MULTIPARTON INTERACTIONS AND PRODUCTION OF MINIJETS IN HIGH ENERGY HADRONIC COLLISIONS

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ABSTRACT

We discuss the inclusive cross section to produce two minijets with a large separation in rapidity in high energy hadronic collisions. The contribution to the inclusive cross section from the exchange of a BFKL Pomeron is compared with the contribution from the exchange of two BFKL Pomerons, which is induced by the unitarization of the semi-hard interaction. The effect of the multiple exchange is studied both as a function of the azimuthal correlation and as a function of the transverse momentum of the observed minijets.

1. Introduction

One of the main topics in perturbative QCD is presently represented by semi-hard hadronic interactions, namely by hadron interactions with momentum transfer constant with energy but large enough to apply perturbation theory. One of the characteristic features of this kinematical regime is the large size of the corresponding cross sections, which, although in the perturbative domain, rise rapidly with energy. In fact, already at the energies of present hadron colliders, one may easily obtain semi-hard cross sections whose size is comparable to the total hadronic cross section[1,2]. At the partonic level, in a typical interaction configuration, one of the two interacting partons has a finite fraction of the parent's hadron momentum while the other one has a momentum fraction close to zero. The separation in rapidity of the two partons is therefore increasingly large with energy and, in the parton-parton c.m. system, the transverse momentum exchange is small with respect to the longitudinal momenta. The Regge limit is then approached in semi-hard interactions not only in the whole hadron-hadron process but also in the underlying parton-parton interactions.

When considering the large p_t regime the momentum exchange is of the order of the incoming partons momenta. At the parton level such a large scale factor can be transferred only in a few interaction vertices and, as a result, the elastic two body parton collision is a good first order approximation to the elementary partonic interaction. In the semi-hard regime, since the semi-hard scale is small with respect to the total energy available, there are several parton vertices with momentum exchange of the order of the semi-hard scale. A consequence is that all semi-hard radiated gluons are to be taken explicitly into account for a proper factorization of the semi-hard component of the interaction. When the $2 \to n$, rather than the $2 \to 2$, is the parton subprocess relevant to the semi-hard component of the hadronic interaction, a difficulty arises in constructing an inclusive cross section, where only few of the radiated partons are actually detected as mini-jets in the final state. In fact one is not allowed any more to use the lowest order tree diagram to represent the parton amplitude, since the tree level amplitude is singular in the soft and collinear limit. To avoid the infrared problem one faces when evaluating an inclusive cross section, one needs to keep virtual corrections explicitly into account and, as a consequence, the elementary subprocess acquires a non trivial structure. The problem has been addressed already several years ago in a series of papers by Lipatov and collaborators[3] . Lipatov's solution is the BFKL Pomeron: the partonic interaction is described by the exchange of a gluon ladder structure with vacuum quantum numbers in the t-channel. The s-channel discontinuity of the BFKL Pomeron represents the production of the semi-hard gluons. In the limit in which the transverse momenta are always negligible with respect to the longitudinal ones, the steps of the ladder are ordered in rapidity and dynamics is greatly simplified. Indeed the simplified kinematics lets one to isolate the two basic elements which build up the ladder:

a- The gauge independent non-local vertices, which keep into account the dominant term, in the $t/s \to 0$ limit, of the diagrams with gluon emission from all near-by lines, and

b- the Reggeization of the t-channel gluons, which is the virtual correction that allows a solution to the infrared problem.

The ladder structure can be iterated in the t-channel, which may be expressed as an integral equation, the Lipatov's equation. Lipatov's equation allows an analytic solution free from infrared (and ultraviolet) singularities. One obtains in this way an explicit expression for the cross section where two gluons interact producing many gluons and two of them, the ones nearby in rapidity to the interacting partons, are observed. If y is the separation in rapidity of the interacting gluons and k_a, k_b are the transverse momenta of the observed ones, the inclusive cross section can be expressed as

$$\frac{d\hat{\sigma}}{d^2k_a d^2k_b} = \left[\frac{C_A\alpha_s}{k_a^2}\right] f(k_a,k_b,y) \left[\frac{C_A\alpha_s}{k_b^2}\right] \tag{1}$$

where $C_A = N_c$ is the number of colors, α_s is the strong coupling constant and $f(k_a,k_b,y)$ is the inverse Laplace transform of the solution to Lipatov's equation. Actually:

$$f(k_a,k_b,y) = \frac{1}{(2\pi)^2 k_a k_b} \sum_{\mathrm{n}=-\infty}^{+\infty} e^{i\mathrm{n}\phi} \int_{-\infty}^{+\infty} d\nu e^{\omega(\nu,\mathrm{n})y} e^{i\nu \ln(k_a^2/k_b^2)} \tag{2}$$

where ϕ is the azimuthal angle between the observed gluons,

$$\omega(\nu,\mathrm{n}) = -2\frac{\alpha_s N_c}{\pi} \Re\left[\psi\Big(\frac{|\,\mathrm{n}\,|+1}{2}+i\nu\Big) - \psi(1)\right] \tag{3}$$

and

$$\psi(z) = \frac{d\ln\Gamma(z)}{dz} \tag{4}$$

is the Digamma function. The inclusive cross section for production of two mini-jets, as a result of a BFKL Pomeron exchange, is obtained by folding Eq.(1) with the structure functions of the interacting hadrons A and B:

$$\frac{d\sigma}{dx_A dx_B d^2k_a d^2k_b} = f_{eff}(x_A,k_a^2) f_{eff}(x_B,k_b^2) \frac{d\hat{\sigma}}{d^2k_a d^2k_b} \tag{5}$$

where f_{eff} is the effective structure function

$$f_{eff}(x) = G(x) + \frac{4}{9}\sum_f \Big[Q_f(x) + \bar{Q}_f(x)\Big] \tag{6}$$

namely the gluon structure function plus 4/9 of the quark and anti-quark structure functions with flavour f. The expression for the cross section in Eq.(1) correlates the azimuthal angle ϕ with the distance in rapidity of the observed partons. The differential cross section in Eq.(1) may be easily integrated, at ϕ fixed, on k_a and k_b down to the lower cut off k_m, which represents the threshold in transverse momentum that allows a parton to be observed as a minijet in the final state. This yields:

$$\frac{d\hat{\sigma}}{d\phi} = \frac{(C_A\alpha_s)^2}{2\pi}\frac{1}{k_m^2}\sum_{\mathrm{n}=-\infty}^{+\infty} e^{i\mathrm{n}\phi}\int_{-\infty}^{+\infty} d\nu \frac{e^{\omega(\nu,\mathrm{n})y}}{1+4\nu^2} \tag{7}$$

which is a suitable expression to study the azimuthal correlation of the observed partons as a function of the rapidity difference y. Simple expressions may also be obtained for the cross section where the momentum of one of the two observed gluons has been integrated down to the lower limit k_m:

$$\frac{d\hat{\sigma}}{d^2k_a} = \frac{(C_A\alpha_s)^2}{2\pi}\frac{1}{k_a^3k_m}\int_{-\infty}^{+\infty} d\nu \frac{e^{\omega(\nu,0)y}}{2i\nu+1}\left(\frac{k_a}{k_m}\right)^{2i\nu} \tag{8}$$

and for the cross section where both observed gluons momenta have been integrated down to k_m:

$$\hat{\sigma} = \frac{(C_A\alpha_s)^2}{k_m^2}\int_{-\infty}^{+\infty} d\nu \frac{e^{\omega(\nu,0)y}}{1+4\nu^2} \tag{9}$$

The high energy behaviour of the integrated cross section is estimated by evaluating the asymptotic limit of the integral in Eq.(9) for large y[4]:

$$\hat{\sigma} \to \frac{\pi(C_A\alpha_s)^2}{2k_m^2}\frac{exp\big[4\ln2 N_c\alpha_s y/\pi\big]}{\big[7\zeta(3)N_c\alpha_s y/2\big]^{1/2}} \tag{10}$$

where ζ is the Riemann zeta function. Eq.(10) shows a rapid growth, roughly linear with the parton-parton c.m. energy, of the partonic cross section corresponding to the exchange of a BFKL Pomeron and gives a justification to the large size of the semi-hard cross section.

The possibility to describe the elementary parton process by means of Lipatov's dynamics has been considered recently in a series of papers[5]. One of the main points of interest is the search for clear signatures of the underlying parton dynamics in the final state of high energy hadronic collisions. Correlations in transverse momentum and azimuthal angle, as a function of the distance in rapidity y of final state minijets, have been therefore estimated according to the expectations of the Lipatov's picture of the interaction as expressed by Eq.(7) and (8)[6]. On the other hand, to approach Lipatov's limit, one needs to keep the lower threshold of the transverse momenta of the observed minijets as small as possible, compatibly with the requirement of being still in the perturbative regime. As shown by Eq.(10) smaller values of k_m correspond to larger values of $\hat{\sigma}$. As a consequence of the larger probability of the elementary partonic intercourse one is therefore forced to take into account the possibility of having several elementary partonic collisions in each inelastic hadronic event, in order to implement unitarity.

In the present paper, by assuming the validity of the AGK cutting rules in semi-hard interactions, we unitarize the semi-hard cross section and we derive the most general correction term to the inclusive cross section in Eq.(5). The correlations among the minijets observed in the final state are then estimated, considering the simplest possibility of multiple parton interaction, and are compared with the expectation from the single BFKL Pomeron exchange. The paper is organized as follows: in the next section the unitarity correction to the single scattering term is derived. In the following paragraph a few numerical estimates are presented, with the purpose of giving an indication on the kinematical region where corrections can be expected to become sizeable. In the last section the general features of the unitarization of the single scattering term are summarized and a few general conclusions are drawn.

2. General framework for muliparton interactions

In order to approach the problem of multiparton interactions, with the purpose of obtaining for the inclusive cross section an expression which is more general with respect to Eq.(5), we find it appropriate to introduce a functional formalism, which keeps to a minimum level the occurrence of cumbersome expressions. As a preliminary step, we show how to derive all the relevant inclusive cross sections in the simple case of a single parton-parton collision with fixed fractional longitudinal momenta (x, x'). Let us introduce the functional

$$\hat{\Theta}[x, x'; z] \equiv \sum_n \int \frac{d\hat{\sigma}_n(x, x')}{dk_1 \dots dk_n} z(k_1) \dots z(k_n) dk_1 \dots dk_n \tag{11}$$

where z is the argument of the functional and $d\hat{\sigma}_n$ is the differential cross section to produce n partons with momenta $(k_1, \dots, k_n)$. Obviously the value of the functional for $z = 1$ is the semi-hard parton cross section $\hat{\sigma}(x, x')$. Actually

$$\hat{\Theta}[x, x'; 1] = \hat{\sigma}(x, x') \tag{12}$$

All the inclusive cross sections are generated by taking an appropriate number of functional derivatives of the generating functional with respect to z[7]:

$$\frac{d\hat{\sigma}(x, x')^{incl}}{dk_1 \dots dk_n} = \frac{\delta\hat{\Theta}[x, x'; z]}{\delta z(k_1) \dots \delta z(k_n)} \bigg|_{z=1} . \tag{13}$$

To obtain the inclusive cross section in the case of the actual hadronic collision a more elaborate analysis is needed. In the case of soft interactions multi-Reggeon exchanges are conveniently taken into account by making use of the AGK cutting rules[8]. Although no general proof of their validity is available in the case of semi-hard interactions, it has nevertheless been possible to show that the cutting rules hold for one of the components of the interaction which is leading in the large-$\hat{s}$ fixed-$\hat{t}$ limit[9]. If one assumes the validity of the cutting rules for semi-hard interactions, one is allowed to represent the semi-hard cross section σ_H as a probabilistic distribution of multiple semi-hard parton collisions[10]. The most general expression for σ_H requires however the introduction of the whole infinite set of multiparton distributions[11], which keep into account hadron fluctuations in the parton number:

$$\sigma_H = \int d^2\beta \sigma_H(\beta)$$

$$\begin{aligned} \sigma_H(\beta) = \int \sum_n \sum_m \frac{1}{n!} W_A^{(n)}(u_1 \dots u_n) \frac{1}{m!} W_B^{(m)}(u_1' - \beta \dots u_m' - \beta) \\ \times \Big\{ 1 - \prod_{i=1}^{n} \prod_{j=1}^{m} \big[1 - \hat{\sigma}(u_i, u_j') \big] \Big\} \prod du du' \end{aligned} \tag{14}$$

Here the $W^{(k)}(u_1 \dots u_k)$ are the exclusive k-body parton distribution, namely the probabilities to find a hadron in a fluctuation with k partons with coordinates $u_1 \dots u_k$, $u_i \equiv (b_i, x_i)$ standing for the transverse partonic coordinate (b_i) and longitudinal fractional momentum (x_i). β is the impact parameter between the two interacting hadrons and $\hat{\sigma}(u_i, u_j')$, represents the probability for the parton i of the A-hadron to have an hard interaction with the parton j of the B-hadron. The semi-hard cross section is constructed by summing over all possible partonic configurations of the two interacting hadrons (the sums over n and m) and, for each configuration with n A-partons and m B-partons, summing over all possible multiple partonic interactions. This last sum is constructed by asking for the probability of no interaction between the two configurations (actually $\prod_{i=1}^{n}\prod_{j=1}^{m}[1-\hat{\sigma}_{i,j}]$). The difference from one of the probability of no interaction gives the sum over all semi-hard interactions. $\sigma_H(\beta)$ is then the probability to have at least one semihard parton interaction when the impact parameter in the hadronic collision is equal to β. The semi-hard cross section is obtained by integrating the probability $\sigma_H(\beta)$ on the impact parameter. Analogously, the elementary semi-hard cross section $\hat{\sigma}(x,x')$ is obtained by integrating the elementary interaction probability $\hat{\sigma}(u,u')$ on the relative transverse coordinate $\mathbf{b}-\mathbf{b}'$. The expansion of $\sigma_H(\beta)$ as a sum on multiple interactions reads:

$$\begin{aligned}\sigma_H(\beta)=\int\sum_n\sum_m\frac{1}{n!}W_A^{(n)}(u_1\dots u_n)\frac{1}{m!}W_B^{(m)}(u_1'-\beta\dots u_m'-\beta)\\ \times\mathcal{S}\sum_{N=1}^{Q}\binom{Q}{N}\hat{\sigma}_1\dots\hat{\sigma}_N(1-\hat{\sigma}_{N+1})\dots(1-\hat{\sigma}_Q)\end{aligned}\tag{15}$$

$\mathcal{S}$ is a symmetrizing operator, which one may conveniently introduce taking advantage of the symmetry of $W^{(k)}$ for permutations of the arguments[12], and the index N counts the interactions which, for a given configuration with n A-partons and m B-partons, range in number from 1 to $Q=nm$. As a matter of fact, the main advantage of Eq.(15) is the clear separation between real and virtual contributions to the semihard cross section. More precisely, after summing, according with the AGK cutting rules, over all discontinuities of the semi-hard amplitudes, which contribute to the inelastic process of interest, the product $\hat{\sigma}_1\dots\hat{\sigma}_N$ is the remnant of the contribution from the real production terms. The product $(1-\hat{\sigma}_{N+1})\dots(1-\hat{\sigma}_Q)$ is, on the contrary, the remnant of the contribution of the virtual corrections[13]. The replacement $\hat{\sigma}_k \to \hat{\Theta}_k[z]$ in the former product, corresponding to the real production process, allows one to generalize the functional in Eq.(11) and to obtain the inclusive cross sections in the most general case of multiple parton interactions. One may therefore write

$$\begin{aligned}\Theta_H[\beta;z] = \int \sum_n \sum_m \frac{1}{n!} W_A^{(n)}(u_1 \ldots u_n) \frac{1}{m!} W_B^{(m)}(u'_1 \ldots u'_m) \\ \times \mathcal{S} \sum_{N=1}^{Q} \binom{Q}{N} \hat{\Theta}_1[z] \ldots \hat{\Theta}_N[z](1-\hat{\sigma}_{N+1}) \ldots (1-\hat{\sigma}_Q) \prod du du'\end{aligned} \tag{16}$$

which gives the required inclusive cross sections via the relation

$$\frac{d\sigma_H^{incl}}{dk_1 \ldots dk_n} = \int d^2\beta \frac{\delta \Theta_H[\beta;z]}{\delta z(k_1) \ldots \delta z(k_n)} \bigg|_{z=1} . \tag{17}$$

For later convenience $\Theta_H[\beta;z]$ can also be expressed as

$$\begin{aligned}\Theta_H[\beta;z] = \int \sum_n \sum_m \frac{1}{n!} W_A^{(n)}(u_1 \ldots u_n) \frac{1}{m!} W_B^{(m)}(u'_1 - \beta \ldots u'_m - \beta) \\ \times \Big\{ \prod_{i=1}^{n} \prod_{j=1}^{m} [1 + \hat{\Theta}[u_i, u'_j; z] - \hat{\sigma}(u_i, u'_j)] \\ - \prod_{i=1}^{n} \prod_{j=1}^{m} [1 - \hat{\sigma}(u_i, u'_j)] \Big\} \prod du du'\end{aligned} \tag{18}$$

We are now in a position to discuss the processes we are interested in, namely the events in which only two mini-jets are tagged. By setting $n = 2$ in Eq. (17) and using the second expression for $\Theta_H[\beta;z]$, a lengthy but simple algebra yields

$$
\begin{aligned}
\frac{d\sigma_H^{incl}(\beta)}{dk_1 dk_2} =& \int D_A^{(1)}(u) D_B^{(1)}(u'-\beta) \frac{d\hat{\sigma}(u,u')}{dk_1 dk_2} du du' \\
&+ \int D_A^{(2)}(u,v) D_B^{(2)}(u'-\beta, v'-\beta) \frac{d\hat{\sigma}(u,u')}{dk_1} \frac{d\hat{\sigma}(v,v')}{dk_2} du du' dv dv' \\
&+ \int D_A^{(1)}(u) D_B^{(2)}(u'-\beta, v'-\beta) \frac{d\hat{\sigma}(u,u')}{dk_1} \frac{d\hat{\sigma}(u,v')}{dk_2} du du' dv' \\
&+ \int D_A^{(2)}(u,v) D_B^{(1)}(u'-\beta) \frac{d\hat{\sigma}(u,u')}{dk_1} \frac{d\hat{\sigma}(v,u')}{dk_2} du dv du'
\end{aligned}
\tag{19}
$$

where $D^{(1)}(u)$ and $D^{(2)}(u,v)$ are the one-body and two-body inclusive distributions [10]:

$$
\begin{aligned}
D^{(1)}(u) &= W^{(1)}(u) + \int W^{(2)}(u,u')du' + \frac{1}{2}\int W^{(3)}(u,u',u'')du'du'' + \dots \\
D^{(2)}(u_1,u_2) &= W^{(2)}(u_1,u_2) + \int W^{(3)}(u_1,u_2,u')du' \\
&\quad + \frac{1}{2}\int W^{(4)}(u_1,u_2,u',u'')du'du'' \dots
\end{aligned}
\tag{20}
$$

In the r.h.s. of Eq. (19) every term has a clear physical interpretation. The first convolution is nothing but the usual single-collision contribution to the semi-hard cross section. The second term corresponds to two disconnected partonic collisions; finally, the last two entries correspond to those events in which a parton from hadron A or B has suffered a rescattering on hadron B or A respectively.

From ref.[14] we know that the average number of rescatterings can be safely neglected in a typical hadron-hadron collision and for values of k_m which allow the final state parton to be observed as an actual minijet in the final state. We are therefore allowed to neglect the last two terms in the r.h.s. of Eq. (19). The two-body inclusive distribution $D^{(2)}$ may be expressed by introducing the two body parton correlation $C^{(2)}$:

$$D^{(2)}(u_1,u_2) \equiv D^{(1)}(u_1)D^{(1)}(u_2) + \frac{1}{2}C^{(2)}(u_1,u_2) \tag{21}$$

If one neglects both rescatterings and correlations in Eq.(19), one is left with the following simplified expression for the inclusive cross section:

$$\begin{aligned}\frac{d\sigma_H^{incl}}{dk_1 dk_2} = \int d^2\beta \Big[& D_A^{(1)} \otimes \frac{d\hat{\sigma}}{dk_1 dk_2} \otimes D_B^{(1)} \\ & + \big(D_A^{(1)} \otimes \frac{d\hat{\sigma}}{dk_1} \otimes D_B^{(1)}\big)\big(D_A^{(1)} \otimes \frac{d\hat{\sigma}}{dk_2} \otimes D_B^{(1)}\big)\Big]\end{aligned} \tag{22}$$

where $\otimes$ is a compact notation for the convolutions appearing in Eq. (19). A possible further simplification follows from the assumption that $D^{(1)}(u)$ has the factorized form

$$D^{(1)}(x,b) = f_{eff}(x)F(\mathbf{b}) \tag{23}$$

with the obvious normalizing condition

$$\int d^2bF(\mathbf{b}) = 1 \tag{24}$$

By substituting Eq. (23) in Eq. (22) one obtains

$$\frac{d\sigma_H^{incl}}{dk_1 dk_2} = \frac{d\sigma_s}{dk_1 dk_2} + \frac{1}{\sigma_{eff}}\frac{d\sigma_s}{dk_1}\frac{d\sigma_s}{dk_2} \tag{25}$$

where

$$\frac{1}{\sigma_{eff}} \equiv \int d^2\beta \Big[\int d^2bF(\mathbf{b})F(\mathbf{b}-\beta)\Big]^2 \tag{26}$$

and $d\sigma_s$ is the single collision expression, obtained by convoluting the elementary cross section with the usual one-body parton distribution $f_{eff}(x)$.

3. Numerical estimates

The formalism described in the previous section is a rather general approach to the problem of unitarity corrections in semi-hard interactions. Indeed the expression for the inclusive cross section in Eq.(19) is completely general in the probabilistic picture of the semi-hard hadronic interaction. It is an exact consequence of the cross section as expressed in Eq.(14), which finds its justification in the AGK cutting rules[8]. In the inclusive cross section given by Eq.(19) all possible multiple parton collisions are kept into account and multiparton correlations are treated at all orders. Consistently with the general principles, namely with the AGK cancellation[8,15], the double inclusive cross section depends only on the single and double scattering terms. For a quantitative estimate of the role of unitarity corrections to the single scattering term, the required non perturbative input is represented both by the one-body parton distribution $D^{(1)}$ and by the two-body parton distribution $D^{(2)}$. The two-body parton distribution contains an independent information on the hadron structure with respect to $D^{(1)}$, actually the two-body parton correlation $C^{(2)}$. While no experimental information is presently available on $C^{(2)}$ an indication is available from CDF on the scale factor σ_{eff} which characterizes the double parton interactions[16]. We will therefore limit our numerical analysis to the simplified case where $C^{(2)}$ is neglected and only disconnected parton collisions are taken into account, in such a way that the inclusive cross section is expressed by Eq.(25). All unitarity corrections to single scattering are therefore expressed by the second term in Eq.(25), which is obtained with the same input needed to evaluate the single scattering term, apart from the scale factor σ_{eff}, that summarizes all the geometrical details which enter in the unitarity correction.

A few qualitative considerations are appropriate before illustrating the results of a quantitative analysis. By introducing the jet rapidities (y_a, y_b) and integrating in the transverse momenta down to the lower cut off k_m, while keeping fixed the azimuthal angle between the observed minijets ϕ, the inclusive cross section is expressed as:

$$\frac{d\sigma_H^{incl}}{d\phi dy_a dy_b} = \frac{d\sigma_s}{d\phi dy_a dy_b} + \frac{1}{\sigma_{eff}}\frac{1}{2\pi}\frac{d\sigma_s}{dy_a}\frac{d\sigma_s}{dy_b} \tag{27}$$

In the limit of small relative rapidities $y = y_a - y_b$, a parton-parton interaction produces only two final state partons. Since they are back-to -back in ϕ, the single collision expression $d\sigma_s/d\phi dy_a dy_b$ is proportional to a Dirac delta $\delta(\phi - \pi)$. This can be easily verified by setting $y = 0$ in Eq.(7). On the opposite side, that is, for large values of y, the leading contribution to the r.h.s. of Eq.(7) comes from the $n = 0$ term, for which the partons are decorrelated in ϕ. Physically, this is due to the large number of gluons radiated in the parton-parton interaction. Indeed, the flattening of the ϕ distribution with increasing dijet rapidity gap was suggested[6] as a signature for the BFKL dynamics. From this point of view, a multiple partonic collision represents a background process which mimics the effect of multigluon emission. In the r.h.s. of Eq. (27), this background is described by the term weighted by the scale factor $1/\sigma_{eff}$. The experimental indication on the scale factor is $5.4 < \sigma_{eff} < 29mb$ (90% C.L.)[16]. Unfortunately σ_{eff} is not the only input variable which is still rather uncertain for a numerical computation. Indeed there is a large ambiguity already to compute the single scattering term. In fact to obtain the Lipatov's solution one needs to neglect the running of the strong coupling constant, in such a way that α_s has to be considered as a parameter in the actual evaluation of $\hat{\sigma}$. Since the dependence of $\hat{\sigma}$ on α_s, as it may be seen in Eq.(10), is exponential a numerical comparison of the two terms in Eq.(27) is rather uncertain.

To have a quantitiative feeling of the importance of the unitarity correction we have tried to estabilish a possible sensible choice of the input values of σ_{eff} and α_s by making a comparison with available experimental data. The experimental points in fig.1 are the values of the cross section for production of minijets with $k_m \geq 5GeV$ measured by UA1[2]. The dashed curves refer to the single scattering integrated cross section with $\alpha_s = .34$ (upper curve) and $\alpha_s = .29$ (lower curve), corresponding to the values of the running coupling constant at the scale $\sqrt{Q^2} = k_m/3$ and $\sqrt{Q^2} = k_m/2$ respectively. The structure functions are the HMRS(B) structure functions[17]. The unitarized expression for the semihard cross section has a simple analytical representation when semi-hard rescatterings and multi-parton correlations are neglected[10]. Actually:

$$\sigma_H = \int d^2\beta \Big(1 - exp\Big[\sigma_s \int F_A(\mathbf{b})F_B(\mathbf{b}-\beta)d^2b\Big]\Big) \tag{28}$$

where σ_s is the integrated single scattering inclusive cross section. The continuous curves in fig.1 refer to the unitarized cross section σ_H, as expressed in Eq.(28). For $F(b)$ we have taken a gaussian, the width corresponding to a value of $\sigma_{eff} = 20mb$. The two curves refer to the two different choices of α_s mentioned above. The region identified by the two continuous lines contains the experimental points and therefore gives an indication on possible meaningful input parameters. One may also observe in fig.1 how the rise of the experimental cross section is much closer to the rise of the unitarized curves than to the rise of the single scattering term alone.

Before moving to different values of energy it is worthy to briefly comment on k_m, which, to some extent, is a free parameter. A low value of k_m corresponds to semi-hard cross sections that are well above σ_{eff} (in the single collision approximation). In this conditions the contribution from multiple scatterings is largely dominant and the ϕ distribution is practically flat. On the contrary, large values of k_m correspond to semi-hard cross sections that are negligible with respect to σ_{eff} and no unitarity correction is required. Keeping this in mind, we realize that the interesting values of k_m are those for which the total semi-hard cross section is comparable to σ_{eff}. This criterion yields $k_m \simeq 5.2 \div 6.1 GeV$ at $\sqrt{s} = 1.8TeV$ and $k_m \simeq 11.2 \div 12.7GeV$ at $\sqrt{s} = 18TeV$, depending on the two different choices of values for α_s which we have considered and for $\sigma_{eff} = 20mb$.

In order to have some quantitative indication on the effect that unitarization produces on the expectations based on the BFKL dynamics, we have studied the azimuthal correlation of the observed minijets, which, according with the BFKL dynamics has a distinctive dependence on the distance in rapidity. In fig. (2-a,b) we have plotted the differential cross section Eq.(27) as a function of ϕ, for fixed rapidities (y_a, y_b) at $\sqrt{s} = 1.8TeV$ (a) and $\sqrt{s} = 18TeV$ (b) (the normalization is such that the curves take a value equal to unity at $\phi = 0$ and $\phi = 2\pi$). The naive ϕ distribution, obtained by considering one elementary interaction only, is represented by the dashed line, while the continuous line describes the corrected distribution which takes into account an arbitrary number of parton-parton collisions. The flattening caused by the unitarity corrections is clearly visible: at $\sqrt{s} = 1.8TeV$, fig. (2-a), the height of the central peak at $\phi = \pi$ is reduced by a factor three approximately; the same trend, but with a stronger suppression of the correlation, is present at higher energies, see fig. (2-b). Figure (3) shows how the effect of unitarity corrections depends on the cutoff k_m. By lowering this threshold we increase the semi-hard cross section and, accordingly, we enhance the probability of having several elementary parton collisions in each inelastic hadronic event. As a consequence, we expect the tagged minijets to become less and less correlated in the azimuthal angle ϕ. This is confirmed by our plot which corresponds to $\sqrt{s} = 1.8TeV$ and to a rapidity gap $y = 5$, actually $y_a = 2.5$ and $y_b = -2.5$. The different choices of the cut off k_m are $k_m = 7GeV$ (solid line), $k_m = 6GeV$ (dashed line) and $K_m = 5GeV$ (dotted line). It is worthwhile to stress that, for the lower choice $k_m = 5GeV$, we cannot distinguish the ϕ distribution from a uniform one, unless we perform a quite accurate measure at the 3% level. Finally, fig. (4) shows how the correlation in the azimuthal angle of the tagging jets fades away as the rapidity interval is increased.

4. Conclusions

Minijet physics is the ideal tool to study BFKL dynamics. Indeed one comes closer and closer to the BFKL limiting case by keeping the lower threshold in transverse momentum k_m of the observed minijets as small as possible. However the region of small k_m is also the region where unitarity corrections become increasingly important. In the present paper we have made an attempt to estimate the unitarity corrections to the inclusive cross section for producing two minijets. After assuming the validity of the AGK cutting rules in semihard interactions, we have kept into account unitarity corrections by representing the hadronic process as a probabilistic superposition of multiple BFKL Pomeron exchanges. In the case of the inclusive cross section for producing two minijets, only the single and the double scattering terms contribute. With the purpose of making a quantitative estimate, we have considered the simplest possibility for the double scattering contribution. Actually we have neglected semi-hard parton rescatterings in the interaction, and two-body parton correlations in the two-body inclusive distributions. In this simplified case the unitarity correction depends on one single parameter only, namely σ_{eff}, that is the scale factor one needs to introduce in order to obtain the probability of the double interaction. For a quantitative illustration of the effect of the correction term, a second parameter which has to be fixed is the strong coupling constant α_s, whose value is not determined by the BFKL dynamics. Keeping into account the experimental suggestion on σ_{eff}[16], we have fixed the input parameters by comparing with the UA1 measurement of the semihard cross section for production of minijets[2]. Having selected in this way a possible range of values for the parameters, the indication we obtain from our numerical estimate is that at Tevatron energy the correction term to the single BFKL Pomeron exchange, depending on the actual quantity one is considering, may be larger than 100% for minijets with $k_m \simeq 6GeV$. When moving at LHC energies the same correction applies with values of $k_m \simeq 12GeV$. It is worthwhile noticing that, at Tevatron energy and with $k_m \simeq 6GeV$ the average invariant mass of a partonic interaction is $\simeq .2TeV$, while at LHC energies and with $k_m \simeq 12GeV$, the average invariant mass is $\simeq 1TeV$. The expectation is therefore that a secondary BFKL Pomeron is exchanged in a large fraction of parton interactions at those values of invariant mass and at the corresponding hadron-hadron c.m. energy. A detailed experimental analysis of minijet production at Tevatron would therefore be of great importance both as a test of the BFKL approach, and to access the non perturbative information on the hadron structure which enters in the multiple parton interactions, whose detailed knowledge is of growing importance to understand hadron dynamics at higher energies.

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Figure captions

Fig. 1: Cross section for production of minijets with $k_m \geq 5GeV$. Experimental data from UA1[2]. Dashed curves: single BFKL Pomeron exchange with $\alpha_s = .34$ (upper curve) and $\alpha_s = .29$ (lower curve). Continuous curves: unitarized cross section, Eq.(28) in the text, same values of α_s as in the previous case and $\sigma_{eff} = 20mb$.

Fig. 2: ϕ distribution with unitarity corrections included (solid line) and in the single collision approximation (dashed line). $N(\phi)$ is proportional to the differential cross section given by eq. (27), with minijet rapidities kept fixed at $y_a = 2.5$ and $y_b = -2.5$. The normalization is such that $N(0) = N(2\pi) = 1$.

Fig. 3: ϕ distribution for several choices of the cutoff: $k_{min} = 7GeV$ (solid line), $k_{min} = 6GeV$ (dashed line), $k_{min} = 5GeV$ (dotted line). $N(\phi)$ is defined as in Fig. (2) and unitarity corrections are included.

Fig.4: ϕ distribution for different choices of the rapidity gap. The cutoff is $k_{min} = 7GeV$ and the minijet rapidities are fixed at $y_{a,b} = \pm 2.5$ (solid line) and $y_{a,b} = \pm 3.5$ (dashed line).