PARTICLE CREATION AMPLIFICATION IN CURVED SPACE DUE TO THERMAL EFFECTS

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A physical system composed by a scalar field minimally coupled to gravity and a thermal reservoir, as in thermo field dynamics, all of then in curved space time, is considered. When the formalism of thermo field dynamics is generalized to the above mentioned case, an amplification in the number of created particles is predicted.

1. **Introduction**

As is shown in ref.[1] Thermo Field Dynamics (TFD) gives us, for thermal equilibrium at a given temperature, an elegant method to obtain mean values of physical magnitudes resulting at this level equivalent to the Schwinger- Keldysh closed time path formalism (SKF). However as is shown in ref.[2] TFD has advantages on the SKF formalism when the generalization to nonequilibrium situations is performed, giving different results.

The main idea in TFD is the duplication of the degrees of freedom, introducing the tilde operators $\tilde{a}_{\mathbf{k}}$ and $\tilde{a}^{\dagger}_{\mathbf{k}}$ which operate on the quantum states of the reservoir [3]. In our generalization the system and the reservoir are in a curved background. The operators $a_{\mathbf{k}}$, ${a^{\dagger}}_{\mathbf{k}}$, $\tilde{a}_{\mathbf{k}}$ and $\tilde{a}^{\dagger}_{\mathbf{k}}$ are all related to the thermal operators $a_{\mathbf{k}}(\beta)$, ${a^{\dagger}}_{\mathbf{k}}(\beta)$, $\tilde{a}_{\mathbf{k}}(\beta)$ and $\tilde{a}^{\dagger}_{\mathbf{k}}(\beta)$ by means of a Bogoliubov transformation. Moreover due to the curvature of the space-time we have a set of vacua related to the folliation. The operators related with those vacua are indicated by $a_{\mathbf{k}}(t)$, ${a^{\dagger}}_{\mathbf{k}}(t)$, $\tilde{a}_{\mathbf{k}}(t)$ and $\tilde{a}^{\dagger}_{\mathbf{k}}(t)$. The parameter "$t$" labels the folliation. We have also a Bogoliubov transformation that relate those operators with the primitive ones (see ref.[4]). The particle creation will be due to two effects, one of them the interaction with the geometry and the other one related to the interaction with the thermal bath. The first one is a dynamical effect and the second one is a spontaneous phenomenon.

2. Thermal creation of particles

Similarly as in ref.[5], we can relate operators at zero temperature with the ones at temperature $T = T(\beta)$ by means of the following time dependent transformation matrix:

$$\begin{pmatrix} a(\beta) \\ \tilde{a}^{\dagger\dagger}(\beta) \end{pmatrix} = (1 + n_{0\beta})^{1/2} \begin{pmatrix} 1 & -F \\ -fF^{-1} & 1 \end{pmatrix} \begin{pmatrix} a \\ \tilde{a}^{\dagger} \end{pmatrix} \tag{2.1}$$

where $a_{\mathbf{k}}$ ($a^{\dagger}{}_{\mathbf{k}}$) are quasiparticle annihilation (creation) operators for bosonic fields (in equation (2.1) we omitted the subindex $\mathbf{k}$ asociated with the mode of the quasiparticle), defined by

$$a_{\mathbf{k}}|0>= 0.$$

$$a^{\dagger}{}_{\mathbf{k}}|0>= |1_{\mathbf{k}}>, etc$$

$$[a_{\mathbf{k}}, a^{\dagger}{}_{\mathbf{k'}}] = \delta_{\mathbf{k},\mathbf{k'}}$$

The tilde operators $\tilde{a}_{\mathbf{k}}$, $\tilde{a}^{\dagger}_{\mathbf{k}}$ represent the quantum effect of the reservoir. We have also

$$\tilde{a}_{\mathbf{k}}|\tilde{0}>= 0.$$

$$\tilde{a}^{\dagger}_{\mathbf{k}}|\tilde{0}>= |\tilde{1}_{\mathbf{k}}>, \ etc$$

$$[\tilde{a}_{\mathbf{k}}, \tilde{a}^{\dagger}_{\mathbf{k'}}] = \delta_{\mathbf{k},\mathbf{k'}}$$

We will use $\{|n, \tilde{n}>\} = \{|n>\} \otimes \{|\tilde{n}>\}$ as space of states. In the following we will call $|\mathbf{0}> := |0, \tilde{0}>$.

The operators $a(\beta(t))$, $a^{\dagger}(\beta(t))$, $\tilde{a}(\beta(t))$ and $\tilde{a}(\beta(t))$ operate on the thermal vacuum:

$$a_{\mathbf{k}}(\beta)|\mathbf{0}, \beta>= 0.$$

$$a^{\dagger\dagger}{}_{\mathbf{k}}(\beta)|\mathbf{0}, \beta>= |1_{\mathbf{k}}, \beta>, \ etc$$

and satisfy

$$[a_{\mathbf{k}}(\beta), a^{\dagger\dagger}{}_{\mathbf{k'}}(\beta)] = \delta_{\mathbf{k}\mathbf{k'}}$$

and similarly the operators $\tilde{a}_{\mathbf{k}}(\beta)$ and $\tilde{a}^{\dagger\dagger}_{\mathbf{k}}(\beta)$. The symbol $\dagger\dagger$ is used because $(a^{\dagger\dagger})^{\dagger} \neq a$ i.e. in general the transformation is not unitary (only when $F = f^{1/2}$).

Now we will define the function $n_{0\beta}(t)$ as the mean value of created particles due to the interaction with the reservoir, in the form

$$n_{0\beta} =< \mathbf{0}|a^{\dagger\dagger}(\beta)a(\beta)|\mathbf{0} > \tag{2.2}$$

As we can see using eq. (2.1)

$$n_{0\beta} = \frac{f}{1-f}$$

f is in principle an arbitrary time dependent function, in particular when $f = \exp -\beta\epsilon$ we have planckian spectrum.

The time dependent function F is also arbitrary. One choise used in the literature [6] is known as thermal state condition, which corresponds to $F = f^{\alpha}$ with $0 < \alpha < 1$. In particular when $\alpha = 1/2$, $(a^{\dagger\dagger})^{\dagger} = a$ (then $\dagger\dagger = \dagger$) and the transformation is unitary.

It is easier to prove also, using the inverse transformation:

$$< \mathbf{0}|a^{\dagger\dagger}(\beta)a(\beta)|\mathbf{0} >=< \mathbf{0}, \beta|a^{\dagger\dagger}a|\mathbf{0}, \beta >$$

3. **Particle creation in curved space**

Following Parker (ref. [4]) we can relate the annihilation-creation operators at different times, by means of the following Bogoliubov transformation:

$$a_{\mathbf{k}}(t) = e^{i\gamma_{\alpha}(\mathbf{k},t)}cosh\theta(\mathbf{k},t)a_{\mathbf{k}} + e^{i\gamma_{\beta}(\mathbf{k},t)}sinh\theta(\mathbf{k},t)a^{\dagger}{}_{-\mathbf{k}}$$

$$a^{\dagger}{}_{\mathbf{k}}(t) = e^{-i\gamma_{\beta}(\mathbf{k},t)}sinh\theta(\mathbf{k},t)a_{\mathbf{k}} + e^{-i\gamma_{\alpha}(\mathbf{k},t)}cosh\theta(\mathbf{k},t)a^{\dagger}{}_{-\mathbf{k}} \tag{3.1}$$

Where γ_{α}, γ_{β} and θ are determined by the particle model used and by the field equation. When an isotropic Robertson-Walker metric is considered the functions introduced in eq. (3.1) satisfy the eqs. (see eqs (27) and (39) of ref.[4]):

$$\dot{\gamma}_{\beta}\tanh\theta\cos\gamma + \dot{\theta}\sin\gamma - \dot{\gamma}_{\alpha}\cos\mu + \dot{\theta}\tanh\theta\sin\mu = 0$$

$$-\dot{\gamma}_{\beta}\tanh\theta\sin\gamma + \dot{\theta}\cos\gamma - \dot{\gamma}_{\alpha}\sin\mu + \dot{\theta}\tanh\theta\cos\mu = 0$$

with $\mu := 2\int_{t_0}^{t} W(\mathbf{k}, t')dt'$ and $\gamma := \gamma_\alpha + \gamma_\beta$. Therefore in eqs (3.1) we have six unknowed functions: $\dot{\gamma}_\beta$, $\dot{\gamma}_\alpha$, γ, θ, $\dot{\theta}$ and μ. Two initial conditions are added to the two equations (3.1). In reference [4] those conditions are the following

$$\theta(\mathbf{k}, 0) = 0$$

$$\gamma_\alpha(\mathbf{k}, 0) = 0 \tag{3.2}$$

which give us as initial conditions a particular functional form (similar to WKB) for the field modes. To completely solve the problem two additional conditions must be introduced which determine definitively, at all instant, the particle model.

The annihilation-creation operators of eq. (3.1) satisfy the bosonic commutation relations, i.e. :

$$[a_{\mathbf{k}}(t), a_{\mathbf{k}'}(t)] = 0, [{a^\dagger}_{\mathbf{k}}(t), {a^\dagger}_{\mathbf{k}'}(t)] = 0,$$

and

$$[a_{\mathbf{k}}(t), {a^\dagger}_{\mathbf{k}'}(t)] = \delta_{\mathbf{k},\mathbf{k}'} \quad \forall \quad t$$

as we can see from eqs. (3.1) and the initial condition (3.2)

$$a_{\mathbf{k}}(t = 0) = a_{\mathbf{k}}$$

$${a^\dagger}_{\mathbf{k}}(t = 0) = {a^\dagger}_{\mathbf{k}}$$

$$a_{\mathbf{k}}|0>= 0, \quad {a^\dagger}_{\mathbf{k}}|0>= |1_{\mathbf{k}}>$$

$$a_{\mathbf{k}}(t)|0, t>= 0, \quad {a^\dagger}_{\mathbf{k}}(t)|0, t>= |1_{\mathbf{k}}> .$$

We also define the mean value of created particles with $\mathbf{k}$ mode by ${n^{\mathbf{k}}}_{0t} =< 0|{a^\dagger}_{\mathbf{k}}(t)a_{\mathbf{k}}(t)|0>$. In the following, the $\mathbf{k}$ index is supressed.

4. **Introduction of tilde operators in curved space-time**

Next we will assume that the system and the reservoir are both in a curved background.

In order to use the transformations given by eqs. (2.1) and (3.1) on the same operators, we will introduce the quadrivectorial operators $A_{\mathbf{k}}$, $A_{\mathbf{k}}(\beta)$ and $A_{\mathbf{k}}(t)$, defined by (using l as generic variable)

$$\mathbf{A}_{\mathbf{k}}(l) := \begin{pmatrix} a_{\mathbf{k}}(l) \\ {a^\dagger}_{\mathbf{k}}(l) \\ \tilde{a}_{\mathbf{k}}(l) \\ \tilde{a}^\dagger_{\mathbf{k}}(l) \end{pmatrix} \tag{4.1}$$

Let us also introduce the 4×4 matrices $\mathbf{\Omega}$ and $\mathbf{\Upsilon}$ so that the two transformations can be written as

$$\mathbf{A_k}(t) = \mathbf{\Omega}(\mathbf{k},t)\mathbf{A_k} \tag{4.2a}$$

$$\mathbf{A_k}(\beta) = \mathbf{\Upsilon}(\mathbf{k},\beta)\mathbf{A_k} \tag{4.2b}$$

where

$$\mathbf{\Omega} = \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} e^{i\gamma_\alpha(\mathbf{k},t)}\cosh\theta(\mathbf{k},t), & e^{i\gamma_\beta(\mathbf{k},t)}\sinh\theta(\mathbf{k},t)\mathbf{P} \\ e^{-i\gamma_\beta(\mathbf{k},t)}\sinh\theta(\mathbf{k},t)\mathbf{P}, & e^{-i\gamma_\alpha(\mathbf{k},t)}\cosh\theta(\mathbf{k},t) \end{pmatrix}$$

$$\mathbf{\Upsilon} = \begin{pmatrix} \mathbf{I} & \mathbf{L} \\ \mathbf{L} & \mathbf{I} \end{pmatrix}(1+n_{0\beta})^{1/2}$$

$$\mathbf{L} = \begin{pmatrix} 0 & -F \\ -fF^{-1} & 0 \end{pmatrix}$$

Moreover $\mathbf{I}$ is the 2×2 identity matrix and $\mathbf{0}$ the zero 2×2 matrix. $\mathbf{P}$ is a parity operator, such that

$$\mathbf{P}a_{\mathbf{k}} = a_{-\mathbf{k}}$$

The total number of created particles comes from two sources, a geometric one due to the curvature of the space-time and the other one due to the thermal effects. Therefore we can define $n_{0t\beta}$ as the mean value of the total number of created particles, by

$$n_{0t\beta} =< \mathbf{0}|a^{\dagger\dagger}(t,\beta(t))a(t,\beta(t))|\mathbf{0} > \tag{4.3}$$

Then we need the transformation

$$\mathbf{A}(t,\beta(t)) = \mathbf{\Upsilon}(t)\mathbf{\Omega}(t)\mathbf{A} := \mathbf{\Lambda}(t)\mathbf{A}$$

therefore

$$\mathbf{\Lambda} = (1+n_{0\beta})^{1/2}(1+n_{0t})^{1/2}\begin{pmatrix} \mathbf{R} & \mathbf{T} \\ \mathbf{T} & \mathbf{R} \end{pmatrix}$$

with

$$\mathbf{R} = \begin{pmatrix} e^{i\gamma_\alpha} & e^{i\gamma_\beta}\tanh\theta\mathbf{P} \\ e^{-i\gamma_\beta}\tanh\theta\mathbf{P} & e^{-i\gamma_\alpha} \end{pmatrix},$$

$$\mathbf{T} = \begin{pmatrix} -Fe^{-i\gamma_\beta}\tanh\theta\mathbf{P} & -Fe^{-i\gamma_\alpha} \\ -fF^{-1}e^{i\gamma_\alpha} & -fF^{-1}e^{i\gamma_\beta}\tanh\theta\mathbf{P} \end{pmatrix}$$

When the calculation in eq. (4.3) is performed the following result is obtained

$$n_{0t\beta} = n_{0t} + 2n_{0\beta}n_{0t} \tag{4.4}$$

This result is independent of the particle model. From eq.(4.4) we see that if at the begining we had a curved geometry at zero temperature there would be a number n_{0t} of created particles at time "t". When all the system is in interaction with a reservoir at temperature T the number of particles increases in agreement with eq. (4.4).

5. **Concluding remarks.**

As was shown in ref.[4] the transformation given by eq.(3.1) is a consistent way to obtain Heinsenberg annihilation-creation operators for a Robertson-Walker metric. For other metrics the Bogoliubov transformation may be different. However the result obtained in eq.(4.4) is independent of the particle model used.

The increment in the number of created particles due to the temperature is equal to the one produced in curved space at zero temperature, when there is a particle distribution in the initial state, as is proved in ref.[4]. In other words this fact is related with the analogy, which is shown in ref.[7], between the role of the tilde operator $\tilde{a}_{\mathbf{k}}$ and the operator $a_{-\mathbf{k}}$ in curved space.

In our generalization of TFD to a curved space we said nothing about what the tilde modes are. We think that the role of tilde modes is played by the quantum perturbations of the metric, this possibility is presently being studied by us. As a first approach we found that the conformal fluctuation of the metric can satisfy all the hypothesis in order to act as a natural tilde field (see ref.[8]).

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