

# SU(3) LATTICE GAUGE THEORY IN THE FUNDAMENTAL-ADJOINT PLANE AND SCALING ALONG THE WILSON AXIS<sup>1</sup>

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## Abstract

We present further evidence for the bulk nature of the phase transition line in the fundamental-adjoint action plane of SU(3) lattice gauge theory. Computing the string tension and some glueball masses along the thermal phase transition line of finite temperature systems with  $N_t = 4$ , which was found to join onto the bulk transition line at its endpoint, we find that the ratio  $\sqrt{\sigma}/T_c$  remains approximately constant. However, the mass of the  $0^{++}$  glueball decreases as the endpoint of the bulk transition line is approached, while the other glueball masses appear unchanged. This is consistent with the notion that the bulk transition line ends in a critical endpoint, with the continuum limit there being a  $\phi^4$  theory with a diverging correlation length only in the  $0^{++}$  channel. We comment on the implications for the scaling behavior along the fundamental or Wilson axis.

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# 1 Introduction

The study of the phase diagram of fundamental–adjoint pure gauge systems,

$$S = \beta_f \sum_P [1 - \frac{1}{N} \text{ReTr}U_P] + \beta_a \sum_P [1 - \frac{1}{N^2} \text{Tr}U_P^\dagger \text{Tr}U_P], \quad (1)$$

in the early 80's revealed a non-trivial phase structure with first order (bulk) transitions in the region of small  $\beta_f$  [1, 2]. In particular a line of first order transitions points towards the fundamental axis and, after terminating, extends as a roughly straight line of bulk crossovers to the fundamental axis and beyond. This non-trivial phase structure, and in particular the critical endpoint, has been argued to be associated with the dip in the discrete  $\beta$ -function of the theory with standard Wilson action, which occurs in the region where the bulk crossover line crosses the fundamental axis.

In a couple of recent papers Gavai, Grady and Mathur [3] and Mathur and Gavai [4] returned to investigating the behavior of pure gauge SU(2) theory in the fundamental–adjoint plane at finite, non-zero temperature. They raised doubts about the bulk nature of the phase transition and claimed that their results were consistent with the transitions, for lattices with temporal extent  $N_t = 4, 6$  and 8, to be of thermal, deconfining nature, displaced toward weak coupling with increasing  $N_t$ . On the transition line for a fixed  $N_t$  there is then a switch from second order behavior near the fundamental axis to first order behavior at larger adjoint coupling. In a Landau Ginzburg model of the effective action in terms of Polyakov lines, Mathur [5] reported that he could reproduce the claimed behavior seen in the numerical simulations. These results, should they be confirmed, are rather unsettling, since they contradict the usual universality picture of lattice gauge theory with a second order deconfinement transition for gauge group SU(2).

Puzzled by the finding of Ref. [3], we studied the finite temperature behavior of pure gauge SU(3) theory in the fundamental–adjoint plane [6, 7]. We obtained results in agreement with the usual universality picture: there is a first order bulk transition line ending at

$$(\beta_f^*, \beta_a^*) = (4.00(7), 2.06(8)). \quad (2)$$

The thermal deconfinement transition lines for fixed  $N_t$  (being of first order for gauge group SU(3)) in the fundamental adjoint plane are ordered such that the thermal transition for smaller  $N_t$  occurs to the left, at smaller  $\beta_f$ , than that for a larger  $N_t$ . In this order the thermal transition lines join on to the bulk transition line. The thermal transition line for  $N_t = 4$  joins the bulk transition line very close to the critical endpoint. This is shown in Figure 4 of Ref. [7], reproduced here as Figure 1.

To solidify this picture, which is in agreement with the usual universality scenario, we have continued the investigation studying zero-temperature observables. We computed the string tension and the masses of some glueballs, in particular the  $0^{++}$  glueball, along the thermal transition line for  $N_t = 4$ . For universal continuum behavior  $\sqrt{\sigma}$  and the glueball masses should be constant along the thermal transition line for a fixed  $N_t$ , leading to constant ratios  $T_c/\sqrt{\sigma}$  and  $m_g/\sqrt{\sigma}$ .

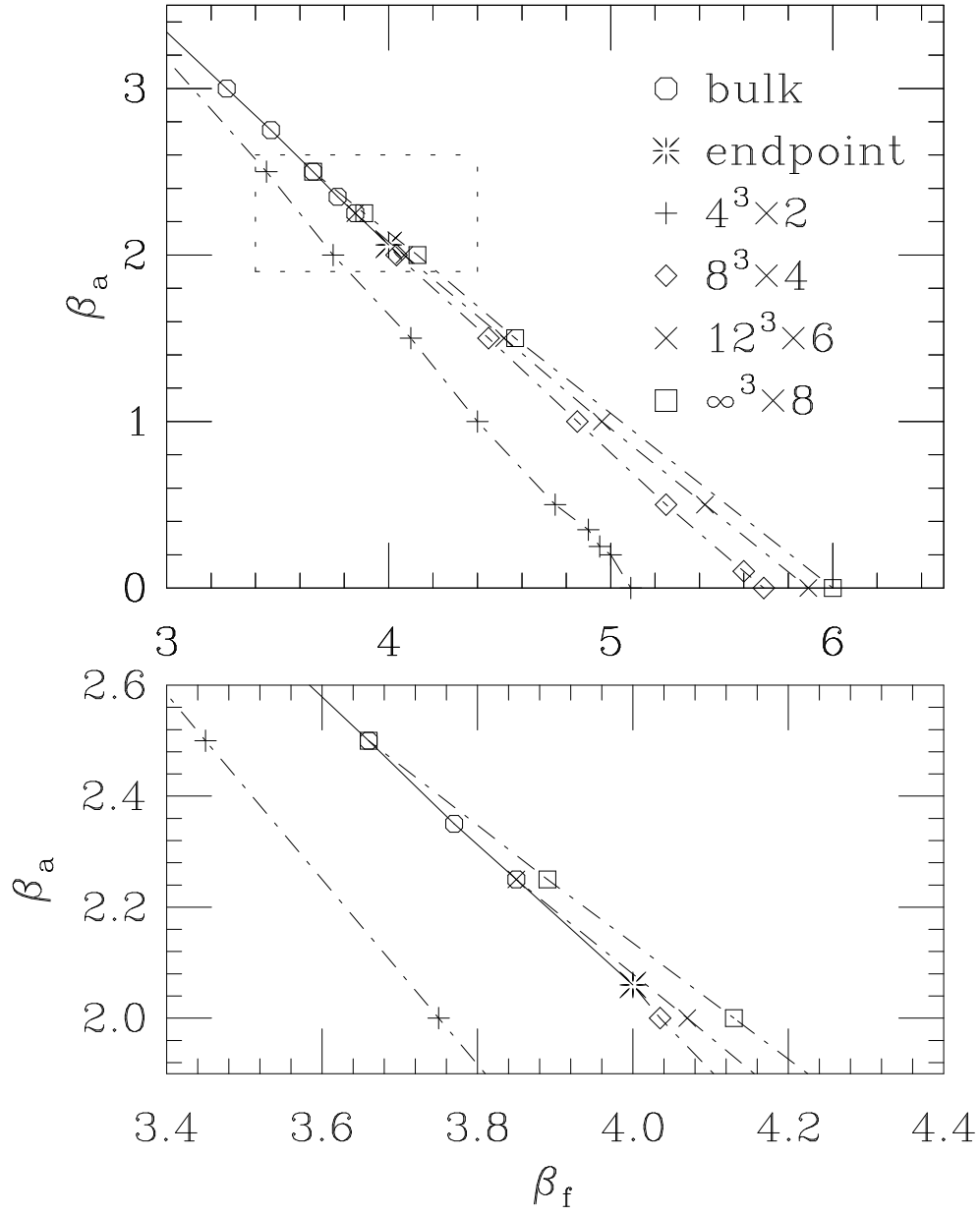


Figure 1: The phase diagram together with the thermal deconfinement transition points for  $N_t = 2, 4, 6$  and  $8$  from Ref. [7]. The lower plot shows an enlargement of the region around the end point of the bulk transition.

Of course, at small  $N_t$ , corresponding to a large lattice spacing  $a$ , we expect to see some deviations from this constant behavior. However, we find large deviations for  $m_{0^{++}}/\sqrt{\sigma}$  as we approach the critical endpoint of the bulk transition along the  $N_t = 4$  thermal transition line. The scalar glueball mass decreases dramatically, much more than what could be expected from simple scaling violations at a large lattice spacing. On the other hand, this is not really a surprise since at the critical endpoint at least one mass in the  $0^{++}$ -channel has to vanish. We elaborate on our findings in the next sections and then discuss the implications for the scaling behavior along the fundamental (or Wilson) axis.

## 2 Observables and Analysis

We have made simulations of the model with action (1) along the thermal transition line for  $N_t = 4$ , and continued along the bulk transition line, on a  $12^4$  lattice. Experience indicates this size to be sufficient to avoid significant finite size effects. The simulations were carried out with a 10-hit Metropolis algorithm tuned to an acceptance rate of about 50%. Observables were measured every 20 sweeps after thermalization. Close to the fundamental axis this resulted in essentially statistically independent measurements, while closer to the critical endpoint the autocorrelation time was significantly increased. We computed finite  $T$  approximants to the potential from time-like Wilson loops constructed with ‘APE’-smeared spatial links [8] to increase the overlap to the ground state potential. On and off axis spatial paths were considered with distances  $R = n, \sqrt{2}n, \sqrt{3}n$  and  $\sqrt{5}n$ , with  $n = 1, 2, \dots$  an integer. The string tension was then extracted from the usual fit

$$V(\vec{R}) = V_0 - \frac{e}{R} + l \left( G_L(\vec{R}) - \frac{1}{R} \right) + \sigma R. \quad (3)$$

Here  $G_L(\vec{R})$  is the lattice Coulomb potential, included in the fit to take account of short distance lattice artefacts. Our fits are fully correlated  $\chi^2$ -fits with the correlations estimated by bootstrap, after binning to alleviate autocorrelation effects. The results of the best fits are listed in Table 1.

We also computed glueball correlation functions in the  $0^{++}$ ,  $2^{++}$  and  $1^{+-}$  channel that can be built from simple plaquette operators. In an attempt to improve the signals we built these plaquettes not only from the original, but also from the smeared links, already used for the computation of the potential. Not surprisingly for computations done around the critical coupling for the  $N_t = 4$  thermal phase transition, we did not obtain a significant signal in the  $1^{+-}$  channel and only an effective mass from time distances  $t = 0/1$  in the  $2^{++}$  channel – we had 500 measurements everywhere, except for  $\beta_a = 2.0$ , near the critical endpoint where the number was increased, as given in Table 1. In the  $0^{++}$  channel we got a signal at distance  $t = 1/2$  at small  $\beta_a$  and out to  $t = 3/4$  at  $\beta_a = 2.0$ . Our best results are also given in Table 1.

The quantities  $\sqrt{\sigma}$  and  $m_{0^{++}}$  are shown in Figure 2 plotted versus  $\beta_a$ . As can be seen,  $\sqrt{\sigma}$  remains approximately constant along the thermal transition line for  $N_t = 4$  – the errors shown in the figure are statistical only; no estimate of the error from the uncertainty in the

$\beta_a$	$\beta_{fc}(N_t = 4)$	$\beta_f$	$L$	$N_{meas}$	$\sqrt{\sigma}$	$m_{0^{++}}$	$m_{2^{++}}(t = 1)$
0.0	5.6925(2)	5.7 <sup>a</sup>			0.4099( 12)	0.97( 2)	2.39(13)
0.5	5.25(5)	5.25	12	500	0.4218( 28)	0.93(11)	2.29(13)
1.0	4.85(5)	4.85	12	500	0.4024( 82)	0.78(28)	2.45(19)
1.5	4.45(5)	4.45	12	500	0.3743( 51)	0.56(17)	2.13( 9)
2.0	4.035(5)	4.03	12	1000	0.555 ( 11)	0.34( 6)	3.11(16)
2.0	4.035(5)	4.035	12	2000	0.4725(128)	0.20( 4)	3.17(14)
2.0	4.035(5)	4.04	12	1000	0.3750( 24)	0.27( 8)	2.37( 9)
2.25	3.8475(25)	3.8475 <sup>b</sup>	12	500	0.619 ( 18)	0.37(10)	3.15(37)
2.25	3.8475(25)	3.8475 <sup>c</sup>	12	500	0.3005( 22)	0.61( 8)	1.59( 7)
2.25	3.8475(25)	3.8475 <sup>c</sup>	16	500	0.2965( 19)	0.62( 5)	1.72( 9)

Table 1: The results in the neighborhood of the  $N_t = 4$  thermal transition line. Comments: (a)  $\sqrt{\sigma}$  at  $\beta_a = 0.0$  comes from [9],  $m_{0^{++}}$  from [10]; we did not take their best value, but rather the effective mass from the same distance as at  $\beta_a = 0.5$ ;  $m_{2^{++}}$  is from [11]; (b) in the disordered phase and (c) in the ordered phase on the bulk transition.

determination of  $\beta_{fc}$  has been attempted except for  $\beta_a = 2.0$ . There, the computation has been repeated for two nearby couplings, also listed in Table 1; the variation with  $\beta_f$  becomes so rapid that the error in the determination of  $\beta_{fc}$  becomes the dominating factor.

While our estimate for  $m_{2^{++}}$ , albeit an unreliable estimate since we had to use distances  $t = 0$  and 1 to obtain enough of a signal to extract an effective mass, also remains approximately constant,  $m_{0^{++}}$  shows a remarkable decrease as the critical endpoint is approached. This observed behavior suggests that the mass in the  $0^{++}$  channel vanishes at the critical endpoint, thereby giving strong additional evidence for the existence of this critical endpoint, since at a critical point at least one mass, in the  $0^{++}$  channel to have a rotationally invariant continuum limit, must vanish.

Since no other observable seems to be dramatically affected by the critical endpoint, we conjecture that the continuum theory one would obtain there is simply the (trivial)  $\phi^4$  theory. To substantiate this claim somewhat, we made a fit to the scalar mass of the form

$$m_{0^{++}} = A(\beta_a^* - \beta_a)^p \quad (4)$$

expected near a critical point. A 3-parameter fit gave  $A = 0.76(11)$ ,  $p = 0.35(20)$  and  $\beta_a^* = 2.02(6)$  with a  $\chi^2$  of 0.29 for 2 dof. Note that the estimate for  $\beta_a^*$  is in agreement with the previous estimate (2) obtained in [7] from fits to the jump in the plaquette across the bulk transition line. Within its large error, the exponent  $p$  is compatible with the mean field value 0.5 of  $\phi^4$  theory, up to logarithmic corrections. Since the errors of the fit parameters are rather large we also made a fit with  $\beta_a^*$  held fixed at its value 2.06 of (2). This fit gave  $A = 0.71(3)$  and  $p = 0.44(5)$  with a  $\chi^2$  of 0.47 for 3 dof. Again,  $p$  is compatible with the mean field value. Finally, a fit with  $\beta_a^* = 2.06$  and  $p = 0.5$  both held fixed gave  $A = 0.68(1)$  with a  $\chi^2$  of 1.50 for 4 dof. This last, still very acceptable fit is included in Figure 2.

While we believe to have assembled impressive further evidence for the bulk nature of

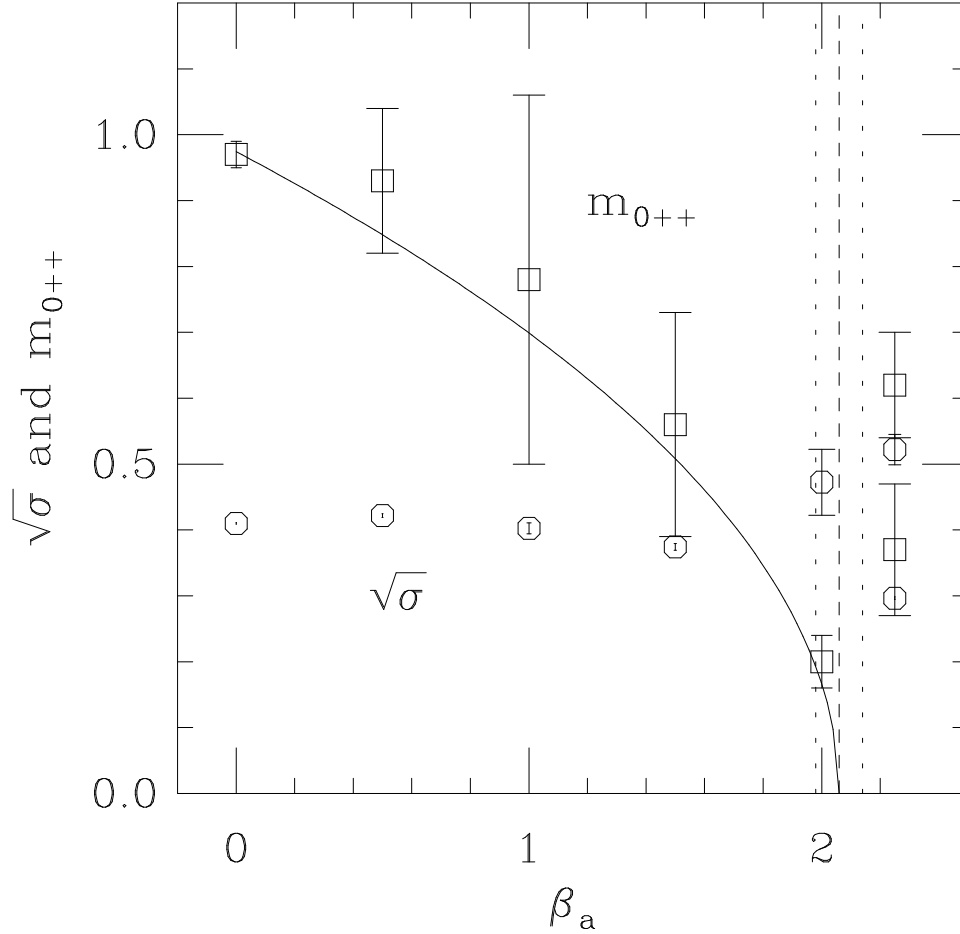


Figure 2:  $\sqrt{\sigma}$  (octagons) and  $m_{0++}$  (squares) as a function of  $\beta_a$  along the thermal transition line for  $N_t = 4$ . At  $\beta_a = 2.25$  we show the results from both phases at the bulk transition. The dashed vertical line gives the location of the critical endpoint, (2), with the dotted lines indicating the error band. The curve is a fit to  $m_{0++} = A(2.06 - \beta_a)^{1/2}$ .

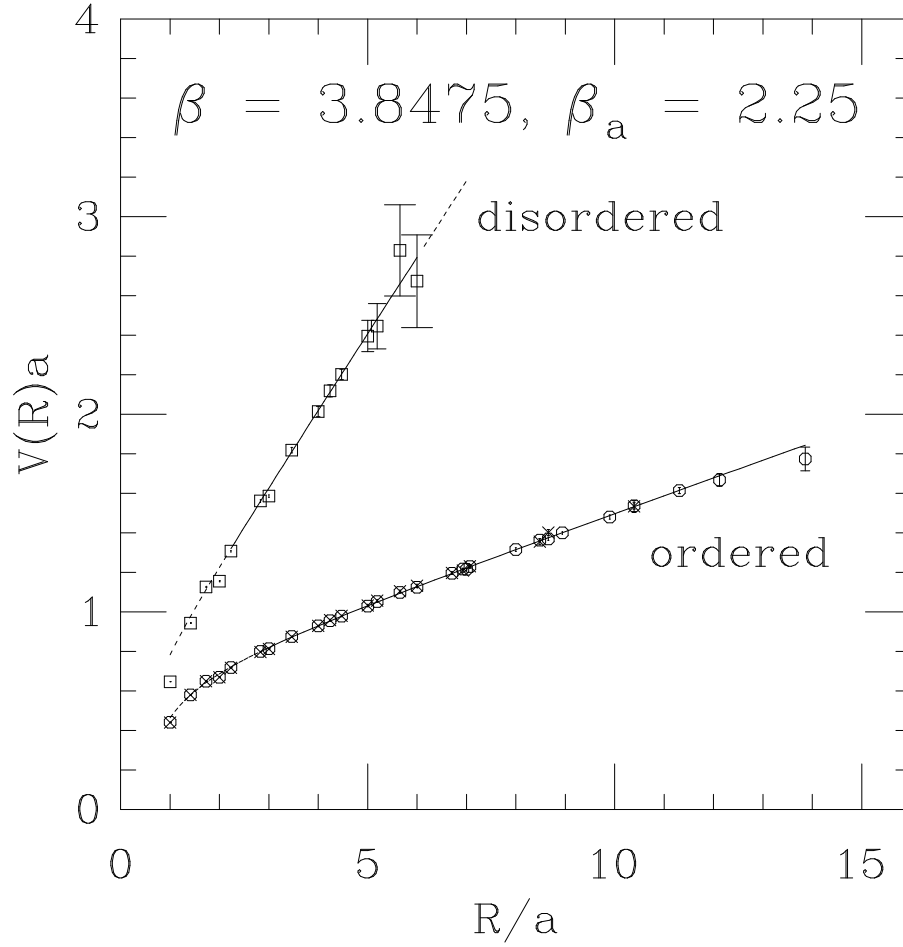


Figure 3: The potential in both phases on the bulk transition line at  $(\beta_f, \beta_a) = (3.8475, 2.25)$  on  $12^4$  lattices ( $\square$  and  $\times$ ) and on a  $16^4$  lattice ( $\circ$ ).

the phase transition line in the fundamental–adjoint plane and of its critical endpoint, we decided to make one further test. We computed the potential in both phases on the bulk transition line at  $\beta_a = 2.25$ . This is about the location where the  $N_t = 6$  thermal transition line joins onto the bulk transition line according to Ref. [7]. Hence, even in the ordered phase a lattice of size  $12^4$  should be large enough to obtain a reliable potential. To make sure that this is indeed the case, we repeated the computation in the ordered phase on a  $16^4$  lattice. As can be seen in Table 1 the finite size effect on the  $12^4$  lattice does indeed appear negligible. On both sides of the bulk transition – at identical couplings: we did not observe even an attempt at tunneling from one phase to the other – we find a clearly confining potential. Therefore the bulk transition does not, as expected, affect confinement, provided the lattices are large enough.

Of course, the string tension (and glueball masses) jump from one side of the bulk transition to the other. Indeed, as the bulk transition line is approached from the fundamental axis the string tension varies more and more rapidly – and the thermal deconfinement transition lines for different  $N_t$  come closer together – *i.e.*, the dip in the step  $\beta$ -function becomes deeper until a jump is developed through the bulk transition line.

### 3 Implications for Scaling

In the previous section we have corroborated the existence of a first order bulk phase transition line ending in a critical endpoint. We have provided evidence that physical observables are little affected by this phase transition line, except for a stronger dependence on  $\beta_f$  at fixed  $\beta_a$  – a deepening of the dip in the step  $\beta$ -function – and eventually a jump across the bulk transition line. The notable exception to this is the glueball mass in the  $0^{++}$  channel which decreases as the critical endpoint is approached and vanishes there.

The influence of the critical endpoint on the  $0^{++}$  glueball appears still visible in the crossover region on the fundamental (Wilson) axis. This has first been argued in Ref. [12]. The  $0^{++}$  glueball is lighter in the crossover region, leading to a different scaling behavior than other observables. This can be seen in Figure 4 where we show the latest data of  $T_c/\sqrt{\sigma}$  from Ref. [13] and  $m_{0^{++}}/\sqrt{\sigma}$  with  $\sqrt{\sigma}$  taken from Refs. [9, 14] and the glueball mass from Refs. [10, 15, 16]. While  $T_c/\sqrt{\sigma}$  stays approximately constant in the  $\beta$  interval shown,  $m_{0^{++}}/\sqrt{\sigma}$  decreases visibly in the crossover region around  $\beta = 5.7$ .

It has long been known that the scalar glueball mass scales differently in the crossover region than  $T_c$  and the string tension, whose scaling behavior as a function of  $\beta$  deviates from asymptotic scaling but agrees with the step  $\beta$ -function found in MCRG computations. The scalar glueball seemed more compatible with asymptotic scaling. However, in view of our findings that the different behavior of the scalar glueball mass comes from the influence of the nearby critical endpoint this asymptotic scaling behavior appears to be accidental. It would be interesting to determine the scaling behavior of other (pure gauge) observables. We conjecture that they would behave more like  $T_c$  or  $\sqrt{\sigma}$  than like the scalar glueball mass. Unfortunately no reliable, large volume, glueball masses for  $J^{PC}$  quantum numbers other than  $0^{++}$  for couplings in the crossover region exist in the literature to check this conjecture.



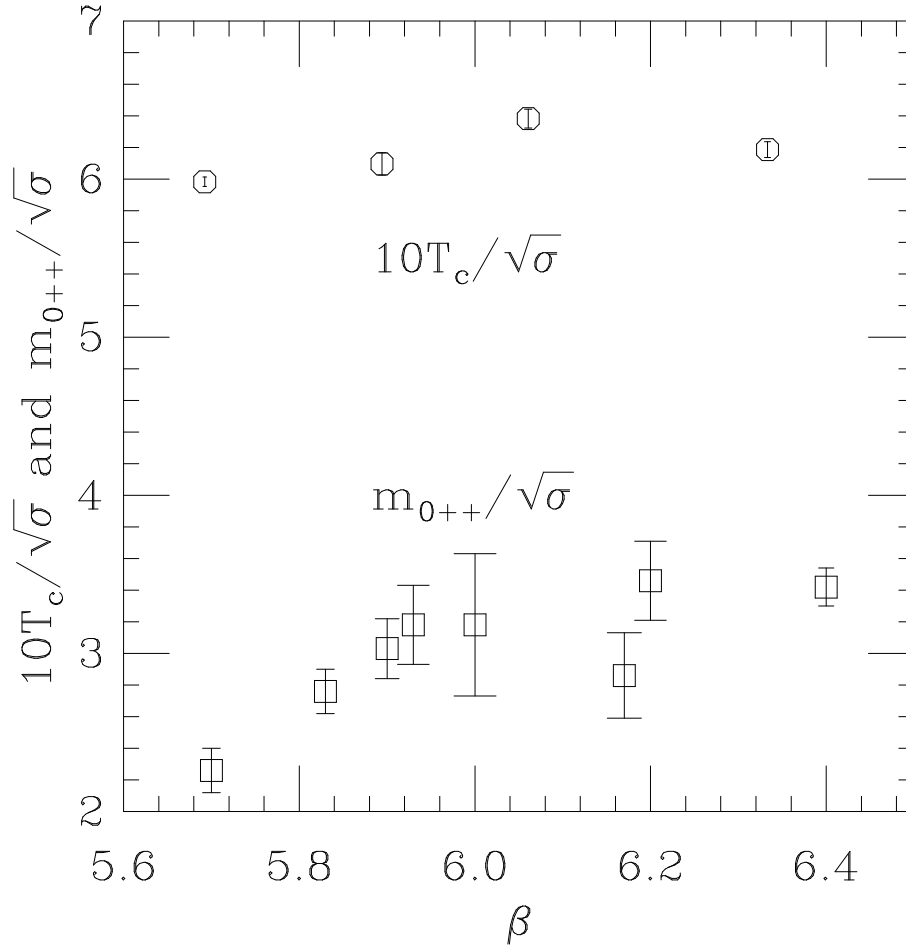


Figure 4:  $10T_c/\sqrt{\sigma}$  (octagons) and  $m_{0++}/\sqrt{\sigma}$  (squares) as a function of  $\beta$  for the fundamental (Wilson) action.

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