{oi} which transform as $a_{oi} \to a_{oi} e^{i\epsilon_i\alpha}$ under chiral rotation (1). This will be in conflict with the chiral charge of $\chi(x)$. As a result, the fermion action S_F is devoid of the zero mode generators

$$S_F \;=\; \int \chi^+(x)(\not{D} - iM)q(x)d^4x$$

$$= \sum_r [\lambda_r (a_r^* a_r + a_{-r}^* a_{-r}) - iM(a_r^* a_r - a_{-r}^* a_{-r})] \tag{21}$$

The measure appropriate for this action

$$d\mu = \Pi_r da_r^* da_r da_{-r}^* da_{-r}$$

leads to the partition function

$$Z_F = \Pi_{\lambda_r > o} (\lambda_r^2 + M^2) \tag{22}$$

whose chiral limit (M=0) does not vanish in the nontrivial sector ($\nu \neq 0$) of gauge fields.

The two-point correlation function, obtained in the usual path integral approach, coincides with the familiar formula

$$< q(x)\bar{q}(y) >=< x| \frac{1}{\not{D} - iM} |y > - \frac{\sum_i \varphi_{oi}(x)\varphi_{oi}^+(y)}{-iM} \tag{23}$$

except that the zero mode contributions are subtracted out. In the limit of zero coupling $g = 0$, the momentum representation of the correlation function coincides with the Wick-rotated relativistic Feynman propagator

$$< q(x)\bar{q}(y) >_{g=0} = \frac{1}{(2\pi)^4} \int d^4p \frac{\not{p} + iM}{p^2 + M^2} e^{-ip(x-y)} \tag{24}$$

The remaining 2n-point correlation functions follow, in path integral framework, from the anticommutation of $q(x)$ and $\bar{q}(y)$

$$< q(x_1)...q(x_m)\bar{q}(y_1)...\bar{q}(y_n) >= \delta_{mn} det[< q(x_i)\bar{q}(y_i)] \tag{25}$$

The 'local' chiral anomaly is given by an expression analogous to that in the OS formulation (16) except that zero modes are excluded from the sum on the right hand side,

$$\begin{aligned} A^{II(x)} &= 2 \sum_{\lambda_r > 0} \varphi_r^+(x)\gamma_5\varphi_r(x) \\ &= \frac{g^2}{16\pi^2} \int tr F_{\mu\nu} \bar{F}_{\mu\nu} d^4x - \epsilon_i \varphi_{oi}^+(x)\varphi_{oi}(x) \end{aligned} \tag{26}$$

Thus the four divergence of the U(1) axial current has a nontrivial anomaly $A^{II}(x)$

$$\partial_\mu J_{\mu 5}(x) = 2\bar{q}(x)M\gamma_5 q(x) + A^{II}(x) \tag{27}$$

which coincides, as it must, with the perturbative (absence of zero modes) ABJ anomaly. However, the 'global' chiral anomaly vanishes

$$\begin{aligned} lnJ^{II}(\alpha) &= -2i\alpha \int A^{II}(x) d^4x \\ &= 0 \end{aligned} \tag{28}$$

and our desired goal is achieved. The vanishing is the direct consequence of the orthogonality of nonzero eigenmodes $\varphi_r(x)$ and $\gamma_5\varphi_r(x)$, or, if one prefers, the index theorem.

The novel representation (20) is equivalent to the functional relation

$$\chi(x) = \frac{1}{[\not{D}^2]^{\frac{1}{2}}} \not{D} q(x) \tag{29}$$

which is nonlocal. This is not a disability in Euclidean field theory. Locality is a field theoretic axiom only in rleativistic metric. This translates in Euclidean field theory into the axiom of (anti) symmetry of correlation functions under permutations [6]. The fact that all the correlation functions are reproduced correctly through the eqs.(24) and (25) assures us that the novel representation (20, 29) is consistent not only with the axiom of symmetry but with the other axioms of Euclidean field theory, e.g., reflection positivity, cluster decomposition etc., as well.

4 Resolution of strong CP and U(1) problems

In the scenario corresponding to the novel representation (20, 29) the QCD action is invariant under 'global' chiral rotation in the chiral (M = 0) limit

$$S_{QCD} \rightarrow S^{II}_{QCD}(\alpha) = S_G + S_F + (\theta_{ew} + 2\alpha) Delta^{'} S + \theta_{QCD} \Delta S \tag{30}$$

In effective Lagrangians, the 'anomaly' term, which is invariant under 'global' chiral rotation but reproduces the ABJ anomaly in axial Ward identity, is easily constructed

$$\Delta S^{II}_{eff} = -m^2_{\eta'} f^2_\pi \int [trln(\frac{U}{U^+}) - < trln(\frac{U}{U^+}) >]^2 d^4x \tag{31}$$

The controversial features of the popular construction (19) thus disappear and the U(1) problem satisfactorily resolved [5].

The law of transformation (30) shows that the parameter θ_{QCD}, the coefficient of ΔS, is invariant, while θ_{ew} is unphysical and can be eliminated trivially by 'global' chiral rotation (1). There is no longer any problem of fine tuning and CP symmetry of QCD is simply ensured through the 'natural' choice $\theta_{QCD} = 0$ [5].

We conclude that the strong CP and the U(1) problems both are legacies of the OS recipe. The alternative scenario of QCD with the novel representation (20, 29) for the conjugate Dirac field is not afflicted with these blemishes [5].

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