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Stainless Super p -branes

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Abstract

The elementary and solitonic supersymmetric p -brane solutions to supergravity theories form families related by dimensional reduction, each headed by a maximal ('stainless') member that cannot be isotropically dimensionally oxidized into higher dimensions. We find several new families, headed by stainless solutions in various dimensions $D \leq 9$. In some cases, these occur with dimensions (D, p) that coincide with those of descendants of known families, but since the new solutions are stainless, they are necessarily distinct. The new stainless supersymmetric solutions include a 6-brane and a 5-brane in $D = 9$, a string in $D = 5$, and particles in all dimensions $5 \leq D \leq 9$.

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1 Introduction

Since the discovery of p -brane theories with manifest spacetime supersymmetry [1, 2], it has become increasingly clear that there is a close relationship between such theories and the set of soliton-like solutions to supergravity theories [3]. All the known supersymmetric p -brane theories achieve a matching of the on-shell world-volume bosonic and fermionic degrees of freedom, by virtue of a local fermionic symmetry known as κ symmetry. This symmetry compensates for the excess of fermionic over bosonic degrees of freedom by gauging away half of the former. The consistency of κ symmetry with spacetime supersymmetry places severe constraints on the spacetime dimension D and the world volume dimension $d = p + 1$ [4]. Four classic families of super p -branes were found to satisfy the consistency criterion. The members within each family are related by a process of double dimensional reduction [5], in which both the spacetime and the world volume are simultaneously compactified on a circle, and the dependence on the extra direction is dropped in each space. Thus the classic super p -branes may be classified by giving the maximal-dimensional member of each of the four families. These occur in $(D, d) = (11, 3)$, $(10, 6)$, $(6, 4)$ and $(4, 3)$. On a plot or ‘brane scan’ of D vs d , the additional p -branes obtained by double dimensional reduction lie on the North-east/South-west diagonal lines descending from the maximal cases.

The idea that a super p -brane could be viewed as a long-wavelength description of a topological defect in a supersymmetric theory originated in the construction of the supermembrane in $D = 4$ [1]. This supermembrane occurs as a kink solution of a $D = 4$ chiral scalar supermultiplet theory with a potential giving a degenerate vacuum. A crucial feature of this solution is that half the original supersymmetry is left unbroken. This partial breaking of supersymmetry is also a general feature of all the subsequently-discovered p -brane solitons.

Another feature of super p -branes became clear with the curved-superspace construction of the $D = 11$ supermembrane action in [2], and its generalisations to the other classic super p -branes. This new feature was the occurrence of integrability conditions on the supergravity background that are required for the existence of the world-volume κ symmetry. In the case of the $D = 11$ supermembrane, and of the type IIA string, related to it by double dimensional reduction, these integrability conditions imply the full set of supergravity field equations [2, 5].

The association of super p -branes to supergravity is also natural because the supersymmetric p -branes can be viewed as the natural ‘matter’ sources for the corresponding supergravity theories. A very specific rôle in this association is played by the antisymmetric tensor field

strengths, whose gauge potentials couple directly to the $(p + 1)$ -dimensional world volumes. In the coupled solutions of super p -branes and their corresponding supergravity backgrounds, the backgrounds are naturally singular on the p -brane world volumes, which can act like delta-function sources. These singularities may or may not be clothed by horizons, depending upon the circumstances. Such singular supergravity solutions are called ‘elementary,’ in distinction to the non-singular ‘solitonic’ solutions described previously.

The association of p -branes with singular supergravity solutions was made concrete with the explicit construction of superstring solutions in the case of $N = 1, D = 10$ supergravity [6]. These solutions preserve half of the original supersymmetry, and consequently they saturate a Bogomol’ny bound on the energy density. Subsequently, an analogous elementary membrane solution of $D = 11$ supergravity was found [7]. Many further solutions of supergravity theories have also been found, both for elementary p -branes [8] and for solitonic p -branes [9]. (There are also solitonic solutions in supergravity theories coupled to Yang-Mills, such as that based upon Yang-Mills instantons, and corresponding to the heterotic string [10].)

The multiplicity of elementary and solitonic p -brane solutions to supergravity theories, covering many more values of (D, d) than the classic κ -symmetric points on the brane scan, suggests that the original classification needs to be generalised. Leaving aside for the moment the problem of formulating more general κ -symmetric actions, it is worthwhile to try to find the general pattern of elementary and solitonic p -brane solutions in supergravity theories.

Many supergravity theories in $D \leq 10$ dimensions can be obtained from $D = 11$ supergravity by Kaluza-Klein dimensional reduction, in which a consistent truncation of the higher-dimensional to the lower-dimensional theory is made. Since the truncation is consistent, it follows that solutions of the lower-dimensional theory are also solutions of the higher-dimensional one. This lifting of solutions to the higher dimension is known as dimensional oxidation. In some cases, an elementary or solitonic brane solution in the lower dimension oxidizes to another elementary or solitonic brane solution in the higher dimension. The ability to view an oxidized brane solution as itself being a brane solution depends upon whether the isotropicity of the lower-dimensional solution extends to an isotropicity in the higher-dimensional sense. For the isotropicity to extend, the extra coordinate of the higher-dimensional spacetime must either become isotropically grouped with the p -brane coordinates of the lower dimension, making a $(p + 1)$ -brane, or else it must become isotropically grouped with the coordinates of the transverse space, making a p -brane in the higher dimension. As we shall show later, the latter can never happen within the framework of Kaluza-Klein dimensional reduction. The former, on the other

hand, can occur under certain circumstances. This is the direct analogue, at the level of solutions to supergravity theories, of the process of double dimensional reduction of p -brane actions [5]. Just as for those actions, it is useful in classifying the brane solutions to distinguish between the ones that can be oxidized to isotropic brane solutions of a higher-dimensional supergravity theory, and those that cannot be isotropically oxidized. We shall call the former solutions ‘rusty,’ and the latter solutions ‘stainless.’ Thus when constructing a brane scan of supergravity solutions, one may omit the rusty solutions, which are simply the Kaluza-Klein descendants of stainless solutions in some higher dimension. The full solution set is thus characterised by the stainless solutions.

A frequently-encountered contention in the recent literature is that the only fundamental brane solutions occur in $D = 11$ and $D = 10$ supergravities [11], and that all the others are simply obtained by dimensional reduction. In this paper, we shall show that this is not the case, given our requirement of isotropicity in the oxidation process.¹ In particular, we shall find new stainless brane solutions to supergravity theories in all $5 \leq D \leq 9$. (We shall not be concerned in the present paper with supersymmetric p -brane solutions to super Yang-Mills or other rigid supersymmetric theories.) Amongst other stainless examples, we shall find a 6-brane and a 5-brane in $D = 9$, and a string in $D = 5$, none of which are obtainable from $D = 11$ or $D = 10$ p -brane solutions by dimensional reduction.

2 Solutions and Kaluza-Klein dimensional reduction

2.1 p -brane solutions

We are concerned with elementary and solitonic solutions of supergravity theories that admit interpretations as p -branes embedded in spacetime. These solutions will in general involve the metric tensor g_{MN} , a dilaton ϕ and an n -index antisymmetric tensor $F_{M_1 \dots M_n}$ in D dimensions. The Lagrangian for these fields takes the form

$$\mathcal{L} = eR - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{2n!}e e^{-a\phi} F^2, \quad (1)$$

¹An opposite viewpoint has been put forward in [16], in which all oxidations of brane solutions are regarded as branes in the higher dimension. We prefer not to adopt this viewpoint since, if the isotropicity requirement on the world volume is dropped, the solutions are not ordinary extended objects, and moreover it would not then be clear what degree of anisotropicity should be regarded as acceptable.

where $e = \sqrt{-g}$ is the determinant of the vielbein. The equations of motion are

$$\begin{aligned}\square\phi &= -\frac{a}{2n!} e^{-a\phi} F^2, \\ R_{MN} &= \frac{1}{2} \partial_M \phi \partial_N \phi + S_{MN}, \\ \partial_{M_1} (e e^{-a\phi} F^{M_1 \dots M_n}) &= 0,\end{aligned}\tag{2}$$

where S_{MN} is a symmetric tensor given by

$$S_{MN} = \frac{1}{2(n-1)!} e^{-a\phi} \left(F_{MN}^2 - \frac{n-1}{n(D-2)} F^2 g_{MN} \right).\tag{3}$$

The ansatz for the metric for the D dimensional spacetime is given by [6, 11]

$$ds^2 = e^{2A} dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B} dy^m dy^n \delta_{mn},\tag{4}$$

where x^μ ($\mu = 0, \dots, d-1$) are the coordinates of the $(d-1)$ -brane world volume, and y^m are the coordinates of the $(D-d)$ -dimensional transverse space. The functions A and B depend only on $r = \sqrt{y^m y^m}$. Note that the form of the metric ansatz is preserved under the replacement $r \rightarrow 1/r$. The Ricci tensor for the metric (4) is given by

$$\begin{aligned}R_{\mu\nu} &= -\eta_{\mu\nu} e^{2(A-B)} \left(A'' + dA'^2 + \tilde{d}A'B' + \frac{\tilde{d}+1}{r} A' \right), \\ R_{mn} &= -\delta_{mn} \left(B'' + dA'B' + \tilde{d}B'^2 + \frac{2\tilde{d}+1}{r} B' + \frac{d}{r} A' \right) \\ &\quad - \frac{y^m y^n}{r^2} \left(\tilde{d}B'' + dA'' - 2dA'B' + dA'^2 - \tilde{d}B'^2 - \frac{\tilde{d}}{r} B' - \frac{d}{r} A' \right),\end{aligned}\tag{5}$$

where $\tilde{d} = D-d-2$ and a prime denotes a derivative with respect to r . A convenient choice of vielbein basis for the metric (4) is $e^\mu = e^A dx^\mu$ and $e^m = e^B dy^m$, where underlined indices denote tangent space components. The corresponding spin connection is

$$\begin{aligned}\omega^{\underline{\mu}\underline{n}} &= e^{-B} \partial_n A e^\mu, \quad \omega^{\underline{\mu}\underline{\nu}} = 0, \\ \omega^{\underline{m}\underline{n}} &= e^{-B} \partial_n B e^m - e^{-B} \partial_m B e^n.\end{aligned}\tag{6}$$

For the elementary p -brane solutions, the ansatz for the antisymmetric tensor is given in terms of its potential, and takes the form [6]

$$A_{\mu_1 \dots \mu_{n-1}} = \epsilon_{\mu_1 \dots \mu_{n-1}} e^C,\tag{7}$$

and hence

$$F_{m\mu_1 \dots \mu_{n-1}} = \epsilon_{\mu_1 \dots \mu_{n-1}} \partial_m e^C,\tag{8}$$

where C is a function of r only. Here and throughout this paper $\epsilon_{M\dots N}$ and $\epsilon^{M\dots N}$ are taken to be the tensor densities of weights -1 and 1 respectively, with purely numerical components ± 1 or 0 . Note in particular that they are not related just by raising and lowering indices using the metric tensor. The dimension of the world volume is given by $d = n - 1$ for the elementary p -brane solutions.

For the solitonic $(d - 1)$ -brane solutions, the ansatz for the antisymmetric tensor is [9]

$$F_{m_1\dots m_n} = \lambda \epsilon_{m_1\dots m_n p} \frac{y^p}{r^{n+1}} , \quad (9)$$

where λ is a constant. The power of r is governed by the requirement that F should satisfy the Bianchi identity. The dimension of the world volume is given by $d = D - n - 1$ for the solitonic p -brane solutions.

For both types of solution, the symmetric tensor S_{MN} takes the form

$$\begin{aligned} S_{\mu\nu} &= -\frac{\tilde{d}}{2(D-2)} S^2 e^{2(A-B)} \eta_{\mu\nu} , \\ S_{mn} &= \frac{d}{2(D-2)} S^2 \delta_{mn} - \frac{1}{2} S^2 \frac{y^m y^n}{r^2} . \end{aligned} \quad (10)$$

The function S is given in the two cases by

$$\begin{aligned} \text{elementary :} \quad S &= e^{-\frac{1}{2}a\phi - dA + C} C' & d = n - 1 , \\ \text{solitonic :} \quad S &= \lambda e^{-\frac{1}{2}a\phi - \tilde{d}B} r^{-\tilde{d}-1} & d = D - n - 1 . \end{aligned} \quad (11)$$

With these ansätze, the equations of motion for the dilaton and the metric tensor in (2) become

$$\begin{aligned} \phi'' + dA'\phi' + \tilde{d}B'\phi' + \frac{\tilde{d}+1}{r}\phi' &= \frac{1}{2}\epsilon a S^2 , \\ A'' + dA'^2 + \tilde{d}A'B' + \frac{\tilde{d}+1}{r}A' &= \frac{\tilde{d}}{2(D-2)}S^2 , \\ B'' + dA'B' + \tilde{d}B'^2 + \frac{2\tilde{d}+1}{r}B' + \frac{d}{r}A' &= -\frac{d}{2(D-2)}S^2 , \\ \tilde{d}B'' + dA'' - 2dA'B' + dA'^2 - \tilde{d}B'^2 - \frac{\tilde{d}}{r}B' - \frac{d}{r}A' + \frac{1}{2}\phi'^2 &= \frac{1}{2}S^2 , \end{aligned} \quad (12)$$

where $\epsilon = 1$ for the elementary ansatz and $\epsilon = -1$ for the solitonic ansatz. The equation of motion for the field strength F in (2) is automatically satisfied by the solitonic ansatz (9), whilst for the elementary ansatz (7) it gives rise to the equation

$$C'' + C'(C' + \tilde{d}B' - dA' - a\phi') + \frac{\tilde{d}+1}{r}C' = 0 . \quad (13)$$

Solutions to the equations of motion (12) and (13) can be obtained by making the following ansatz:

$$A' = \frac{\epsilon}{\Lambda} S, \quad \phi' = \frac{\epsilon(D-2)a}{\tilde{d}} A', \quad (14)$$

where Λ is a constant. By choosing Λ such that

$$\Lambda^2 = \frac{(D-2)^2 a^2}{\tilde{d}^2} + \frac{2d(D-2)}{\tilde{d}}, \quad (15)$$

one can eliminate the non-linear terms A'^2 , B'^2 and $A'B'$ from a linear combination of the last three equations in (12).² Then it is a simple matter to solve the equations; the solution is given by

$$\begin{aligned} B &= -\frac{d}{\tilde{d}} A, & \phi &= \frac{a(D-2)}{\epsilon\tilde{d}} A, \\ e^{-cA} &= 1 + \frac{k}{r\tilde{d}}, \end{aligned} \quad (16)$$

where $k = \epsilon\Lambda\lambda/(2(D-2))$ and $c = d + a^2(D-2)/(2\tilde{d})$. In the elementary case, the function C satisfies the equation

$$(e^C)' = \lambda e^{2cA} r^{-\tilde{d}-1}. \quad (17)$$

In presenting these solutions we have chosen simple values for some integration constants where no loss of generality is involved. The solutions (16) are valid when $d\tilde{d} > 0$. For the cases $d = 0$ or $\tilde{d} = 0$, the solutions can also be straightforwardly obtained; an example will be given in section 4.2. Note that the forms of the metrics for both elementary and solitonic $(d-1)$ -branes are the same, although, as we saw earlier, the solutions are obtained from a $(d+1)$ -form antisymmetric tensor field strength in the former case, and from a $(D-d-1)$ -form antisymmetric tensor field strength in the latter case.

So far, we have obtained solutions for the bosonic theory described by the Lagrangian (1) for arbitrary values of the constant a , and with an antisymmetric tensor of arbitrary degree. In supergravity theories, however, there occur antisymmetric tensors of certain specific degrees only, each with its corresponding specific value of the constant a . We may summarise the a values arising in supergravity theories as follows. Without loss of generality, we may discuss all theories in versions where all antisymmetric tensor field strengths have degrees $n \leq D/2$. The a values are given by

$$a^2 = \Delta - \frac{2d\tilde{d}}{D-2}, \quad (18)$$

²There are more general solutions of the equations (12) than those that follow from the ansatz (14,15). However, as we shall see later, when one considers supergravity theories the equations implied by requiring that half the supersymmetry be preserved are equivalent to (14,15).

where

$$d\tilde{d} = (n - 1)(D - n - 1) . \quad (19)$$

Some examples of values of Δ that arise in supergravity theories are $\Delta = 4$ for $n \neq 2$, and $\Delta = 4$ and 2 for $n = 2$. We shall discuss the set of Δ values in more detail in section 4.1. Note that in cases where there is no dilaton, the solution for the A and B functions that appear in the metric ansatz is precisely given by (16) with the value of a taken to be zero. In this sense we can assign the value $\Delta = 2d\tilde{d}/(D - 2)$, which, by eqn (18), sets $a = 0$, in a supergravity theory where there is no dilaton. For example $\Delta = 4$ for the 4-form field strength in $D = 11$ supergravity, $\Delta = 2$ for the 3-form field strength in $D = 6$ self-dual supergravity, and $\Delta = \frac{4}{3}$ for the 2-form field strength in $D = 5$ simple supergravity.

It follows from eqn (16) that the metrics for the brane solutions are given by

$$ds^2 = \left(1 + \frac{k}{r^{\tilde{d}}}\right)^{-\frac{4\tilde{d}}{(D-2)\Delta}} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1 + \frac{k}{r^{\tilde{d}}}\right)^{\frac{4\tilde{d}}{(D-2)\Delta}} dy^m dy^m . \quad (20)$$

This coincides with the results given in ref. [11] for the case of $\Delta = 4$. Note from (15) and (16) that in terms of Δ , the functions A , B and ϕ satisfy

$$A' = \frac{\epsilon\tilde{d}}{(D-2)\sqrt{\Delta}} S , \quad B' = -\frac{\epsilon d}{(D-2)\sqrt{\Delta}} S , \quad \phi' = \frac{a}{\sqrt{\Delta}} S . \quad (21)$$

As we shall see in detail in the next section, some of the $(d-1)$ -brane solutions that we have obtained in a D -dimensional supergravity can be isotropically oxidized to d -brane solutions in a $(D+1)$ -dimensional supergravity. The degree of the antisymmetric tensor involved in a p -brane solution, and the value of the constant a , play crucial rôles in determining whether the solution can or cannot be isotropically oxidized in this way.

At this point, a remark about supersymmetry is in order. In order for the solutions that we have obtained above to acquire an interpretation as *super* $(d-1)$ -branes embedded in D -dimensional spacetime, we shall have to verify that these solutions preserve half of the supersymmetry of the corresponding supergravity theories. We have verified, case by case, that this is indeed true, at least as long as the antisymmetric tensor is part of the supergravity multiplet. In fact, the conditions arising from the requirement of preserving half of the supersymmetries turn out to be precisely equivalent to those that we imposed in the ansatz (14).

In concluding this subsection, we return to a more detailed discussion of a point to which we alluded earlier, namely that we may choose, when discussing the solution set of elementary and solitonic branes in supergravity theories, to restrict our attention to the versions of the

various supergravity theories in which all antisymmetric tensors F_n have degrees n that do not exceed $D/2$. The reason why we may do this without losing generality is that an elementary or solitonic solution of a version of a supergravity theory in which the antisymmetric tensor participating in the solution is dualised is *precisely* the same as the solitonic or elementary solution, respectively, of the undualised form of the supergravity theory. To see this, consider the solitonic solution of (2), with F_n given by the ansatz (9). This has

$$F_n = \frac{1}{n!} F_{m_1 \dots m_n} dy^{m_1} \wedge \dots \wedge dy^{m_n} = \frac{\lambda}{n!} e^{-nB} \epsilon_{m_1 \dots m_n p} \frac{y^p}{r^{n+1}} e^{m_1} \wedge \dots \wedge e^{m_n} . \quad (22)$$

Thus the Hodge dual of this n -form is given by

$$*F_n = \frac{\lambda}{(D-n)!} \frac{y^m}{r^{n+1}} e^{-nB} \epsilon_{\mu_1 \dots \mu_d} e^m \wedge e^{\mu_1} \wedge \dots \wedge e^{\mu_d} . \quad (23)$$

In the dual version of the theory, the $(D-n)$ -form \tilde{F} whose Bianchi identity implies the field equation for F_n given in (2) is $\tilde{F} = e^{-a\phi} *F_n$, which, from (23), has components given by

$$\tilde{F}_{m\mu_1 \dots \mu_d} = \frac{\lambda y^m}{r^{n+1}} e^{dA - \tilde{d}B - a\phi} \epsilon_{\mu_1 \dots \mu_d} . \quad (24)$$

Hence by using (16), with $\tilde{d} = n-1$, we see that $\tilde{F}_{m\mu_1 \dots \mu_d}$ is precisely of the form of the elementary ansatz (7) for a $(d+1)$ -index field strength, where the function C satisfies its equation of motion (17). Thus we see that the solitonic solution of the dualised theory is precisely the same thing as the elementary solution of the undualised theory, and *vice versa*, with the antisymmetric tensor written in different variables. We may therefore, without loss of generality, consider all supergravity theories in their versions where the degrees of their antisymmetric tensors F_n satisfy $n \leq D/2$. The set of all elementary and solitonic brane solutions of these theories spans the entire set of inequivalent brane solutions of these theories together with their dualised versions.

2.2 Kaluza-Klein dimensional reduction

In order to describe the processes of oxidation and reduction, we need to set up the Kaluza-Klein procedure for dimensional reduction from $(D+1)$ to D dimensions. Let us denote the coordinates of a $(D+1)$ -dimensional spacetime by $x^{\tilde{M}} = (x^M, z)$, where z is the coordinate of the extra dimension. The $(D+1)$ -dimensional metric $d\hat{s}^2$ is related to the D -dimensional metric ds^2 by

$$d\hat{s}^2 = e^{2\alpha\varphi} ds^2 + e^{2\beta\varphi} (dz + \mathcal{A}_M dx^M)^2 , \quad (25)$$

where φ and \mathcal{A} are taken to be independent of the extra coordinate z . The constants α and β will be determined shortly. A convenient choice for the vielbein $\hat{e}^{\hat{A}}_{\hat{M}}$ of the $(D+1)$ -dimensional spacetime is

$$\begin{aligned}\hat{e}^A_M &= e^{\alpha\varphi} e^A_M, & \hat{e}^z_M &= e^{\beta\varphi} \mathcal{A}_M, \\ \hat{e}^A_z &= 0, & \hat{e}^z_z &= e^{\beta\varphi}.\end{aligned}\tag{26}$$

Note that M and z denote world indices, whilst A and \underline{z} denote tangent-space indices.

The spin connection is given by

$$\begin{aligned}\hat{\omega}^{AB} &= \omega^{AB} + \alpha e^{-\alpha\varphi} \left(\partial^B \varphi \hat{e}^A - \partial^A \varphi \hat{e}^B \right) - \frac{1}{2} \mathcal{F}^{AB} e^{(\beta-2\alpha)\varphi} \hat{e}^z, \\ \hat{\omega}^{Az} &= -\beta e^{-\alpha\varphi} \partial^A \varphi \hat{e}^z - \frac{1}{2} \mathcal{F}^A_B e^{(\beta-2\alpha)\varphi} \hat{e}^B,\end{aligned}\tag{27}$$

where $\partial_A = E_A^M \partial_M$ is the partial derivative with a tangent-space index, and $\mathcal{F}_{MN} = 2\partial_{[M} \mathcal{A}_{N]}$. Here, E_A^M is the inverse vielbein in D dimensions. Choosing $\beta = -(D-2)\varphi$, we find that the $(D+1)$ -dimensional Einstein-Hilbert action $\hat{e} \hat{R}$ reduces to

$$\hat{e} \hat{R} = eR - (D-1)(D-2)\alpha^2 e(\partial\varphi)^2 - \frac{1}{4} e e^{-2(D-1)\alpha\varphi} \mathcal{F}^2.\tag{28}$$

The Kaluza-Klein dilaton φ may be given its canonical normalisation by choosing the constant α such that

$$\alpha^2 = \frac{1}{2(D-1)(D-2)}.\tag{29}$$

It is sometimes useful to have expressions for the $(D+1)$ -dimensional Ricci tensor. Its tangent-space components are given, after setting $\beta = -(D-2)\alpha$, by

$$\begin{aligned}\hat{R}_{AB} &= e^{-2\alpha\varphi} \left(R_{AB} - (D-1)(D-2)\alpha^2 \partial_A \varphi \partial_B \varphi - \alpha \square \varphi \eta_{AB} \right) - \frac{1}{2} e^{-2D\alpha\varphi} \mathcal{F}_A^C \mathcal{F}_{BC}, \\ \hat{R}_{Az} &= \frac{1}{2} e^{(D-3)\alpha\varphi} \nabla^B \left(e^{-2(D-1)\alpha\varphi} \mathcal{F}_{AB} \right), \\ \hat{R}_{zz} &= (D-2)\alpha e^{-2\alpha\varphi} \square \varphi + \frac{1}{4} e^{-2D\alpha\varphi} \mathcal{F}^2.\end{aligned}\tag{30}$$

Let us now apply the above formalism to the case of a bosonic Lagrangian of the form (1), but in $(D+1)$ rather than D dimensions:

$$\mathcal{L} = \hat{e} \hat{R} - \frac{1}{2} \hat{e} (\partial \hat{\phi})^2 - \frac{1}{2 n!} \hat{e} e^{-\hat{a}\hat{\phi}} \hat{F}_n^2,\tag{31}$$

where we add a subscript index n to indicate that F is an n -form. The Kaluza-Klein ansatz for $\hat{\phi}$ is simply $\hat{\phi} = \phi$, where ϕ is independent of the extra coordinate z . For \hat{F}_n , which is written locally in terms of a potential \hat{A}_{n-1} as $\hat{F}_n = d\hat{A}_{n-1}$, the ansatz for \hat{A}_{n-1} is

$$\hat{A}_{n-1} = B_{n-1} + B_{n-2} \wedge dz,\tag{32}$$

where B_{n-1} and B_{n-2} are potentials for the n -form field strength $G_n = dB_{n-1}$ and the $(n-1)$ -form field strength $G_{n-1} = dB_{n-2}$ in D dimensions. Defining

$$G'_n = G_n - G_{n-1} \wedge \mathcal{A}, \quad (33)$$

where $\mathcal{A} = \mathcal{A}_M dx^M$, one finds

$$\hat{F}_n = G'_n + G_{n-1} \wedge (dz + \mathcal{A}). \quad (34)$$

The tangent-space components of \hat{F}_n in $(D+1)$ dimensions are therefore given by $\hat{F}_{A_1 \dots A_n} = G'_{A_1 \dots A_n} e^{-n\alpha\varphi}$ and $\hat{F}_{A_1 \dots A_{n-1} z} = G_{A_1 \dots A_{n-1}} e^{-(n-1)\alpha\varphi - \beta\varphi}$. Substituting into (31), and using $\beta = -(D-2)\alpha$, we obtain the reduced D -dimensional Lagrangian

$$\begin{aligned} \mathcal{L} = & eR - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{2}e(\partial\varphi)^2 - \frac{1}{4}ee^{-2(D-1)\alpha\varphi} \mathcal{F}^2 \\ & - \frac{e}{2n!} e^{-2(n-1)\alpha\varphi - \hat{a}\phi} G_n'^2 - \frac{e}{2(n-1)!} e^{2(D-n)\alpha\varphi - \hat{a}\phi} G_{n-1}^2, \end{aligned} \quad (35)$$

where α is given by (29). As one sees, different combinations of φ and ϕ appear in the exponential prefactors of $G_n'^2$ and G_{n-1}^2 . Nonetheless, each of these prefactors may easily be seen to be of the form $e^{-a_n \tilde{\phi}}$, where $\tilde{\phi}$ is an $SO(2)$ rotated combination of φ and ϕ . In these prefactors, the coefficients a_n satisfy the formula (18) in D dimensions, with $d\tilde{d}$ given by (19), and with the *same* value of Δ as for \hat{a} in $(D+1)$ dimensions. (Note that $d\tilde{d}$ in (18) is n -dependent, so one obtains different values for the $G_n'^2$ and G_{n-1}^2 prefactors.) The 2-form field strength \mathcal{F} has an a value given by (18) with $\Delta = 4$.

Most supergravity theories can be obtained from 11-dimensional supergravity *via* Kaluza-Klein dimensional reduction. Any such dimensional reduction can be viewed as a sequence of reductions by one dimension at a time, of the kind we are discussing here. Any solution of a lower-dimensional supergravity theory in such a sequence can therefore be reinterpreted as a solution of any one of the higher theories in the sequence by use of the Kaluza-Klein ansatz (25). In particular, this implies that any elementary or solitonic p -brane solution is also a solution in the higher dimensions. However, it is important to realise that the resulting higher dimensional solution may not necessarily preserve the isotropic form of the p -brane ansatz (4). In this paper, we are using the term ‘stainless’ to describe the property of a brane solution of a lower-dimensional supergravity that cannot be oxidized into an isotropic brane solution in any supergravity in the next higher dimension.³ On the other hand, a $(p+1)$ -brane solution

³We note that in defining a stainless p -brane to be one that cannot be oxidized to an isotropic brane in a

in $(D + 1)$ dimensions necessarily gives rise under dimensional reduction to an isotropic p -brane solution in D dimensions. This automatic preservation of isotropicity for solutions under dimensional reduction corresponds directly to the process of double dimensional reduction [5] of p -brane actions.

The above ideas can be illustrated in our example of the bosonic Lagrangians (31) and (35). First, we shall show that the elementary and solitonic solutions in $(D + 1)$ dimensions reduce respectively to elementary and solitonic solutions in D dimensions. In the case of an elementary solution, the n -index antisymmetric tensor in $(D + 1)$ dimensions leads to an elementary brane with world volume dimension $\hat{d} = n - 1$. The elementary ansatz for the $(D + 1)$ -dimensional field strength \hat{F}_n in (31) is $\hat{F}_{m\mu_1\dots\mu_{n-2}z} = \epsilon_{\mu_1\dots\mu_{n-2}z}\partial_m e^C$. It follows from eqn (34) that the corresponding D dimensional fields G_{n-1} , G'_n and \mathcal{A} become

$$\begin{aligned} G_{m\mu_1\dots\mu_{n-2}} &= \epsilon_{\mu_1\dots\mu_{n-2}}\partial_m e^C, \\ G'_{M_1\dots M_n} &= 0, \quad \mathcal{A}_M = 0. \end{aligned} \tag{36}$$

This is nothing but the usual elementary-type ansatz for an $(n - 1)$ -index antisymmetric tensor in D dimensions, and thus gives rise to an elementary brane solution (16) with world volume dimension $d = n - 2$.

The metric ansatz in $(D + 1)$ dimensions is given by $d\hat{s}^2 = e^{2\hat{A}}(dx^\mu dx^\nu \eta_{\mu\nu} + dz^2) + e^{2\hat{B}}dy^m dy^m$. In the elementary solution in $(D + 1)$ dimensions, it follows from (16) that $\phi = \hat{a}(D - 1)\hat{A}/\tilde{d}$, and $\hat{B} = -(d + 1)\hat{A}/\tilde{d}$. (Note that \tilde{d} is the same for both D and $(D + 1)$ dimensions since, by definition, $\tilde{d} + 2$ is the codimension of the world volume of the brane.) On the other hand in D dimensions, we see from (35) that the combination of scalar fields $-2(D - n)\alpha\varphi + \hat{a}\phi = a\tilde{\phi}$, with $a^2 = \hat{a}^2 + 4(D - n)^2\alpha^2$, defines the $SO(2)$ -rotated D -dimensional dilaton $\tilde{\phi}$, whilst the orthogonal combination $2(D - n)\alpha\phi + \hat{a}\varphi$ is set to zero. Since $n = d + 2$, it then follows from (29) that \hat{a} and a are related by

$$\hat{a}^2 = a^2 - \frac{2\tilde{d}^2}{(D - 2)(D - 1)}. \tag{37}$$

higher dimension, we have not wanted to prejudge what a non-stainless p -brane may oxidize into. *A priori*, one could envisage that the extra dimension acquired upon oxidation could either become isotropically included into the world-brane dimensions, giving a $(p + 1)$ -brane in $(D + 1)$ dimensions, or that the extra dimension could be isotropically included into the transverse dimensions, in which case one would still have a p -brane in the $(D + 1)$ dimensions. The latter possibility, however, can never be realised within the scheme of Kaluza-Klein dimensional reduction because all fields are by construction taken to be independent of the extra coordinate, and this would be inconsistent with our ansatz (4).

Thus we find that $\tilde{\phi} = a(D-2)A/\tilde{d}$, $B = -dA/\tilde{d}$ and $e^{cA} = e^{\hat{c}\hat{A}}$, since, from the Kaluza-Klein ansatz (25) for the metric, we have $\hat{A} = A + \alpha\varphi$ and $\hat{B} = B + \alpha\varphi$. But these expressions for $\tilde{\phi}$ and B are precisely of the form given in (16) for the elementary $(d-1)$ -brane in D dimensions. Thus we conclude that under dimensional reduction, an elementary d -brane in $(D+1)$ reduces to an elementary $(d-1)$ -brane in D dimensions.

In the case of solitonic solutions, the analysis is parallel. The ansatz for the n -index antisymmetric tensor, which leads to a solitonic brane solution with world volume dimension $d = D - n$ in $(D+1)$ dimensions, takes the form $\hat{F}_{m_1 \dots m_n} = \lambda \epsilon_{m_1 \dots m_n p} y^p r^{-n-1}$. It follows from eqn (34) that the corresponding D dimensional fields G'_n , G_{n-1} and \mathcal{A} become

$$\begin{aligned} G'_{m_1 \dots m_n} &= \lambda \epsilon_{m_1 \dots m_n p} y^p r^{-n-1} , \\ G_{M_1 \dots M_{n-1}} &= 0 , \quad \mathcal{A}_M = 0 . \end{aligned} \tag{38}$$

This is indeed just the field configuration for a solitonic $(d-1)$ -brane in D dimensions. The analysis of the relation between the metrics in $(D+1)$ and D dimensions is very similar to that in the elementary case.

It is of interest to note that in the reduction of a d -brane in $(D+1)$ dimensions to a $(d-1)$ brane in D dimensions, the degree of the antisymmetric tensor involved in the solution reduces by one in the elementary case, but remains unchanged in the solitonic case. Note also that the relation between \hat{a} and a in eqn (37) is always satisfied in the dimensional reduction of a brane solution in $(D+1)$ to one in D dimensions. This implies, conversely, that eqn (37) is a *necessary* condition for the reverse procedure to be possible. It is easy to verify that the relation (37) is uniquely satisfied with \hat{a} and a given by eqn (18), provided that Δ is the same for both \hat{a} and a .

We have seen that brane solutions in higher dimensions can be reduced to those in lower dimensions *via* the Kaluza-Klein procedure; however, the inverse procedure is not necessarily possible. For example the D -dimensional bosonic Lagrangian (35) that is derived from the $(D+1)$ -dimensional Lagrangian (31) admits six brane solutions, namely an elementary and a solitonic solution for each of the three antisymmetric tensors G_n , G_{n-1} and \mathcal{F} . Two of these solutions are isotropically oxidizable to brane solutions in $(D+1)$ dimensions, by reversing the procedure discussed above. The remaining four solutions are stainless because they cannot be oxidized to isotropic brane solutions of the $(D+1)$ dimensional theory defined by eqn (31). To illustrate this, consider the elementary solution that uses the antisymmetric tensor G_n in the D -dimensional Lagrangian (35). The solution for the metric in D dimensions is given by

(4) with A and B given in eqn (16). This solution can be oxidized into a solution in $(D + 1)$ dimensions, whose metric is given by

$$d\hat{s}^2 = e^{2\hat{A}} dx^\mu dx^\nu \eta_{\mu\nu} + e^{2\hat{B}} (dy^m dy^m + dz^2), \quad (39)$$

where

$$\hat{A} = \frac{(D-2)(\tilde{d}+1)}{(D-1)\tilde{d}} A, \quad \hat{B} = -\frac{(D-2)d}{(D-1)\tilde{d}} A. \quad (40)$$

From the form of this $(D+1)$ -dimensional metric, we can see that it does not describe an isotropic d -brane, since the different r -dependent prefactor for dz^2 prevents z from being grouped together with the coordinates x^μ . Note also that, although dz^2 does have the same prefactor as $dy^m dy^m$, this metric is still not isotropic in the transverse directions because the prefactors $e^{2\hat{A}}$ and $e^{2\hat{B}}$ are functions of $r = \sqrt{y^m y^m}$ and not of $\sqrt{y^m y^m + z^2}$.

To summarise, we have seen that an elementary or solitonic $(p+1)$ -brane solution in $(D+1)$ dimensions can always be reduced respectively to an elementary or solitonic p -brane solution in D dimensions. On the other hand, the inverse process of dimensional oxidation to an isotropic brane solution is not always possible. Thus in a brane scan of elementary and solitonic solutions, we may factor out the rusty solutions and characterise the full solution set by the stainless p -branes only.

There are three cases in which a p -brane solution can turn out to be stainless. The first case is when a brane solution arises in a supergravity theory that cannot be obtained by dimensional reduction, such as $D = 11$ supergravity or type IIB supergravity in $D = 10$. In the remaining two cases, the supergravity theory itself can be obtained by dimensional reduction, but oxidation to an isotropic brane solution is nonetheless not possible. In the second case, no $(D+1)$ -dimensional supergravity theory has the necessary antisymmetric tensor for an isotropic brane solution. Specifically, if the D -dimensional solution is elementary, the $(D+1)$ -dimensional theory would need an antisymmetric tensor of degree one higher than that in the D -dimensional theory. If it is instead a solitonic solution, the $(D+1)$ -dimensional theory would need an antisymmetric tensor of the same degree as in the D -dimensional theory. In the third case, an antisymmetric tensor of the required degree exists in the $(D+1)$ -dimensional theory, but the exponential dilaton prefactor has a coefficient \hat{a} that does not satisfy eqn (37). We shall meet examples of all three cases in the subsequent sections.

3 $D \geq 10$ supergravity

$D = 11$ is the highest dimension for any supergravity theory, and hence all the $D = 11$ p -brane solutions are necessarily stainless. Since there is only one antisymmetric tensor field strength in the theory, namely a 4-index field, there is just one elementary membrane solution [7] and one solitonic 5-brane solution [12].

Dimensional reduction of $D = 11$ supergravity to $D = 10$ yields type IIA supergravity. The type IIA theory contains: a 2-form field strength giving rise to a particle and a 6-brane; a 3-form giving rise to a string and a 5-brane; and a 4-form giving rise to a membrane and a 4-brane. In each case we have listed first the elementary and then the solitonic solution. All of these solutions break half of the $D = 10$, $N = 2$ supersymmetry. Of the six solutions two, namely the elementary string and the solitonic 4-brane, can be oxidized to the corresponding elementary membrane and solitonic 5-brane in $D = 11$. The remaining four solutions are stainless since $D = 11$ supergravity lacks the necessary antisymmetric tensors. Note that the $11 \rightarrow 10$ situation corresponds precisely to the bosonic example we discussed in section 2.2.

In addition, in $D = 10$, there is the type IIB supergravity, which cannot be obtained by dimensional reduction from $D = 11$. This theory contains a complex 3-form field strength giving rise to an elementary string and a solitonic 5-brane solution; and a self-dual 5-form field strength giving rise to a self-dual 3-brane [13]. The string and 5-brane are in fact also solutions of $D = 10$, $N = 1$ supergravity, and are hence identical to the string and 5-brane solutions of the type IIA theory. Thus, although the type IIB theory cannot itself be obtained by dimensional reduction from $D = 11$, these particular solutions of the IIB theory do have an oxidation pathway up to isotropic solutions in $D = 11$. In such situations, we do not consider brane solutions to be stainless. The remaining solution, the self-dual 3-brane, is the only solution that belongs exclusively to the IIB theory. It is stainless and breaks half of the $N = 2$ supersymmetry.

4 $D = 9$ supergravity

4.1 $N = 1$, $D = 9$ supergravity

$N = 1$ supergravity in $D = 9$ [14] contains a 2-form field strength giving rise to an elementary particle and a solitonic 5-brane; and a 3-form field strength giving rise to an elementary string and a solitonic 4-brane. The solitonic 4-brane solution can be isotropically oxidized to the solitonic 5-brane of $N = 1$, $D = 10$ supergravity. The situation is somewhat more complicated

for the oxidation of the elementary string solution. Obviously, this solution cannot be oxidized isotropically to an elementary membrane solution of $N = 1$, $D = 10$ supergravity because this theory lacks the necessary 4-form field strength, and thus no elementary membrane exists in the $N = 1$, $D = 10$ theory. Nonetheless, the $D = 9$ string solution is not stainless because there is a different oxidation pathway available to it. The $D = 9$ string can also be viewed as a solution of $N = 2$, $D = 9$ supergravity. In this guise, it can oxidize isotropically to a solution of type IIA $D = 10$ supergravity, which *does* have a 4-form field strength.

The elementary particle and solitonic 5-brane solution that arise from the 2-form field strength are stainless. Naïvely, one might expect these solutions could oxidize up to the elementary string and solitonic 6-brane solutions of type IIA $D = 10$ supergravity. However, as we showed in section 2.2, even when the necessary forms are present in the higher-dimensional theory an isotropic oxidation is possible only when the coefficient a appearing in the dilaton prefactor $e^{-a\phi}$ satisfies the relation (37). In the case of $N = 1$, $D = 9$ supergravity, the coefficient a is given by eqn (18) with $\Delta = 2$. On the other hand, the coefficient a in the type IIA, $D = 10$ theory is given by eqn (18) with $\Delta = 4$. Since the Δ value has to be preserved under dimensional reduction, it follows that the particle and 5-brane solutions in $D = 9$ are stainless.

There *are* elementary particle and solitonic 5-brane descendants in $D = 9$, nonetheless. These *are* obtained by dimensional reduction from the type IIA $D = 10$ elementary membrane and solitonic 6-brane. From the $D = 9$ point of view, these are obtained as solutions to $N = 2$ supergravity using a 2-form field strength whose dilaton prefactor indeed has an a coefficient given by (18) with the necessary $\Delta = 4$. The difference in Δ values establishes the distinctness of the stainless particle and 5-brane discussed above from those obtained by dimensional reduction. The metrics for the stainless elementary particle and solitonic 5-brane are given by

$$\begin{aligned} \text{elementary:} \quad ds^2 &= -\left(1 + \frac{k}{r^6}\right)^{-12/7} dt^2 + \left(1 + \frac{k}{r^6}\right)^{2/7} dy^m dy^m, \\ \text{solitonic:} \quad ds^2 &= \left(1 + \frac{k}{r}\right)^{-2/7} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1 + \frac{k}{r}\right)^{12/7} dy^m dy^m. \end{aligned} \quad (41)$$

By contrast, the metrics for the elementary particle and solitonic 5-brane that can oxidize to an elementary string and a solitonic 6-brane in $D = 10$ are given by

$$\begin{aligned} \text{elementary:} \quad ds^2 &= -\left(1 + \frac{k}{r^6}\right)^{-6/7} dt^2 + \left(1 + \frac{k}{r^6}\right)^{1/7} dy^m dy^m, \\ \text{solitonic:} \quad ds^2 &= \left(1 + \frac{k}{r}\right)^{-1/7} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1 + \frac{k}{r}\right)^{6/7} dy^m dy^m. \end{aligned} \quad (42)$$

Let us now examine in detail the new stainless $D = 9$ solutions. In particular, we need to verify that they preserve half of the supersymmetry. Since these solutions cannot be obtained

from isotropic solutions in $D = 10$, we do not have an automatic guarantee that half of the supersymmetry will be preserved. To investigate this, we first give the bosonic sector of the Lagrangian and the supersymmetry transformations. The bosonic sector of the Lagrangian is

$$\mathcal{L} = eR - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{12}ee^{-\sqrt{\frac{8}{7}}\phi} G_{MNP}G^{MNP} - \frac{1}{4}ee^{-\sqrt{\frac{2}{7}}\phi} F_{MN}F^{MN} , \quad (43)$$

where $F_{MN} = 2\partial_{[M}A_{N]}$ and $G_{MNP} = 3\partial_{[M}B_{NP]} + \frac{3}{2}A_{[M}F_{NP]}$ [14]. By comparison with eqn (18) it is easy to verify that the Δ value for the 3-form G is 4, but the value for the 2-form F is 2. The supersymmetry transformation rules for the bosonic fields are:

$$\begin{aligned} \delta e^A_M &= -i\bar{\epsilon}\Gamma^A\psi_M , & \delta\phi &= i\sqrt{2}\bar{\epsilon}\chi , \\ \delta A_M &= -\frac{2}{\sqrt{14}}e^{\sqrt{\frac{1}{14}}\phi}\bar{\epsilon}\Gamma_M\chi + \sqrt{2}e^{\sqrt{\frac{1}{14}}\phi}\bar{\epsilon}\psi_M , \\ \delta B_{MN} &= -2ie^{\sqrt{\frac{2}{7}}\phi}\bar{\epsilon}\Gamma_{[M}\psi_{N]} + \frac{2i}{\sqrt{7}}e^{\sqrt{\frac{2}{7}}\phi}\bar{\epsilon}\Gamma_{MN}\chi + A_{[M}\delta A_{N]} . \end{aligned} \quad (44)$$

For the fermionic fields, the supersymmetry transformations are:

$$\begin{aligned} \delta\chi &= -\frac{1}{2\sqrt{2}}\Gamma^M\epsilon\partial_M\phi + \frac{1}{12\sqrt{7}}e^{-\sqrt{\frac{2}{7}}\phi}G_{MNP}\Gamma^{MNP}\epsilon - \frac{i}{4\sqrt{14}}e^{-\sqrt{\frac{1}{14}}\phi}F_{MN}\Gamma^{MN}\epsilon , \\ \delta\psi_M &= D_M\epsilon + \frac{1}{84}e^{-\sqrt{\frac{2}{7}}\phi}G_{NPQ}\left(\Gamma_M^{NPQ} - \frac{15}{2}\delta_M^N\Gamma^{PQ}\right)\epsilon \\ &\quad - \frac{i}{28\sqrt{2}}e^{-\sqrt{\frac{1}{14}}\phi}F_{NP}\left(\Gamma_M^{NP} - 12\delta_M^N\Gamma^P\right)\epsilon . \end{aligned} \quad (45)$$

The elementary particle and solitonic 5-brane in $D = 9$ dimensions are obtained from the ansätze for the 2-index antisymmetric tensor field strength F_{MN} given in (7) and (9) respectively. The solutions are given by (16). We shall first verify that the solitonic 5-brane solution preserves half of the supersymmetry. We begin by making a 6 + 3 split of the gamma matrices:

$$\Gamma^\mu = \gamma^\mu \otimes \mathbf{1} , \quad \Gamma^m = \gamma_7 \otimes \gamma^m , \quad (46)$$

where $\gamma_7 = \gamma_0\gamma_1\dots\gamma_5$ on the world volume and $\gamma_1\gamma_2\gamma_3 = i$ in the transverse space. Here, and throughout the paper, we adopt the convention that γ_μ and γ_m are purely numerical matrices, with flat indices. The transformation rules for the fermionic fields in (45) become

$$\begin{aligned} \delta\chi &= -\frac{1}{2\sqrt{2}}e^{-B}\partial_m\phi\gamma_7\otimes\gamma_m\epsilon + \frac{\lambda}{2\sqrt{14}}e^{-2B-\sqrt{\frac{1}{14}}\phi}\frac{y^m}{r^3}\mathbf{1}\otimes\gamma_m\epsilon , \\ \delta\psi_\mu &= \frac{1}{2}\partial_m A e^{A-B}\gamma_\mu\gamma_7\otimes\gamma_m\epsilon + \frac{\lambda}{14\sqrt{2}}e^{A-2B-\sqrt{\frac{1}{14}}\phi}\frac{y^m}{r^3}\gamma_\mu\otimes\gamma_m\epsilon , \\ \delta\psi_m &= \partial_m\epsilon + \frac{i}{2}\partial_n B\epsilon_{mnp}\mathbf{1}\otimes\gamma_p + \frac{\lambda}{14\sqrt{2}}e^{-B-\sqrt{\frac{1}{14}}\phi}\frac{y^m}{r^3}\gamma_7\otimes\mathbf{1}\epsilon \\ &\quad - \frac{3i\lambda}{7\sqrt{2}}e^{-B-\sqrt{\frac{1}{14}}\phi}\epsilon_{mnp}\frac{y^n}{r^3}\gamma_7\otimes\gamma_p\epsilon . \end{aligned} \quad (47)$$

Substituting the solitonic solution (16) and noting that ϕ' and A' satisfy (14) and (15), we find that these variations all vanish provided that

$$\varepsilon = e^{\frac{1}{2}A} \varepsilon_0, \quad \gamma_7 \otimes \mathbf{1} \varepsilon_0 = \varepsilon_0, \quad (48)$$

where ε_0 is a constant spinor. Thus our solitonic 5-brane solution preserves half of the supersymmetry.

We shall now verify that the elementary particle solution also preserves half the supersymmetry. We make a 1 + 8 split of the gamma matrices:

$$\Gamma^0 = i\gamma_9, \quad \Gamma^m = \gamma^m, \quad (49)$$

where $\gamma_9 = \gamma_1\gamma_2 \cdots \gamma_8$. The transformation rules (45) for the fermionic fields become

$$\begin{aligned} \delta\chi &= -\frac{1}{2\sqrt{2}} e^{-B} \partial_m \phi \gamma_m \varepsilon + \frac{1}{2\sqrt{14}} e^{-A-B+C-\sqrt{\frac{1}{14}}\phi} \partial_m C \gamma_m \gamma_9 \varepsilon, \\ \delta\psi_0 &= \frac{i}{2} e^{A-B} \partial_m A \gamma_m \gamma_9 \varepsilon - \frac{3i}{7\sqrt{2}} e^{-B+C-\sqrt{\frac{1}{14}}\phi} \partial_m C \gamma_m \varepsilon, \\ \delta\psi_m &= \partial_m \varepsilon + \frac{i}{2} \partial_n B \gamma_{mn} \varepsilon + \frac{1}{14\sqrt{2}} e^{-A+C-\sqrt{\frac{1}{14}}\phi} \partial_m C \gamma_{mn} \gamma_9 \varepsilon \\ &\quad - \frac{3}{7\sqrt{2}} e^{-A+C-\sqrt{\frac{1}{14}}\phi} \partial_m C \gamma_9 \varepsilon. \end{aligned} \quad (50)$$

Analogously to the solitonic case, the elementary particle solution also preserves half of the supersymmetry provided that

$$\varepsilon = e^{\frac{1}{2}A} \varepsilon_0, \quad \gamma_9 \varepsilon_0 = \varepsilon_0, \quad (51)$$

So far we have obtained a stainless elementary particle and stainless solitonic 5-brane. Both solutions break half of the supersymmetry. The reason why these two solutions cannot be isotropically oxidized into $D = 10$ dimensions is that both are obtained from the 2-index antisymmetric tensor field strength with the dilaton prefactor $e^{-a\phi}$ where the a coefficient is given by (18) with $\Delta = 2$, instead of the value $\Delta = 4$ that characterises the prefactors of antisymmetric tensor field strengths in $D = 10$. At first sight the occurrence of this new value of Δ may seem paradoxical since, Kaluza-Klein dimensional reduction preserves the value of Δ , as we discussed for the scalar field $\tilde{\phi}$ defined below eqn (35). Since $N = 1$, $D = 9$ supergravity can be obtained by dimensional reduction of $N = 1$, $D = 10$ supergravity, which has a single 3-form field strength, with a $\Delta = 4$ prefactor, it follows that all the antisymmetric tensors in $D = 9$ will have $\Delta = 4$ prefactors.

The resolution of this apparent paradox involves details of the truncation of dimensionally reduced $N = 1$, $D = 10$ supergravity to the pure $N = 1$ supergravity multiplet in $D = 9$. The

truncation removes a single $D = 9$ Maxwell multiplet. The Lagrangian of the bosonic sector of $N = 1, D = 10$ supergravity is

$$\mathcal{L} = \hat{e}\hat{R} - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{12}\hat{e}e^{-\phi}\hat{F}_{MNP}\hat{F}^{MNP} . \quad (52)$$

Following the Kaluza-Klein dimensional reduction scheme discussed in section 2.2, this leads to the $D = 9$ Lagrangian

$$\begin{aligned} \mathcal{L} = & eR - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{2}e(\partial\varphi)^2 - \frac{1}{12}ee^{-\phi-\sqrt{\frac{1}{7}}\varphi}G_3'^2 \\ & - \frac{1}{4}ee^{-\frac{4}{\sqrt{7}}\varphi}\mathcal{F}^2 - \frac{1}{4}ee^{-\phi+\frac{3}{\sqrt{7}}\varphi}G_2^2 . \end{aligned} \quad (53)$$

As it stands, one cannot consistently truncate out either of the 2-form field strengths or either of the two scalars. Nonetheless, it is possible to make a consistent truncation to the bosonic sector of pure $N = 1, D = 9$ supergravity.⁴ In order to do this, we must first rotate the basis for the scalar fields:

$$\phi = \sqrt{\frac{7}{8}}\phi_1 - \sqrt{\frac{1}{8}}\phi_2 , \quad \varphi = \sqrt{\frac{1}{8}}\phi_1 + \sqrt{\frac{7}{8}}\phi_2 . \quad (54)$$

In terms of this rotated basis, the Lagrangian (53) becomes

$$\begin{aligned} \mathcal{L} = & eR - \frac{1}{2}e(\partial\phi_1)^2 - \frac{1}{2}e(\partial\phi_2)^2 - \frac{1}{12}ee^{-\sqrt{\frac{8}{7}}\phi_1}G_3'^2 \\ & - \frac{1}{4}ee^{-\sqrt{\frac{2}{7}}\phi_1-\sqrt{2}\phi_2}\mathcal{F}^2 - \frac{1}{4}ee^{-\sqrt{\frac{2}{7}}\phi_1+\sqrt{2}\phi_2}G_2^2 . \end{aligned} \quad (55)$$

Now we can see that it is consistent with the equation of motion for ϕ_2 to set $\phi_2 = 0$ provided that at the same time we set \mathcal{F} equal to G_2 . Defining then $F = \sqrt{2}\mathcal{F} = \sqrt{2}G_2$, we obtain the Lagrangian for the bosonic sector of pure $N = 1, D = 9$ supergravity:

$$\mathcal{L} = eR - \frac{1}{2}e(\partial\phi_1)^2 - \frac{1}{12}ee^{-\sqrt{\frac{8}{7}}\phi_1}G_3'^2 - \frac{1}{4}ee^{-\sqrt{\frac{2}{7}}\phi_1}F^2 , \quad (56)$$

where $dG_3' + \frac{1}{2}F \wedge F = 0$. This result coincides with the Lagrangian given in ref. [14], which appears in eqn (43). Thus although the value of Δ for the combinations $-\sqrt{\frac{2}{7}}\phi_1 \pm \sqrt{2}\phi_2$ occurring in the 2-form field strength prefactors before truncation is $\Delta = 4$, the value after the truncation in which ϕ_2 is set equal to zero is $\Delta = 2$.⁵

⁴The possibility of making a consistent truncation to the $N = 1, D = 9$ supermultiplet may be shown using arguments similar to those in ref. [15]

⁵Another point of view for resolving the apparent paradox is to regard the stainless particle and 5-brane as solutions of the full dimensionally reduced $N = 1, D = 10$ supergravity, *i.e.* $N = 1, D = 9$ supergravity plus the Maxwell multiplet. From this point of view, these solutions fall outside out p -brane ansätze (7) and (9) because more than one antisymmetric tensor field strength takes a non-vanishing value. The solutions arising from this new ansatz are equivalent to those in the truncated $N = 1$ theory with $\Delta = 2$.

Having studied this example in detail, we are now in a position to be more precise about the possible values of Δ that can arise in supergravity theories. We have seen that we may treat $D = 11$ supergravity, which has no dilaton, as having the value $\Delta = 4$ for its 4-form field strength, since this value corresponds, by virtue of eqn (18), to $a = 0$. We have also seen that pure Kaluza-Klein dimensional reduction, where one performs no truncation on the lower-dimensional theory, preserves the values of Δ from the higher dimension. Thus in the absence of any truncation, all supergravity theories that are obtained by dimensional reduction from $D = 11$ will have $\Delta = 4$ for all dilaton couplings. However, as we demonstrated in the case of $N = 1, D = 9$ supergravity above, if a supergravity theory in a lower dimension is obtained by a process of *truncation* as well as dimensional reduction, then the values of Δ for the coupling of the particular combinations of dilaton fields that survive the truncation to the antisymmetric tensor combinations that survive the truncation can differ from 4. For example, one can have $\Delta = 2$ for 2-form field strengths in $D \leq 9$ supergravities.

Before ending this section, it is of interest to investigate the warped metrics that one does obtain in $D = 10$ if one oxidizes the stainless elementary particle and solitonic 5-brane from $D = 9$, so as to compare them with the isotropic metrics of the elementary string and solitonic 6-brane occurring in $D = 10$. The metrics obtained by oxidizing the stainless $D = 9$ solutions are given by

$$\begin{aligned} \text{elementary :} \quad d\hat{s}^2 &= \left(1 + \frac{k}{r^6}\right)^{-7/4} dx^\mu dx^\nu \eta_{\mu\nu} + \left(1 + \frac{k}{r^6}\right)^{1/4} \left(dy^m dy^m + (dz + \mathcal{A})^2\right), \\ \text{solitonic :} \quad d\hat{s}^2 &= \left(1 + \frac{k}{r}\right)^{-1/4} \left(dx^\mu dx^\nu \eta_{\mu\nu} + (dz + \mathcal{A})^2\right) + \left(1 + \frac{k}{r}\right)^{7/4} dy^m dy^m. \end{aligned} \quad (57)$$

Here we see that we have pushed oxidation too far: neither of these two metrics describes isotropic brane solutions in $D = 10$. In both cases there is a non-vanishing gauge potential \mathcal{A} , which describes a topologically non-trivial field configuration, implying that z is a coordinate on a non-trivial $U(1)$ fibre bundle, and thus the metric is ‘twisted.’ Furthermore, in order for this coordinate to be well defined, it must be taken to be periodic with period $\Delta z = \int \mathcal{F}$ (or $\int \mathcal{F}$ divided by any integer). In the elementary case, as we also saw in the general example given in eqn (39), the metric would not be isotropic even if \mathcal{A} were equal to zero, for the reasons we discussed. By contrast, the metrics for the *isotropic* elementary string and solitonic 6-brane are given by (20) with $D = 10$ and $\Delta = 4$, by taking $d = 2$ and $d = 7$ respectively.

4.2 $N = 2, D = 9$ supergravity

$N = 2$ supergravity in $D = 9$ contains three 2-form, two 3-form and one 4-form field strengths. In addition there are three scalar fields. Two of these behave like dilatons and appear undifferentiated in exponential prefactors multiplying the kinetic terms for the antisymmetric tensors. The third scalar does not appear in exponential prefactors in the Lagrangian; furthermore, its kinetic term itself has a dilaton prefactor. Thus we may view this scalar field as the 0-form potential for a 1-form field strength. We can use this field strength to obtain a solitonic 6-brane in $D = 9$.

$N = 2, D = 9$ supergravity has not yet been constructed; however, it could be easily obtained by dimensional reduction of type IIA supergravity in $D = 10$. We expect that the elementary and solitonic brane solutions that are obtained from the 2-form, 3-form and 4-form field strengths are either obtainable by dimensional reduction from those in $D = 10$ or are equivalent to the stainless solutions we constructed in $N = 1, D = 9$ supergravity. However, the solitonic 6-brane that is associated with the 1-form field strength is necessarily stainless, since the 1-form field strength appears first in $D = 9$ supergravity in the descent from eleven dimensions. We shall first obtain the solution and then shall verify that it preserves half of the supersymmetry.

The Lagrangian of the relevant part of the bosonic sector of $N = 2, D = 9$ supergravity can be obtained by Kaluza-Klein dimensional reduction of the metric, dilaton and 2-form field strength in type IIA supergravity in $D = 10$, whose Lagrangian is given by eqn (31) with $n = 2$ and $\hat{a} = 3/2$. The reduced 9-dimensional Lagrangian is given by (35), again with $n = 2$ and $\hat{a} = 3/2$. In order to obtain a solitonic 6-brane solution, we can consistently set $\mathcal{F} = 0, G'_2 = 0$ and furthermore truncate out one of the two scalar field degrees of freedom by setting

$$\frac{3}{2}\phi - 14\alpha\varphi = 2\tilde{\phi}, \quad 14\alpha\phi + \frac{3}{2}\varphi = 0. \quad (58)$$

Thus the Lagrangian for the relevant bosonic fields in $D = 9$ is

$$\mathcal{L} = eR - \frac{1}{2}e(\partial\tilde{\phi})^2 - \frac{1}{2}ee^{-2\tilde{\phi}}G_1^2. \quad (59)$$

This construction, which precisely parallels the previous discussions for general values of n , emphasises that $G_M = \partial_M b$ should properly be thought of as the field strength for the 0-form gauge potential $b = \hat{A}_z$, since it has its origin in the gauge field \hat{F}_2 in $D = 10$. Thus it is legitimate for G_1 to take the necessary topologically non-trivial form in the solitonic 6-brane solution in $D = 9$, in which its 0-form potential is well-defined only in patches.

Using the ansatz for G_1 given by eqn (9), we can obtain the 6-brane solution. However in this case $\tilde{d} = 0$, and hence the general solution given by eqn (16) no longer applies. Nonetheless the equations (12) are easy to solve; the metric of the solitonic 6-brane in $D = 9$ is given by

$$ds^2 = dx^\mu dx^\nu \eta_{\mu\nu} + (1 + k \log r) dy^m dy^m, \quad (60)$$

and the dilaton field $\tilde{\phi}$ is given by $e^{\tilde{\phi}} = 1 + k \log r$. It satisfies $\tilde{\phi}' = S$, where S is given in eqn (11).

If $N = 2$, $D = 9$ supergravity had been constructed, it would have been a simple matter to check whether the above solution preserved half of the supersymmetry. In lieu of this, we may exploit the fact that Kaluza-Klein dimensional reduction preserves unbroken supersymmetry, and carry out the computation for the corresponding oxidized brane solution in $D = 10$. Of course, since the 6-brane in $D = 9$ is stainless, the resulting oxidized metric will not be an isotropic 7-brane. In fact it takes the form

$$d\hat{s}^2 = e^{-\frac{1}{8}\tilde{\phi}} dx^\mu dx^\nu \eta_{\mu\nu} + e^{\frac{7}{8}\tilde{\phi}} (dy^m dy^m + dz^2). \quad (61)$$

The relevant terms in the fermionic transformation rules of type IIA, $D = 10$ supergravity, involving the non-vanishing 2-form field strength \hat{F}_2 , are:

$$\begin{aligned} \delta\chi &= \frac{\sqrt{2}}{4} \partial_{\hat{M}} \phi \hat{\Gamma}^{\hat{M}} \hat{\Gamma}^{11} \varepsilon - \frac{3}{16\sqrt{2}} e^{-\frac{3}{4}\phi} \hat{F}_{\hat{M}\hat{N}} \hat{\Gamma}^{\hat{M}\hat{N}} \varepsilon, \\ \delta\psi_{\hat{M}} &= \hat{D}_{\hat{M}} \varepsilon - \frac{1}{64} e^{-\frac{3}{4}\phi} \hat{F}_{\hat{N}\hat{P}} (\hat{\Gamma}_{\hat{M}}^{\hat{N}\hat{P}} - 14 \delta_{\hat{M}}^{\hat{N}} \hat{\Gamma}^{\hat{P}}) \hat{\Gamma}^{11} \varepsilon. \end{aligned} \quad (62)$$

It follows from (34) that the 2-form field strength is given by $\hat{F}_{mz} = G_m = \lambda \varepsilon_{mn} y^n / r^2$. The functions A and B appearing in the $D = 9$ solitonic 6-brane metric, the Kaluza-Klein scalar φ , and the $D = 10$ dilaton ϕ are given in terms of $\tilde{\phi}$ by $A = 0$, $B = \frac{1}{2}\tilde{\phi}$, $\alpha\varphi = -\frac{1}{16}\tilde{\phi}$, $\phi = \frac{3}{4}\tilde{\phi}$. The Kaluza-Klein vector potential \mathcal{A}_M is equal to zero. With these, and the expressions (27) for the $D = 10$ spin connection appearing in $\hat{D}_{\hat{M}}$ in terms of the $D = 9$ spin connection and φ , it is now straightforward to substitute the oxidized solution into the fermionic transformation rules given in eqn (62). We find that half the supersymmetry is preserved if ε satisfies the conditions

$$\varepsilon = e^{-\frac{1}{32}\tilde{\phi}} \varepsilon_0, \quad \hat{\Gamma}_{\underline{mn}} \varepsilon_0 = -\varepsilon_{mn} \hat{\Gamma}_{\underline{z}} \hat{\Gamma}^{11} \varepsilon_0, \quad (63)$$

where ε_0 is a constant spinor. Having demonstrated in $D = 10$ that the non-isotropic oxidation (61) of the 6-brane preserves half of the type IIA, $D = 10$ supersymmetry, it follows that the 6-brane solution itself in $D = 9$ also preserves half of the $N = 2$, $D = 9$ supersymmetry.

5 $D \leq 8$ supergravity

As one descends through the dimensions, starting at $D = 11$, one encounters various stainless brane solutions. First of all, they occur if the supergravity theory in a given dimension cannot be obtained from dimensional reduction. This happens in $D = 11$, and $D = 10$ for type IIB supergravity. A second reason for the occurrence of stainless brane solutions is if no supergravity theory in the next higher dimension has the necessary antisymmetric tensor field strength. The above two reasons account for all the stainless brane solutions in $D = 11$ and $D = 10$, and the stainless solitonic 6-brane in $D = 9$. By the time one has reached $D = 9$, all possible degrees $n \leq D/2$ for antisymmetric tensor field strengths have occurred. Because of this, any further stainless brane solutions in $D \leq 8$ will arise only for the third of the reasons we discussed in section 2.2, namely, that the Δ values for the exponential dilaton prefactors of the relevant antisymmetric tensors in the higher and lower dimensions differ. This phenomenon already occurred for the 2-form field strength in $D = 9$, giving rise to the stainless elementary particle and solitonic 5-brane, as we discussed in the previous section.

In view of the above considerations, it is not surprising that further stainless brane solutions in $D \leq 8$ are relatively sparse. In fact in $D = 8$ and $D = 7$, the only stainless solutions are elementary particles in each case. These solutions arise using the 2-form field strength with $\Delta = 2$. They are stainless since all the 3-form field strengths in one dimension higher have $\Delta = 4$.

In $D = 6$, analogously to the cases of $D = 8$ and $D = 7$, there is also a stainless elementary particle obtained from the 2-form field strength with $\Delta = 2$, which is part of the supergravity multiplet in $N = 2$, $D = 6$ supergravity. In $N = 1$, $D = 6$ supergravity, on the other hand, there exists a self-dual 3-form field strength, and there is no dilaton. As we discussed in section 2.2, brane solutions are still given by eqn (16), with a set to zero, even in the absence of the dilaton. Thus this self-dual string solution is equivalent to the case where $\Delta = 2$, with the metric given by (20). Since there is no supergravity theory in $D = 7$ that contains a 3-form or 4-form field strength with $\Delta = 2$, the self-dual string in $D = 6$ is stainless.

The existence of a 3-form with $\Delta = 2$ in $D = 6$ implies that there is no further stainless elementary particle in $D \leq 5$ that arises from the 2-form field strength with $\Delta = 2$. However, in $N = 1$, $D = 5$ supergravity, a new value of Δ for the 2-form field strength arises, namely $\Delta = 4/3$. This reflects the fact that there is no dilaton in the theory. This 2-form field strength accordingly gives rise to a stainless elementary particle and a stainless solitonic string, with

metrics given by eqn (20). To see how this works, we can carry out the Kaluza-Klein dimensional reduction of $N = 1$, $D = 6$ supergravity. Its bosonic sector comprises just the metric tensor and the self-dual 3-form field strength mentioned above. Since there is no covariant action for this theory, we must instead implement the dimensional reduction on the equations of motion themselves. The bosonic equations of motion are given by

$$\hat{R}_{\hat{M}\hat{N}} = \frac{1}{4}\hat{F}_{\hat{M}\hat{P}\hat{Q}}\hat{F}_{\hat{N}}^{\hat{P}\hat{Q}} , \quad F_{\hat{M}\hat{N}\hat{P}} = *F_{\hat{M}\hat{N}\hat{P}} . \quad (64)$$

The Kaluza-Klein ansatz for the metric and the antisymmetric tensor are given by (25) and (34) as usual, but now, the self-duality condition $\hat{F} = *\hat{F}$ implies that the lower-dimensional 2-form and 3-form field strengths G_2 and G'_3 are related:

$$G'_{ABC} = \frac{1}{2}e^{4\alpha\varphi} \epsilon_{ABCDE} G^{DE} , \quad (65)$$

where α , given by (29), takes the value $\alpha = 1/(2\sqrt{6})$. Substituting these ansätze into the 6-dimensional equations of motion (64), and making use of the expressions (30) for the Ricci-tensor components, we obtain the 5-dimensional equations of motion

$$\begin{aligned} R_{AB} &= \frac{1}{2}\partial_A\varphi\partial_B\varphi + \frac{1}{2}e^{-8\alpha\varphi}(\mathcal{F}_{AB}^2 - \frac{1}{6}\mathcal{F}^2\eta_{AB}) + e^{4\alpha\varphi}(G_{AB}^2 - \frac{1}{6}G^2\eta_{AB}) , \\ \nabla^B(e^{-8\alpha\varphi}\mathcal{F}_{AB}) &= \frac{1}{4}e^{3\alpha\varphi}\epsilon_{ABCDE}G^{BC}G^{DE} , \\ \square\varphi &= 2\alpha e^{4\alpha\varphi}G^2 - 2\alpha e^{-8\alpha\varphi}\mathcal{F}^2 . \end{aligned} \quad (66)$$

We see that we may consistently truncate these fields to those of minimal $D = 5$ supergravity, whose bosonic sector comprises just the metric and a 2-form field strength, by setting $\varphi = 0$ and $\mathcal{F}_{AB} = G_{AB}$. Defining $F_{AB} \equiv \sqrt{3}\mathcal{F}_{AB} = \sqrt{3}G_{AB}$, we find that the equations of motion for the remaining fields can be derived from the Lagrangian

$$\mathcal{L} = eR - \frac{1}{4}eF^2 - \frac{1}{12\sqrt{3}}\epsilon^{MNPQR}F_{MN}F_{PQ}A_R , \quad (67)$$

where $F_{MN} = 2\partial_{[M}A_{N]}$. This Lagrangian describes the bosonic sector of minimal $D = 5$ supergravity. We see that a 2-form field strength with a new value of Δ has emerged in the descent to five dimensions, namely $\Delta = \frac{4}{3}$ and hence $a = 0$. It follows that brane solutions in minimal $D = 5$ supergravity, which make use of this 2-form field strength, cannot be oxidized to give isotropic brane solutions in any higher dimension. In this way, we obtain the stainless elementary particle and solitonic string solutions referred to above. Their metrics are given by (20), with $d = 1$, $\tilde{d} = 2$ and $d = 2$, $\tilde{d} = 1$ respectively, where $\Delta = \frac{4}{3}$.

To check the unbroken supersymmetry of these solutions, we need the gravitino transformation rule in $D = 5$ simple supergravity, which reads

$$\delta\psi_M = D_M\varepsilon - \frac{i}{8\sqrt{3}}F_{NP}\left(\Gamma_M^{NP} - 4\delta_M^N\Gamma^P\right)\varepsilon. \quad (68)$$

For the solitonic string, we decompose the $D = 5$ gamma matrices in the $2 + 3$ split $\Gamma^\mu = \gamma^\mu \otimes \mathbf{1}$, $\Gamma^m = \gamma_3 \otimes \gamma^m$, where $\gamma_0\gamma_1 = \gamma_3$ on the brane volume, and $\gamma_1\gamma_2\gamma_3 = i$ in the transverse space. Thus we have

$$\begin{aligned} \delta\psi_\mu &= \frac{1}{2}\partial_m A e^{A-B} \gamma_\mu\gamma_3 \otimes \gamma_m\varepsilon + \frac{\lambda}{4\sqrt{3}}e^{A-2B} \frac{y^m}{r^3} \gamma_\mu \otimes \gamma_m\varepsilon, \\ \delta\psi_m &= \partial_m\varepsilon + \frac{i}{2}\partial_n B \epsilon_{mnp} \mathbf{1} \otimes \gamma_p\varepsilon + \frac{\lambda}{4\sqrt{3}}e^{-B} \frac{y^m}{r^3} \gamma_3 \otimes \mathbf{1}\varepsilon - \frac{i\lambda}{2\sqrt{3}}e^{-B} \frac{y^n}{r^3} \gamma_3 \otimes \gamma_p\varepsilon. \end{aligned} \quad (69)$$

Substituting the solitonic string solution, which, from (21) in the limit $a = 0$ has $A' = -\frac{1}{2}B' = -\lambda/(2\sqrt{3})e^{-B}r^{-2}$, we find that half of the supersymmetry is preserved provided that

$$\varepsilon = e^{\frac{1}{2}A} \varepsilon_0, \quad \gamma_3 \otimes \mathbf{1} \varepsilon_0 = \varepsilon_0, \quad (70)$$

where ε_0 is a constant spinor.

For the elementary particle, we decompose the $D = 5$ gamma matrices in the $1 + 4$ split $\Gamma^0 = i\gamma_5$, $\Gamma^m = \gamma^m$, where $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. The supersymmetry transformation rule (68) becomes

$$\begin{aligned} \delta\psi_0 &= \frac{i}{2}e^{A-B} \partial_m A \gamma_m\gamma_5\varepsilon - \frac{i}{2\sqrt{3}}e^{C-B} \partial_m C \gamma_m\varepsilon, \\ \delta\psi_m &= \partial_m\varepsilon + \frac{1}{2}\partial_n B \gamma_{mn}\varepsilon + \frac{1}{4\sqrt{3}}e^{C-A} \partial_n C \gamma_{mn}\gamma_5\varepsilon - \frac{1}{2\sqrt{3}}e^{C-A} \partial_m C \gamma_5\varepsilon. \end{aligned} \quad (71)$$

Substituting the elementary particle solution, which, from (21) in the limit $a = 0$ has $A' = -2B' = (1/\sqrt{3})e^{C-A}C'$, we find that half of the supersymmetry is preserved provided that

$$\varepsilon = e^{\frac{1}{2}A} \varepsilon, \quad \gamma_5 \varepsilon_0 = \varepsilon_0, \quad (72)$$

where ε_0 is a constant spinor.

It is interesting to note that there are in total *three* inequivalent solitonic string solutions in $D = 5$, namely the stainless example we have just derived, a rusty string that oxidizes to our stainless 5-brane in $D = 9$, and another rusty string that oxidizes to the stainless 6-brane in $D = 10$. Their metrics in $D = 5$ are given by (20) with $d = 2$ and $\tilde{d} = 1$, by taking $\Delta = \frac{4}{3}$, $\Delta = 2$ and $\Delta = 4$ respectively. Upon dimensional reduction to $D = 4$, they give rise to elementary particles with $a = 1/\sqrt{3}$, 1 and $\sqrt{3}$ respectively. These correspond to the black hole solutions of $D = 4$ string theory (see, for example, [11]).⁶

⁶We are grateful to J. Rahmfeld for drawing our attention to the black hole solutions in the $D = 4$ string.

6 Zero modes

In the previous sections, we described stainless p -brane solutions in various dimensions. The complete set of brane solutions is thus given by those solutions together with their descendants *via* Kaluza-Klein double dimensional reduction. All of these solutions break half of the supersymmetry.

Each broken supersymmetry transformation in a p -brane solution gives rise to a corresponding fermionic Goldstone zero mode. There will also be bosonic zero modes associated with the breaking of local bosonic gauge symmetries by the non-vanishing p -brane background solution. These will certainly include the translational zero modes corresponding to the broken constant general coordinate transformations $\delta y^m = c^m$ in the space transverse to the p -brane world volume. Thus there will be $D - d = \tilde{d} + 2$ such bosonic zero modes. Since supersymmetry remains partially unbroken by the solution, it follows that the fermionic and bosonic zero modes must form supermultiplets under the remaining unbroken supersymmetry. In particular, there must be equal numbers of fermionic and bosonic zero-mode degrees of freedom.

The matching of the zero modes for the bosonic and fermionic fields is straightforward in the case of the elementary membrane in $D = 11$ and the solitonic 5-brane in $D = 10$, and also for all their descendants *via* dimensional reduction. In all of these cases, the number of translational zero modes is precisely the same as that of the fermionic zero modes, *i.e.* the number of on-shell fermionic zero-mode degrees of freedom. Thus for the supermembrane in $D = 11$, there are $8 = 32/2/2$ fermionic zero modes, where the original 32 components of the supersymmetry parameter in $D = 11$ are halved once to arrive at the number of on-shell degrees of freedom, and halved again because half of the supersymmetries are broken. The membrane solution breaks translational invariance in the y^m directions, giving rise to $8 = 11 - 3$ bosonic zero modes. The same counting of $8 + 8$ degrees of freedom holds for the dimensional reduction to the string in $D = 10$. For the solitonic 5-brane in $N = 1$, $D = 10$ supergravity, there are $4 = 16/2/2$ fermionic zero modes, and $4 = 10 - 6$ bosonic translational zero modes. This matching of $4 + 4$ degrees of freedom holds for the various stages of dimensional reduction all the way down to the string in $D = 6$ and the superparticle in $D = 5$.

In all the other brane solutions, the number of translational zero modes is less than the number of fermionic zero modes. Since we know that the remaining unbroken symmetry guarantees a matching of the bose and fermi zero modes, it follows that there must be further bosonic zero modes associated with these solutions. They arise from the breaking of antisym-

metric tensor gauge symmetries. The simplest way to find these additional bosonic zero modes is first to construct the fermionic zero modes, and then to obtain their bosonic partners by transforming them under the remaining unbroken supersymmetries.

We shall carry out this procedure first for the stainless solitonic 5-brane in $N = 1$, $D = 9$ supergravity, which we constructed in section 4.1. This solution has four fermionic zero modes; however, it has only $9 - 6 = 3$ translational zero modes. As we shall see, there is one further bosonic zero mode associated with the breaking of the gauge invariance of the 2-form field strength that takes a non-zero value in the background solution.

The fermionic supersymmetry transformations in the background of the solitonic 5-brane are given by eqn (47). As we discussed in section 4.1, these variations vanish for spinors ε satisfying (48), which includes a chirality condition. They correspond to the unbroken supersymmetry generators. The broken generators, on the other hand, correspond to supersymmetry parameters η that have the opposite chirality under the γ_7 matrix on the world volume. Specifically, we shall consider spinors η given by

$$\eta = e^{-\frac{1}{2}A} \eta_0 , \quad \gamma_7 \otimes \mathbf{1} \eta_0 = -\eta_0 , \quad (73)$$

where η_0 is constant. This choice is motivated by the simplifications to the fermionic zero-mode structure that result. Note that any other asymptotically constant spinors with the same γ_7 eigenvalue could equally well have been chosen. These would lead to zero-modes differing from ours by pure gauge transformations whose parameters die off at infinity. With our choice, it follows from (47) that the purely bosonic 5-brane soliton background varies into the following fermionic configuration:

$$\begin{aligned} \chi &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}A-B} \partial_m \phi \mathbf{1} \otimes \gamma_m \eta_0 , \\ \psi_\mu &= -\partial_m A e^{\frac{1}{2}A-B} \gamma_\mu \otimes \gamma_m \eta_0 , \\ \psi_m &= \partial_n B e^{-\frac{1}{2}A} \mathbf{1} \otimes \gamma_{mn} \eta_0 . \end{aligned} \quad (74)$$

These, then, describe the four fermionic zero modes, parametrised by the eight independent spinors η_0 that satisfy the chirality condition given in (73). (Recall that for $d \geq 3$, the count of on-shell fermionic degrees of freedom is half that of the off-shell spinor fields.) We can substitute these spinors into the bosonic transformation rules (44), taking the supersymmetry parameter ε to be one of the eight unbroken generators given by (48), in order to obtain the bosonic superpartners of the fermionic zero modes.

Carrying out this procedure, we find the following non-vanishing results for the bosonic zero modes:

$$\begin{aligned}
\delta\phi &= \delta_{\text{Diff}} \phi \\
\delta e^\mu{}_\nu &= \delta_{\text{Diff}} e^\mu{}_\nu + \Omega^\mu{}_\rho e^\rho{}_\nu , \\
\delta e^m{}_n &= \delta_{\text{Diff}} e^m{}_n + \Omega^m{}_p e^p{}_n , \\
\delta A_m &= \delta_{\text{Diff}} A_m + \partial_m \tilde{\Lambda} , \\
\delta B_{mn} &= \delta_{\text{Diff}} B_{mn} - \frac{1}{2} \tilde{\Lambda} F_{mn} + \partial_{[m} \tilde{\Lambda}_{n]} ,
\end{aligned} \tag{75}$$

where δ_{Diff} denotes a diffeomorphism transformation, $\delta_{\text{Diff}} V_m = c^n \partial_n V_m + \partial_m c^n V_n$, *etc.*, and

$$\begin{aligned}
c^m &\equiv i e^{-B} \varepsilon_0 \mathbf{1} \otimes \gamma_m \eta_0 , \\
\Lambda &\equiv \sqrt{2} e^{-A} \varepsilon_0 \eta_0 , \\
\Lambda_m &\equiv -\frac{7}{4} e^{-2A} c_m , \\
\tilde{\Lambda} &\equiv \Lambda - c^m A_m , \quad \tilde{\Lambda}_m \equiv \Lambda_m - \Lambda A_m + 2c^p B_{mp} \\
\Omega^\mu{}_\rho &\equiv i \bar{\varepsilon}_0 \gamma^\mu{}_\rho \otimes \gamma_m \eta_0 e^{-B} \partial_m A , \\
\Omega^m{}_p &\equiv \partial_m c^p - \partial_p c^m - \epsilon^m{}_{pq} \bar{\varepsilon}_0 \eta_0 e^{-B} \partial_q B .
\end{aligned} \tag{76}$$

The quantities $\tilde{\Lambda}$ and $\tilde{\Lambda}_m$ are gauge transformation parameters for the potentials A_m and B_{mn} respectively; $\tilde{\Lambda}$ appears also in the B_{mn} transformation because its field strength is given by $G = dB + \frac{1}{2} A \wedge F$. As one chooses different constant spinors ε_0 and η_0 , satisfying their respective ± 1 chirality conditions under γ_7 , the diffeomorphism parameter c^m and gauge transformation parameter Λ_m jointly fill out a 3-dimensional space of gauge transformations that are constant and non-vanishing as r tends to infinity. Likewise, Λ describes a further independent asymptotically constant non-vanishing gauge transformation. Taken together, we have the four independent bosonic zero modes of the solitonic 5-brane in $D = 9$ dimensions. Note that $\Omega^\mu{}_\rho$ and $\Omega^m{}_p$ are parameters of Lorentz transformations, which die off at infinity and thus do not contribute to the true zero modes, which correspond to the broken generators of the global asymptotic symmetry group.⁷

⁷Another way to view the zero modes is through consideration of the $D = 9$ local supersymmetry algebra. For example,

$$\begin{aligned}
[\delta_2, \delta_1] e^A{}_M &= -i \nabla_M (\bar{\varepsilon}_1 \Gamma^A \varepsilon_2) - \frac{i}{42} e^{-\sqrt{\frac{2}{7}} \phi} G^{NPQ} \bar{\varepsilon}_1 \Gamma^A{}_{MNPQ} \varepsilon_2 + \frac{5i}{14} e^{-\sqrt{\frac{2}{7}} \phi} G_M{}^{AQ} \bar{\varepsilon}_1 \Gamma_Q \varepsilon_2 , \\
&\quad - \frac{1}{14\sqrt{2}} e^{-\sqrt{\frac{1}{14}} \phi} F^{NP} \bar{\varepsilon}_1 \Gamma^A{}_{MNP} \varepsilon_2 + \frac{6}{7\sqrt{2}} e^{-\sqrt{\frac{1}{14}} \phi} F_M{}^A \bar{\varepsilon}_1 \varepsilon_2 .
\end{aligned}$$

The zero-modes of the 5-brane in $D = 9$ are thus properly balanced between the boson and fermion sectors. With respect to the unbroken $N = 1$, $D = 6$ supersymmetry, they form a hypermultiplet, in the form where the four scalars occur as an $SU(2)$ triplet plus a singlet. The spinor zero-modes form an $SU(2)$ -Majorana doublet. Note that while the zero-modes form a supermultiplet under the unbroken supersymmetry in $d = 6$, as they must, not all of the scalars correspond to translational zero-modes. In the reduction from 9 to 6 dimensions, only three translational modes occur, leaving one more to arise from a different broken gauge symmetry. In the present case, this extra scalar mode arises from the broken gauge symmetry of the A_M potential, *i.e.* the Λ zero mode in (76).⁸

The other p -brane solutions discussed in this paper all leave half the original D -dimensional supersymmetry unbroken, and form appropriate supermultiplets of the unbroken supersymmetry. As another example, consider the string solution in $N = 1$, $D = 5$ supergravity that we discussed in section 5. Of the original 8 real components of the supersymmetry transformation, 4 are unbroken by the solution and 4 are broken, giving rise to 4 fermionic zero-mode *fields*. Of the bosonic zero-modes, one obviously has 3 corresponding to the broken translations. One more scalar zero-mode arises from the broken gauge symmetry of the 2-form field strength. In order to organise these into a supermultiplet of the unbroken $d = 2$ supersymmetry, one needs to recall one of the characteristic features of $d = 2$ supersymmetry. As one may see from eqn (70), the surviving $d = 2$ supersymmetry is *holomorphic*; it is in fact a $(4,0)$ supersymmetry. This supersymmetry relates the 4 fermionic zero-mode fields to 4 holomorphic bosonic modes. The usual style of counting zero-modes in $d \geq 3$, in which the count of fermionic zero-modes is taken to be half of the number of fermionic fields, is not particularly convenient in $d = 2$. In $d = 2$, the bosons also need to be split into holomorphic and antiholomorphic parts. Although our solitonic string solution clearly will have both holomorphic and antiholomorphic components of the bosonic zero-modes propagating according to the worldsheet equations of motion, only one

The covariant derivative $\nabla_M c^A$, where $c^A = i \bar{\varepsilon}_1 \Gamma^A \varepsilon_2$, gives the expected diffeomorphism. At infinity in the transverse y^m directions, this reduces to the translational part of the asymptotic symmetry. All the other terms in this commutator represent Lorentz gauge transformations whose field-dependent parameters fall off sufficiently fast at transverse infinity so as not to be included into the asymptotic symmetry group. The zero-modes of our solution are obtained by substituting the solution into this general commutator. Upon making this substitution, one reobtains precisely the vielbein parts of the zero-modes given in (75). The Lorentz gauge transformations in (75) arise from the terms involving F_{MN} in the commutator.

⁸Note also that giving an expectation value to an Abelian field strength can cause symmetry breaking in the present context, unlike in an ordinary Yang-Mills context, because of the structure of the linked gauge symmetry involving A_M and B_{MN} .

of these sectors becomes paired with the fermionic zero modes in the (4,0) supermultiplet. The other sector remains unpaired as a set of supersymmetric singlets.

7 Discussion

In this paper, we have searched for supersymmetric p -brane solutions of supergravity theories in diverse dimensions. As these solutions occur in families related by dimensional reduction, we have concentrated on the maximal, or stainless, solution in each family. In addition to the previously-known examples, we have found a number of new solutions in $5 \leq D \leq 9$ dimensions. (The lower dimensional bound arises here because we have restricted our attention to solutions of supergravity theories, and have not considered p -branes in theories with rigid supersymmetry.) Our new stainless solutions cannot oxidize isotropically to brane solutions in higher dimensions. Put another way, this means that these new solutions cannot simply be viewed as dimensional reductions of previously-known brane solutions in $D = 11$ or $D = 10$ dimensions.

A question that we have not addressed so far concerns the world brane actions that should describe the zero-mode fluctuations around the static, isotropic solutions considered here. From supersymmetry, one has detailed knowledge of the supermultiplet structure of the zero modes that would appear in a gauge-fixed world brane action. In general, one expects that such gauge-fixed actions should be extendable to spacetime supersymmetric and Lorentz invariant actions by adding the appropriate additional unphysical degrees of freedom and their associated local world volume gauge symmetries. For the four classic sequences of super p -branes, these actions are generalisations of the $D = 10$ superstring [17, 1] or $D = 11$ supermembrane [2] actions. Very little is known about the structure of the covariant world volume actions for any of the other p -brane cases; this subject remains an important open problem.

In the absence of detailed knowledge of the world volume action, some information can be extracted if one assumes that the bosonic sector of the action takes the general form of a Nambu-Goto action coupled to the spacetime metric, dilaton, and a d -form gauge potential, with the non-polynomiality removed by the introduction of a world-volume metric γ_{ij} as an auxiliary field [18]:

$$I_{\text{brane}} = \int d^d \xi \left(-\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N g_{MN} e^{b\phi} + \frac{d-2}{2} \sqrt{-\gamma} - \frac{1}{d!} \epsilon^{i_1 \dots i_d} \partial_{i_1} X^{M_1} \dots \partial_{i_d} X^{M_d} A_{M_1 \dots M_d} \right). \quad (77)$$

The exponent b of the dilaton coupling in the first term is *a priori* unknown. One way of

selecting a value for it in a number of cases is by resorting to an argument based on a scaling symmetry of the pure supergravity theory [11]. Under the constant rescalings

$$e^\phi \longrightarrow \lambda^\alpha e^\phi, \quad g_{MN} \longrightarrow \lambda^{2\beta} g_{MN}, \quad A_{M_1 \dots M_d} \longrightarrow \lambda^\gamma A_{M_1 \dots M_d}, \quad (78)$$

the action given by (1) scales by an overall factor $\lambda^{\beta(D-2)}$, provided that we choose $\gamma = \beta d + \alpha$. Requiring that I_{brane} scale in the same way, one finds that the parameters must be related by

$$\alpha = \frac{d\tilde{d}}{a(D-2)}, \quad \beta = \frac{d}{D-2}, \quad \gamma = d, \quad b = \frac{a}{d}. \quad (79)$$

Substituting the ansätze (4) and (7) into (77), one finds that the branewave equation following from (77) implies [11]

$$(e^C)' = (e^{dA+a\phi/2})'. \quad (80)$$

Comparing with the solution given by (16) and (17), one finds that a must satisfy

$$a^2 = 4 - \frac{2d\tilde{d}}{D-2}. \quad (81)$$

Thus requiring that the elementary brane solution should also satisfy the branewave equation, with the parameter b in (77) determined by requiring the above scaling symmetry, the value of a appearing in (1) is in all cases given by (18) with $\Delta = 4$.

All of the antisymmetric tensors in $D = 11$ and $D = 10$ supergravities have $\Delta = 4$, as we have seen. Thus the elementary p -brane solutions associated to these antisymmetric tensors accord with the above discussion. However, as we have seen, there are other values of Δ that also occur in lower dimensional supergravity theories. Elementary p -brane solutions in such cases cannot have zero modes that are described by the action (77) with the choice of parameter b dictated by the scaling symmetry. An example is provided by the self-dual string in $D = 6$, for which the self-dual 3-form has $\Delta = 2$. In this case, since there is no action for $N = 1$, $D = 6$ supergravity, one has to implement the scaling argument at the level of the equations of motion. The supergravity equations of motion themselves (64) are invariant under the scaling $g_{MN} \longrightarrow \lambda^{2\beta} g_{MN}$ and $G_{MNP} \longrightarrow \lambda^{2\beta} G_{MNP}$, for arbitrary β . However, the energy-momentum tensor $T_{MN}(x) = -\int d^2\xi \sqrt{-\gamma} \gamma^{ij} \partial_i X^P \partial_j X^Q g_{MP} g_{NQ} \delta^6(x - X) / \sqrt{-g}$ scales with a factor $\lambda^{-2\beta}$. Thus the coupled supergravity-string equations break the scaling symmetry. In fact all of the new stainless elementary p -branes, which have $\Delta = 2$ or $\Delta = \frac{4}{3}$, exhibit a similar breaking of the scaling symmetry in their couplings.

The ultimate significance of the scaling symmetry used in the above arguments remains unclear to us. Are elementary p -brane solutions that respect the scaling symmetry in their couplings more fundamental than others? We do not have an answer to this question at present.

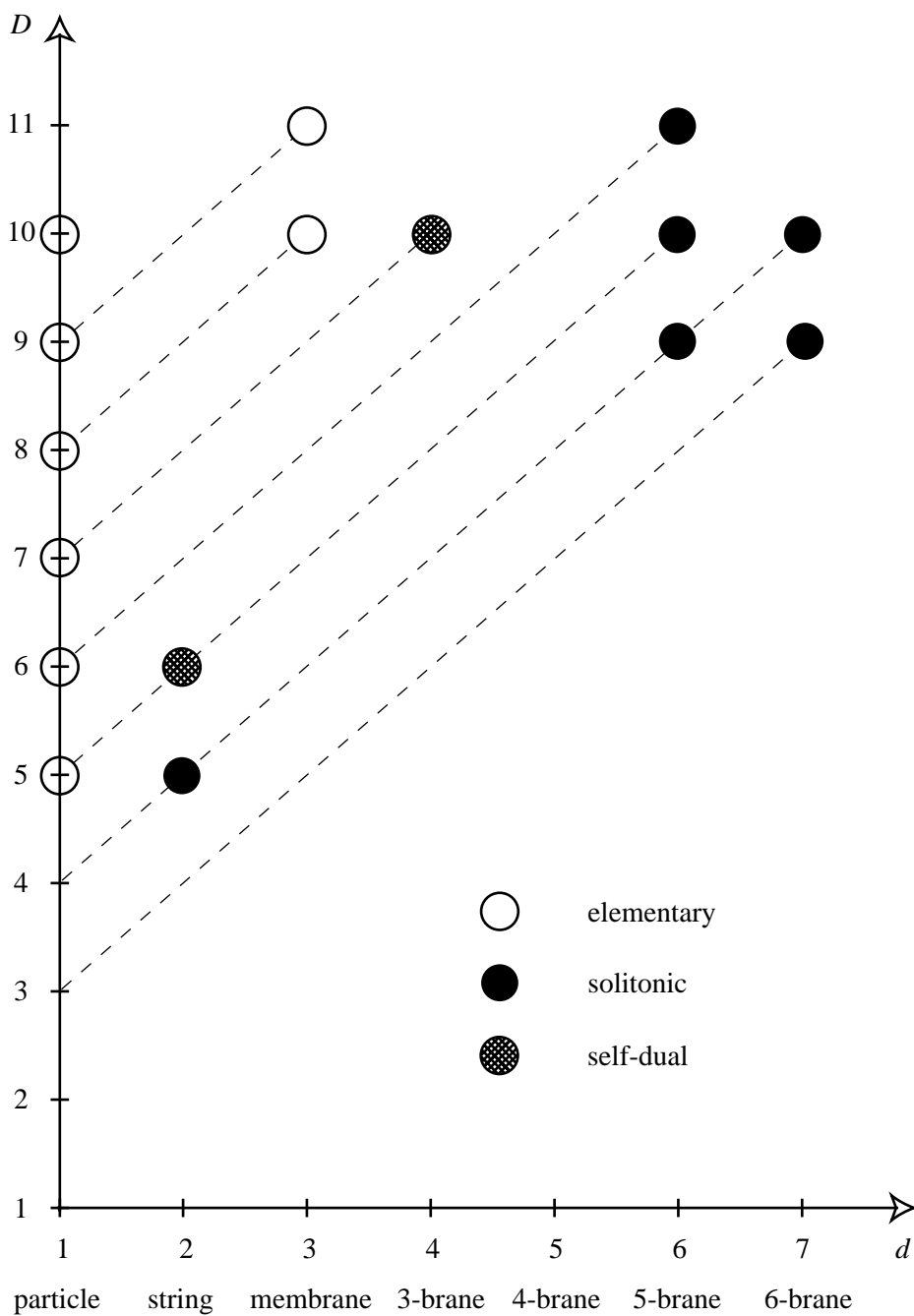
We would remark, however, that couplings that break scaling symmetries present in pure uncoupled gravity and supergravity theories are not at all uncommon. Take, for example, a massless charged particle coupled to Einstein-Maxwell theory in $D = 4$. The Einstein-Maxwell action $I = \int d^4x (eR - \frac{1}{4}eF^2)$ in the absence of the particle coupling scales as $I \rightarrow \rho^2 I$ under $g_{\mu\nu} \rightarrow \rho^2 g_{\mu\nu}$, $A_\mu \rightarrow \rho A_\mu$. But once the particle is coupled in, *via* the standard worldline-reparametrisation invariant action $\int d\tau (e^{-1} \dot{X}^\mu \dot{X}^\nu g_{\mu\nu} + A_\mu \dot{X}^\mu)$, the scaling symmetry is broken by the electromagnetic coupling $A_\mu \dot{X}^\mu$.

Another reason why the status of the scaling symmetries remains in doubt comes from quantisation. The supergravity theories arising as low-energy effective field theories of superstring theories have field equations determined *via* the beta functions from the requirement that the string's conformal invariance be preserved. The leading terms of these effective field equations reproduce standard supergravity field equations, but there are also an infinite series of quantum corrections, all of which break the scaling symmetries.

Nonetheless, it is intriguing that a purely bosonic discussion based upon the coupling of Nambu-Goto-type actions and preservation of scaling symmetries fixes dilaton couplings to antisymmetric tensor gauge fields in a way that agrees with many of the couplings actually found in supergravity theories (*i.e.* the $\Delta = 4$ couplings).

In concluding, we shall summarise the results that we have obtained in this paper in a revised brane-scan, in which we plot only the stainless members of each p -brane family. In accordance with our discussion at the end of section 2.1, we may, without loss of generality, consider only the versions of the various supergravity theories where all the antisymmetric tensors have degrees $n \leq D/2$, since no further inequivalent elementary or solitonic brane solutions arise from dualised versions of the theories. Accordingly, in the stainless brane-scan, we denote solutions of the $n \leq D/2$ theories that are elementary by open circles, solutions that are solitonic by solid circles, and self-dual solutions by cross-hatched circles. The dashed lines extending diagonally downwards from the various points on the brane scan indicate that each stainless solution gives rise to its own set of dimensionally-reduced descendants.

Brane Scan of Stainless Supergravity Solutions



(For supergravity theories in their $n \leq D/2$ versions)

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