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DECAYING VACUUM ENERGY AND DEFLATIONARY COSMOLOGY IN OPEN AND CLOSED UNIVERSES

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Abstract

We consider a nonsingular deflationary cosmological model with decaying vacuum energy density in universes of arbitrary spatial curvature. Irrespective of the value of k, the models are characterized by an arbitrary time scale H_I^{-1} which determines the initial temperature of the universe and the largest value of the vacuum energy density, the slow decay of which generates all the presently observed matter-energy of the universe. If H_I^{-1} is of the order of the Planck time, the models begin with the Planck temperature and the present day value of the cosmological constant satisfies $\Lambda_I/\Lambda_0 \simeq 10^{118}$ as theoretically suggested. It is also shown that all models allow a density parameter $\Omega_0 < 2/3$ and that the age of the universe is large enough to agree with observations even with the high value of H_0 suggested by recent measurements.

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1 Introduction

A great deal of attention has recently been paid to cosmological models with a nonvanishing vacuum energy density, or equivalently a nonzero cosmological Λ-term. The revival of interest in these models is physically compelling on both observational and physical grounds[1]-[10]. A large class of recent observations (the age of the universe, dynamical estimates of the density parameter, kinematical tests,...etc) consistently point to the probable existence of an effective vacuum component which, although incredibly small in comparison with common microscopic scales, is expected to contribute appreciably to the present large-scale structure of the universe (for a recent review see[10]). From a theoretical standpoint there is also a widespread belief that the early universe evolved through a cascade of phase transitions, thereby yielding a present vacuum energy density that is smaller than its value at Planck times by a factor of at least 118 orders of magnitude[3, 5].

On the other hand, since the value of the cosmological "constant" Λ_0 (a subscript 0 denotes the present day value of a quantity) may be viewed as a remnant of a primordial inflationary stage, it seems natural to address the following question: Is it possible to describe the history of the universe accounting for a vacuum energy density that is high enough to drive inflation at early times and is small enough to be compatible with observations at late times?

To the best of our knowledge there is no formulation (from first principles) that provides a satisfactory description of the time-dependence of Λ which presumably occurs as the universe evolves. In such a situation the classical, phenomenological approach seems to be a good tool with which to gain some insight into this question. In fact, models with $\Lambda = \Lambda(t)$ have been the subject of numerous papers in recent years[11]-[18]. Indeed, since the basic motivation is to understand the present day smallness of the cosmological constant, most scenarios do not attempt to provide any natural relation between the magnitude of Λ at the beginning of inflation and the present day observational upper bound.

In a previous paper[19], we investigated some consequences of a phenomenological decay law for Λ which yielded a partial solution to the above question. However, since that model was formulated in the framework of a *flat* Friedmann, Robertson-Walker (FRW) geometry, the results were crucially dependent on that particular spacetime[20].

In the present paper we wish to demonstrate that the main results of the previous work remain valid in spacetimes of arbitrary spatial curvature. To be more precise, there exists a large class of nonsingular deflationary cosmologies, beginning from the decay of a pure de Sitter vacuum and subsequently evolving smoothly to a quasi-FRW stage at late times. The models in this class seem to agree with present cosmological observations for all values of the curvature parameter k. As a general feature, the process of vacuum decay generates all the matter-radiation of the present day universe and has the added attraction of simultaneously solving the same problems that inflation aims to explain. In addition, as theoretically suggested, the maximum allowed value for the vacuum energy density is naturally larger than its present value by about 118 orders of magnitude.

2 The Models

We shall consider metrics described by the general FRW line element

$$ds^2 = dt^2 - R(t)^2 \left(\frac{1}{1-kr^2} dr^2 + r^2 d\Sigma^2 \right) \ , \tag{1}$$

where $R(t)$ is the scale factor, $d\Sigma^2$ is the area element on the unit 2-sphere, $k = 0, \pm 1$ is the curvature parameter and we have adopted the metric signature convention (+,−,−,−). Throughout we use units such that $c = 1$.

In such a background the Einstein field equations (EFE) for the nonvacuum component plus a cosmological Λ-term are

$$8\pi G\rho + \Lambda = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} \ , \tag{2}$$

$$8\pi G p - \Lambda = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} \ , \tag{3}$$

where ρ and p are the energy density and pressure respectively of the nonvacuum component which is assumed to obey the γ-law equation of state

$$p = (\gamma - 1)\rho \ , \qquad \gamma \in [1, 2] \ . \tag{4}$$

As we shall see, regardless of the value of k, a primordial inflationary scenario will automatically be generated at early times if the vacuum decays according to the following phenomenological decay ansatz

$$\rho_V = \frac{\Lambda}{8\pi G} = \beta\rho_T \left(1 + \frac{1-\beta}{\beta}\frac{H}{H_I}\right) \ , \tag{5}$$

where ρ_V and $\rho_T = \rho_V + \rho$ are the vacuum and total energy densities respectively, $H \equiv \dot{R}/R$ is the Hubble parameter, H_I^{-1} is the arbitrary time scale of inflation and β is a dimensionless parameter of order unity. For $H = H_I$ equation (5) reduces to $\rho_V = \rho_T$ so that we have inflation with no matter-radiation component ($\rho = 0$), while for late times ($H \ll H_I$), $\rho_V \sim \beta\rho_T$ as is required by recent observations[1]-[10]. Since at all times $H \leq H_I$, equation (5) can be viewed as the first two terms of a power series expansion of ρ_V in the parameter $y \equiv H/H_I$. The ansatz (5) together with equations (2) and (3) generalize the model of Freese et. al.[12] by including the curvature terms and by introducing a time dependence in the parameter $x \equiv \rho_V/(\rho_V + \rho)$ which here is given by $x = \beta + (1-\beta)H/H_I$. Of course, at late times $H \ll H_I$ and this parameter reduces to $x \simeq \beta$ as assumed in[12]. Note also that in the flat case $8\pi G\rho_T = 3H^2$ and the flat decaying Λ-model of Ref.[19] is readily recovered, since in this case (5) reduces to (see equation (1) of Ref.[19])

$$\Lambda(H) = 3\beta H^2 + 3(1-\beta)\frac{H^3}{H_I} \; .$$

Let us now consider the evolution of the scale factor in these models. Combining equations (4) and (5) with the EFE we obtain the following differential equation for R and expression for ρ

$$R\ddot{R} + \Delta(\dot{R}^2 + k)\left(1 - \frac{(\Delta+1)}{\Delta}\frac{H}{H_I}\right) = 0 \; , \tag{6}$$

$$8\pi G\rho = 3(1-\beta)\left(H^2 + \frac{k}{R^2}\right)\left(1 - \frac{H}{H_I}\right) \; , \tag{7}$$

where

$$\Delta \equiv \frac{3\gamma(1-\beta) - 2}{2} \; . \tag{8}$$

Thus, in the very beginning, where $H = H_I$, (7) gives $\rho = 0$ in accordance with the above qualitative arguments and at late times, where $H \ll H_I$, the universe is in a quasi-FRW epoch characterized by $\rho = \rho_T(1-\beta)$ and $\rho_V = \beta\rho_T$ (see equations (5) and (7)). Note that $\beta \in [0,1]$ parametrizes the extent to which our model departs from the standard FRW picture in this phase.

To analyze the solutions of (6) in its various asymptotic regimes it proves convenient to introduce an *effective "adiabatic index"*

$$\tilde{\gamma} = \gamma(1-\beta)\left(1 - \frac{H}{H_I}\right) \; , \tag{9}$$

so that (6) assumes the general FRW-type form, namely

$$R\ddot{R} + \left(\frac{3\tilde{\gamma} - 2}{2}\right)\dot{R}^2 + \left(\frac{3\tilde{\gamma} - 2}{2}\right)k = 0 \; . \tag{10}$$

For $H = H_I$, equation (9) gives $\tilde{\gamma} = 0$ with (10) reducing to

$$R\ddot{R} - \dot{R}^2 - k = 0 \ , \tag{11}$$

which yields the well known de Sitter solutions

$$R(t) = \begin{cases} H_I^{-1}\cosh(H_I t) & k = +1 \\ R_* e^{H_I t} & k = 0 \ , \\ H_I^{-1}\sinh(H_I t) & k = -1 \end{cases} \tag{12}$$

Hence, unlike in the standard FRW model, the present scenario begins in a pure non-singular de Sitter vacuum with Hubble parameter $H = H_I$. Accordingly, equation (7) gives $\rho = 0$ as discussed earlier. Note also that in this limit the initial value of the Λ-parameter is $\Lambda_I = 3H_I^2$ corresponding to a vacuum energy density of $\rho_V = 3H_I^2/8\pi G$, regardless of the value of k. In this way, the initial evolution is such that the singularity, flatness and horizon problems are simultaneously eliminated. Analytically, the ansatz (5) can be viewed as the simplest vacuum decay law which destabilizes the initial de Sitter configurations given by (12). As should be expected, no dynamic privilege can be associated with a particular choice of the curvature parameter of the initial vacuum state. All these solutions have constant curvature and are unstable in the future. Of course, closed ($k = 1$) solutions are not of the "bouncing" type, rather the universe begins its evolution from a closed de Sitter universe.

In the opposite limit, $H \ll H_I$, equation (9) reduces to $\tilde{\gamma} = \gamma(1 - \beta)$ so that equation (6) takes the form

$$R\ddot{R} + \Delta\dot{R}^2 + \Delta k = 0 \ , \tag{13}$$

which is the general equation for a slightly modified FRW model. There exists a first integral to this equation, namely

$$\dot{R}^2 = AR^{-2\Delta} - k \ , \tag{14}$$

where the constant $A > 0$ in order that ρ be positive definite in this phase (see equation (7)). Parenthetically, such a condition also guarantees the positivity of the vacuum (and consequently the total) energy density.

Inserting (14) into (5) and (7), the vacuum and the matter energy density can be expressed for $H \ll H_I$ as

$$\begin{aligned} \rho_V &= \beta \rho_{T_0} \left(\frac{R_0}{R}\right)^{3\gamma(1-\beta)} = \beta \rho_T \ , \\ \rho &= (1-\beta)\rho_{T_0} \left(\frac{R_0}{R}\right)^{3\gamma(1-\beta)} \equiv (1-\beta)\rho_T \ , \end{aligned} \tag{15}$$

where $\rho_{T_0} = 3A/8\pi G R_0^{3\gamma(1-\beta)}$. For $\gamma = 4/3$ it follows from (15) that the radiation energy density scales as $\rho_r \sim R^{-4(1-\beta)}$ while for a dust filled universe ($\gamma = 1$) the energy density satisfies $\rho_d \sim R^{-3(1-\beta)}$. Hence, there is a natural transition from a vacuum-radiation to a vacuum-dust dominated phase as the universe expands, just as in the standard FRW model with no-vacuum component. For the sake of completeness, we remark that in the flat case the evolution of the scale factor can be analytically described (see Ref. [19], eq. (10)). In the present notation this is given by

$$H_I t = \ln\left(\frac{R}{R_*}\right) + \frac{2(H_I - H_0)A^{-1/2}}{3\gamma(1-\beta)} R^{3\gamma(1-\beta)/2} \ . \tag{16}$$

Hence, in the very beginning when the logarithm term is dominant, we obtain to a high degree of approximation $R \simeq R_* e^{H_I t}$ in accordance with our equation (12). At late times ($R \gg R_*$ or $H \ll H_I$) one obtains from (14) that $A = H_0^2 R_0^{3\gamma(1-\beta)}$ with (16) reducing to

$$R \sim R_0 \left(3\gamma(1-\beta)\frac{H_0 t}{2}\right)^{2/3\gamma(1-\beta)} \quad ,$$

as expected (see equation (15) of Ref. [19]). Note also from (5) and (7) that, irrespective of k, both ρ_V and ρ always satisfy the weak energy condition (e.g. positiveness of the energy density) during the course of the evolution (see Fig. 1).

It is also worth mentioning that in this scenario there is no preinflationary stage as in most inflationary variants presented in the literature[23]-[26]. In such models the universe emerges from a radiation dominated FRW-type phase and enters a de Sitter epoch at a critical temperature due to vacuum domination. In particular, the existence of such a hot radiation-dominated phase preceding the vacuum stage means that inflation does not evade the singularity problem. In connection with this we note that Narlikar and Padmanabhan proposed a new variant on the "Creation-field cosmology" in order to avoid the singularity problem and other difficulties of the standard big-bang model[22]. However, unlike the scenario with vacuum decay presented here, in such a model the singularity is removed at the expense of a "C-field" of negative energy density which leads to matter creation.

The initial state of our scenario is the simplest one (constant curvature) and is physically appealing from a quantum theoretical point of view. It resembles the early inflationary model proposed by Starobinskii where the initial de Sitter configurations are supported by quantum one-loop corrections to the vacuum energy-momentum tensor[27]. However, unlike the Starobinskii model which evolves directly from de Sitter to dust domination, the scenario proposed here contains the same phases of the standard FRW picture and, as we shall see, has interesting concrete cosmological consequences for the present vacuum-dust dominated phase (see next section). As a matter of fact, there have been many suggestions in the literature that the de Sitter spacetime may be destabilized and decay to ordinary FRW universes[28]-[30]. Of particular interest for us is the scenario proposed by Gott[28]. In such a model the universe begins with the Hawking temperature evolving, at late times, to the standard FRW model with negative curvature parameter. As we shall see (see section 4), this connection with the Hawking temperature will be preserved in our scenario for all values of k since it will define, in a natural way, the highest values of Λ and of the temperature at the beginning of the universe.

3 Deflation Confronts Observations

Time varying Λ models usually modify the predictions of the standard FRW picture at both early and late times, thereby leading to the possibility of constraining the free parameters of any vacuum decaying universe. In the last section we saw that the deflationary process driven by the vacuum decay ansatz (5) has H_I and β as free parameters. However, as we shall see next, the former does not play any role at late times so that all predictions of the model concerning the present universe depend only on the parameter β.

In order to constrain β, we shall discuss some dynamical tests. Following the standard development we define the usual observational parameters $\Omega_0 \equiv 8\pi G\rho_0/3H_0^2$ (the matter density parameter), $q_0 \equiv -R\ddot{R}/\dot{R}^2$ (the decceleration parameter) and $\Omega_{V_0} \equiv \Lambda_0/3H_0^2$ (the vacuum density parameter). Using equations (2), (6) and (7) we obtain the following expressions for these quantities

$$\Omega_{V_0} = \beta\left(1 + \frac{k}{R_0^2 H_0^2}\right) + \mathcal{O}\left(\frac{H_0}{H_I}\right) \;, \tag{17}$$

$$\Omega_0 = (1-\beta)\left(1 + \frac{k}{R_0^2 H_0^2}\right) + \mathcal{O}\left(\frac{H_0}{H_I}\right) \;, \tag{18}$$

$$q_0 = \frac{1-3\beta}{2}\left(1+\frac{k}{R_0^2 H_0^2}\right) + \mathcal{O}\left(\frac{H_0}{H_I}\right) \ . \tag{19}$$

As in the flat case (see equations (11)-(13) of Ref. 17) the last term on the right hand side of the above expressions may always be neglected. More precisely, if the deflationary process begins at the Planck time, $H_I^{-1} \sim 10^{-43}s$ and since $H_0^{-1} \sim 10^{17}s$ it thus follows that $H_0/H_I \sim 10^{-60}$ while the remaining terms are of order unity. Even if deflation begins much later, say at $H_I^{-1} \sim 10^{-35}s$ or $H_I^{-1} \sim 10^{-15}s$ (the respective scales of grand and electroweak unification in the standard model) one obtains $H_0/H_I \sim 10^{-52}$ and $H_0/H_I \sim 10^{-32}$ respectively. Hence, to a high degree of accuracy, H_I is unimportant today and equations (17)-(19) may be written in the simplified forms

$$\Omega_{V_0} = \beta\Omega_{T_0} \ , \tag{20}$$

$$\Omega_0 = (1-\beta)\Omega_{T_0} \ , \tag{21}$$

$$q_0 = \frac{1-3\beta}{2}\Omega_{T_0} \ , \tag{22}$$

where we have introduced the present day total energy density parameter $\Omega_{T_0} = 1 + k/R_0^2 H_0^2$. For $\beta = 0$ the above expressions reduce to the ones of the standard FRW model ($\Omega_V = 0$), whereas for $\beta \neq 0$ but $k = 0$ ($\Omega_{T_0} = 1$), the results of Ref.[19] are readily recovered.

The consistency of the above approximations is easily established by adding equations (20) and (21) to obtain $\Omega_{T_0} = \Omega_0 + \Omega_{V_0}$. Further, by eliminating β from (21) and (22) it follows that

$$\Omega_0 = \frac{2}{3}\Omega_T + \frac{2}{3}q_0 \ , \tag{23}$$

which reduces to the well known result ($\Omega_T = 1$) for zero-curvature, (see, for instance, Ref. [15]). As a matter of fact, one can show that the above relation is quite general, remaining valid for any decaying Λ model. In particular, for $\beta > 1/3$ and $\Omega_{T_0} \leq 1$, equations (22) and (23) imply that flat and open universes satisfy $\Omega_0 < 2/3$, whereas for closed models this holds only if the additional constraint $1 < \Omega_{T_0} < 2/3(1-\beta)$ is imposed. Note also that (21) can be rewritten as

$$\frac{k}{R_0^2} = \left(\frac{\Omega_0}{1-\beta} - 1\right) H_0^2 \; , \tag{24}$$

explaining how the low-energy problem is alleviated in such a scenario, since this is the same as the usual FRW expression but with an effective matter density parameter $\Omega_{eff} = \Omega_0/(1-\beta)$. As we show below, this fact allows us to easily solve the age problem in this context.

The most physically appealing observational data calling for the investigation of cosmological "constant" models involves the so-called "*age problem*". In short, the ages of the oldest globular clusters are estimated to be 16 ± 3 Gyr while, paradoxically, a large value of the Hubble parameter (the natural inverse time scale of the FRW geometries) centered at $H_0 = 80 \pm 17 \; kms^{-1}Mpc^{-1}$ is favored by recent measurements[32]. The root of the conflict is that in the standard flat FRW model this value of H_0 corresponds to an expansion age ($t_0 = 2/3H_0$) of nearly 8.3 Gyr. The situation is even worse if the data of Pierce et.al.[33] ($H_0 = 87 \pm 7kms^{-1}Mpc^{-1}$) are considered. In this case the age is only 7.3Gyr.

Such a paradox is easily resolved in the present decaying Λ-model. As in the flat case[19], the time required by the deflationary process is much longer than the corresponding quasi-FRW phase. Note that, even in the open case, the spacetime is regular at the horizon ($t = 0$) and can be continued beyond this point[27]. Computing the value of the constant A in terms of the observational parameters (see equation (14)), it is straightforward to conclude that a lower bound for the age of the universe is given by

$$t_0 = H_0^{-1} \int_{x_{min}}^{1} \frac{dx}{\sqrt{1 - \frac{\Omega_0}{1-\beta} + \frac{\Omega_0}{1-\beta} x^{-(1-3\beta)}}} \ , \tag{25}$$

where x_{min} is the smallest value of x for which the integrand remains real. In particular, for flat models ($\Omega_{T_0} = 1$, $\Omega_0 = 1 - \beta$, $x_{min} = 0$) this expression yields

$$t_0 = \frac{2}{3(1-\beta)} H_0^{-1} \ , \tag{26}$$

in agreement with Ref. [19]. In what follows all estimates will be made using the somewhat more conservative data of Friedman et.al.[32]. Figure 2 shows the age of the universe (in units of H_0^{-1}) as a function of Ω_0 for some selected values of β. The above mentioned observations restrict the dimensionless age parameter $H_0 t_0$ (which is 2/3 in the standard flat FRW model) to the interval

$$0.85 \leq H_0 t_0 \leq 1.91 \ , \tag{27}$$

which should be compared with the rather conservative bounds ($0.6 \leq H_0 t_0 \leq 1.4$) adopted in Ref. [19]. From (26) and (27) it is easily seen that deflationary models solve the age conflict if the allowed values of β are constrained to be $0.21 \leq \beta \leq 0.64$. It is interesting that for β in this range the values of our observational parameters are restricted to satisfy (see equations (20)-(22))

$$0.63 H_0^2 \leq \Lambda_0 \leq 1.92 H_0^2 \ , \tag{28}$$

$$0.36 \leq \Omega_0 \leq 0.79 \ , \tag{29}$$

$$-0.46 \leq q_0 \leq 0.18 \ , \tag{30}$$

It is worth noting that not only is Λ_0 below the presently accepted upper bound ($\Omega_V \leq 0.8$, $\Lambda_0 \leq 2.4 H_0^2$), but the low-energy problem becomes much less serious. As a matter of fact, if the "best-fit" model consists of $\Omega_T = 1$ with $\Omega_{V_0} = 0.7 \pm 0.1$ and $\Omega_0 = 0.3 \pm 0.1$, as claimed by some authors [1, 10], then $\beta = 0.8 \pm 0.1$ and from (25) the age problem is more easily resolved.

As is well known, vacuum decay Λ-models predict both matter and entropy production[11]-[19]. The present day rate of the former is readily obtained from the energy conservation law $T^{\mu\nu}{}_{;\nu} = 0$ expressed as

$$\dot{\rho} + 3H(\rho + p) = -\frac{1}{8\pi G}\dot{\Lambda} \ , \tag{31}$$

or equivalently, from (4)

$$\frac{1}{R^{3\gamma}}\frac{d}{dt}(\rho R^{3\gamma}) = -\frac{1}{8\pi G}\dot{\Lambda} \ . \tag{32}$$

At the present time ($H \ll H_I$, $\gamma = 1$), the matter production rate is easily computed. Combining equations (5) and (14) it follows that

$$\dot{\Lambda}(t_0) = -9(1-\beta)\beta H_0 \left(H_0^2 + \frac{k}{R_0^2}\right) + \mathcal{O}\left(\frac{H}{H_I}\right) \ ,$$

(in Ref. [19] the factor β is absent) and using (21) we have

$$\left.\frac{1}{R_0^3}\frac{d}{dt}(\rho R^3)\right|_{t_0} = 3\beta H_0 \rho_0 \ , \tag{33}$$

as previously obtained (see equation (17) of Ref. [19]). Therefore the present matter creation rate does not depend explicitly on the curvature parameter. Observe that the factor $3\rho_0 H_0 \sim 10^{-41}\ gcm^{-3}yr^{-1}$ is merely the creation rate appearing in the steady state model and thus lies far below detectable limits. Note also that (31) may be rewritten to yield an expression for the rate of entropy production in this model[11, 14] as

$$T\frac{dS}{dt} = -\frac{\dot{\Lambda}R^3}{8\pi G} \ .$$

In particular, for $H = H_I$ we have $\dot{S} = 0$ and at late times ($H \ll H_I$) it is easy to see that

$$\frac{dS}{dt} = \frac{3\beta H_0 \rho_0 R_0^3}{T_0} \ .$$

At this point it is appropriate to make a remark concerning baryogenesis in these models. The important observational quantity for baryogenesis is the baryon to entropy ratio $\eta \equiv n_b/s$ where n_b is the excess number density of baryons over antibaryons and s is the entropy density. Since in our models both the temperature-scale factor relationship and the entropy density at a given temperature differ from those in the standard FRW picture we expect there to be implications for all baryogenesis scenarios. Naturally, similar remarks can also be made concerning the predictions of light element abundances from primordial nucleosynthesis. In this context we note that the results of Freese et al.[12] indicate very tight bounds on the parameter β, thereby leading to the conclusion that the universe cannot be vacuum dominated for times later than about $t \sim 1s$. However, such a result is in conflict with a wealth of observational indications of a vacuum component in the presently observed universe[10]. This issue will be addressed elsewhere.

4 Final Comments

The study of cosmological models with decaying vacuum energy density has at least a twofold motivation: to determine how the high value of the vacuum energy density that drove inflation became so small at present and to solve the age problem which, by the latest measurements, plagues the standard model for all values of the curvature parameter.

In this paper, the FRW-flat cosmological scenario driven by decaying vacuum energy density as proposed in Ref. [19] has been extended to include the curvature terms. Our deflationary model provides an interesting cosmological history that evolves in three stages: First, an unstable de Sitter configuration is supported by the largest values of the vacuum energy density $\rho_V = 3H_I^2/8\pi G$. Initially, for all values of k, there is no matter or radiation in the usual sense. This happens because H_I is the maximum allowed value for the Hubble parameter and at $H = H_I$ the model yields $\rho = 0$ (see equation (7) and Fig. 1). As we shall see in a moment, this de Sitter initial state is an indispensable ingredient in harmonizing the scenario with the so-called "*cosmological constant problem*". Secondly, the de Sitter configuration evolves to a quasi-FRW vacuum-radiation dominated phase, thereby naturally solving the horizon and other well-known problems in the same manner as in inflation. This is achieved simply by taking $\gamma = 4/3$ in all equations at early times. There genuinely is no flatness problem in this scenario. Such a problem appears in the standard FRW model because the total entropy ($S \sim T^3R^3$) is constant with $T \propto t^{-1/2}$ and $R \propto t^{1/2}$ at times of order the Planck time[34]. As we have shown, these conditions are not satisfied in our model. The burst of entropy and matter is provided by the decay of the vacuum which is solely responsible by the initial de Sitter configurations for $k = 0, \pm1$. The status of the FRW class of geometries is recovered in the sense that only observations can decide if the universe is flat, negatively curved or positively curved nowadays. In other words, the flat ($k = 0$) geometry is no longer theoretically favored. Such an evolution, which for $k = 0$ is exactly described by equation (16), can also be viewed as a noteworthy solution to the "graceful exit" problem of old inflation[35]. Finally, the transition from the vacuum-radiation to the vacuum-dust stage occurs in the same manner as in the standard cosmology.

The ansatz (5) can mathematically be considered as the simplest $\Lambda(t)$ which destabilizes the initial de Sitter configurations. As is well known, in the spirit of quantum cosmology it seems natural to expect negligibly small deviations from such a highly symmetric spacetime at the beginning of the universe (see [36] and references therein). In connection with this we recall that quantum effects in the de Sitter spacetime give rise to a geometrothermodynamic equilibrium state characterized by the Gibbons-Hawking temperature $k_B T = \hbar(\Lambda/12\pi^2)^{1/2}$ [37]. In the present case $\Lambda_I = 3H_I^2$ so that the initial temperature of our scenario is given by

$$T_I = \frac{\hbar H_I}{2\pi k_B} \ , \tag{34}$$

where H_I^{-1}, the arbitrary time scale of the de Sitter state is not fixed by the model. This allows us to make the natural choice that H_I^{-1} be of the order of the Planck time. Indeed, in the framework of quantum cosmology, many authors have suggested that the spontaneous birth of the universe leads naturally to a de Sitter stage with $H^{-1} \sim t_p$ or equivalently $\rho_V = \rho_{\rm PLANCK}$ (see for example [38]). It is remarkable that such a choice, say $H_I = 2\pi t_p^{-1}$, has two interesting consequences: First, from (34) the initial temperature of the universe is just the Planck temperature

$$T_I = \frac{1}{k_B}\sqrt{\frac{\hbar}{G}} \ .$$

Further, since our model essentially predicts $\Lambda_I/\Lambda_0 \sim (H_I/H_0)^2$ we obtain $\Lambda_0 \sim 10^{-118}\Lambda_I$ as theoretically expected. This generalizes the results of Ref. [19] for all values of the curvature parameter. The vacuum energy density decays from $\rho_V = \rho_{\rm PLANCK}$ to the present value $\rho_V \simeq \beta H_0^2$, thereby generating all the matter-energy filling the observable universe. Presumably, the specific form of the constants H_I

and β will be furnished by a fundamental particle physics model of decaying vacuum energy density.

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Figure Captions

Figure 1: The vacuum (full line) and matter (dashed line) energy densities as a function of the Hubble parameter in units of H_I. Note that in these units the present value, H_0, is essentially zero.

Figure 2: The age of the universe in units of H_0^{-1} as a function of Ω_0 for selected values of β. The two horizontal lines on the plot are the allowed range of the age from observations (see equation 27). Note that for $0.21 \leq \beta \leq 0.64$ the age problem is solved.