

Improving the Measurement of the Top Quark Mass

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Abstract

The mass resolution for observing hadronic top quark decay $t \rightarrow b W \rightarrow 3$ jets can be improved by about 20% by defining the jets using fixed cones in the rest frames of the t and W , in place of the traditional cones defined in the rest frame of the detector. This improved mass resolution can be used to make a more accurate measurement of the top quark mass, and to improve the discrimination between $t\bar{t}$ events and background for studies of the production mechanism.

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The discovery of the top quark in $p\bar{p}$ collisions at $\sqrt{s} = 1.8 \text{ TeV}$ [1,2] represents a milestone for the Standard Model. The next steps beyond that milestone will be to improve the measurement of the top mass, and to test our understanding of QCD further by studying the details of its production and decay.

We focus here on the “single-lepton” channel defined by $p\bar{p} \rightarrow t\bar{t}X$ with one top decaying hadronically ($t \rightarrow W^+ b \rightarrow 3 \text{ jets}$, or its charge conjugate) and the other decaying leptonically ($t \rightarrow W^+ b \rightarrow \ell^+ \nu_\ell + 1 \text{ jet}$, or its charge conjugate, with $\ell = e$ or μ). This channel is experimentally favorable because the high p_\perp lepton and large missing p_\perp due to the neutrino provide a clean event trigger that naturally discriminates against backgrounds.

The top mass determination relies on measuring the energies and directions of jets, and inferring from them the energies and directions of the original quarks. In addition to instrumental effects and calibrations, this requires corrections for the hard-scale branching of partons, and for non-perturbative hadronization effects. These QCD effects can be modeled by event generators such as HERWIG [3], which have been shown to describe the appropriate physics in Z^0 decay at LEP [4,5], where the original $q\bar{q}$ energy is precisely known.

Part of the top quark analysis involves measuring the mass of the hadronically decaying W boson. An eventual goal of the analysis should be to confirm the QCD effects in W decay that are similar to those seen in great detail for Z^0 decay. In the meantime, we can assume that the QCD effects are understood, and attempt to use the W mass to help in determining the correct jet assignments in $t\bar{t}$ events. The decay $W \rightarrow 2 \text{ jets}$ is also a potentially useful tool to calibrate the detector, since the decay jets probe the detector at a wide range of angles and jet transverse momenta, and the W mass is accurately known. This calibration is unavailable outside of top quark events, because hadronic W decays are normally obscured by large QCD backgrounds [6].

The point of this paper can be seen most clearly by considering the W mass measurement. Since the W is color neutral, its decay products have nothing to do with other jets present in the $t\bar{t}$ final state. In the W rest frame, the decays will be very similar to Z^0 decays as observed at LEP. In particular, they will almost always appear as two back-to-back jets that

can readily be defined by fixed cones of half-angle θ_0 oriented in opposite directions. We will choose $\theta_0 = 30^\circ$. This cone size is large enough to contain a good fraction of the energies of the jets, which are broadened by the collinear radiation singularity of QCD. Meanwhile, $\theta_0 = 30^\circ$ is small enough that it leaves 87% of the full 4π of solid angle outside the two cones, so not too much “background” is included inside them. We will also try $\theta_0 = 45^\circ$, which leaves 71% of the full 4π solid angle outside the two cones. Although cone algorithms are not the traditional jet definitions in e^+e^- physics, they have been shown to work, and even to provide superior resolution in jet angle and energy [5].

Our sensible decision to use fixed cones in the W rest frame is quite different from the current experimental procedure for top analysis, in which jets are defined by cones $\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < R$ in the Lego variables of the detector. These variables, $\eta = -\log \tan \theta/2 =$ pseudorapidity and $\phi =$ azimuthal angle, are Lorentz invariant only under boosts along the beam direction; whereas the W can have a rather large component of boost transverse to the beam. For example, in $t\bar{t}$ events with typical acceptance cuts, the middle 60% of the W transverse momentum distribution extends from 40 GeV/c to 110 GeV/c. Hence p_\perp^W is frequently either large or small compared to m_W . Lego-defined jet cones therefore correspond to a wide variety of sizes and shapes of “cone” in the W rest frame, depending on details of decay angles that have nothing to do with the W decay.

Similarly, the jet cone for the b -quark jet from $t \rightarrow W b$ is most reasonably defined by a fixed angle in the t rest frame. Since the b carries color, the argument for a cylindrically symmetric cone in this rest frame is weaker than in the case of the W decay. But the t rest frame is nevertheless at least more sensible than using the rest frame of the detector. We will show by Monte Carlo simulation that defining the b -jet cone in the t rest frame indeed improves the mass resolution for t significantly.

It would similarly be logical to define the jet cone for the b -jet from the other, semileptonically decaying, top in the rest frame of that top. That should be done when the method advocated in this paper is applied to real data. But we ignore it here, in order to avoid bringing in the unrelated issue of how best to estimate the longitudinal momentum of the

neutrino, which is needed to find that frame.

The suggestion of this paper can most simply be applied as an add-on to the traditional top quark analysis with “kinematic reconstruction” [1], *i.e.*, with explicit matching of the observed jets to decay partons. The procedure for each event is as follows:

1. Identify the four-momenta of the jets corresponding to $t \rightarrow j_1 j_2 j_3$ and $t \rightarrow j_4 \ell \nu$, where j_1, j_2 come from W decay and j_3, j_4 are b -quark jets, using the usual procedures of CDF or DØ. b -jet tagging is helpful for this, but not crucial. There may of course be additional jets observed in the event.
2. Define j_F and j_B to be four-momenta directed in the forward and backward beam directions: $j_F = (1; 0, 0, 1)$ and $j_B = (1; 0, 0, -1)$ for beams in the z -direction.
3. Redefine the 4-momentum of j_1 as the sum of the 4-momenta observed in the calorimeter detector in all cells (“towers”) that lie within θ_0 of the old j_1 in the rest frame of the old $j_1 + j_2$. However, do not include cells that lie closer in angle to j_2, j_3, j_4, j_F , or j_B than to j_1 .

Similarly redefine the 4-momentum of j_2 as the sum of the 4-momenta of all cells that lie within θ_0 of the old j_2 in the rest frame of the old $j_1 + j_2$, and are not closer in angle to j_1, j_3, j_4, j_F , or j_B .

Similarly redefine the 4-momentum of j_3 as the sum of the 4-momenta in cells that lie within θ_0 of the old j_3 in the rest frame of the old $j_1 + j_2 + j_3$, and are not closer in angle to j_1, j_2, j_4, j_F , or j_B .

4. Replace the old jet 4-momentum estimates by the new ones and repeat Step 3 a few times. This iteration converges completely for all but 0.2% of events. A maximum of 10 iterations is sufficient, since the momenta cease changing before that in all but 0.5% of events.

The essential new work is in Step 3, which is very easy to implement in the following way. Treat the energy in each cell of the detector that receives energy above its noise level

as if it came from a zero-mass particle. There will typically be a few hundred such cells in each event. Make a list of their four-momenta. The appropriate elements of this list to be added in the various parts of Step 3 can be found without making any explicit Lorentz transformations, using the exact formula for the angle θ between \vec{p} and \vec{q} in the rest frame of \vec{r} :

$$\cos \theta = \frac{1 - p \cdot q r^2 / (p \cdot r q \cdot r)}{\sqrt{1 - q^2 r^2 / (q \cdot r)^2}}, \quad (1)$$

which holds for any massless four-vector p .

To test this idea, HERWIG 5.7 [3] was used to simulate the single-leptonic top channel at the Tevatron energy $\sqrt{s} = 1.8 \text{ TeV}$. Typical experimental cuts were approximated by $|\eta^\ell| < 2.0$, $p_\perp^\ell > 20 \text{ GeV}/c$, $p_\perp^\nu > 25 \text{ GeV}/c$. The four hadron jets were required, at the partonic level, to have $p_\perp^{\text{jet}} > 20 \text{ GeV}/c$ and to be isolated in Lego from the lepton by $R \equiv \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} > 0.5$ and from each other by $R > 0.8$. The detector was simulated by an array of 0.1×0.1 cells in $\eta \times \phi$, covering the region $|\eta| < 4.5$. Gaussian energy resolutions with $\Delta E/E = 0.55/\sqrt{E}$ for charged hadrons and $\Delta E/E = 0.15/\sqrt{E}$ for leptons and photons were assumed (in $\text{GeV} = 1$ units). Constant terms in the energy resolution were not included. A small amount of shower spreading was included: the energy of each final hadron was spread out over a disk of radius 0.1 in the (η, ϕ) plane before being deposited in the appropriate calorimeter cells.

A cone-type jet finding algorithm described previously [7] was used to search for jets in the simulated calorimeter. A small cone size $R = 0.3$ was used, as is standard practice in top quark analysis to reduce the loss of events from overlapping jets, since one is asking for 4 jets within a small area (frequently $-1 < \eta < +1$) in Lego. A minimum ‘‘observed’’ jet p_\perp of $15 \text{ GeV}/c$ was required. A feature of the jet-finder used is that it finishes with an iteration in which each cell of the detector that lies within $R = 0.3$ of at least one jet axis is assigned to the nearest jet axis. The jet momenta are then recomputed and this procedure is iterated. This feature tends to improve the mass resolution for objects decaying into jets. It thus provides an honest comparison to our method, which is similar except that the nearest

jets are defined by angles in appropriate rest frames.

To obtain a sample that could be cleanly interpreted, the “observed” jets in the simulation were matched to their original quark partons, using a criterion based mainly on the agreement in Lego variables η and ϕ , together with a small contribution from the agreement in p_{\perp} . A good unique match was obtained for 84% of the events, which were used to produce the following Figures.

Figure 1 shows the histogram of the mass observed for $W \rightarrow jj$. The solid curve is for jets defined in the usual way, by $R < 0.3$ in the Lego variables of the detector. Note that the center of the peak is well below the actual $m_W = 82 \text{ GeV}/c^2$ assumed in the simulation, and that the peak is asymmetrical, with a tail toward lower jj masses. This skewing of the mass distribution toward low mass is a direct result of the soft gluon radiation that is characteristic of QCD, and which has been verified by the LEP data. It demonstrates that making significant QCD corrections is a necessary part of the top quark mass measurement, especially when a cone size as small as 0.3 is used.

The dashed curve in Figure 1 shows the result of redefining the jets in the appropriate rest frames, using the iteration described above. Note that the qualitative QCD feature of skewing toward lower mass is still visible, but much less pronounced. Also, the center of the peak is at a higher mass — much closer to the true partonic mass. This can be understood from that fact that jet cones defined by $\theta < 30^\circ$ are generally larger than the traditional $R < 0.3$ cones in the detector frame. More importantly, the overall width of the peak is narrower. For example, if one defines the width of the peak by the range ΔM that contains the middle 60% of the probability, the new jet definition has reduced $\Delta M/M$ by a factor of 0.80.

Figure 2 similarly shows the histogram of the mass observed for $t \rightarrow W b \rightarrow jjj$. One again sees a QCD skewing of the peak toward lower jjj masses, relative to the value $m_t = 175 \text{ GeV}/c^2$ assumed in the simulation. (This mass shift includes a contribution of $\sim -5 \text{ GeV}/c^2$ from losses in jet energy due to neutrinos.) One sees that the new jet definition again clearly improves the mass resolution — this time reducing $\Delta M/M$ for the middle

60% of the distribution by a factor of 0.82 .

The choice of cone size $\theta_0 = 30^\circ$ used in the rest frames is not crucial. In fact, θ_0 can be increased to as much as 45° without spoiling the convergence of the iteration, and with an even greater improvement in mass resolution. This is shown by the dotted curves in Figs. 1 and 2.

In conclusion, we have seen that redefining the jets using fixed angle cones in the rest frames of the decaying t and W systems can be used to improve the mass resolution $\Delta M/M$ by approximately 20%. The new jet definition substantially reduces the shift toward lower masses of the centers of the peaks, which will reduce the magnitude — and hence also the associated error — of the QCD corrections needed to infer the peak mass at the partonic level. This technique should therefore be used to improve the mass measurement of the top quark. The improved resolution will help by reducing the statistical uncertainty in the location of the mass peak, which will arise from the relatively small number of events (~ 100) that will be available in each experiment at the end of the current Tevatron run. It will help in addition with isolating the signal from the non-top QCD background, since the narrower peak will produce a larger ratio of signal/background. The improved resolution for the $W \rightarrow jj$ mass measurement will further help to isolate the single-leptonic $t\bar{t}$ signal. Making the hadronic W peak more visible will also help in facilitating a direct calibration of jet energy measurements using the known W mass.

The improvement in mass resolution described in this paper could also be applied to the difficult problem of observing doubly hadronic top events $t\bar{t} \rightarrow (Wb)(Wb) \rightarrow 6 \text{ jets}$ [8].

The procedure advocated here has a solid basis in QCD, and has been shown to be effective using a reasonable Monte Carlo simulation. The next steps, which can only be carried out by the experimenters of CDF and DØ, must be to try the procedure out using full detector simulations, and then real data.

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FIGURES

FIG. 1. Simulated dijet invariant mass distribution for $W \rightarrow jj$ from semileptonic top events. The *solid* curve is for jets defined by the usual $\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < 0.3$ cones in the rest frame of the detector. The *dashed* curve is for jets defined by $\theta < 30^\circ$ cones in the W rest frame, via the iteration advocated in this paper. The *dotted* curve is similar, but for $\theta < 45^\circ$ cones.

FIG. 2. Simulated trijet invariant mass distribution for $t \rightarrow W b \rightarrow jjj$ from semileptonic top events. As in Fig. 1, the *solid* curve is for jets defined by $\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < 0.3$ cones, while the *dashed* (*dotted*) curves are for $\theta < 30^\circ$ ($\theta < 45^\circ$) cones defined in the t rest frame for the b -jet and in the W rest frame for the other two jets.