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Information Loss in Quantum Gravity Without Black Holes

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Abstract

We use the weak field approximation to show that information is lost in principle in quantum gravity.

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1.Introduction

In fields theories which do not involve gravitation, the information that a macroscopic system has is the *log* of the number of orthogonal states with the same macroscopic properties (E, V, N, \dots). There is, however, a hidden assumption in this definition. The hidden assumption is that one can distinguish at least in principle between all the microscopic orthogonal states which define the same macroscopic properties. Fortunately, in field theories which do not involve gravitation the assumption is correct.

In this paper we examine this assumption in the context of quantum gravity. We suggest that the assumption is incorrect and that there is information loss in quantum gravity. This result, in itself, is not new; Hawking has shown in [1] that information is lost in the context of black holes. Since then there have been many suggestions how to recover the information lost in a black hole, most of them based on the argument that one of the assumptions that Hawking made on quantum gravity is incorrect. However, it seems fair to say that the description of quantum black holes is still a puzzle. Further more, the meaning of the black hole entropy, $\frac{A}{4}$, is not clear and the connection between black hole entropy and entropy in fields theories which do not involve gravitation is not well understood [3]. Our new point is that we do not consider black holes, but use only general properties of quantum gravity and the weak field approximation; in particular there is no horizon in our system. This is quite surprising since it is usually claimed that the

basic reason for information loss in black holes is the horizon. The general properties of quantum gravity which we assume are the following:

1-At large distances, general relativity equation is a good approximation to quantum gravity. We denote the minimal scale for which general relativity equation is a good approximation to quantum gravity by x_c . In other words, at distances much larger than x_c the first order of the gravitational effects can be described by general relativity.

2-At large distances, quantum gravity can be described by means of local quantum field theory. In other words, if one explores the theory only at large scales one finds only local effects. Let us denote the minimal scale for which the theory is local by x_d . In particular, any measurement at scales larger than x_d is carried out by a local interaction between the measurement apparatus and the measured system.

Notice that these two assumptions are the easiest way to describe the correspondence principle, which we know to exist, between quantum gravity and general relativity and between quantum gravity and local quantum field theory. In fact this correspondence principle is the only experimental data that we have on quantum gravity. Still, those are only assumptions on quantum gravity and not general properties of quantum gravity, since there are more complicated ways to describe the correspondence principle. For example, one can speculate that x_d is not just a constant of nature, but a function of the state of the system. Notice further that those assumptions are usually

made in any attempt to construct a quantum theory of gravitation, where usually $x_c \approx x_d \approx L_p$ (where L_p is Planck length); in particular, Hawking made those assumptions in his description of black hole evaporation [2]. In our discussion we do not assume that x_c and/or x_d are of the order of L_p but we only assume that they are finite. The outline of the paper is as follows: In sec.2 we present the system to be discussed and we show that the information assumption is correct in the absence of gravitation. In sec.3 we show that in the presence of gravitation, one can not distinguish between all the orthogonal states of the system, therefore, the information assumption is incorrect.

2.The system

Let us describe the system that we want to investigate. There are n^3 objects in the system, each object is constructed from m^3 cells whose size is a ,

$$a \gg \max[x_c, x_d]. \quad (1)$$

The cells are attached to each other so the size of each object is ma , (Figure 1). There are also n^3 huge cells whose size is b ($b \gg ma$); the huge cells are also attached to each other, so the size of the system is nb , (Figure 2). At each little cell there is exactly one particle with spin J at the ground state so there are $n^3 m^3$ particles. The center of each object is located at one of the huge cells. At each huge cell there is one object.

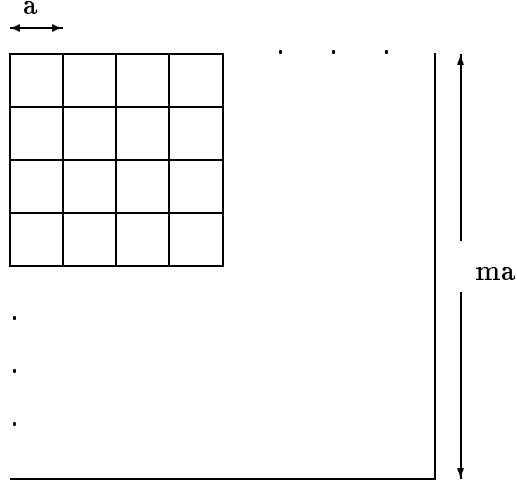


Figure 1: The object

The basis of orthogonal states for this system is:

$$| J_{z_1}, J_{z_2}, \dots, J_{z_n}, \vec{r}_1, \vec{r}_2, \dots, \vec{r}_n \rangle \quad (2)$$

where J_{z_i} is the spin of the i particle in the z direction so there are $2J + 1$ possibilities for each particle. \vec{r}_i is the location of the center of the i 'th object at the huge cell. If we move one object a distance equal or bigger then a we obtain a new orthogonal state. Thus, there are k^3 possibilities for each huge cell, where $k = \frac{b}{a}$. Hence, $[(2J + 1)^{m^3} k^3]^{n^3}$ local orthogonal states are required to describe the system. Let us examine the validity of the information assumption in the absence of gravitation. Is there a physical apparatus which can distinguish, at least in principle, between all the orthogonal states? The answer to this question is obviously yes (even in the presence of gravitation)

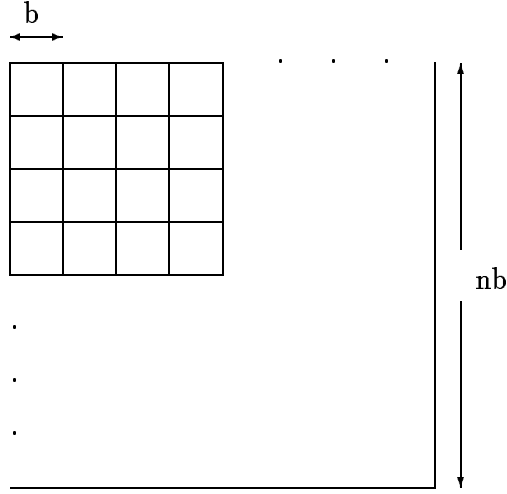


Figure 2: The system

if we use the Von-Neumann measurement interaction

$$H_{int} = f(t)q \sum \alpha_i | \phi_i \rangle \langle \phi_i |, \quad (3)$$

where the sum is taken over all the orthogonal states and q is the canonical position of the apparatus which measures the state of the system. $\alpha_i \neq \alpha_j$ for $i \neq j$ so one can find the state of the system according to the difference in the momentum of the apparatus. The crucial point is that we can not use the general Von-Neumann interaction but we must use a local H_{int} . The correct question is therefore: Is there a physical apparatus which can distinguish, at least in principle, between all the orthogonal states by means of a local interaction? If one wishes to measure J_{z_i} then one should put a detector which measures the spin of the particle at the i 'th cell. In order

to distinguish between all the orthogonal states, one must also measure the distances between the objects or between an object and the edge of the huge cell. In any case the measurement can be carried in the following way: A clock at one object measures the time t_i when a photon is sent towards the other object. At the other object there is a mirror which reflects the photon back to the first object. From the reading of the clock when the photon arrives, t_f , one obtains that the distance between the objects is

$$r = \frac{(t_i - t_f)}{2} \quad (4)$$

(in units where $c = 1$). Notice that in the absence of gravitation the energy momentum tensor of the detectors does not affect the photon thus there is no difficulty to get

$$\Delta r < a, \quad \forall a \quad (5)$$

and therefore, to distinguish between all the orthogonal states . The information assumption is therefore correct in the absence of gravitation.

3.The information loss

In this section we study the hidden assumption in the definition of information in the presence of gravitation. The special property of gravitation which suggests that something dramatic might occur in quantum gravity is the fact that in general relativity, unlike in any other theory, distances are defined by the fields of the theory ($g_{\mu\nu}$). Gravitation is, therefore, a geometric force, or in the spirit of [4] where gravitation is described as a regular

force, it has a universal interaction with all fields (with the energy momentum tensor as the source of interaction). Thus, the gravitational influence on the particle which is sent in order to measure the distance between the objects is universal. If for instance, one wishes to measure distances in the presence of a strong fluctuation in the electromagnetic field then one should use a chargeless particle which does not interact with the electromagnetic field, so the information assumption is correct in Q.E.D. However, all particles interact with gravitation. This implies that in quantum gravity the uncertainty in the distance between the object is larger than the standard uncertainty given by the Heisenberg relations [5].

Consider the same physical apparatus which is supposed to distinguish between all the orthogonal states of the system. Recall that by the definition of the system $a \gg \max[x_c, x_d]$, thus according to assumption 2 we get (in units where $\hbar = 1$)

$$\Delta p_i \geq \frac{1}{a} \quad (6)$$

where Δp_i is the uncertainty of the momentum of the detector which measures the spin of the i 'th particle. In the weak field approximation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} , \quad |h_{\mu\nu}| \ll 1 \quad (7)$$

and

$$P_\mu = \int d^3x T_{0\mu} \quad (8)$$

where $T_{\mu\nu}$ is the energy momentum tensor. Thus

$$\Delta \int_{cell} d^3x T_{0\mu} \geq \frac{1}{a} \quad (9)$$

Since $a \gg x_c$ we can also use assumption 1 and the well known solution of the field equations in the weak field approximation, to find that (in units where $G = 1$)

$$g_{\mu\nu}(x, t) = \eta_{\mu\nu} + 4 \int d^3x' \frac{S_{\mu\nu}(x', t - |x - x'|)}{|x - x'|} \quad (10)$$

where

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^\gamma_\gamma. \quad (11)$$

There must be no correlation between different detectors so the uncertainty in the momentum of one object is:

$$\Delta P \geq \frac{\sqrt{m^3}}{a}. \quad (12)$$

Using eq.(10) we get,

$$\Delta^2 g_{oi}(x, t) \geq 16 \sum_{objects} \frac{m^3}{a^2(x - x_{object})^2}. \quad (13)$$

The average distance between nearest objects is b and thus:

$$\Delta g_{oi} \approx \frac{\sqrt{m^3 n}}{ab} \quad (14)$$

Now, when one tries to measure the distance between nearest objects one finds that the velocity of the photon is

$$v = \frac{1}{2}(-g_{oi} + \sqrt{4 + g_{oi}^2}) \quad (15)$$

Since $g_{oi} \ll 1$ we get

$$\Delta v \approx \frac{1}{2} \Delta g_{oi} \gtrsim \frac{\sqrt{m^3 n}}{2ab} \quad (16)$$

Now, if we write explicitly the speed of light in eq.(4) we obtain

$$\Delta r = \Delta v \frac{t_f - t_i}{2v^2} = r \frac{\Delta v}{v} \quad (17)$$

Note that the first equality is due to the fact that the average distance between nearest objects is b so the correlation distance of Δg_{oi} is also b . The uncertainty of the distance between nearest objects $r \approx b$ is therefore

$$\Delta r \gtrsim \frac{\sqrt{nm^3}}{2a}. \quad (18)$$

One can no longer claim that Δr is as small as one wishes. If $\Delta r > a$ then one can not distinguish between all the $[(2J + 1)^{m^3} k^3]^{n^3}$ orthogonal states by means of local measurements. In fact one can only distinguish between $[(2J + 1)^{m^3} (\frac{k}{l})^3]^{n^3}$ orthogonal states, where $l \gtrsim \frac{\sqrt{nm^3}}{2a^2}$. An unavoidable conclusion is that the hidden assumption in the definition of information and the general properties of quantum gravity which we assume can not coexist. The loss of information is due to the fact that the physical information that one can obtain about this system (by physical information we mean the *log* of the maximal number of orthogonal distinguishable states) is smaller than the mathematical information of the system (the *log* of the number of orthogonal states). Denoting by ΔS the information loss, we find:

$$\Delta S \equiv S_{math} - S_{phy} \gtrsim n^3 \log[(2J + 1)^{m^3} k^3] - n^3 \log[(2J + 1)^{m^3} (\frac{k}{l})^3] = 3n^3 \log l \quad (19)$$

Note that ΔS does not depend on J as Bekenstein-Hawking entropy is independent of the number of fields.

Another way to illustrate the information loss is to notice that one can change a . One can start with $a \gtrsim (nm^3)^{\frac{1}{4}}$ so one can distinguish between all the $[(2J+1)^{m^3} k^3]^{n^3}$ orthogonal states; if the system is changed adiabatically until $a \lesssim (nm^3)^{\frac{1}{4}}$ then one can only distinguish between $[(2J+1)^{m^3} (\frac{k}{7})^3]^{n^3}$ orthogonal states. If a is changed back to a region where $a \gtrsim (nm^3)^{\frac{1}{4}}$ then there are subsets of states which are distinguishable at the beginning and at the end but at the middle (when $a \lesssim (nm^3)^{\frac{1}{4}}$) they are indistinguishable. This suggests that there is no complete correlation between initial and final distinguishable states so there is no S matrix and unitarity is violated. In general, a similar process occurs in black hole evaporation. At the beginning when there is a collapsing star one can distinguish, in principle, between almost all the possible initial states of the star which lead to a black hole. At the end there is a thermal radiation and again one can distinguish between all the possible final states. However in the middle, when there is a black hole one can only distinguish between subsets which are characterized by the total mass, charge and angular momentum.

A few remarks are in order: First there is no local information loss in this system- an observer who is located at one of the objects will have no difficulty to obtain all the information about the object. There is only global information loss in this system. The part which is lost is the information

about the distances between the objects. Second, we must make sure that there are no black holes in the system. In order to do so we should compare the size of the object to its energy and the size of any subsystem to its energy. Since the minimal energy of a particle/detector with uncertainty a is $\frac{1}{a}$ we find that the total energy of the object is $E_{ob} \geq \frac{m^3}{a}$ and that the total energy of the system is $E_{sys} \geq \frac{n^3 m^3}{a}$. Recall that the size of the object is ma and that the size of the system is nb so in order to avoid black holes we must impose

$$a > m, \quad ba > m^3 n^2, \quad (20)$$

Note that b does not appear in ΔS so there is no difficulty to fulfill eq.(20) and avoid black holes. Third, the loss of information concerning the location of the huge cell Δr is small comparing to its possible location b . This can be seen easily from the following arguments:

$$\frac{\Delta r}{b} = \frac{\sqrt{nm^3}}{ab} < b^{-\frac{3}{4}}, \quad (21)$$

where we use eq.(20). Since $b \gg 1$ we obtain

$$\frac{\Delta r}{b} \ll 1. \quad (22)$$

It seems ,therefore that in order to obtain $\Delta r \approx b$ we must include black holes. Our goal in this paper is only to present the conceptual problem in quantum gravity and for this purpose $\Delta r > a$ is enough.

4.Conclusions

For the last few hundred years physics has been described by mathematics. This description is meaningless unless there is a dictionary which connects between the mathematical description of physics and physics. Such a dictionary is absent in quantum gravity if the two assumptions that we made in the introduction are correct, because although the system which we study appears to have (according to our assumption) a well defined mathematical description (eq.(2)) the physical information contained in the system is smaller than the mathematical information (eq.(19)). In our opinion, assumption (2) is incorrect and quantum gravity is a non-local theory at any scale though there are states with local interpretation. In other words, d is not a constant of nature, but rather a function of the state of the system. There are states for which d is smaller than any of the scales in the system (in general this is the state of the universe now), but as we see there are states for which d is bigger than some of the scales in the system although all the scales in the system are much larger than Planck scale.

A natural question which arises now is whether or not this problem is relevant to the description of gravitational effects in particle physics. On the one hand the systems which we consider are so large and complex that one probably should not take them into account in particle physics as virtual states, so the non-locality of those states at large distances should not affect the large distance locality of particle physics. On the other hand the system which we consider is so large and complex only because we want to stay on

solid ground by using the weak field approximation. When one considers states for which the weak field approximation is not valid one might obtain information loss in smaller and simpler systems. In particular, the understanding of this conceptual problem might be crucial for the description of quantum black holes. Furthermore, if indeed assumption two is incorrect then the whole description of quantum gravity is different than what we are used to in local ² quantum field theory. Hence the questions one can ask in quantum gravity might be different from the questions one asks in local fields theory, much as the questions in quantum mechanics are not similar to the questions in classical mechanics.

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