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# THE NEUTRINO MASSES IN SUSY GUT

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## **Abstract**

The neutrino mass problem in  $SU(4) \times SU(2)_L \times SU(2)_R$  SUSY GUT obtained from the compactification of  $E_8 \times E_8$  heterotic string is analyzed. The estimated values of the neutrino masses and mixing angles can explain the experimental data on solar neutrino flux.

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Recent solar neutrino experiments give an evidence for nonzero neutrino masses. The solar neutrino deficit can be explained in terms of the neutrino resonant oscillations if the neutrino mass difference is of order:  $\Delta m^2 \sim (0.3 - 1.2) \cdot 10^{-5} eV^2$  or vacuum oscillations if  $\Delta m^2 \sim (0.5 - 1.1) \cdot 10^{-10} eV^2$  [1, 2, 3].

It is well-known, that in  $SO(10)$  GUT small neutrino masses can be obtained via seesaw mechanism [4]. The neutrino mass matrix for three left  $\nu_e, \nu_\mu, \nu_\tau$  and three right  $\nu_e^c, \nu_\mu^c, \nu_\tau^c$  neutrinos has the following form:

$$\begin{pmatrix} 0 & M_D \\ M_D & R \end{pmatrix} \quad (1)$$

In (1)  $M$  is a Dirac type  $3 \times 3$  mass matrix (usually it is equal to mass matrix of  $u, c, t$ , quarks),  $R$ - is the right neutrino  $3 \times 3$  mass matrix with entries much greater than the electroweak symmetry breaking scale. After the diagonalization of (1) one obtains three heavy Majorana states (their masses practically coincide with the eigenvalues of matrix  $R$ ) and three light Majorana states with the mass matrix:

$$M_\nu = \frac{M_D^2}{R} \quad (2)$$

The scale of matrix  $R$  entries can be of order of the  $SO(10)$  subgroup  $G = SU(4) \times SU(2)_L \times SU(2)_R$  breaking scale, if  $G$  is broken by the vacuum expectation value (v.e.v.) of Higgs field in the **126** representation of  $SO(10)$ . For the superstring inspired models, however,  $G$  can be broken only by the v.e.v. of Higgs field in **16** representation of  $SO(10)$ . In this case the masses of right neutrinos can arise only due to the radiative corrections [5]. For the SUSY GUT, however, this mechanism does not work [6] and the right neutrinos can obtain masses only due to the nonrenormalizable interactions which can arise in the superstring models [7].

Our aim is consider the problem of the neutrino masses for the supersymmetric model  $G = SU(4) \times SU(2)_L \times SU(2)_R$ , which may (or may not) be considered as a subgroup of  $SO(10)$ . The particle content of the model is the following [8]: the sixteen fermions for each generation (including the right neutrino) belong to the representations  $\mathbf{F}$  and  $\overline{\mathbf{F}}$ , where

$$\begin{aligned} \mathbf{F} &= (4, 2, 1) = (u, d, \nu, e) \\ \overline{\mathbf{F}} &= (\overline{4}, 1, 2) = (u^c, d^c, \nu^c, e^c); \end{aligned}$$

and the Higgs fields- to the representations:  $\mathbf{H} = (4, \mathbf{1}, \mathbf{2})$  and  $\bar{\mathbf{H}} = (\bar{4}, \mathbf{1}, \mathbf{2})$ ,  $\mathbf{h} = (\mathbf{1}, \mathbf{2}, \mathbf{2})$ ,  $\mathbf{D} = (\mathbf{6}, \mathbf{1}, \mathbf{1})$ . The vacuum expectation values (v.e.v.) of the fields  $\mathbf{H}$  and  $\bar{\mathbf{H}}$  are connected with the breaking of the group  $G$  and the v.e.v. of the field  $\mathbf{h}$  -with the breaking of the group  $G_{ew} = SU(2)_L \times U(1)_Y$ . In addition, there is a set of the  $G$  singlet scalar fields  $\phi_m$  ( $m=1,2,\dots$ ). For the supersymmetric models, derived from the  $E_8 \times E_8$  heterotic string compactification over Calabi-Yau manifolds with  $SU(3)$  holonomy, the maximal gauge group in the four dimension is  $E_8 \times E_6$  (by embedding the spin connection of the manifold in the gauge group,  $E_8 \times E_8$  can be broken to  $E_8 \times E_6$ ) and chiral superfields belong to the  $\underline{27}, \underline{27}, \underline{1}$  representations of  $E_6$  [9]. In this case, the minimal set would consist of  $n_g + 1$   $SO(10)$  (or  $G$ ) singlets, where  $n_g$  is the number of fermion generations. One of the singlets develops the v.e.v. at the electroweak scale and generates the masses of the singlet fields [8].

Then most general superpotential for the supersymmetric  $G = SU(4) \times SU(2)_L \times SU(2)_R$  model has the form [7, 8] :

$$\begin{aligned}
W &= \lambda_1^{ij} F_i \bar{F}_j + \lambda_2^{im} \bar{F}_i H \phi_m + \lambda_3 H H D + \\
&+ \lambda_4 \bar{H} \bar{H} D + \lambda_5^m \phi_m h h + \lambda_6^{mnl} \phi_m \phi_n \phi_l + \\
&+ \lambda_7^{ij} F_i F_j D + \lambda_8^{ij} \bar{F}_i \bar{F}_j + \lambda_9^m D D \phi_m
\end{aligned} \tag{3}$$

The  $9 \times 9$  mass matrix of the three left, three right neutrinos and three singlets has the following form

$$\begin{pmatrix} 0 & M_D & 0 \\ M_D & R & M_G \\ 0 & M_G & M_\phi \end{pmatrix}, \tag{4}$$

where  $M_G$  is  $3 \times 3$  matrix of right neutrino-singlets mixing,  $M_\phi$  is  $3 \times 3$  mass matrix of singlets. As a result one obtains that (in the case of absence of mixing) the light neutrino masses are proportional to  $m_q^2 m_\phi / m_G^2$ , where  $m_q^2$  for  $q = u, c, t$  are the masses of  $u, c, t$  -quarks,  $m_\phi$  is the typical singlet mass  $\sim M_W$  (electroweak breaking scale),  $m_G$  is of order of  $G$  breaking scale  $\sim 10^{16} GeV$  [7]. This gives the ultralight neutrino masses:  $m_{\nu_1} \sim 10^{-16} eV$ ,  $m_{\nu_2} \sim 10^{-12} eV$ ,  $m_{\nu_3} \sim 10^{-8} eV$ . The similar problems exist in  $SU(5)$  -flipped model. The various possibilities to obtain more acceptable neutrino masses are considered in [6, 7, 10]. Here we want to consider the problem of the neutrino masses for the superstring derived models proposed by Witten

[11]. Witten [11] has shown that it is possible to construct stable, irreducible and holomorphic  $SU(4)$  or  $SU(5)$  vector bundles over Calabi-Yau manifolds. This means that one can obtain an  $SO(10)$  or  $SU(5)$  supersymmetric gauge theories in four dimension by the embedding the structure group of the bundle in  $E_8$ . For the  $SU(4)$  vector bundle, when the maximal gauge group in the four dimension is  $SO(10)$ , the content of the chiral superfields is the following [11, 12, 13, 14]:

$$n_g \mathbf{16} + \delta(\mathbf{16} + \overline{\mathbf{16}}) + \epsilon \mathbf{10} + \eta \mathbf{1}, \quad (5)$$

where  $n_g$  is the number of generations ( $n_g=3$ ),  $\delta, \epsilon, \eta$  are the integer numbers  $\delta, \epsilon, \eta \geq 1$ . As for ordinary case of tangent bundle with the  $E_6$  as a maximal gauge group, in this case also it is possible to obtain models with the gauge symmetries which are subgroups of  $SO(10)$  via Hosotani mechanism [11, 13, 15]. Let us consider the neutrino mass problem in such a model with gauge symmetry  $G = SU(4) \times SU(2)_L \times SU(2)_R$ . For the simplest case  $\epsilon = \delta = 1$  and  $\eta = 2$  one has only two  $SO(10)$  singlets. One of these singlets developed the v.e.v. of order of electroweak symmetry breaking scale. The second singlet is mixed with the neutrinos. The  $7 \times 7$  mass matrix for three left, three right neutrinos and this singlet has the following form:

$$\mathcal{M} = \begin{pmatrix} M_L & M_D & 0 \\ M_D & R & V \\ 0 & V^T & m_\phi \end{pmatrix} \quad (6)$$

where  $M_L, M_D, R$  are  $3 \times 3$  matrices,  $V$  is a three dimensional column. The elements of the left neutrino mass matrix  $M_L$  can arise due to the nonrenormalizable interactions which are allowed in the string models [16]:

$$\lambda \frac{FFHHhh}{M^3},$$

where  $M$  is the typical scale connected with nonrenormalizable interactions, it must be of order of Plank scale or string unification scale,  $\lambda$  - some constant. Then the matrix elements of  $M_L$  are of order  $m_L \sim \frac{M_W^2 m_G^2}{M^3}$ . The  $3 \times 3$  matrix  $R$  also arises due to the nonrenormalizable interactions [6, 7]:

$$\lambda' \frac{\bar{F}\bar{F}HH}{M},$$

The matrix elements of R are of order  $m_R \sim \frac{m_G^2}{M}$ . The 3x3 neutrino Dirac mass matrix  $M_D$  (which we assume to be equal to the up quark mass matrix),  $V_i$  ( $i=1,2,3$ )- the mixing between singlet and right neutrinos and the mass  $m_\phi$  of the singlet arise from the usual interaction terms (3).

To obtain the estimates for the neutrino masses one must determine G breaking scale  $m_G$ . We will consider two cases: the symmetry  $G = SU(4) \times SU(2)_L \times SU(2)_R$  with and without the left-right discrete symmetry. The renormalization group equations for one loop gives the following solutions for the coupling constants  $\alpha_1, \alpha_2, \alpha_3$  [13]:

$$\alpha_i^{-1}(M_Z) = \alpha_X^{-1} - \frac{b_i}{2\pi} \ln \frac{M_S}{M_Z} - \frac{b_{is}^-}{2\pi} \ln \frac{M_R}{M_S} - \frac{\hat{b}_{is}}{2\pi} \ln \frac{M_X}{M_R} \quad (7)$$

where  $M_S$  is the supersymmetry breaking scale (we assume that it is between 100GeV and 10000GeV), the values of  $b_i, b_{is}^-, \hat{b}_{is}$  ( $i=1,2,3$ ) are given in [13]. The more precise results can be obtained in two loop approximation. The renormalization group equations analysis in two loop approximation gives the following results for  $m_G$

$$m_G \sim (1.6 \cdot 10^{16} - 2.2 \cdot 10^{17}) GeV \quad (8)$$

for the case of the presence of the discrete left-right symmetry and

$$m_G \sim (1.5 \cdot 10^{15} - 2 \cdot 10^{16}) GeV \quad (9)$$

for the case of absence of such a symmetry. The results of (8) and (9) are obtained for the initial values of the electroweak coupling constants

$$\begin{aligned} \alpha_3(M_Z) &= 0.120 \pm 0.006 \\ \sin^2 \theta_W &= 0.2328 \pm 0.0007 \\ \alpha^{-1}(M_Z) &= 128.8 \pm 0.9 \end{aligned} \quad (10)$$

Let us consider now the mass matrix (6). After the diagonalization of the matrix (6) we obtain 7 Majorana neutrinos (three of which are light). One can estimate the light neutrino mass values without specifying the exact form of matrix (6). The only assumption we made, is the following: all the matrix elements of the matrix  $M_L$  are of the same order of magnitude  $m_L \sim \frac{M_W^2 m_G^2}{M^3}$ . The same statement must be valid for the  $3 \times 3$  matrix R

and three dimensional column V separately: the elements of R are of order  $m_R \sim \frac{m_G^2}{M}$  and the elements of V are of order of  $m_G$ . Then, it is easy to estimate the value of the determinant of the matrix (6) and the sums of its diagonal minors of 6x6, 5x5, 4x4 order:

$$\begin{aligned}
\det \mathcal{M} &\sim m_c^2 m_t^2 \frac{M_W^2 m_G^4}{M^3} \\
\det \mathcal{M}_6 &\sim m_c^2 m_t^2 m_G^2 \\
\det \mathcal{M}_5 &\sim m_t^2 \frac{m_G^4}{M} \\
\det \mathcal{M}_4 &\sim \frac{m_G^6}{M^2}
\end{aligned} \tag{11}$$

Two eigenvalues of the matrix (6) are of order  $m_R$  and the other two- of order  $m_G$ . One can obtain a simple formulae for the masses of three light neutrinos for the case  $m_{\nu_3} \gg m_{\nu_2} \gg m_{\nu_1}$  :

$$\begin{aligned}
m_{\nu_1} &\sim \frac{\det \mathcal{M}}{\det \mathcal{M}_6} \sim \frac{M_W^2 m_G^2}{M^3} \\
m_{\nu_2} &\sim \frac{\det \mathcal{M}_6}{\det \mathcal{M}_5} \sim r_c^{-1} \frac{m_c^2 M}{m_G^2} \\
m_{\nu_3} &\sim \frac{\det \mathcal{M}_5}{\det \mathcal{M}_4} \sim r_t^{-1} \frac{m_t^2 M}{m_G^2}
\end{aligned} \tag{12}$$

In (11), (12)  $r_c, r_t$  -are the factors connected with the quark mass renormalization from the unification scale to ordinary energies. These factors depend on the ratio of vacuum expectation values of scalar doublets connected with the electroweak symmetry breaking ( $tg\theta$ ) and the t-quark mass. For the t-quark mass  $m_t = (175 \pm 15)\text{GeV}$  and  $0.1 < tg\theta < 10$  one obtains:

$$\begin{aligned}
20 &> r_c > 3 \\
7 &> r_t > 1
\end{aligned} \tag{13}$$

In (13) the larger smaller of  $r_c, r_t$  correspond to the larger values of  $m_t$  .

What about the mixing angles? To obtain the estimates for the mixing angles between the left  $\nu_e, \nu_\mu, \nu_\tau$  neutrinos one has to specify the form of Dirac-type mass matrix  $M_D$  . Let us consider the case when the mass matrix of

u, c, t- quarks (which we consider to be equal to  $M_D$ ) has a form, proposed by Fritzsch [17]:

$$M_D = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix} \quad (14)$$

where  $a = \sqrt{m_u m_c}$ ,  $b = \sqrt{m_c m_t}$ ,  $c = m_t$ . We must made some assumptions for the matrix  $R$  and  $M_L$  also: let us consider the case when they have a diagonal form:

$$R = \begin{pmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{pmatrix} \quad M_L = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \quad (15)$$

It is possible to estimate the mixing angles for three light neutrino states  $\nu_1, \nu_2, \nu_3$  by solving the equation for eigenvectors and eigenvalues of the matrix  $\mathcal{M}$  for the case  $m_{\nu_3} \gg m_{\nu_1}, m_{\nu_2}$ . Using the following formulae for the light neutrino masses:

$$\begin{aligned} m_{\nu_3} &\simeq \frac{\det \mathcal{M}_5}{\det \mathcal{M}_4} \\ m_{\nu_2} m_{\nu_1} &\simeq \frac{\det \mathcal{M}}{\det \mathcal{M}_5} \\ m_{\nu_2} + m_{\nu_1} &\simeq \frac{\det \mathcal{M}_6}{\det \mathcal{M}_5} \end{aligned} \quad (16)$$

and calculating the determinant of matrix  $\mathcal{M}$  and sums of its main minors of 6, 5, and 4 order

$$\begin{aligned} \det \mathcal{M} &\simeq -m_1 m_c^2 m_t^2 M_1^2 + m_1 m_2 m_t^2 (R_1 M_2^2 + R_2 M_1^2) \\ &\quad - 2m_1 \sqrt{m_u m_c} m_c m_t^2 M_1 M_2 - 2m_1 m_2 \sqrt{m_t m_c} m_t R_1 M_2 M_3 \\ &\quad - m_1 m_u m_c m_t^2 M_2^2 - m_2 m_u m_c m_t^2 M_2^2 \\ \det \mathcal{M}_6 &\simeq -m_c^2 m_t^2 M_1^2 + (m_1 + m_2) m_t^2 (R_1 M_2^2 + R_2 M_1^2) \\ &\quad - 2\sqrt{m_u m_c} m_c m_t^2 M_1 M_2 - 2(m_1 + m_2) \sqrt{m_t m_c} m_t R_1 M_2 M_3 \\ \det \mathcal{M}_5 &\simeq m_t^2 (R_1 M_2^2 + R_2 M_1^2) - 2\sqrt{m_c m_t} m_t R_1 M_2 M_3 \\ \det \mathcal{M}_4 &\simeq M_1^2 R_2 R_3 + M_2^2 R_1 R_3 + M_3^2 R_1 R_2 \end{aligned} \quad (17)$$

(where  $M_1, M_2, M_3$  are the elements in  $\mathcal{M}$  connected with the mixing of right neutrinos and singlet:  $V^T = (M_1, M_2, M_3)$ ) one can obtain the estimates for

the light neutrino masses. In (17) we omit the terms not relevant for our consideration. We will consider two alternatives for the  $\nu_1, \nu_2$ :

$$m_{\nu_1} \ll m_{\nu_2} \quad (18)$$

$$m_{\nu_1} \sim m_{\nu_2} \quad (19)$$

The conditions (18), (19) are equivalent to the conditions

$$m_G \ll \sqrt{\frac{m_c}{M_W}} M \quad (20)$$

$$m_G \sim \sqrt{\frac{m_c}{M_W}} M \quad (21)$$

For the first case (18), (20) one obtains the same formulae for neutrino masses as previously

$$m_{\nu_1} \sim m_L \quad m_{\nu_2} \sim \frac{m_c^2}{r_c R} \quad m_{\nu_3} \sim \frac{m_t^2}{r_t R} \quad (22)$$

where we assume that  $R \sim R_1 \sim R_2 \sim R_3$  and  $m_1 \sim m_2 \sim m_3$ . The three eigenstates  $\nu_i$ ,  $i=1,2,3$  of matrix (6) with three lightest masses one can express via weak eigenstates  $\nu_\alpha$ ,  $\alpha = e, \mu, \tau$  by means of unitary transformation:  $\nu_i = a_{i\alpha} \nu_\alpha$ , where  $a_{i\alpha}$  is the unitary matrix. For the mixing angles between  $\nu_e$  and  $\nu_\mu$  ( $\theta$ ) and  $\nu_e$  and  $\nu_\tau$  ( $\theta'$ ) one obtains:

$$\begin{aligned} \tan\theta &\sim \frac{a_{1\mu}}{a_{1e}} \sim \sqrt{\frac{m_u}{m_c}} \\ \tan\theta' &\sim \frac{a_{1\tau}}{a_{1e}} \sim \sqrt{\frac{m_u}{m_t}} \end{aligned} \quad (23)$$

Taking into account the results (8), (9) for the G symmetry breaking scale one obtains the following estimates for the three light neutrino masses:

$$\begin{aligned} m_{\nu_1} &\sim (10^{-12} - 2 \cdot 10^{-10}) \left(\frac{M_{Pl}}{M}\right)^3 eV \\ m_{\nu_2} &\sim (4 \cdot 10^{-8} - 10^{-5}) \frac{M}{M_{Pl}} eV \\ m_{\nu_3} &\sim (8 \cdot 10^{-3} - 0.6) \frac{M}{M_{Pl}} eV \end{aligned} \quad (24)$$



for case of the presence of discrete left-right (case (a)) symmetry and

$$\begin{aligned}
m_{\nu_1} &\sim (10^{-14} - 10^{-12}) \left(\frac{M_{Pl}}{M}\right)^3 eV \\
m_{\nu_2} &\sim (6 \cdot 10^{-6} - 2.5 \cdot 10^{-3}) \frac{M}{M_{Pl}} eV \\
m_{\nu_3} &\sim (0.3 - 150) \frac{M}{M_{Pl}} eV
\end{aligned} \tag{25}$$

for the case of the absence of left-right discrete symmetry (case (b)). In (24), (25)  $M_{Pl} = 1.2 \cdot 10^{19} GeV$  is the Plank mass. Of course, all the estimates (24), (25) are correct with accuracy of order of magnitude.

To explain the solar neutrino deficit via resonant oscillations the neutrino mass difference and mixing angles must be of order

$$\begin{aligned}
\sqrt{\Delta m^2} &\sim (1.7 - 3.5) 10^{-3} eV \\
\sin^2 2\theta &\sim (0.6 - 1.4) 10^{-2} \quad or \quad (0.65 - 0.85)
\end{aligned} \tag{26}$$

Then for the case (a) it is possible to obtain such a mass difference between third and second and first neutrinos if the fraction  $\frac{M}{M_{Pl}}$  is of order  $\sim (1 - 0.01)$  which is reasonable value [13]. The problem arises with mixing angle: the formula (23) gives small value for  $\sin^2 2\theta \sim 10^{-4}$ .

For the case (b) such a mass difference it is possible to obtain between the second and the first neutrinos if the fraction  $\frac{M}{M_{Pl}}$  is order of unity. In this case the condition (20) is valid. The mixing angle is of order  $\sin^2 \theta \sim 1.3 \cdot 10^{-2}$  which is a reasonable value. Thus, in this case obtained values for the mixing angles and neutrino mass difference allow one to explain the solar neutrino deficit via resonant oscillations in the sun.

To explain the solar neutrino deficit via longwave vacuum oscillations the neutrino mass difference and the mixing angles must be of order

$$\begin{aligned}
\sqrt{\Delta m^2} &\sim (0.5 - 1.1) 10^{-5} eV \\
\sin^2 2\theta &> 0.75
\end{aligned} \tag{27}$$

It is clear that it is impossible for the considering case as our mixing angles are too small.

Let as proceed to the case (19), (21). For the case (a) the condition (21) gives for M:

$$\frac{M}{M_{Pl}} = (0.018 - 0.22) \tag{28}$$

For such a values of  $M$  the light neutrino masses are of order

$$\begin{aligned} m_{\nu_1} \sim m_{\nu_2} &\sim 4 \cdot (10^{-8} - 10^{-5}) \frac{M}{M_{Pl}} eV \sim (7.1 \cdot 10^{-10} - 2.2 \cdot 10^{-6}) eV \\ m_{\nu_3} &\sim (8 \cdot 10^{-3} - 0.6) \frac{M}{M_{Pl}} eV \sim (1.4 \cdot 10^{-4} - 0.13) eV \end{aligned} \quad (29)$$

It is clear that the mass difference of second and first family is too small. The mass difference between third and first family can achieve acceptable value but the mixing angle as in previous case will be small  $\sin^2 2\theta \sim \cdot 10^{-4}$ . Let us consider the case (b). From (21) one obtains for  $M$

$$\frac{M}{M_{Pl}} = (0.0014 - 0.02) \quad (30)$$

For such a values of  $M$  the light neutrino masses are of order

$$\begin{aligned} m_{\nu_1} \sim m_{\nu_2} &\sim (6 \cdot 10^{-6} - 2.5 \cdot 10^{-3}) \frac{M}{M_{Pl}} eV \sim (8.4 \cdot 10^{-9} - 5 \cdot 10^{-5}) eV \\ m_{\nu_3} &\sim (0.3 - 150) \frac{M}{M_{Pl}} eV \sim (4.2 \cdot 10^{-3} - 3.0) eV \end{aligned} \quad (31)$$

The value  $(0.5 - 1.1)10^{-5} eV$  which is necessary to explain the solar neutrino deficit via longwave vacuum oscillations is achieved for the mass difference of first two neutrinos for the following values of  $M$ :

$$\frac{M}{M_{Pl}} = (0.02 - 0.002) \quad (32)$$

What about the mixing angles for the considering case? As the masses of two lightest neutrinos are close to each other, the mixing angle between them can be relatively large.

The angle relevant for the neutrino oscillations is given by (if we neglect the mixing with third light neutrino):

$$\tan \theta = \frac{a_{1\mu}}{a_{1e}} \simeq \sqrt{\frac{m_c m_{\nu_1} - m_1}{m_u m_{\nu_1} - m_2}} \quad (33)$$

where

$$m_{\nu_1} = m_1 + \frac{1}{2} m \left( 1 + \frac{m_1 - m_2 + m'}{m_1 - m_2 - m'} \right)$$

$$\begin{aligned}
m_{\nu_2} &= m_2 + m' + \frac{1}{2}m \left( 1 - \frac{m_1 - m_2 + m'}{m_1 - m_2 - m'} \right) \\
m &= -\frac{m_u m_c m_t^2 M_1 M_2}{\det \mathcal{M}_5} \\
m' &= -\frac{m_c^2 m_t^2 M_1^2 + m_u m_c m_t^2 M_2^2 + 2\sqrt{m_u m_c} m_c m_t^2 M_1 M_2}{\det \mathcal{M}_5}
\end{aligned} \tag{34}$$

To explain the solar neutrino deficit via vacuum longwave oscillations  $\tan\theta$  must satisfy the condition

$$0.58 < \tan\theta < 1.75 \tag{35}$$

(as it follows from (27)). This gives :

$$(m_1 - m_2 - m')^2 \sim mm' \tag{36}$$

or, with the same accuracy:

$$(m_{\nu_1} - m_{\nu_2})^2 \sim mm' \tag{37}$$

Thus, it is possible to obtain large (35) mixing angles in this case if the condition (36) takes place. This means that the masses of first two light neutrinos must be very close to each other: the difference of masses must be  $\sim 100$  times smaller than the masses.

Thus, our analyze shows that it is possible to explain the solar neutrino deficit for the considering theory for the case of a unification group  $G = SU(4) \times SU(2)_L \times SU(2)_R$  without the left-right discrete symmetry can be explained in two ways. 1) Via resonant oscillations in the sun if the value of which is characteristic scale of nonrenormalizable interactions is of order of Plank mass  $M_{Pl} = 1.2 \cdot 10^{19} GeV$ . Generally speaking, it is more natural, that such a interactions arises in string scale  $M_s = \frac{g_{string} M_{Pl}}{\sqrt{8\pi}} = 1.7 \cdot 10^{18} GeV$  [18, 19] however the difference of  $\sim 7$  can be explained by means of coupling constants  $\lambda, \lambda'$  in nonrenormalizable terms:

$$\lambda \frac{FFHHhh}{M^3}, \quad \lambda' \frac{\bar{F}\bar{F}HH}{M}$$

The mixing angle in this case  $\sim 0.06$  which is the value we need. 2) Via longwave vacuum oscillations: in this case the masses of two light neutrinos are very close to each other and the mixing angle is large as needed.

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