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Some Issues in Soft SUSY-breaking Terms from Dilaton/Moduli sectors*

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Abstract

We study the structure of the soft SUSY-breaking terms obtained from some classes of 4-D strings under the assumption of dilaton/moduli dominance in the process of SUSY-breaking. We generalize previous analyses in several ways and in particular consider the new features appearing when several moduli fields contribute to SUSY breaking (instead of an overall modulus T). Some qualitative features indeed change in the multimoduli case. A general discussion for symmetric Abelian orbifolds as well as explicit examples are given. Certain general sum-rules involving soft terms of different particles are shown to apply to large classes of models. Unlike in the overall modulus T case, gauginos may be lighter than scalars even at the tree-level. However, if one insists in getting that pattern of soft terms, these sum rules force some of the scalars to get negative mass^2. These tachyonic masses could be a problem for standard model 4-D strings but an advantage in the case of string-GUTs. We also discuss the possible effects of off-diagonal metrics for the matter fields which may give rise to flavour-changing neutral currents. Different sources for the bilinear B soft term are studied. It is found that the Giudice-Masiero mechanism for generating a "μ-term", as naturally implemented in orbifolds, leads to the prediction $|tg\beta| = 1$ at the string scale, independently of the Goldstino direction.

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1 Introduction

Recently there has been some activity in trying to obtain information about the structure of soft Supersymmetry (SUSY)-breaking terms in effective $N = 1$ theories coming from four-dimensional strings. The basic idea is to identify some $N = 1$ chiral fields whose auxiliary components could break SUSY by acquiring a vacuum expectation value (vev). No special assumption is made about the possible origin of SUSY-breaking. Natural candidates in four-dimensional strings are 1) the complex dilaton field $S = \frac{4\pi}{g^2} + ia$ which is present in any four-dimensional string and 2) the moduli fields T^i, U^i which parametrize the size and shape of the compactified variety in models obtained by compactification of a ten-dimensional heterotic string. It is not totally unreasonable to think that some of these fields may play an important role in SUSY-breaking. To start with, if string models are to make any sense, these fields should be strongly affected by non-perturbative phenomena. They are massless in perturbation theory and non-perturbative effects should give them a mass to avoid deviations from the equivalence principle and other phenomenological problems. Secondly, these fields are generically present in large classes of four-dimensional models (the dilaton in all of them). Finally, the couplings of these fields to charged matter are suppressed by powers of the Planck mass, which makes them natural candidates to constitute the SUSY-breaking "hidden sector" which is assumed to be present in phenomenological models of low-energy SUSY.

The important point in this assumption of locating the seed of SUSY-breaking in the dilaton/moduli sectors, is that it leads to some interesting relationships among different soft terms which could perhaps be experimentally tested. In ref.[1] three of the authors presented a systematic discussion of the structure of soft terms which may be obtained under the assumption of dilaton/moduli dominated SUSY breaking in some classes of four-dimensional strings, with particular emphasis on the case of Abelian $(0, 2)$ orbifold models [2]. We mostly considered a situation in which only the dilaton S and an "overall modulus T" field contribute to SUSY-breaking. In fact, actual four-dimensional strings like orbifolds contain several T_i moduli. Generic $(0, 2)$ orbifold models contain three T_i moduli fields (only Z_3 has 9 and Z_4, Z_6' have 5) and a maximum of three ("complex structure") U_i fields. The use of an overall modulus T is equivalent to the assumption that the three T_i fields of generic orbifold models contribute exactly the same to SUSY-breaking. In the absence of further dynamical information it is reasonable to expect similar contributions from the three moduli although not necessarily exactly the same. In any case it is natural to ask what changes if one relaxes the overall modulus hypothesis and works with the multimoduli case. This is one of the purposes of the present paper.

In section 2 we present an analysis of the effects of relaxing the overall modulus assumption on the results obtained for soft terms. In the multimoduli case several parameters are needed to specify the Goldstino direction in the dilaton/moduli space, in contrast with the overall modulus case where the relevant information is contained in just one angular parameter θ. The presence of more free parameters leads to some loss of predictivity for the soft terms. However, we show that in some cases there are certain sum-rules among soft terms which hold independently of the Goldstino direction. The presence of these sum rules cause that, *on average* the *qualitative* results in ref.[1] still apply. Specifically, if one insists e.g. in obtaining scalar masses heavier than gauginos (something not possible at the tree-level in the approach of ref.[1]) , this is possible in the multimoduli case, but the sum-rules often force some of the scalars to get negative mass^2. If we want to avoid this, we have to stick to gaugino masses bigger than (or of order) the scalar masses. This would lead us back to the qualitative results obtained in ref.[1]. In the case of standard model 4-D strings this tachyonic behaviour may be particularly problematic, since charge and/or colour could be broken. In the case of GUTs constructed from strings, it may just be the signal of GUT symmetry breaking. We exemplify the different type of soft terms which may be obtained in the multimoduli case in some particular examples, including an $SO(10)$ String-GUT.

Section 3 addresses another simplifying assumption in ref.[1]. There only the case of diagonal kinetic terms for the charged fields was considered. Indeed this is the generic case in most orbifolds, where typically some discrete symmetries (or R-symmetries) forbid off-diagonal metrics for the matter fields. On the other hand there are some orbifolds in which off-diagonal metrics indeed appear and one expects that in other compactification schemes such metrics may also appear. This question is not totally academic since, in the presence of off-diagonal metrics, the soft terms obtained upon SUSY-breaking are also in general off-diagonal. This may lead to flavour changing neutral current (FCNC) effects in the low energy effective $N = 1$ softly broken Lagrangian.

A third topic of interest is the B-parameter, the soft mass term which is associated to a SUSY mass term $\mu H_1 H_2$ for the pair of Higgsses $H_{1,2}$ in the Minimal Supersymmetric Standard Model (MSSM). Compared to the other soft terms, the result for the B-parameter is more model-dependent. Indeed, it depends not only on the dilaton/moduli dominance assumption but also on the particular mechanism which could generate the associated "μ-term". An interesting possibility to generate such a term is the one suggested in ref.[3] in which it was pointed out that in the presence of certain bilinear terms in the Kähler potential an effective μ-term of order the gravitino mass, $m_{3/2}$, is naturally generated. Interestingly enough, such bilinear terms in the Kähler potential do appear in string models and particularly in Abelian orbifolds. In section 4 we compute the μ and B parameters as well as the soft scalar masses of the charged fields which could play the role of Higgs particles in such Abelian orbifold schemes. We find the interesting result that, independently of the Goldstino direction in the dilaton/moduli space, one gets the prediction $|tg\beta| = 1$ at the string scale. In other words, the direction $\langle H_1 \rangle = \langle H_2 \rangle$ remains flat *even after SUSY-breaking*. The results for B corresponding to other sources for the μ-term are also presented in the multimoduli case under consideration. In particular, the possibility of generating a small μ-term from the superpotential [4] is studied. We leave some final comments and conclusions for section 5.

2 Soft terms: the multimoduli case

We are going to consider $N = 1$ SUSY 4-D strings with m moduli T_i, $i = 1, .., m$. Such notation refers to both T-type and U-type (Kähler class and complex structure in the Calabi-Yau language) fields. In addition there will be charged matter fields C_α and the complex dilaton field S. In general we will be considering $(0, 2)$ compactifications and thus the charged fields do not need to correspond to 27s of E_6.

Before further specifying the class of theories that we are going to consider a comment about the total number of moduli is in order. We are used to think of large numbers of T and U-like moduli due to the fact that in $(2,2)$ (E_6) compactifications there is a one to one correspondence between moduli and charged fields. However, in the case of $(0,2)$ models with arbitrary gauge group (which is the case of phenomenological interest) the number of moduli is drastically reduced. For example, in the standard $(2,2)$ Z_3 orbifold there are 36 moduli T_i, 9 associated to the untwisted sector and 27 to the fixed points of the orbifold. In the thousands of $(0,2)$ Z_3 orbifolds one can construct by adding different gauge backgrounds or doing different gauge embeddings, only the 9 untwisted moduli remain in the spectrum. The same applies to models with U-fields. This is also the case for compactifications using $(2,2)$ minimal superconformal models. Here all singlets associated to twisted sectors are projected out when proceeding to $(0,2)$ [5]. So, as these examples show, in the case of $(0,2)$ compactifications the number of moduli is drastically reduced to a few fields. In the case of generic Abelian orbifolds one is in fact left with only three T-type moduli T_i $(i=1,2,3)$, the only exceptions being Z_3, Z_4 and Z_6', where such number is 9, 5 and 5 respectively. The number of U-type fields in these $(0,2)$ orbifolds oscillates between 0 and 3, depending on the specific example. Specifically, $(0,2)$ $Z_2\times Z_2$ orbifolds have 3 U fields, the orbifolds of type Z_4, Z_6,Z_8, $Z_2\times Z_4$,$Z_2\times Z_6$ and Z_{12}' have just one U field and the rest have no untwisted U-fields. Thus, apart from the three exceptions mentioned above, this class of models has at most 6 moduli, three of T-type (always present) and at most three of U-type. In the case of models obtained from Calabi-Yau type of compactifications a similar effect is expected and only one T-field associated to the overall modulus is guaranteed to exist in $(0,2)$ models.

We will consider effective $N=1$ supergravity (SUGRA) Kähler potentials of the type:

$$\begin{aligned} K(S,S^*,T_i,T_i^*,C_\alpha,C_\alpha^*) \ &= \ -\log(S+S^*) \ + \ \hat{K}(T_i,T_i^*) \ + \ \tilde{K}_{\overline{\alpha}\beta}(T_i,T_i^*)C^{*\overline{\alpha}}C^\beta \\ &\quad + \ (\ Z_{\alpha\beta}(T_i,T_i^*)C^\alpha C^\beta \ + \ h.c. \ ) \ . \end{aligned} \tag{1}$$

The first piece is the usual term corresponding to the complex dilaton S which is present for any compactification whereas the second is the Kähler potential of the moduli fields, where we recall that we are denoting the T- and U-type moduli collectively by T_i. The greek indices label the matter fields and their kinetic term functions are given by $\tilde{K}_{\overline{\alpha}\beta}$ and $Z_{\alpha\beta}$ to lowest order in the matter fields. The last piece is often forbidden by gauge invariance in specific models although it may be relevant in some cases as discussed in section 4. In this section we are going to consider the case of diagonal metric both for the moduli and the matter fields and leave the off-diagonal case for the next section. Then $\hat{K}(T_i,T_i^*)$ will be a sum of contributions (one for each T_i), whereas $\tilde{K}_{\overline{\alpha}\beta}$ will be taken of the diagonal form $\tilde{K}_{\overline{\alpha}\beta}\equiv\delta_{\overline{\alpha}\beta}\tilde{K}_\alpha$. The complete $N=1$ SUGRA Lagrangian is determined by the Kähler potential $K(\phi_M,\phi_M^*)$, the superpotential $W(\phi_M)$ and the gauge kinetic functions $f_a(\phi_M)$, where ϕ_M generically denotes the chiral fields S,T_i,C_α. As is well known, K and W appear in the Lagrangian only in the combination $G=K+\log|W|^2$. In particular, the (F-part of the) scalar potential is given by

$$V(\phi_M,\phi_M^*) \ = \ e^G\left(G_M K^{M\bar{N}}G_{\bar{N}} \ - \ 3\right) \ , \tag{2}$$

where $G_M\equiv\partial_M G\equiv\partial G/\partial\phi_M$ and $K^{M\bar{N}}$ is the inverse of the Kähler metric $K_{\bar{N}M}\equiv\partial_{\bar{N}}\partial_M K$.

The crucial assumption now is to locate the origin of SUSY-breaking in the dilaton/moduli sector. It is perfectly conceivable that other fields in the theory, like charged matter fields, could contribute in a leading manner to SUSY-breaking. If that is the case, the structure of soft SUSY-breaking terms will be totally model-dependent and we would be able to make no model-independent statements at all about soft terms. On the contrary, assuming the seed of SUSY-breaking originates in the dilaton-moduli sectors will enable us to extract some interesting results. We will thus make that assumption without any further justification. Let us take the following parametrization for the vev's of the dilaton and moduli auxiliary fields $F^S = e^{G/2} G^{-1}_{\bar{S}S} G_{\bar{S}}$ and $F^i = e^{G/2} G^{-1}_{\bar{i}i} G_{\bar{i}}$:

$$G^{1/2}_{\bar{S}S} F^S \;=\; \sqrt{3} m_{3/2} \sin\theta e^{-i\gamma_S} \;\; ; \;\; G^{1/2}_{\bar{i}i} F^i \;=\; \sqrt{3} m_{3/2} \cos\theta \; e^{-i\gamma_i} \Theta_i \;\; , \qquad (3)$$

where $\sum_i \Theta_i^2 = 1$ and $e^G = m_{3/2}^2$ is the gravitino mass-squared. The angle θ and the Θ_i just parametrize the direction of the goldstino in the S, T_i field space. We have also allowed for the possibility of some complex phases γ_S, γ_i which could be relevant for the CP structure of the theory. This parametrization has the virtue that when we plug it in the general form of the SUGRA scalar potential eq.(2), its vev (the cosmological constant) vanishes by construction. Notice that such a phenomenological approach allows us to 'reabsorb' (or circumvent) our ignorance about the (nonperturbative) S- and T_i- dependent part of the superpotential, which is responsible for SUSY-breaking. It is now a straightforward exercise to compute the bosonic soft SUSY-breaking terms in this class of theories. Plugging eqs.(3) and (1) into eq.(2) one finds the following results (we recall that we are considering here a diagonal metric for the matter fields):

$$m_\alpha^2 = \; m_{3/2}^2 \; \left[1 \; - \; 3\cos^2\theta \; (\hat{K}_{\bar{i}i})^{-1/2} \Theta_i e^{i\gamma_i} (\log \tilde{K}_\alpha)_{\bar{i}j} (\hat{K}_{\bar{j}j})^{-1/2} \Theta_j e^{-i\gamma_j} \; \right] \; ,$$
$$A_{\alpha\beta\gamma} = \; -\sqrt{3} m_{3/2} \; [e^{-i\gamma_S} \sin\theta$$
$$- \; e^{-i\gamma_i} \cos\theta \; \Theta_i (\hat{K}_{\bar{i}i})^{-1/2} \; \left(\hat{K}_i - \textstyle\sum_{\delta=\alpha,\beta,\gamma} (\log \tilde{K}_\delta)_i + (\log h_{\alpha\beta\gamma})_i \; \right) \;] \; . \qquad (4)$$

The above scalar masses and trilinear scalar couplings correspond to charged fields which have already been canonically normalized. Here $h_{\alpha\beta\gamma}$ is a renormalizable Yukawa coupling involving three charged chiral fields and $A_{\alpha\beta\gamma}$ is its corresponding trilinear soft term.

Physical gaugino masses M_a for the canonically normalized gaugino fields are given by $M_a = \frac{1}{2}(Ref_a)^{-1} e^{G/2} f_{aM} K^{M\bar{N}} G_{\bar{N}}$. Since the tree-level gauge kinetic function is given for any 4-D string by $f_a = k_a S$, where k_a is the Kac-Moody level of the gauge factor, the result for tree-level gaugino masses is independent of the moduli sector and is simply given by:

$$M \equiv M_a \;\; = \;\; m_{3/2} \sqrt{3} \sin\theta e^{-i\gamma_S} \; . \qquad (5)$$

As we mentioned above, the parametrization of the auxiliary field vev's was chosen in such a way to guarantee the automatic vanishing of the vev of the scalar potential ($V_0 = 0$). If the value of V_0 is not assumed to be zero the above formulae are modified in the following simple way. One just has to replace $m_{3/2} \to C m_{3/2}$, where $|C|^2 \; = 1 + V_0/3m_{3/2}^2$. In addition, the formula for m_α^2 gets an additional contribution given by $2m_{3/2}^2(|C|^2 - 1) = 2V_0/3$.

The soft term formulae above are in general valid for any compactification as long we are considering diagonal metrics. In addition one is tacitally assuming that the tree-level Kähler potential and f_a-functions constitute a good aproximation. The Kähler potentials for the moduli are in general complicated functions. To illustrate some general features of the multimoduli case we will concentrate here on the case of generic $(0,2)$ symmetric Abelian orbifolds. As we mentioned above, this class of models contains three T-type moduli and (at most) three U-type moduli. We will denote them collectively by T_i, where e.g. $T_i = U_{i-3}$; $i = 4,5,6$. For this class of models the Kähler potential has the form [6]

$$K(\phi,\phi^*) \ = \ -\log(S+S^*) \ - \ \sum_i \log(T_i+T_i^*) \ + \ \sum_\alpha |C_\alpha|^2 \Pi_i (T_i+T_i^*)^{n_\alpha^i} \ . \qquad (6)$$

Here n_α^i are fractional numbers usually called "modular weights" of the matter fields C_α. For each given Abelian orbifold, independently of the gauge group or particle content, the possible values of the modular weights are very restricted. For a classification of modular weights for all Abelian orbifolds see ref.[7]. Using the particular form (6) of the Kähler potential and eqs.(4,5) we obtain the following results[1] for the scalar masses, gaugino masses and soft trilinear couplings:

$$\begin{aligned} m_\alpha^2 &= \ m_{3/2}^2(1 \ + \ 3\cos^2\theta \ \vec{n_\alpha}.\vec{\Theta}^2) \ , \\ M &= \ \sqrt{3} m_{3/2} \sin\theta e^{-i\gamma_S} \ , \\ A_{\alpha\beta\gamma} &= \ -\sqrt{3} m_{3/2} \ (\sin\theta e^{-i\gamma_S} \ + \ \cos\theta \textstyle\sum_{i=1}^6 e^{-i\gamma_i} \Theta^i \omega_{\alpha\beta\gamma}^i) \ , \end{aligned} \qquad (7)$$

where we have defined :

$$\omega_{\alpha\beta\gamma}^i \ = \ (1+n_\alpha^i+n_\beta^i+n_\gamma^i - Y_{\alpha\beta\gamma}^i) \ \ ; \ \ Y_{\alpha\beta\gamma}^i \ = \ \frac{h_{\alpha\beta\gamma}^i}{h_{\alpha\beta\gamma}} 2ReT_i \ . \qquad (8)$$

Notice that neither the scalar nor the gaugino masses have any explicit dependence on S or T_i, they only depend on the gravitino mass and the goldstino angles. This is one of the advantages of a parametrization in terms of such angles. In the case of the A-parameter an explicit T_i-dependence may appear in the term proportional to $Y_{\alpha\beta\gamma}^i$. This explicit dependence disappears in three interesting cases: 1) In the dilaton-dominated case ($\cos\theta = 0$). 2) When the Yukawa couplings involve only untwisted (**U**) particles, i.e couplings of the type **UUU**, in which case the coupling is a constant. 3) When the particles involved in the coupling have all overall modular weight $n_\alpha = -1$ (again, the coupling is constant). This is possible for couplings of the type $\mathbf{U}\mathbf{T}_{-1}\mathbf{T}_{-1}$, $\mathbf{T}_{-1}\mathbf{T}_{-1}\mathbf{T}_{-1}$, where the subindex indicates the value of the overall modular weight of the twisted (**T**) particle (see below). This is for example the case of any $Z_2 \times Z_2$ orbifold. There is a fourth case in which the $Y_{\alpha\beta\gamma}^i$-term does not disappear but is suppressed for large radii. This happens when the coupling $h_{\alpha\beta\gamma}$ links twisted fields, **TTT**, associated to the same fixed point. In this case one has $h_{\alpha\beta\gamma} \simeq (constant + O(e^{-T}))$ [11] and then $Y_{\alpha\beta\gamma}^i \to 0$. In all the first three cases discussed above the soft terms obtained are independent of the values of S and T_i.

It is appropriate at this point to recall some information about the "modular weights" n_α^i appearing in these expressions. For particles belonging to the untwisted sectors one has

$$n_\alpha^i \ = \ -\delta_\alpha^i \ ; \ i=1,2,3; \ \ n_\alpha^i \ = \ -\delta_\alpha^{i-3} \ ; \ i=4,5,6 \ . \qquad (9)$$

[1]This analysis was also carried out, for the particular case of the three diagonal moduli T_i, in refs.[8] and [9], in order to obtain unification of gauge coupling constants and to analyze FCNC constraints, respectively. Some particular multimoduli examples were also considered in ref.[10] .

Here $i = 1, 2, 3$ labels the three T-type moduli and $i = 4, 5, 6$ the three (maximum) U-type moduli, whereas $\alpha = 1, 2, 3$ labels the three untwisted sectors of the orbifold. Each twisted sector is associated to an order N twist vector $\vec{v} = (v^1, v^2, v^3)$ defined so that $0 \leq v^i < 1$, $\sum_{i=1}^{3} v^i = 1$. In terms of the v_i one finds the following modular weights for particles in twisted sectors:

$$
\begin{aligned}
n_\alpha^i &= -(1 - v^i + p^i - q^i) \ ; \ i = 1, 2, 3; \quad (v^i \neq 0) \ , \\
n_\alpha^{i+3} &= -(1 - v^i + q^i - p^i) \ ; \ i = 1, 2, 3; \quad (v^i \neq 0) \ , \\
n_\alpha^i = n_\alpha^{i+3} &= 0 \quad (v^i = 0) \ ,
\end{aligned}
\tag{10}
$$

where p^i and q^i denote the number of (left-handed) oscillator operators of each chirality in the i-th complex direction (see ref.[7] for details). The "overall T modular weights" corresponding to the "overall modulus" T field considered in ref.[1] are given by $n_\alpha = \sum_{i=1}^{3} n_\alpha^i$. Twisted sectors with all $v^i \neq 0$ (and no oscillators) have overall modular weights $n_\alpha = -2$ due to the property $\sum_{i=1}^{3} v^i = 1$. Twisted sectors with one of the v^i vanishing have the form $\vec{v} = (1/r, (r-1)/r, 0)$ (plus permutations) with $r = 2, 3, 4, 6$. Such sectors obviously have overall modular weights $n_\alpha = -1$. If the twisted particle has also p (q) positive (negative) chirality oscillators, the overall T modular weight gets an extra addition $= p - q$. Particles with oscillators normally correspond to small representations of the gauge group (e.g., singlets) so that one expects the interesting charged particles to be associated to either untwisted sector or twisted sectors with no oscillators (or perhaps at most one or two oscillators).

With the above information we can now analyze the different structure of soft terms available for each Abelian orbifold. The results obtained in ref.[1] corresponded to the assumption that only S and the overall modulus T were the seed of SUSY breaking. Within the more general framework here described, those results correspond to the particular goldstino direction

$$
\vec{\Theta}^2 \ = \ (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0) \tag{11}
$$

and can be recovered from eq.(7) and eq.(8) (assuming also $\gamma_i = \gamma_T$, $h^i_{\alpha\beta\gamma} = h^T_{\alpha\beta\gamma}/3$):

$$
\begin{aligned}
m_\alpha^2 &= \ m_{3/2}^2 (1 \ + \ n_\alpha \cos^2\theta) \ , \\
M &= \ \sqrt{3} m_{3/2} \sin\theta e^{-i\gamma_S} \ , \\
A_{\alpha\beta\gamma} &= \ -\sqrt{3} m_{3/2} \ (\sin\theta e^{-i\gamma_S} \ + \ \frac{1}{\sqrt{3}} \cos\theta e^{-i\gamma_T} \omega_{\alpha\beta\gamma}) \ ,
\end{aligned}
\tag{12}
$$

where we have defined :

$$
\omega_{\alpha\beta\gamma} \ = \ (3 + n_\alpha + n_\beta + n_\gamma - Y^T_{\alpha\beta\gamma}) \ \ ; \ \ Y^T_{\alpha\beta\gamma} \ = \ 2ReT \frac{h^T_{\alpha\beta\gamma}}{h_{\alpha\beta\gamma}} \ . \tag{13}
$$

In that case one could extract a number of generic qualitative properties of soft terms with regard to three important issues : the existence or not of negative mass^2 for some matter fields, the universality of soft scalar masses, and the relative sizes of gaugino versus scalar masses. In the case of an overall T modulus one finds (see the above formulae):

1) Scalars in untwisted and in twisted sectors with overall T-modular weight $n_\alpha = -1$ have always masses-squared ≥ 0.

2) Scalars in twisted sectors with $n_\alpha \leq -2$ are always lighter than those with $n_\alpha = -1$. The condition $\cos^2\theta \leq 1/|n_\alpha|$ is required for a particle C_α not to become tachyonic.

3) Universal soft scalar masses are obtained in two cases: First, in the dilaton-dominated SUSY-breaking ($\cos\theta = 0$) which implies that the whole soft terms are universal (see eq.(7)) [12, 1]. Second, if all scalars have the same overall modular weight $n_\alpha = n$ [1]. For example, this always occurs for any $Z_2 \times Z_2$ orbifold.

4) Due to the above constraints, all scalars C_α verify $M^2 \geq m_\alpha^2$.

We would like now to study to what extent these general conclusions change in the multimoduli case. We will discuss them in turn.

1) Soft masses for $n_\alpha = -1$ particles

Let us start with the first of these issues, the masses of $n_\alpha = -1$ sectors. There are two types of such sectors, the untwisted sector (which is present in any orbifold) and the twisted sectors with $n_\alpha = -1$. We will discuss them in turn. Using the formulae above one finds the following expressions for scalars in the three untwisted sectors of any orbifold:

$$\begin{aligned} m_1^2 &= m_{3/2}^2\,(1 - 3\cos^2\theta(\Theta_1^2 + \Theta_4^2))\ , \\ m_2^2 &= m_{3/2}^2\,(1 - 3\cos^2\theta(\Theta_2^2 + \Theta_5^2))\ , \\ m_3^2 &= m_{3/2}^2\,(1 - 3\cos^2\theta(\Theta_3^2 + \Theta_6^2))\ . \end{aligned} \tag{14}$$

One immediately observes that the only way to avoid the presence of tachyons for *any* choice of goldstino direction in all three sectors is imposing the condition $\cos^2\theta \leq 1/3$. This is to be compared to the overall modulus case (12) in which positive mass^2 was obtained for any θ. Notice the following important sum-rule which is valid for the untwisted particles of any orbifold:

$$m_1^2\ +\ m_2^2\ +\ m_3^2\ =\ |M|^2\ . \tag{15}$$

Furthermore, since $\vec{n_1} + \vec{n_2} + \vec{n_3} = -(1,1,1,1,1,1)$ and the **UUU** Yukawa couplings do not depend on the moduli one also has

$$A_{123}\ =\ -M\ . \tag{16}$$

Let us consider now the case of twisted sectors with $n_\alpha = -1$. As we said, the associated twist vectors have the form $\vec{v} = (1/r, (r-1)/r, 0)$ (plus permutations) with $r = 2,3,4,6$. Looking at the first of the eqs.(7) one sees that one has guaranteed a positive mass^2 if $\cos^2\theta \leq r/3(r-1)$. The tighter bound is obtained when $r = 6$ which yields $\cos^2\theta \leq 2/5$. A generalization of eqs.(15) and (16) apply also in this case. Consider three particles C_α,C_β,C_γ all with overall modular weight $= -1$ coupling through a Yukawa $h_{\alpha\beta\gamma}$. They may belong both to the untwisted sector or to a twisted sector with $n = -1$, i.e. couplings of the type $\mathbf{U}\mathbf{T}_{-1}\mathbf{T}_{-1}$, $\mathbf{T}_{-1}\mathbf{T}_{-1}\mathbf{T}_{-1}$. Then it is easy to convince oneself that again for any possible twist $\vec{n_\alpha} + \vec{n_\beta} + \vec{n_\gamma} = -(1,1,1,1,1,1)$. Then one finds that for *any choice* of goldstino direction

$$m_\alpha^2\ +\ m_\beta^2\ +\ m_\gamma^2\ =\ |M|^2\ = 3m_{3/2}^2\sin^2\theta \tag{17}$$

and besides

$$A_{\alpha\beta\gamma}\ =\ -M\ . \tag{18}$$

The only difference with eqs.(15), (16) is that eqs.(17), (18) apply to any three $n = -1$ particles linked by a Yukawa coupling (and not only to the three untwisted sectors). Thus, for example, the sum-rule applies to any set of three particles which couple in any $Z_2 \times Z_2$ orbifold. Specific examples will be shown below.

Notice that if we insist in having a vanishing gaugino mass, the sum-rules (15) and (17) force the scalars to be either all massless or at least one of them tachyonic. As we will discuss below, having a tachyonic sector is not necessarily a problem, it may even be an advantage, so one should not disregard this possibility at this point. Of course, in the trivial case when there is no physical particle in that particular sector which would have negative mass^2 the situation is also harmless. Let us show an explicit example of this possibility. Consider the second example of Table 3 of ref.[13]. This is a three-generation Z_3 orbifold model with gauge group $SU(3)_c \times SU(3)_L \times SU(3)_R$. It has the particular property that it has no charged matter in the untwisted sector so that the sum-rule (15) can cause no trouble in the untwisted sector (i.e., no physical tachyons). Consider the goldstino direction e.g. $\vec{\Theta} = (0,0,1)$. The untwisted particles would have had masses $m_1^2 = m_2^2 = m_{3/2}^2$, $m_3^2 = m_{3/2}^2(1 - 3\cos^2\theta)$ whereas the twisted particles would have $m_{\mathbf{T}}^2 = m_{3/2}^2(1 - 2\cos^2\theta)$. The absence of charged massless particles in the untwisted sector would have allowed us to have e.g., $1/3 \leq \cos^2\theta \leq 1/2$, values which would have lead to tachyonic states in the untwisted sector. For the particular value $\cos^2\theta = 1/2$ one gets $m_{\mathbf{T}}^2 = 0$ and gaugino masses $M^2 = 3/2 m_{3/2}^2$.

From the above discussion we conclude that in the multimoduli case, depending on the goldstino direction, tachyons may appear both in the untwisted and $n_\alpha = -1$ twisted sectors unless $\cos^2\theta \leq 1/3$. This is to be compared to the overall modulus T case in which tachyons never appear. For $\cos^2\theta \geq 1/3$, one has to be very careful with the goldstino direction if one is interested in avoiding tachyons. In some sense, a certain amount of fine tuning is required so that the goldstino direction goes more and more in the overall T modulus direction as one increases $\cos^2\theta$. Nevertheless we should not forget that tachyons, as we already mentioned above, are not necessarily a problem, but may just show us an instability.

2) Soft masses for $n_\alpha = -2$ particles

In the absence of oscillators, these are particles originated in twisted sectors $\vec{v} = (v^1, v^2, v^3)$ with all $v^i \neq 0$. Plugging the expressions for the modular weights one finds in this case

$$m_\alpha^2 = \ m_{3/2}^2(1 - 3\cos^2\theta) \ + \ 3m_{3/2}^2\cos^2\theta \vec{v}_\alpha . \vec{\Theta}^2 \ , \tag{19}$$

where $\vec{v}_\alpha = (v^1, v^2, v^3, v^1, v^2, v^3)$. It is obvious from eq.(19) that having $\cos^2\theta \leq 1/3$ will be enough to guarantee the absence of tachyons for any $n = -2$ particle. This is to be compared with the overall modulus case analyzed in ref.[1] in which the weaker condition $\cos^2\theta \leq 1/2$ was required. Notice also that in the overall modulus T case one always had that the $n = -1$ scalar had bigger masses than the $n = -2$ scalars. Here the situation may even be reversed. For any three fields C_α,C_β,C_γ linked through a $\mathbf{T}_{-2}\mathbf{T}_{-2}\mathbf{T}_{-2}$ Yukawa coupling one can check the following sum-rule which is true for any goldstino direction $\vec{\Theta}$:

$$m_\alpha^2 + m_\beta^2 + m_\gamma^2 \ = \ 3m_{3/2}^2(1 - 2\cos^2\theta) \ = \ |M|^2 \ - \ 3m_{3/2}^2\cos^2\theta \ . \tag{20}$$

This shows us that, *on average*, $n = -2$ twisted particles are lighter than $n = -1$ particles but the reverse may be true for some particular fields as long as the above sum-rules are not violated.

It is worth noticing here that twisted Yukawa couplings mixing particles with $n = -1$ and $n = -2$ are also possible (e.g. $\mathbf{T}_{-1}\mathbf{T}_{-2}\mathbf{T}_{-2}$, $\mathbf{T}_{-1}\mathbf{T}_{-1}\mathbf{T}_{-2}$). In this case the sum-rule is

$$m_\alpha^2 + m_\beta^2 + m_\gamma^2 \ = \ |M|^2 \ - \ 3m_{3/2}^2\cos^2\theta \ \delta \tag{21}$$

with

$$\delta \equiv 1 - \sum_k \Theta_k^2 \ , \tag{22}$$

where Θ_k are the auxiliary fields of the moduli associated to the vanishing entry of the $n=-1$ twist vectors (see below eq.(16)) present in the coupling, i.e. those with $n_\alpha^k = 0$. Since $0 < \delta < 1$, the sum-rule (21) is rather in-between the (17) and the (20).

Let us finally comment that if the twisted particle has associated an oscillator operator, the modular weight decreases in as many units as (positive chirality) oscillators. This makes very likely for such particles to have negative mass^2 (unless there is approximate dilaton dominance) . In many cases such particles are just singlets and such tachyonic behaviour may just denote that these fields are forced to aquire vev's.

3) Universality of soft scalar masses

In the dilaton-dominated case ($\cos\theta = 0$) the whole soft terms are universal as in the overall modulus case. Also scalars with different overall modular weights n_α have different masses. However, unlike the overall modulus case, non-universal soft scalar masses for particles with the same n_α are allowed and in fact this will be the most general situation (see e.g. eqs.(14,19)).

4) Gaugino versus scalar masses

In the overall modulus T discussed in ref.[1] the heaviest scalars were the ones with modular weight $n=-1$ which had mass^2 $= |M|^2/3$. So scalars are lighter than gauginos at this level. In the multimoduli case sum-rules like (17) replace the equation $3m^2_{n=-1} = |M|^2$. In some way, on average the scalars are lighter than gauginos but there may be scalars with mass bigger than gauginos. In the case of particles with $n=-1$, eq.(17) tells us that this can only be true at the cost of having some of the other three scalars with *negative* mass^2. This may have diverse phenomenological implications depending what is the particle content of the model, as we now explain in some detail:

4-a) Gaugino versus scalar masses in standard model 4-D strings

Let us consider first the case of string models with gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times G$ and see whether one can avoid the general situation of ref.[1], where scalar masses were found to be always smaller than gaugino masses (at tree-level). In the present more general framework, one can certainly find explicit examples of orbifold sectors where some individual scalar mass is bigger than gaugino masses even at the tree-level. For example, let us consider the case of the Z_8 orbifold with an observable particle in the twisted sector $\mathbf{T}_{\theta^6}$. The modular weight associated to that sector is $\vec{n_{\theta^6}} = (-1/4, -3/4, 0, 0)$ and therefore (see eq.(7))

$$m^2_{\theta^6} \ = \ m^2_{3/2} \left[1 - 3\cos^2\theta \left(\frac{1}{4}\Theta_1^2 + \frac{3}{4}\Theta_2^2 \right) \right] \ . \tag{23}$$

Then, choosing e.g. a goldstino direction with $\cos^2\theta = 5/6$, $\Theta_1 = \Theta_2 = 0$, one gets $m^2_{\theta^6} = m^2_{3/2}$, $M^2 = m^2_{3/2}/2$. Many more examples along these lines can be found of course. In general one finds that it is possible to get $m_\alpha > M$, provided $\sin\theta$ is sufficiently small. Indeed, from the general formulae eq.(7) we see that always $m_\alpha \leq m_{3/2}$ and therefore a necessary (although usually not sufficient) condition to get scalars heavier than gauginos is

$$\cos^2\theta > 2/3 \ . \tag{24}$$

After such preliminary remark one immediately realizes that, especially in the case of standard model 4-D strings, further important restrictions on the possibility of getting scalars heavier than gauginos come from sum-rules like (15,17,20,21), which typically constrain the masses of three particles linked via a Yukawa coupling. Suppose that all the three particles involved are observable particles (squarks, sleptons, Higgses). If we require that the corresponding squared masses be non-negative in order to avoid automatically phenomenological problems such as charge and color breaking or Planck scale Higgs vevs, then the sum rule will immediately imply that such masses are smaller than gaugino masses. Conversely, if we tried to obtain one scalar mass bigger than gaugino masses by an appropriate choice of the goldstino direction, then at least one of the other two scalar masses would become tachyonic. On the other hand, tachyons may be helpful if the particular Yukawa coupling does not involve observable particles. They could break extra gauge symmetries and generate large masses for extra particles. We recall that standard-like models in strings usually have too many extra particles and many extra U(1) interactions. Although the Fayet-Iliopoulos mechanism helps to cure the problem [14], the existence of tachyons is a complementary solution.

Concerning observable particles, we have just seen that the sum rules, supplemented by 'no-tachyon' requirements, typically lead to the conclusion that observable scalars are lighter than gauginos

$$m_\alpha < M \ , \tag{25}$$

similarly to the situation found in the symplified scenario of ref.[1]. Therefore, since gaugino loops play a main role in the renormalization of scalar masses down to low-energy, the gluino, slepton and (first and second generation) squark mass relations at the electroweak scale turn out (again) to be

$$m_l < m_q \simeq M_g \ , \tag{26}$$

where gluinos are slightly heavier than squarks. We recall that slepton masses are smaller than squark masses because they do not feel the important gluino contribution.

It is still possible to ask whether the generic situation described by eqs.(25) and (26) admits exceptions. One possibility is the following. One could get some squark or slepton mass bigger than gaugino masses by allowing a negative soft squared mass for a Higgs field, provided the total squared Higgs mass (including the μ^2 contribution) is non-negative[2]. Another possibility which comes to mind is the case in which a Yukawa coupling among 'observable' particles originates actually from a non-renormalizable (rather than renormalizable) coupling[3], where the extra fields in the coupling get vevs (e.g. $H_2 Q_L u_L^c < \phi ... \phi >$ rather than just $H_2 Q_L u_L^c$). In such a case new sum-rules would apply to the full set of fields in the coupling and the above three-particle sum-rules could be violated. In particular, observable scalars would be allowed to be heavier than gauginos, possibly at the price of having some tachyon among the (standard model singlet) ϕ fields. In both cases mentioned here one could get a violation of (25) for some scalars, i.e.

$$m_\alpha > M_a . \tag{27}$$

[2]Notice that such a possibility can be explored in detail only after specifying the mechanism for generating the μ parameter itself (see e.g. ref.[15]).

[3]Notice however that this is unlikely to be the case for the top Yukawa coupling, which is relevant e.g. for radiative symmetry breaking.

However we recall from our initial discussion that this can happen only for small $\sin\theta$ and special goldstino directions. Moreover, even for small (but not too small) $\sin\theta$, scalar and gaugino masses will be still of the same order, so that the low-energy relation (26) will still hold. The only difference is that now squarks, fulfilling eq.(27), will be slightly heavier than gluinos. In order to reverse the situation and get instead

$$M_g < m_l, m_q \tag{28}$$

one needs one of the above 'mechanisms' and very small $\sin\theta$, so that $m_\alpha >> M_a$. Note that in such a limit additional attention should be payed to avoid that a too large scalar-to-gaugino mass ratio could spoil the solution to the gauge hierarchy problem.

Before concluding, we recall that a pattern like (28) for very small $\sin\theta$ was also obtained in the overall modulus analysis of ref.[1] for different reasons, i.e. as an effect of string loop corrections to K and f_a. After the inclusion of such corrections the masses of gauginos and $n_\alpha = -1$ scalars, which vanish at tree-level for $\sin\theta \to 0$, become nonvanishing and typically satisfy relation (27). One difference with the previous case is that the loop-induced case gives scalar masses smaller than $m_{3/2}$ instead than $\mathcal{O}(m_{3/2})$. In addition, one may consider this possibility of obtaining scalars heavier than gauginos as a sort of fine-tuning. In the absence of a more fundamental theory which tells us in what direction the goldstino angles point, one would naively say that the most natural possibility would be to assume that all moduli contribute to SUSY-breaking in more or less (but *not* exactly) the same[4] amount.

Summarizing the situation concerning standard model strings, we have seen that the overall modulus results are qualitatively confirmed, in the sense that for generic goldstino directions (with not too small $\sin\theta$) the low-energy pattern of eq.(26) typically holds, mainly because of the restrictions coming from mass sum rules and absence of tachyons. Possible exceptions giving rise to patterns like (28) may exist for special goldstino angles, necessarily including a sufficiently small $\sin\theta$.

4-b) Gaugino versus scalar masses in GUT 4-D strings

What it turned out to be a potential disaster in the case of standard model strings may be an interesting advantage in the case of string-GUTs. In this case it could well be that the negative mass^2 may just induce gauge symmetry breaking by forcing a vev for a particular scalar (GUT-Higgs field) in the model. The latter possibility provides us with interesting phenomenological consequences. Here the breaking of SUSY would directly induce further gauge symmetry breaking.

Let us now show an explicit example of the different possibilities discussed above (scalars lighter or heavier than gauginos) in the context of GUTS. We are going to consider a $Z_2 \times Z_2$ orbifold model which is an $SO(10)$ string-GUT recently constructed in ref.[17]. We show in Table 1 the particle content of the model and the quantum numbers of the particles with respect to the gauge group $SO(10) \times (SO(8) \times U(1)^2)$. The three untwisted sectors are denoted by $\mathbf{U}_1$,$\mathbf{U}_2$,$\mathbf{U}_3$ and the three twisted sectors by $\mathbf{T}_\theta$,$\mathbf{T}_\omega$ and $\mathbf{T}_{\theta\omega}$. This model has a GUT-Higgs field transforming as a 54 of $SO(10)$ in the $\mathbf{U}_3$ untwisted sector. Four net generations as well as two pairs $16 + \overline{16}$ are present in the $\mathbf{T}_\theta$,$\mathbf{T}_\omega$ twisted sectors. Finally, 10-plets adequate to do the electro-weak symmetry breaking belong to the $\mathbf{T}_{\theta\omega}$ sector.

[4]For an explicit example of this, using gaugino condensation, see ref.[16].

Yukawa couplings of the following types are present in the model:

$$\mathbf{U}_1\mathbf{U}_2\mathbf{U}_3 \quad , \quad \mathbf{U}_3\mathbf{T}_{\theta\omega}\mathbf{T}_{\theta\omega} \quad , \quad \mathbf{T}_{\theta}\mathbf{T}_{\omega}\mathbf{T}_{\theta\omega} \tag{29}$$

(Not all of the latter two couplings are allowed since the space-group selection rules may forbid some of them.) All Yukawa couplings are constants, do not depend on T_i [11].

The $Z_2 \times Z_2$ orbifold has three T moduli and three U moduli in the untwisted sector but we are considering in this example for simplicity the case in which only S and the $T_i, i = 1, 2, 3$ participate in SUSY-breaking. The modular weights of the different sectors are:

$$\begin{array}{c} \vec{n_1} = (-1,0,0) \ ; \ \vec{n_2} = (0,-1,0) \ ; \ \vec{n_3} = (0,0,-1) \ , \\ \vec{n_\theta} = (0,-1/2,-1/2) \ ; \ \vec{n_\omega} = (-1/2,0,-1/2) \ ; \ \vec{n_{\theta\omega}} = (-1/2,-1/2,0) \ . \end{array} \tag{30}$$

All the sectors in the $Z_2 \times Z_2$ orbifold have overall modular weight =–1 and hence the sum-rule (17) applies for any three set of particles linked by a Yukawa coupling. Notice in particular that $\vec{n_\alpha} + \vec{n_\beta} + \vec{n_\gamma} = -(1,1,1,1,1,1)$ for the sets of particles related by the Yukawas (29). Thus, for *any goldstino angle* one has the constraints:

$$\begin{array}{rcccccl} m_1^2 + m_2^2 + m_3^2 & = & m_\theta^2 + m_\omega^2 + m_{\theta\omega}^2 & = & m_3^2 + m_{\theta\omega}^2 + m_{\theta\omega}^2 & = & M^2 \ , \\ A_{123} & = & A_{\theta\omega(\theta\omega)} & = & A_{3(\theta\omega)(\theta\omega)} & = & -M \ . \end{array} \tag{31}$$

To study the different effects of chosing different goldstino directions let us consider several examples:

A) Dilaton dominance: $\cos^2\theta = 0$. All scalars have masses $m_\alpha^2 = m_{3/2}^2$ and $M^2 = 3m_{3/2}^2$. The same universal M/m_α ratio is mantained in the overall modulus case (i.e., $\vec{\Theta}^2 = (1/3, 1/3, 1/3)$) for any θ. This happens because $n_\alpha = -1$ in eq.(12).

B) Consider the goldstino direction $\vec{\Theta}^2 = (1/2, 1/2, 0)$ and $\cos^2\theta = 2/3$. One finds $|M|^2 = |A|^2 = m_{3/2}^2$ and the scalars get masses as shown in column B of Table 1. The soft masses are no longer universal since e.g. the masses of the electroweak doublets and the generations are different. This is important e.g. in computing electro-weak radiative symmetry breaking.

C) Consider the goldstino direction $\vec{\Theta}^2 = (0, 0, 1)$ and $\cos^2\theta = 2/3$. One still has $|M|^2 = |A|^2 = m_{3/2}^2$ but now the GUT-Higgs 54 and the singlets get negative mass^2 (see column C in Table 1). This will drive a large vev (of order the string scale) $< 54 >$. Although one would naively think that the potential becomes unbounded below, one has to recall that the matter metrics that we are using are correct to leading order on the matter fields and hence for vev's of order of the string scales the potential should be stabilized.

D) Consider finally the direction $\vec{\Theta}^2 = (0, 0, 1)$ but $\cos^2\theta = 1$, i.e., only the modulus T_3 contributes to SUSY-breaking (no dilaton contribution). Now the gauginos are massless, the 10-plets have positive masses but both the 54 and the $16 + \overline{16}$ pairs will tend to get vev's (see column D in Table 1).

As the above examples show, different possibilities are obtained for each given orbifold model depending on the particular goldstino direction. However, not any possibility may be realized within a given class of models. For example, the addition of any combination of soft terms violating the constraints (31) would be inconsistent with the hypothesis of dilaton/moduli induced SUSY-breaking. The reader

$Sector$	$SO(10)\times SO(8)$	Q	Q_A	A	B	C	D
$gauginos$	$(45,1)+(1,28)$	0	0	$3m_{3/2}^2$	$m_{3/2}^2$	$m_{3/2}^2$	0
$\mathbf{U}_1$	(1,8)	1/2	1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	(1,8)	-1/2	-1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
$\mathbf{U}_2$	(1,8)	-1/2	1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	(1,8)	1/2	-1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
$\mathbf{U}_3$	(54,1)	0	0	$m_{3/2}^2$	$m_{3/2}^2$	$-m_{3/2}^2$	$-2m_{3/2}^2$
	(1,1)	0	0	$m_{3/2}^2$	$m_{3/2}^2$	$-m_{3/2}^2$	$-2m_{3/2}^2$
	(1,1)	0	1	$m_{3/2}^2$	$m_{3/2}^2$	$-m_{3/2}^2$	$-2m_{3/2}^2$
	(1,1)	1	0	$m_{3/2}^2$	$m_{3/2}^2$	$-m_{3/2}^2$	$-2m_{3/2}^2$
	(1,1)	-1	0	$m_{3/2}^2$	$m_{3/2}^2$	$-m_{3/2}^2$	$-2m_{3/2}^2$
	(1,1)	0	-1	$m_{3/2}^2$	$m_{3/2}^2$	$-m_{3/2}^2$	$-2m_{3/2}^2$
$\mathbf{T}_\theta$	$3(16,1)$	1/4	1/4	$m_{3/2}^2$	$1/2m_{3/2}^2$	0	$-1/2m_{3/2}^2$
	$(\overline{16},1)$	-1/4	-1/4	$m_{3/2}^2$	$1/2m_{3/2}^2$	0	$-1/2m_{3/2}^2$
$\mathbf{T}_\omega$	$3(16,1)$	-1/4	1/4	$m_{3/2}^2$	$1/2m_{3/2}^2$	0	$-1/2m_{3/2}^2$
	$(\overline{16},1)$	1/4	-1/4	$m_{3/2}^2$	$1/2m_{3/2}^2$	0	$-1/2m_{3/2}^2$
$\mathbf{T}_{\theta\omega}$	$4(10,1)$	0	1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	$4(10,1)$	0	-1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	$3(1,8)$	0	1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	$(1,8)$	0	-1/2	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	$8(1,1)$	1/2	0	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$
	$8(1,1)$	-1/2	0	$m_{3/2}^2$	0	$m_{3/2}^2$	$m_{3/2}^2$

Table 1: Particle content and charges of the string-GUT example discussed in the text. The four rightmost columns desplay four examples of consistent soft masses from dilaton/moduli SUSY breaking.

may check that indeed the four choices of soft terms shown in the Table verify the constraints in (31).

Comparing the conclusions of this section with those found in ref.[1] one certainly finds plenty of differences. However the reader must keep in mind that e.g. the examples B,C,D above correspond to extreme cases in which some modulus does not participate at all in the process of symmetry breaking. On the other hand the overall modulus case is also in some way an extreme case since the different moduli participate in *exactly* the same way, which is also a sort of fine-tuning. As already mentioned above, in the absence of a more fundamental theory which tells us in what direction the goldstino angles point, one would naively say that the most natural possibility would be to assume that all moduli contribute to SUSY-breaking in more or less (but not exactly the same) amount. In this case the conclusions would be half-way in-between the results found in this section and those found in ref.[1]. In this context we must remark the sum-rules discussed above which would be valid for any choice of goldstino directions. Let us finally remark that, in spite of the different possibilities of soft masses in the multimoduli case, the most natural (slepton-squark-gluino) mass relations *at low-energy* will be similar to the ones of the overall modulus case eq.(26) as shown in point *4-a*.

3 Off-diagonal matter metric

In the previous chapter we confined ourselves to the case of diagonal matter metric $\tilde{K}_{\overline{\alpha}\beta} \simeq \delta_{\overline{\alpha}\beta}$. In fact that assumption is justified for most of the Abelian orbifold models. The reason is that, in the case of twisted sectors, each particle has associated space-group discrete quantum numbers which forbid off-diagonal metrics (we are talking here about singular, non-smoothed out $(0,2)$ orbifolds). In the case of matter fields in untwisted sectors, both gauge invariance and discrete R-symmetries from the right-moving sector forbids off-diagonal terms in almost all cases. There are only three exceptions to this general rule, the $(0,2)$ models based on the orbifolds Z_3,Z_4 and Z_6'. They are precisely the only Abelian orbifolds in which there are more than three T_i moduli, 9, 5 and 5 respectively. They also have in common the existence of an enhanced non-Abelian gauge symmetry in their $(2,2)$ versions ($SU(3)$ in the first case, $SU(2)$ in the other two). An off-diagonal metric only appears for fields in the untwisted sectors of those examples. In spite of the relative rareness of off-diagonal metric in orbifolds, it is worth studying what new features can appear in this case compared to the diagonal one, since off-diagonal metrics could be present in other less simple (e.g., Calabi-Yau) compactifications.

First we go back to eq.(1) and compute the scalar soft terms in the most general case where the moduli and matter metrics are not diagonal. Then the soft mass matrix $\mathcal{M}'^2_{\overline{\alpha}\beta}$ (corresponding to unnormalized charged fields) and the soft parameters $A_{\alpha\beta\gamma}$ read

$$\begin{aligned}
\mathcal{M}'^2_{\overline{\alpha}\beta} &= m_{3/2}^2 \tilde{K}_{\overline{\alpha}\beta} - \overline{F}^{\overline{i}} (\partial_{\overline{i}} \partial_j \tilde{K}_{\overline{\alpha}\beta} - \partial_{\overline{i}} \tilde{K}_{\overline{\alpha}\gamma} \tilde{K}^{\gamma\overline{\delta}} \partial_j \tilde{K}_{\overline{\delta}\beta}) F^j && (32)\\
A_{\alpha\beta\gamma} &= F^S K_S h_{\alpha\beta\gamma} + \delta A_{\alpha\beta\gamma} && (33)\\
\delta A_{\alpha\beta\gamma} &= F^i \left[\hat{K}_i h_{\alpha\beta\gamma} + \partial_i h_{\alpha\beta\gamma} - \left(\tilde{K}^{\delta\overline{\rho}} \partial_i \tilde{K}_{\overline{\rho}\alpha} h_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \right] && (34)
\end{aligned}$$

where

$$F^S = e^{G/2} K^{-1}_{\bar{S}S} G_{\bar{S}} \quad , \quad F^i = e^{G/2} \hat{K}^{i\overline{j}} G_{\overline{j}} \qquad (35)$$

A generalization of the usual 'angular parametrization' of the F-field vev's will be introduced below in a representative example. The matrix $\hat{K}^{i\bar{j}}$ is the inverse of the moduli metric $\hat{K}_{\bar{j}k} = \partial_{\bar{j}}\partial_k \hat{K}$, i.e. $\hat{K}^{i\bar{j}}\hat{K}_{\bar{j}k} = \delta^i_k$. Similarly, for the matter metric, we define $\tilde{K}^{\alpha\overline{\beta}}$ so that $\tilde{K}^{\alpha\overline{\beta}}\tilde{K}_{\overline{\beta}\gamma} = \delta^\alpha_\gamma$. Notice that, after normalizing the fields to get canonical kinetic terms, the first piece in eq.(32) will lead to universal diagonal soft masses but the second piece will generically induce off-diagonal contributions. Concerning the A-parameters, notice that in this section we have not factored out the Yukawa couplings as usual, since proportionality is not guaranteed. Indeed, although the first term in $A_{\alpha\beta\gamma}$ is always proportional in flavour space to the corresponding Yukawa coupling, the same thing is not necessarily true for the terms contained in δA. One purpose of this section is to study such 'off-diagonal' effects in the soft terms.

In order to get more concrete and manageable results, we will now particularize the above formulae to the untwisted sectors of Z_3,Z_4 and Z_6' orbifolds. The 9 T^i-moduli of the Z_3 orbifold enter in the Kähler potential as elements of a 3×3 matrix $T^{\alpha\overline{\beta}}$, the role of the index i being played by a pair of indices (with $\alpha,\overline{\beta} = 1,2,3$). Similarly, the 4 T^i-moduli of Z_4 and Z_6' orbifolds associated to (say) the first and second complex planes enter by a 2×2 matrix $T^{\alpha\overline{\beta}}$ (with $\alpha,\overline{\beta} = 1,2$). In addition, Z_4 (Z_6') has two additional moduli T^3 and U^3 (one additional modulus T^3) associated to the third complex plane. Such moduli have diagonal metric, as well as the associated untwisted fields. On the other side, the moduli of 'matrix' type and the associated untwisted charged fields have non-diagonal metric, derivable from a Kähler potential of the form

$$\begin{aligned} \delta K &= -\log\det\left((T+T^\dagger)^{\beta\overline{\alpha}} - C^\beta\overline{C}^{\overline{\alpha}}\right) & (36)\\ &\simeq -\log\det\,(T+T^\dagger)^{\beta\overline{\alpha}} + (T+T^\dagger)^{-1}_{\overline{\alpha}\beta}\overline{C}^{\overline{\alpha}}C^\beta\ . & (37)\end{aligned}$$

It is convenient to define the hermitian matrix

$$t \equiv t^{\alpha\overline{\beta}} \equiv (T+T^\dagger)^{\alpha\overline{\beta}}\ . \tag{38}$$

Then it is easy to find that the metric and inverse metric for moduli and matter fields have the following simple expressions in terms of t:

$$\hat{K}_{\bar{i}j} = t^{-1}_{\overline{\alpha}\gamma}t^{-1}_{\overline{\delta}\beta}\ \ ,\ \ \hat{K}^{j\bar{i}} = t^{\gamma\overline{\alpha}}t^{\beta\overline{\delta}}\ \ (i\equiv\alpha\overline{\beta}\ ,\ j\equiv\gamma\overline{\delta})\ , \tag{39}$$

$$\tilde{K}_{\overline{\alpha}\beta} = t^{-1}_{\overline{\alpha}\beta}\ \ ,\ \ \tilde{K}^{\beta\overline{\alpha}} = t^{\beta\overline{\alpha}}\ . \tag{40}$$

In addition, the F^i's and G_i's in such sectors are also conveniently represented by matrices $F\equiv F^{\alpha\overline{\beta}}$ and $G\equiv\partial G/\partial T^{\alpha\overline{\beta}}$. The relation between the matrices F and G follows from eqs. (35) and (39):

$$F = m_{3/2}tG^*t\ . \tag{41}$$

We first consider the $A_{\alpha\beta\gamma}$ parameters, where the indices can now refer to any untwisted fields of the orbifolds under study. The relevant result is that $\delta A_{\alpha\beta\gamma} = 0$. This follows from the above structure of the metric and from the antisymmetry property of Yukawa couplings with respect to extra indices (understood above), e.g. $SU(3)$ indices in (2,2) Z_3 orbifolds or $SU(2)$ indices in (2,2) Z_4, Z_6' orbifolds. Therefore the result for $A_{\alpha\beta\gamma}$ is simply

$$A_{\alpha\beta\gamma} = F^S K_S h_{\alpha\beta\gamma} = -\sqrt{3}m_{3/2}\sin\theta e^{-i\gamma_S}h_{\alpha\beta\gamma} \tag{42}$$

which is the same result (after factorizing out the Yukawa coupling as usual) as for the untwisted sector of any other orbifold eq.(16). Thus *even in the presence of off-diagonal metrics and multiple moduli* the result in eq.(16) still holds.

We will now consider the soft mass matrix (32) in one of the sectors with off-diagonal metric. The result can be written in the following compact form:

$$\mathcal{M}'^2 = m_{3/2}^2 t^{-1} - t^{-1} F t^{-1} F^\dagger t^{-1} \ . \tag{43}$$

If the matter fields are canonically normalized as $C^\alpha \to \hat{C}^\alpha = (t^{-1/2})^\alpha_\beta C^\beta$, the normalized soft mass matrix can be written as

$$\mathcal{M}^2 = m_{3/2}^2 (1 - \Delta) \ , \tag{44}$$

where 1 stands for the unit matrix and the Δ is the matrix

$$\Delta = \frac{1}{m_{3/2}^2} t^{-1/2} F t^{-1} F^\dagger t^{-1/2} \ . \tag{45}$$

It is interesting to notice that the contribution to SUSY-breaking from the moduli of such a sector is

$$\overline{F}^{\bar{i}} \hat{K}_{\bar{i}j} F^j = m_{3/2}^2 \mathrm{Tr}\Delta \ . \tag{46}$$

To continue the discussion we will focus for definiteness on the case of Z_3, where the 9 moduli $T^{\alpha\overline{\beta}}$ exhaust the set of untwisted moduli. We can consider the following parametrization of the dilaton/moduli SUSY-breaking:

$$(S+S^*)^{-1} F^S = \sqrt{3} m_{3/2} \sin\theta e^{-i\gamma_S} \ \ ; \ \ t^{-1/2} F t^{-1/2} = \sqrt{3} m_{3/2} \cos\theta \Theta \ , \tag{47}$$

where Θ is a 3×3 matrix satisfying

$$\mathrm{Tr}\Theta\Theta^\dagger = 1 \ . \tag{48}$$

Notice that the matrix Δ in $\mathcal{M}^2$ (44) can be written

$$\Delta = 3\cos^2\theta \, \Theta\Theta^\dagger \ . \tag{49}$$

In particular, from this one immediately sees that: 1) Δ is positive definite and $\mathrm{Tr}\Delta = 3\cos^2\theta$; 2) the sum of the three eigenvalues of $\mathcal{M}^2$ satisfies

$$\mathrm{Tr}\mathcal{M}^2 = 3 m_{3/2}^2 \sin^2\theta = |M|^2 \tag{50}$$

which confirms the already stated sum-rule eq.(15) for untwisted matter in orbifolds, *even in the presence of off-diagonal metrics*.

An interesting question related to flavour changing issues[5] concerns the degree of degeneracy among the three eigenvalues of $\mathcal{M}^2$. It is clear that, for generic values (vev's) of the matrices t and F (or Θ), Δ will have a generic matrix structure and therefore the eigenvalues of $\mathcal{M}^2$ will be non-degenerate. The approximately degenerate case occurs only when $\mathcal{M}^2$ is approximately proportional to the unit matrix[6], i.e. $\mathcal{M}^2 \propto 1$. This happens: 1) when $\Delta \ll 1$; 2) when $\Delta \propto 1$.

[5]These were analyzed for the simplest case of diagonal metric in refs.[1, 18].

[6]This corresponds to the simplest way of avoiding FCNC. Another possibility occurs if scalar and fermionic mass matrices happen to be aligned [19]. This and other issues on FCNC would require a detailed analysis of the flavour structure of the models, which go beyond the scope of the present paper.

1) $\Delta \ll 1$. This happens when $\cos^2\theta \ll 1$, i.e. when the contribution of the moduli $T^{\alpha\overline{\beta}}$ to SUSY-breaking is negligible. In the case of Z_3 this just corresponds to the dilaton dominated SUSY-breaking (in the case of Z_4, Z_6' the SUSY-breaking could be shared between S and the third-complex-plane moduli). Actually, when discussing FCNC constraints on soft masses, one should consider the renormalization effects from the string scale to the electroweak scale. Such effects include flavour independent contributions from gauginos. For example, if squarks originated from a sector like the one under study, the low energy mass matrix would read $\mathcal{M}^2(M_Z) \sim m_{3/2}^2((1+24\sin^2\theta)1-\Delta)$, with Δ as in eq.(49) for Z_3. Then the constraint $\cos^2\theta \ll 1$ would be relaxed to $\cos^2\theta \ll 1 + 24\sin^2\theta$ [1] and the moduli would be allowed to participate to some extent to SUSY-breaking. On the other side, no significant relaxation would be obtained for sleptons.

2) $\Delta \propto 1$. This condition guarantees that $\mathcal{M}^2 \propto 1$ even when the moduli participate significantly to SUSY-breaking. Observing eq.(45), we can distinguish two subcases. 2a) If t and F are treated as independent objects, than the only obvious way to satisy that condition is that both $t \propto 1$ and $F \propto 1$. This requires not only that the off-diagonal moduli and F-terms be negligible, but also that the diagonal ones be almost identical, i.e. one is pushed towards the overall modulus limit. 2b) Such conclusion may be evaded if t and F are related in some way, e.g. if $F \propto t$ (giving again $\Delta \propto 1$). If this were the case, the off-diagonal elements of F and t would not need to be negligible with respect to the diagonal ones. An extreme example of this situation happens when W does not depend on the $T^{\alpha\overline{\beta}}$. In that case $F = -m_{3/2}t$ and $\Delta = 1$, implying $\mathcal{M}^2 = 0$ and a no-scale scenario. An example where $\mathcal{M}^2 \neq 0$ can be obtained e.g. if W depends on $T^{\alpha\overline{\beta}}$ only via $\det T^{\alpha\overline{\beta}}$ (and if the vev of $T^{\alpha\overline{\beta}}$ is hermitian).

4 The B-parameter and the μ problem

When an (effective) $N = 1$ SUSY mass $\mu_{\alpha\beta}C^\alpha C^\beta$ appears in the Lagrangian of an $N = 1$ theory, SUSY-breaking also induces an associated SUSY-breaking term $B_{\alpha\beta}\mu_{\alpha\beta}C^\alpha C^\beta + h.c.$. Very often these terms are absent due to gauge invariance. Thus in the MSSM there is only one B-term associated to a possible $\mu H_1 H_2$ SUSY mass term. In fact both a μ-term and a B-term are phenomenologically required in the MSSM in order to, among other things, avoid the presence of a visible axion.

The parameter μ of the MSSM has to be (on phenomenological grounds) of the order of the low-energy SUSY-breaking scale (i.e., of order $m_{3/2}$). The absence of a symmetry reason for such small value for μ is called the "μ-problem" [20]. Thus in order to be able to compute B-term in a given model, we need first a mechanism which might naturally induce a μ-term of order $m_{3/2}$. We will discuss some of the mechanisms proposed within the context of string-models to solve this μ-problem and we will also provide expressions for the associated B-terms in this section.

4.1 B-term from the Kähler potential in orbifold models

It was pointed out in ref.[3] that terms in a Kähler potential like the one proportional to $Z_{\alpha\beta}$ in eq.(1) can naturally induce a μ-term for the C_α fields of order $m_{3/2}$ after SUSY-breaking, thus providing a rationale for the size of μ. Recently it has been realized that such type of terms do appear in the Kähler potential of some Calabi-Yau type compactifications [12] and in orbifold models [21, 22, 10]. Let us consider the case in which e.g., due to gauge invariance, there is only one possible μ-term (and correspondingly one B-term) associated to a pair of matter fields C_1,C_2. From eqs.(1,2,3) and from the fermionic part of the SUGRA lagrangian one can check that a SUSY mass term $\mu C_1 C_2$ and a scalar term $B\mu(C_1C_2)+h.c.$ are induced upon SUSY-breaking in the effective low energy theory (here the kinetic terms for $C_{1,2}$ have been normalized to one). If we introduce the abbreviations

$$L^Z \equiv \log Z \ \ , \ \ L^\alpha \equiv \log \tilde{K}_\alpha \ \ , \ \ X \equiv 1 - \sqrt{3}\cos\theta \ e^{i\gamma_i}\Theta_i(\hat{K}_{\bar{i}i})^{-1/2}L^Z_{\bar{i}} \tag{51}$$

the μ and B parameters (we will call them μ_Z and B_Z) are given by

$$\mu_Z \ = \ m_{3/2}(\tilde{K}_1\tilde{K}_2)^{-1/2} Z X \ , \tag{52}$$

$$\begin{aligned} B_Z \ = \ & m_{3/2}X^{-1}\Big[2+\sqrt{3}\cos\theta(\hat{K}_{\bar{i}i})^{-1/2}\Theta_i\left(e^{-i\gamma_i}(L_i^Z-L_i^1-L_i^2)-e^{i\gamma_i}L^Z_{\bar{i}}\right) \\ & + 3\cos^2\theta(\hat{K}_{\bar{i}i})^{-1/2}\Theta_i e^{i\gamma_i}\ \left(L^Z_{\bar{i}}(L^1_j+L^2_j)-L^Z_{\bar{i}}L^Z_j - L^Z_{\bar{i}j}\right)(\hat{K}_{\bar{j}j})^{-1/2}\Theta_j e^{-i\gamma_j}\Big] \ . \end{aligned} \tag{53}$$

The above formulae apply to the cases where the moduli on which $\tilde{K}_1(T_i,T_i^*)$, $\tilde{K}_2(T_i,T_i^*)$ and $Z(T_i,T_i^*)$ depend have diagonal metric, which is the relevant case we are going to discuss (anyway, the above formulae are easily generalized to more general situations).

If the value of V_0 is not assumed to be zero, one just has to replace $\cos\theta \to C\cos\theta$ in eqs.(51,52,53), where C is given below eq.(5). In addition, the formula for B gets an additional contribution given by $m_{3/2}X^{-1}3(C^2-1)$.

It has been recently shown that the untwisted sector of orbifolds with at least one complex-structure field U possesses the required structure $Z(T_i,T_i^*)C_1C_2+h.c.$ in their Kähler potentials. Specifically, the Z_N orbifolds based on Z_4, Z_6,Z_8, Z_{12}' and the $Z_N\times Z_M$ orbifolds based on $Z_2\times Z_4$ and $Z_2\times Z_6$ do all have a U-type field in (say) the third complex plane. In addition the $Z_2\times Z_2$ orbifold has U fields in the three complex planes. In all these models the piece of the Kähler potential involving the moduli and the untwisted matter fields $C_{1,2}$ in the third complex plane has the form

$$\begin{aligned} K(T_i,T_i^*,C_1,C_2) &= K'(T_l,T_l^*) \\ & - \log\left((T_3+T_3^*)(U_3+U_3^*)-(C_1+C_2^*)(C_1^*+C_2)\right) \end{aligned} \tag{54}$$

$$\simeq K'(T_l,T_l^*) - \log(T_3+T_3^*) - \log(U_3+U_3^*) \ + \frac{(C_1+C_2^*)(C_1^*+C_2)}{(T_3+T_3^*)(U_3+U_3^*)} \tag{55}$$

The first term $K'(T_l,T_l^*)$ determines the (not necessarily diagonal) metric of the moduli $T_l \neq T_3, U_3$ associated to the first and second complex planes. The last term describes an $SO(2,n)/SO(2)\times SO(n)$ Kähler manifold ($n=4$ if we focus on just one component of C_1 and C_2) parametrized by T_3,U_3,C_1,C_2. If the expansion shown in (55) is performed, on one hand one recovers the well known factorization $SO(2,2)/SO(2)\times SO(2) \simeq (SU(1,1)/U(1))^2$ for the submanifold spanned by T_3 and U_3 (which have therefore diagonal metric to lowest order in the matter fields), whereas on the other hand one can easily identify the functions $Z,\tilde{K}_1,\tilde{K}_2$ associated to C_1 and C_2:

$$Z \ = \ \tilde{K}_1 \ = \ \tilde{K}_2 \ = \ \frac{1}{(T_3+T_3^*)(U_3+U_3^*)} \ . \tag{56}$$

Plugging back these expressions in eqs.(52,53,51) one can easily compute μ and B for this interesting class of models:

$$\mu_Z \;=\; m_{3/2}\,\left(1 \;+\; \sqrt{3}\cos\theta(e^{i\gamma_3}\Theta_3 + e^{i\gamma_6}\Theta_6)\right)\;, \tag{57}$$

$$\begin{aligned} B_Z\mu_Z = 2m_{3/2}^2 & \left(1 + \sqrt{3}\cos\theta(\cos\gamma_3\Theta_3 + \cos\gamma_6\Theta_6)\right. \\ & + 3\cos^2\theta\cos(\gamma_3 - \gamma_6)\Theta_3\Theta_6\big)\;. \end{aligned} \tag{58}$$

In addition, we recall from eq.(14) that the soft masses are

$$m_{C_1}^2 \;=\; m_{C_2}^2 \;=\; m_{3/2}^2\,\left(1 \;-\; 3\cos^2\theta(\Theta_3^2 + \Theta_6^2)\right)\;. \tag{59}$$

In general, the dimension-two scalar potential for $C_{1,2}$ (now denoting again normalized fields) after SUSY-breaking has the form

$$V_2(C_1, C_2) \;=\; (m_{C_1}^2 + |\mu|^2)|C_1|^2 + (m_{C_2}^2 + |\mu|^2)|C_2|^2 + (B\mu C_1C_2 + h.c.) \tag{60}$$

In the specific case under consideration, from eqs.(57,58,59) we find the remarkable result, which is also true for any value of C, that the three coefficients in $V_2(C_1, C_2)$ are equal, i.e.

$$m_{C_1}^2 + |\mu_Z|^2 = m_{C_2}^2 + |\mu_Z|^2 = B_Z\mu_Z \tag{61}$$

so that $V_2(C_1, C_2)$ has the simple form

$$V_2(C_1, C_2) \;=\; B_Z\mu_Z\;(C_1 + C_2^*)(C_1^* + C_2)\;. \tag{62}$$

Although the common value of the three coefficients in eq.(61) depends on the Goldstino direction via the parameters $\cos\theta$, Θ_3, Θ_6,... (see expression of $B_Z\mu_Z$ in eq.(58)), we stress that the equality itself and the form of V_2 hold *independently of the Goldstino direction*. The only constraint that one may want to impose is that the coefficient $B_Z\mu_Z$ be non-negative, which would select a region of parameter space. For instance, if one neglects phases, such requirement can be written simply as

$$(1 + \sqrt{3}\cos\theta\;\Theta_3)(1 + \sqrt{3}\cos\theta\;\Theta_6) \geq 0\;. \tag{63}$$

We notice in passing that the fields $C_{1,2}$ appear in the SUSY-breaking scalar potential in the same combination as in the Kähler potential. This particular form may be understood as due to a symmetry under which $C_{1,2} \to C_{1,2} + i\delta$ in the Kähler potential which is transmitted to the final form of the scalar potential.

An important (Goldstino-direction-independent) consequence of the above form (62) is that $V_2(C_1, C_2)$ identically vanishes along the direction $C_1 = -C_2^*$, on which gauge symmetry is broken. If dimension-four couplings respect such flat direction (which is certainly the case for D-terms), we arrive at the important result that along $< C_1 >= - < C_2^* >$ the *flatness is not spoiled by the dilaton/moduli induced SUSY-breaking*. This is certainly a very remarkable property.

This result can be rephrased in terms of the usual parameter $\tan\beta =< C_2 > / < C_1 >$ (we now assume real vev's). It is well known that, for a potential of the generic form (60) (+D-terms), the minimization conditions yield

$$\sin 2\beta \;=\; \frac{-2B\mu}{m_{C_1}^2 + m_{C_2}^2 + 2|\mu|^2}\;. \tag{64}$$

In particular, this relation embodies the boundedness requirement: if the absolute value of the right-hand side becomes bigger than one, this would indicate that the potential becomes unbounded from below. As we have seen, in the class of models under consideration the particular expressions of the mass parameters lead to the equality (61), which in turns implies $\sin 2\beta = -1$. Thus one finds $\tan\beta = -1$ *for any value of* $\cos\theta$,Θ_3,Θ_6 (and of the other Θ_i's of course), i.e. for any Goldstino direction.

It is interesting to relate these results to similar ones obtained in ref.[23] in a slightly different context. In ref.[23] a *specific* SUGRA model was built, where the Higgs-dependent part of the Kähler potential had the form in eq.(54), with $T_3 = U_3$. The geometrical properties of the associated manifold and a simple choice for the superpotential allowed to obtain the simultaneous breaking of SUSY and gauge symmetry, with the cosmological constant identically vanishing along some flat directions which included the $|C_1| = |C_2|$ direction. This also implied a partial participation of charged fields in the process of SUSY-breaking[7]. In the limit of suppressed goldstino components along the Higgsinos, SUSY-breaking was essentially dilaton/moduli dominated. Then such model could be viewed as a very special case of the more general framework here discussed, characterized by specific values of the goldstino angles: $\cos^2\theta = 2/3$, $\Theta_3^2 = \Theta_6^2 = 1/2$ and vanishing values for the remaining Θ_i's. In particular one had $V_2(C_1, C_2) \equiv 0$, the flat direction $|C_1| = |C_2|$ being enforced by the D-term. The remarkable result obtained in this section is that the prediction $|\tan\beta| = 1$ is actually valid for a much broader class of models and holds irrespectively of the goldstino direction in the dilaton/moduli space. Whether the above mechanism can be successfully implemented in the case of the electroweak Higgs fields remains an open question. Flat potentials of the type here considered could be interesting also for the breaking of a grand-unified gauge group (as suggested e.g. in ref.[24]), in particular in the context of models like string-GUTs [17], in which a vev of order the string scale is not problematic.

As an additional comment, it is worth recalling that in previous analyses of the above mechanism for generating μ and B in the string context [12, 25, 1] the value of μ was left as a free parameter since one did not have an explicit expression for the function Z. However, if the explicit orbifold formulae for Z are used, one is able to predict both μ and B reaching the above conclusion. We should add that situations are conceivable where the above result may be evaded, for example if the physical Higgs doublets are a mixture of the above fields with some other doublets coming from other sectors (e.g. twisted) of the theory.

4.2 B-term from the superpotential

There is an alternative mechanism to the one studied in the previous subsection to generate a B-term in the scalar potential. It is well known that if the superpotential W is assumed to have a $\mu C_1 C_2$ SUSY mass term, μ being an initial parameter, then a B-term is automatically generated. We will call it B_μ. If we introduce the abbreviation

$$L^\mu \equiv \log\mu \tag{65}$$

the μ and B parameters are given by

$$\mu' \;=\; \mu e^{K/2}\frac{W^*}{|W|}(\tilde{K}_1\tilde{K}_2)^{-1/2} \;, \tag{66}$$

[7]An elaboration of this idea was later studied in ref.[24].

$$\begin{aligned} B_\mu &= m_{3/2}\Big[-1-\sqrt{3}e^{-i\gamma_S}\sin\theta(1-L^\mu_S 2ReS) \\ &+\sqrt{3}\cos\theta(\hat{K}_{\bar{i}i})^{-1/2}\Theta_i e^{-i\gamma_i}(\hat{K}_i+L^\mu_i-L^1_i-L^2_i)\Big] \ , \end{aligned} \tag{67}$$

where the low-energy SUSY mass μ' is related to μ via the usual SUGRA rescaling, and again the kinetic terms for $C_{1,2}$ have been normalized to one. In the above formulae we have assumed that in general μ will depend on the SUSY-breaking sector fields, i.e. $\mu=\mu(S,T_i)$. These formulae are completely general and valid for any solution to the μ-problem which introduces a small mass term $\mu(S,T_i)C_1C_2$ in W. This type of solutions exists.

In ref.[4] was pointed out that the presence of a non-renormalizable term in the superpotential

$$\lambda W C_1 C_2 \tag{68}$$

characterized by the coupling λ, yields dynamically a μ parameter when W acquires a vev

$$\mu=\lambda W \ . \tag{69}$$

The fact that μ is small is a consequence of our assumption of a correct SUSY-breaking scale $m_{3/2}=e^{G/2}=e^{K/2}|W|$. The superpotential eq.(68) which provides a possible solution to the μ problem can naturally be obtained in the context of strings. A realistic example where non-perturbative SUSY-breaking mechanisms like gaugino-squark condensation induce that superpotential was given in ref.[4], where $\lambda=\lambda(T_i)$ is a non-renormalizable Yukawa coupling between the Higgses and the squarks and after eliminating the gaugino and squarks bound states $W=W(S,T_i)$. In ref.[22] the same kind of superpotential was obtained through pure gaugino condensation in orbifolds with at least one complex-structure field U. This is because in these orbifolds matter field-dependent threshold corrections ($\propto C_1C_2$) appear in the gauge kinetic function f. We recall that after eliminating the gaugino bound states the non-perturbative superpotential $W\sim exp(3f/2b_0)$, where b_0 is the one-loop β-function coefficient of the "hidden" gauge group. After expanding the exponential, the superpotential will have a contribution of the type (68). Again, $\lambda=\lambda(T_i)$, since the above proportionality factor due to threshold corrections depends on Dedekind functions which depend in turn on the moduli.

So with this solution (69) to the μ-problem in strings:

$$\mu(S,T_i)=\lambda(T_i)W(S,T_i) \ . \tag{70}$$

Plugging back this expression in eqs.(66,67) and imposing the vanishing of the cosmological constant V_0, one can easily compute μ and B for this mechanism. We will call them μ_λ and B_λ

$$\mu_\lambda = \lambda m_{3/2}(\tilde{K}_1\tilde{K}_2)^{-1/2} \ , \tag{71}$$

$$B_\lambda = m_{3/2}\Big[2+\sqrt{3}\cos\theta(\hat{K}_{\bar{i}i})^{-1/2}\Theta_i e^{-i\gamma_i}(L^\lambda_i-L^1_i-L^2_i)\Big] \ , \tag{72}$$

where

$$L^\lambda\equiv\log\lambda \ . \tag{73}$$

If the value of V_0 is not assumed to be zero, one just has to replace $\cos\theta\to C\cos\theta$ and $\sin\theta\to C\sin\theta$ in eqs.(67,72), where C is given below eq.(5). In addition, the formula for B_λ, eq.(72), gets an additional contribution given by $m_{3/2}3(C^2-1)$.

Concentrating again on the interesting case of orbifolds, where the Kähler potential eq.(6) is known, we obtain from eq.(72)

$$B_\lambda \;=\; m_{3/2}\left[2-\sqrt{3}\cos\theta\sum_{i=1}^{6}e^{-i\gamma_i}\Theta_i\left(n_1^i+n_2^i-\tfrac{\lambda_i}{\lambda}2ReT_i\right)\right]\;. \qquad (74)$$

Notice that it is conceivable that both mechanisms, the one solving the μ-problem through the Kähler potential (see subsection 4.1) [3] and the other one solving it through the superpotential [4] shown above, could be present simultaneously. In that case the general expressions for B and μ are easily obtained

$$\mu \;=\; \mu_Z+\mu_\lambda\;, \qquad (75)$$
$$B \;=\; \mu^{-1}(B_Z\mu_Z+B_\lambda\mu_\lambda)\;, \qquad (76)$$

where μ_Z, B_Z are given in eqs.(57,58). For example, in the case of orbifolds with at least one complex-structure field U, where the B_Z-term from the Kähler potential is present, if a gaugino condensate is formed, then automatically the B_λ-term from the superpotential is also present as mentioned above. Now, as in the case of B_Z (see eqs.(57,58)), in B_λ (74) only Θ_3 and Θ_6 contribute. We recall that the values of $\tilde{K}_1, \tilde{K}_2$ are given by eq.(56) and besides, $\lambda=\lambda(T_3,U_3)$ (the concrete expression can be found in ref.[22]). However, in this case the last equality of eq.(61) with $Z\to\lambda$ does not hold.

5 Final comments and conclusions

In this paper we have generalized in several directions previous analyses of SUSY-breaking soft terms induced by dilaton/moduli sectors. In particular, we have studied the new features appearing when one goes to the Abelian orbifold multimoduli case. We have found that there are qualitative changes in the general patterns of soft terms. In some way (*on average*) the results are similar to the case in which only S and the "overall modulus" T field are considered. However, if one examines the soft terms for each particle individually one finds different extreme patterns. For example, non-universal soft scalar masses for particles with the same overall modular weight are allowed and in fact this will be the most general situation. Besides, unlike in the case considered in [1], gauginos may be lighter than scalars even at the tree-level. The possibilities are, however, not arbitrary. The fact that *on average* the results are similar to the simple S, T case are embodied in general sum rules like those in eqs.(15,17,20,21) which relate soft terms of different particles in the theory.

Due to the mentioned sum-rules, if we insist in obtaining results qualitatively different from those in ref.[1] (e.g., gauginos lighter than scalars at the tree-level), some scalars may get negative mass^2. This tachyonic behaviour may be just signaling gauge symmetry breaking, which might be a useful possibility in GUT model-building. On the contrary, in the case of standard model 4-D strings, the appearence of this tachyonic behaviour could be dangerous since it could lead to the breaking of charge and/or colour. In order to avoid this problem, one is typically lead to a situation with gauginos heavier than scalars, as in the overall modulus case [1]. We have also commented on possible exceptions to such scenario (involving non renormalizable Yukawa couplings or negative soft mass^2 for the standard model Higgses) which could lead to scalars heavier than gauginos. Such inversion however can take place only for special goldstino directions, and requires necessarily a small $\sin\theta$. We recall that the $\sin\theta \to 0$ limit was also the only one which could produce scalars heavier than gauginos in the overall modulus analysis, for other reasons (i.e. the different effect of string loop corrections on gaugino and scalar masses, vanishing at tree-level).

We have also generalized our study to include the case of orbifolds with off-diagonal untwisted $T^{\alpha\overline{\beta}}$ moduli. In this type of models non-diagonal metrics for the untwisted matter fields appear. In spite of this complication, sum rules analogous to those in eqs.(15,16) still hold (i.e., eqs.(50,42)). Non-diagonal metrics for the matter fields do also in general induce off-diagonal soft-masses for the scalars which in turn can induce flavour-changing neutral currents depending on the size of the off-diagonal moduli, as discussed in section 3.

We have finally considered the μ and B terms obtained in orbifold schemes. We have shown that the scheme in ref.[3] in which a μ-term is generated from a bilinear piece in the Kähler potential, is rather constrained in its orbifold implementation. We find that *irrespective of the Goldstino direction* one always gets $|tg\beta| = 1$ at the string scale. Another way of stating the same result is that the flat direction $\langle H_1 \rangle = \langle H_2 \rangle$ still remains flat after including arbitrary dilaton/moduli-induced SUSY-breaking terms. This is an intriguing result which could have interesting phenomenological applications. The results obtained for the B-parameter in the scheme of ref.[4] in which a μ-term is generated from the superpotential are more model dependent.

A few comments before closing up are in order. First of all we are assuming here that the seed of SUSY-breaking propagates through the auxiliary fields of the dilaton S and the moduli T_i fields. However attractive this possibility might be, it is fair to say that there is no compelling reason why indeed no other fields in the theory could participate. Nevertheless the present scheme has a certain predictivity due to the relative universality of the couplings of the dilaton and moduli. Indeed, the dilaton has universal and model-independent couplings which are there independently of the four-dimensional string considered. The moduli T_i fields are less universal, their number and structure depend on the type of compactification considered. However, there are thousands of different $(0,2)$ models with different particle content which share the same T_i moduli structure. For example, the moduli structure of a given Z_N orbifold is the same for all the thousands of $(0,2)$ models one can construct from it by doing different embeddings and adding discrete Wilson lines. So, in this sense, although not really universal, there are large classes of models with identical T_i couplings. This is not the case of generic charged matter fields whose number and couplings are completely out of control, each individual model being in general completely different from any other. Thus assuming dilaton/moduli dominance in the SUSY-breaking process has at least the advantage of leading to specific predictions for large classes of models whereas if charged matter fields play an important role in SUSY-breaking we will be forced to a model by model analysis, something which looks out of reach.

Another point to remark is that we are using the tree level forms for both the Kähler potential and the gauge kinetic function. One-loop corrections to these functions have been computed in some classes of four-dimensional strings and could be included in the above analysis without difficulty. The effect of these one-loop corrections will in general be negligible except for those corners of the Goldstino directions in which the tree-level soft terms vanish. However, as already mentioned above, this situation would be a sort of fine-tuning. More worrysome are the possible non-perturbative string corrections to the Kähler and gauge kinetic functions. We have made use in our orbifold models of the known tree-level results for those functions. If the non-perturbative string corrections turn out to be important, it would be impossible to make any prediction about soft terms unless we know all the relevant non-perturbative string dynamics, something which looks rather remote (although perhaps not so remote as it looked one year ago!).

One might hope that the relationships obtained among soft terms in the dilaton/moduli dominated schemes could be more general than the original tree-level Lagrangians from which they are derived. In this connection it has been recently realized that the boundary conditions $-A = M_{1/2} = \sqrt{3}m$ of dilaton dominance coincide with some boundary conditions considered by Jones, Mezincescu and Yau in 1984 [26]. They found that those same boundary conditions mantain the (two-loop) finiteness properties of certain $N = 1$ SUSY theories. It has also been noticed [27] that this coincidence could be related to an underlying $N = 4$ structure of the dilaton Lagrangian and that the dilaton-dominated boundary conditions could also appear as a fixed point of renormalization group equations [27, 28]. This could perhaps be an indication that at least some of the possible soft terms obtained in the present scheme could have a more general relevance, not necessarily linked to a particular form of a tree level Lagrangian.

References

[1] A. Brignole, L.E. Ibáñez and C. Muñoz, *Nucl. Phys.* **B422** (1994) 125 [Erratum: **B436** (1995) 747].

[2] L.J. Dixon, J. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* **B261** (1985) 651; **B274** (1986) 285.

[3] G.F. Giudice and A. Masiero, *Phys. Lett.* **B206** (1988) 480.

[4] J.A. Casas and C. Muñoz, *Phys. Lett.* **B306** (1993) 288.

[5] B.R. Greene, M.R. Plesser, E. Rusjan and X. Wang, *Mod. Phys. Lett.* **A6** (1991) 591.

[6] E. Witten, *Phys. Lett.* **155B** (1985) 151; S. Ferrara, C. Kounnas and M. Porrati, *Phys. Lett.* **B181** (1986) 263; M. Cvetic, J. Louis and B. Ovrut, *Phys. Lett.* **B206** (1988) 227; M. Cvetic, J. Molera and B. Ovrut, *Phys. Rev.* **D40** (1989) 1140; L.J. Dixon, V.S. Kaplunovsky and J. Louis, *Nucl. Phys.* **B329** (1990) 27.

[7] L.E. Ibáñez and D. Lüst, *Nucl. Phys.* **B382** (1992) 305.

[8] T. Kobayashi, D. Suematsu, K. Yamada and Y. Yamagishi, *Phys. Lett.* **B348** (1995) 402.

[9] P. Brax and M. Chemtob, *Phys.Rev.* **D51** (1995) 6550.

[10] S. Ferrara, C. Kounnas and F. Zwirner, *Nucl. Phys.* **B429** (1994) 589 [Erratum: **B433** (1995) 255].

[11] For a review, see e.g.: J.A. Casas, F. Gomez and C. Muñoz, *Int. J. Mod. Phys.* **A8** (1993) 455, and references therein.

[12] V.S. Kaplunovsky and J. Louis *Phys. Lett.* **B306** (1993) 269.

[13] L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Phys. Lett.* **187B** (1987) 25.

[14] For a review, see e.g.: J.A. Casas and C. Muñoz *Nucl. Phys. (Proc. Suppl.)* **B16** (1990) 624, and references therein.

[15] A. Brignole, L.E. Ibáñez and C. Muñoz, in preparation.

[16] D. Bailin, A. Love, W.A. Sabra and S. Thomas, *Mod. Phys. Lett.* **A9** (1994) 2543.

[17] G. Aldazabal, A. Font, L.E. Ibáñez and M.A. Uranga, *FTUAM 94/28, hep-th/9410206.*

[18] D. Choudhury, F. Eberlein, A. König, J. Louis and S. Pokorsky, *Phys. Lett.* **B342** (1995) 180; J. Louis and Y. Nir, *Nucl. Phys.* **B447** (1995) 18.

[19] Y. Nir and N. Seiberg, *Phys. Lett.* **B309** (1993) 377.

[20] For a recent review, see: C. Muñoz, *FTUAM 95/20, hep-th/9507108.*

[21] G. Lopes-Cardoso, D. Lüst and T. Mohaupt, *Nucl. Phys.* **B432** (1994) 68.

[22] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, *Nucl. Phys.* **B432** (1994) 187.

[23] A. Brignole and F. Zwirner, *Phys. Lett.* **B342** (1995) 117.

[24] A. Brignole, F. Feruglio and F. Zwirner, *Phys. Lett.* **B356** (1995) 500.

[25] R. Barbieri, J. Louis and M. Moretti, *Phys. Lett.* **B312** (1993) 451.

[26] D.R.T. Jones, L. Mezincescu and Y.P. Yao, *Phys. Lett.* **B148** (1984) 317.

[27] L.E. Ibáñez, Talk given at Strings 95, *FTUAM 95/15.*

[28] P.M. Ferreira, I. Jack and D.R.T. Jones, *LTH 352*, *hep-ph 9506467.*