# The QCD Analysis of the Structure Functions and Effective Nucleon Mass.

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**Abstract**

On the basis of the target mass corrections to structure functions of deep-inelastic scattering of leptons, we evaluate effective nucleon mass that turns out to be twice $M_{nucl.}$ for deep-inelastic scattering on the nucleus target and equals $M_{nucl.}$ for the hydrogen target.

Deep-inelastic scattering of leptons provides a precise information on structure functions (SF) of a nucleon. It is well known that when target mass corrections (TMC) are taken into account, the QCD description of the SF of deep-inelastic scattering is improved. These effect is of the order $M_{nucl.}^2/Q^2$. In this aricle, we are going to consider the question wherher the mass of a nucleon is the best value for the description of data or in order to make the fit better, one has to use another value $M^{eff.}$ which could differ from the mass of nucleon.

The Nachtmann moments [1] of SF $F_2$ and $F_3$ are defined as:

$$M_2^{QCD}(N,Q^2) = \int_0^1 \frac{dx\xi^{N+1}}{x^3} F_2(x,Q^2) \frac{3+3(N+1)V+N(N+2)V^2}{(N+2)(N+3)}, \tag{1}$$

$$M_3^{QCD}(N,Q^2) = \int_0^1 \frac{dx\xi^{N+1}}{x^3} F_3(x,Q^2) \frac{3+(N+1)V}{(N+2)}, \tag{2}$$

where

$$\xi = 2x/(1+V), \qquad V = \sqrt{1+4M_{nucl.}^2 x^2/Q^2} \tag{3}$$

Equations (1,2) could be expanded into a series in powers of $M_{nucl.}^2/Q^2$. Retaining only the terms of the order $M_{nucl.}^2/Q^2$ one could obtain:

$$M_2(N,Q^2) = M_2^{QCD}(N,Q^2) + \frac{N(N-1)}{N+2}\frac{M_{nucl.}^2}{Q^2} M_2^{QCD}(N+2,Q^2), \tag{4}$$

$$M_3(N,Q^2) = M_3^{QCD}(N,Q^2) + \frac{N(N+1)}{N+2}\frac{M_{nucl.}^2}{Q^2} M_3^{QCD}(N+2,Q^2). \tag{5}$$

$M_2(N,Q^2)$ and $M_2(N,Q^2)$ are the Mellin moments of the measured SF $F_2$ and $xF_3$:

$$M_2(N,Q^2) = \int_0^1 dx x^{N-2} F_2(x,Q^2), \tag{6}$$

$$M_3(N,Q^2) = \int_0^1 dx x^{N-2} x F_3(x,Q^2), \tag{7}$$

$$N = 2,3,...$$

The $Q^2$ - evolution of the moments $M_2^{QCD}(N,Q^2)$ and $M_3^{QCD}(N,Q^2)$ is given by QCD [2, 3]. For the nonsinglet SF:

$$M_3^{QCD}(N,Q^2) = \left[\frac{\alpha_S(Q_0^2)}{\alpha_S(Q^2)}\right]^{d_N} M_3^{QCD}(N,Q_0^2), \tag{8}$$

$$N = 2,3,...$$

$$d_N = \gamma^{(0),N}/2\beta_0, \qquad \beta_0 = (11 - \frac{2}{3}f).$$

$$\alpha_s(Q^2)/4\pi = 1/\beta_0 \ln(Q^2/\Lambda_{LO}^2) \tag{9}$$

$$\gamma_N^{(0)NS} = \frac{8}{3}[1 - \frac{2}{N(N+1)} + 4\sum_{j=2}^{N}\frac{1}{j}] \ . \qquad (10)$$

The unknown coefficients $M_3(N,Q_0^2)$ in (9) could be parametrised as a Mellin moments of some function:

$$\begin{aligned} M_3^{QCD}(N,Q_0^2) &= \int_0^1 dx x^{N-2} A x^b (1-x)^c (1+\gamma x), \\ & N = 2,3,... \end{aligned} \qquad (11)$$

where constants A, b, b and $\gamma$ should be determined from the fit of data.

Having in hand the moments (12,9,5, 8) and following the method [4, 5], we can write the structure function $xF_3$ in the form:

$$xF_3^{N_{max}}(x,Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_{j+2}^{NS}\left(Q^2\right), \qquad (12)$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^n(\alpha,\beta)$ are coefficients of the series of $\Theta_n^{\alpha,\beta}(x)$ in powers of x:

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) x^j. \qquad (13)$$

The quantities $N_{max}$, $\alpha$ and $\beta$ have to be chosen so as to achieve the most fast convergence of the series in the r.h.s. of Eq.(12) and to reconstruct $xF_3$ with the accuracy required. Following the results of [5] we use $\alpha = 0.12$ , $\beta = 2.0$ and $N_{max} = 12$ . These numbers guarantee accuracy better than $10^{-3}$ .

Eq. (12) could be applied for reconstructing SF $F_2(x,Q^2)$ for $0.3 \leq x$ and with eq. (1,4) for TMC taken into account.

The parameters A, b, c, $\gamma$ and parameter $\Lambda$ are determined by fitting experimental data. We also consider $M^{eff.}$ as a free parameter. It should be noted that the parameters a, b, c and $\gamma$ depend on $Q_0^2$. We have used experimental points with $Q^2 > 5GeV^2$ for fitting, in order to avoid high–twist effects and chose $Q_0^2 = 10GeV^2$.

The results of concrete calculations made for SF measured in experiments on different targets are presented in Table I.

For the hydrogen target $M^{eff.}$ reproduces the value of the proton mass. For the iron target the effective mass $M^{eff.}$ is twice the nucleon mass. The data of the SKAT collaboration on a target which consists of a mixture of Neon and Hydrogen are not precise enough to determine the value of $\Lambda$. So following [7] we have fixed $\Lambda = 200MeV$ and found the value of $M^{eff.}$ a little bit higher than for the hydrogen target. The increasing effective mass of a nuclon on the nucleus target takes place for a nonsinglet fit both for $F_2$ and $xF_3$ SF. It also takes place both for the leading and next to leading order QCD (see result for $xF_3$ data of CCFR Table 1.). The large value of $M^{eff.}$ found in the QCD fit of data of DIS on nucleon target could be considered as indirect evidence of the existence of multiquark clusters [10, 11, 12, 13] or a few–nucleon correlation in a nucleus [14]. It is also compatible with the measured SF at $x > 1$ on DIS of leptons on the nucleus target [15].

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| Collaboration Reaction | Ref. | | $\Lambda$ $[MeV]$ | $\chi^2_{d.f.}$ | $M^{eff.}$ $[GeV]$ |
|---|---|---|---|---|---|
| BCDMS $\mu p$ $F_2$ | [6] | $0.35 < x$ | $130 \pm 4$ | 183/223 | $0.88 \pm 0.14$ |
| SKAT $\nu Ne, p$ $xF_3$ | [7] | $0.05 \leq x$ | 200 (fix.) | 25.3/30 | $1.42 \pm 0.71$ |
| EMC $\mu Fe$ $F_2$ | [8] | $0.30 < x$ | $106 \pm 26$ | 45.3/45 | $2.08 \pm 0.16$ |
| CCFR $\nu Fe$ $F_2$ | [9] | $0.275 \leq x$ | $146 \pm 12$ | 37.9/81 | $1.76 \pm 0.09$ |
| CCFR $\nu Fe$ $xF_3$ | [9] | $0.015 \leq x$ | $64.7 \pm 21$ | 81.8/81 | $2.04 \pm 0.18$ |
| CCFR $\nu Fe$ $xF_3$ $NLO$ | [9] | $0.015 \leq x$ | $116 \pm 30$ | 73.4/81 | $1.83 \pm 0.20$ |

**Table I.** The summary of various determinations of the $M_n^{eff.}$.