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Assessing Big-Bang Nucleosynthesis

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ABSTRACT

Systematic uncertainties in the light-element abundances and their evolution make a rigorous statistical assessment difficult. However, using Bayesian methods we show that the following statement is robust: the predicted and measured abundances are consistent with 95% credibility only if the baryon-to-photon ratio is between 2×10^{-10} and 6.5×10^{-10} and the number of light neutrino species is less than 3.9. Our analysis suggests that the ^{4}He abundance may have been systematically underestimated.

Big-bang nucleosynthesis occurred seconds after the bang and for this reason offers the most stringent test of the standard cosmology. Comparison of the predicted and measured light-element abundances has evolved dramatically over the past thirty years, beginning with the observation that there was evidence for a significant primeval abundance of ^{4}He which could be explained by the big bang [1] to the present where the abundances of D, ^{3}He, ^{4}He and ^{7}Li are all used to test the big bang.

The predictions of big-bang nucleosynthesis depend upon the baryon-to-photon ratio ($\equiv \eta$) as well as the number of light ($\lesssim 1\,$MeV) particle species, often quantified as the equivalent number of massless neutrino species ($\equiv N_\nu$). For a decade it has been argued that the abundances of all four light elements can be accounted for provided η is between 2.5×10^{-10} and 6×10^{-10} and $N_\nu < 3.1-4$ [2, 3, 4]. The "consistency interval" provides the best determination of the baryon density and is key to the case for the existence of nonbaryonic dark matter. The limit to N_ν provides a crucial hurdle for theories that aspire to unify the fundamental forces and particles.

However, these conclusions were not based upon a rigorous statistical analysis. Because the dominant uncertainties in the light-element abundances are systematic such an analysis is difficult and previous work focussed on concordance intervals. Given the importance of big-bang nucleosynthesis it is worthwhile to try to use more rigorous methods. Here we apply two standard methods, goodness of fit and Bayesian likelihood, and identify the the conclusions which are insensitive to the systematic errors.

We begin by reviewing the general situation. The predictions of standard big-bang nucleosynthesis are shown in Fig. 1. The theoretical uncertainties are statistical, arising from imprecise knowledge of the neutron lifetime and certain nuclear cross sections. Because of 10 Gyr or so of "chemical evolution" since the big bang (nuclear reactions in stars and elsewhere which modify the light-element abundances) determining primeval abundances is not simple and the dominant uncertainties are systematic.

The chemical evolution of ^{4}He is straightforward: stars make additional ^{4}He. Stars also make metals (elements heavier than ^{4}He); by measuring the ^{4}He abundance in metal-poor, extragalactic HII (ionized hydrogen) clouds as a function of some metal indicator (e.g., C, N or O) and extrapolating to zero metallicity a primeval abundance has been inferred: $Y_P = 0.232 \pm 0.003\,(\mathrm{stat}) \pm 0.005\,(\mathrm{sys})$ [5]. Systematic uncertainties arise from trying to convert line strengths to abundances by modeling. The range $Y_P = 0.221 - 0.243$ allows for 2σ statistical + 1σ systematic uncertainty and is consistent with the big-bang prediction provided $\eta \simeq (0.8-4)\times 10^{-10}$ [2]. Others have argued that the systematic uncertainty is a factor of two or even three larger [6]; taking $Y_P \simeq 0.21 - 0.25$ increases the concordance range significantly, $\eta \simeq (0.6-10)\times 10^{-10}$, reflecting the logarithmic dependence of big-bang ^{4}He production upon η [2].

There is a strong case that the ^{7}Li abundance measured in metal-poor, old pop II halo stars, $^7\mathrm{Li/H} = (1.5\pm 0.3)\times 10^{-10}$, reflects the big-bang abundance [7]. However, it is possible that even in these stars the ^{7}Li abundance has been reduced by nuclear burning, perhaps by a factor of two (the presence of ^{6}Li in some of these stars, which is more fragile, provides an upper limit to the amount of astration). Allowing for a 2σ statistical uncertainty and for up to a factor of two astration,[1] leads to the consistency interval $\eta = (1-6)\times 10^{-10}$ [2].

The interpretation of D is particularly challenging because it is burned in virtually all astrophysical situations and its abundance has only been accurately measured in the solar vicinity. Because D is destroyed and not produced [8] and because its abundance is so sensitive to η (D/H $\propto \eta^{-1.7}$), a firm upper limit to η can be obtained by insisting that big-bang production account for the D observed locally, D/H $\gtrsim (1.6\pm 0.1)\times 10^{-5}$ [9]. This leads to the two-decade old bound, $\eta \lesssim 9\times 10^{-10}$, which is the linchpin in the argument that baryons cannot provide closure density [10]. Because D is readily destroyed, it is not possible to use the D abundance to obtain a lower bound to η. The sum of D and ^{3}He is more promising: D is first burned to ^{3}He, and ^{3}He is much more difficult to burn. On the assumption that the ^{3}He survival fraction is greater than 25% the lower limit $\eta \gtrsim 2.5\times 10^{-10}$ has been derived [4]. The D, ^{3}He concordance interval, $\eta \simeq (2.5-9)\times 10^{-10}$.

The overlap of the concordance intervals (see Fig. 1), which occurs for $\eta = (2.5{-}6)\times 10^{-10}$, is the basis for concluding that the light-element abundances are consistent with the big-bang predictions [2].

The dominant uncertainties in comparing the predicted and measured light-element abundances are systematic: the primeval abundance of ^{4}He; the chemical evolution of D and ^{3}He; and whether or not ^{7}Li in the oldest stars has been reduced significantly by nuclear burning. Systematic error is difficult to treat as it is usually poorly understood and poorly quantified. (If it were understood and were well quantified it wouldn't be systematic error!) This is especially true for astronomical *observations*, where the observer has little control over the object being observed.

There are at least three kinds of systematic error. (1) A definitive, but unknown, offset between what is measured and what is of interest. (2) A random source of error whose distribution is poorly known. (3) An important source of error that is unknown. The first kind of systematic error is best treated as an additional parameter in the likelihood function. The second kind of systematic error is best treated by use of a distribution, or by several candidate distributions. The third type of systematic error is a nightmare.

The data themselves can clarify matters. Consider ^{7}Li; its measured abundance in old pop II stars is equal to the primeval abundance with a small statistical error and a larger systematic uncertainty due to astration. This could be a systematic error of the first kind—if all stars reduce their ^{7}Li abundance by the same factor—or of the second kind—if the ^{7}Li abundance in different stars were reduced by different amounts. In the latter case, the measured ^{7}Li abundance should show a large dispersion—which it does not [7]. Thus, we treat astration by considering two limiting possibilities: ^{7}Li/H$=(1.5\pm 0.3)\times 10^{-10}$ (no astration); and second, ^{7}Li/H$=(3.0\pm 0.6)\times 10^{-10}$ (astration by a factor of two).

Several sources of systematic error for ^{4}He have been identified—dust absorption, neutral ^{4}He, stellar absorption, and theoretical emissivities—which can reduce or increase the

[1] Additional smaller, but important, systematic uncertainties arise from the modeling of stellar atmospheres, which is needed to convert line strengths to abundances, and possible enhancement of ^{7}Li by cosmic-ray production [2]. We use astration to illustrate the effects of systematic uncertainty in the ^{7}Li abundance, but it serves to illustrate the point for the other effects too.

measured abundance [6]. If the same effect dominates in each measurement use of an offset parameter in the ^{4}He abundance would be appropriate. On the other hand, if different effects dominate different measurements enlarging the statistical error would be appropriate. We allow for both: the statistical error σ_Y is permitted to be larger than 0.003, and an offset in the ^{4}He abundance, ΔY, is a parameter in the likelihood function ($Y_P = 0.232 + \Delta Y$).

Finally, there is the systematic uncertainty associated with the chemical evolution of D and ^{3}He. Based upon a recent study of the chemical evolution of D and ^{3}He [11] we consider three models that encompass the broadest range of possibilities: Model 0 is the plain, vanilla model; Model 1 is characterized by extreme ^{3}He destruction[2] (average ^{3}He survival factor of about 15%); and Model 2 is characterized by minimal ^{3}He destruction. The likelihood functions for these three models are shown in Fig. 2.

First consider the χ^2 test for goodness of fit. This technique is best suited to situations where the errors are gaussian and well understood and there are many degrees of freedom; neither apply here. Nonetheless, in Fig. 3 we show $\chi^2(\eta)$ for eight different assumptions about the systematic uncertainties: (1,5) $\sigma_Y = 0.003$, $\Delta Y = 0$; (2,6) $\sigma_Y = 0.01$, $\Delta Y = 0$; (3,7) $\sigma_Y = 0.003$, $\Delta Y = 0.01$; (4,8) $\sigma_Y = 0.01$, $\Delta Y = 0.01$. In (1)–(4), ^{7}Li/H $= (3 \pm 0.6) \times 10^{-10}$; and in (5)–(8) ^{7}Li/H $= (1.5 \pm 0.3) \times 10^{-10}$. For clarity, only the results for Model 0 are shown, the results for Models 1 and 2 are similar.

Several conclusions can be drawn from Fig. 3. First, the goodness of fit depends sensitively upon assumptions made about the systematic errors, with the minimum χ^2 ranging from 6 to much less than 1; it is smallest when a systematic shift in ^{4}He is allowed and/or σ_Y is increased. Second, in all cases the the η interval defined by $\Delta\chi^2 = 3$ (from the minimum χ^2) has a lower bound no lower than about 1.5×10^{-10} (in all but (5), no lower than 2.5×10^{-10}) and an upper bound no higher than about 6×10^{-10}.

Next, we turn to Bayesian likelihood, which is best suited to determining parameters of a theory or assessing the relative viability of two or more theories. Since there is no well developed, alternative to the standard theory of nucleosynthesis at present, likelihood is of no use in assessing relative viability. Systematic errors of the first kind are treated as additional (nuisance) parameters in the likelihood function which can be determined by the experiment itself or can be eliminated by marginalization; we treat ΔY as such. We also allow σ_Y to vary to study how results depend upon the assumed uncertainty in the ^{4}He abundance. Because we are interested in setting a limit to N_ν, it too is taken to be a parameter. Values of N_ν greater than three describe extensions of the standard model with additional light degrees of freedom.

In Fig. 4 we show contours of $\mathcal{L}(N_\nu, \Delta Y, \sigma_Y = 0.003)$. The contours are diagonal lines because ΔY and N_ν are not independent parameters—the primary effect of an increase in N_ν is an increase in the predicted ^{4}He abundance ($\Delta Y_P \sim 0.01\, \Delta N_\nu$). A likelihood function that is not compact must be treated with care, because no information about the parameters (here, N_ν and ΔY) can be inferred independently of what was already known (the priors).

For example, the likelihood function $\mathcal{L}(N_\nu)$, which is needed to set limits to N_ν, is

[2]Because the stars that destroy ^{3}He also make metals, it is not possible to destroy ^{3}He to an arbitrary degree without overproducing metals [11].

$\lvert\Delta Y\rvert$	Model 0	Model 1	Model 2
0	3.1/3.1	3.2/3.3	3.1/3.1
0.005	3.2/3.1	3.3/3.3	3.2/3.1
0.010	3.3/3.2	3.5/3.3	3.3/3.2
0.015	3.5/3.4	3.7/3.4	3.4/3.4
0.020	3.7/3.6	3.9/3.7	3.7/3.6

Table 1: Limits to N_ν for Models 0, 1, 2 and Li/H $= (1.5 \pm 0.3) \times 10^{-10}$ (first number) and Li/H $= (3.0 \pm 0.6) \times 10^{-10}$ (second number).

obtained by integrating over ΔY and depends upon the limits of integration. To derive limits to N_ν we do the following: integrate from $-|\Delta Y|$ to $|\Delta Y|$; normalize $\mathcal{L}(N_\nu)$ to have unit likelihood from $N_\nu = 3$ to ∞; the limit is the value of N_ν beyond which 5% of the total likelihood accumulates. The dependence of the limit upon $|\Delta Y|$ is shown in Table 1.

An aside; in a recent paper the likelihood function $\mathcal{L}(N_\nu)$ obtained by integrating from $\Delta Y = -0.005$ to 0.005 was used in an attempt to assess the viability of the standard theory of nucleosynthesis [12]. This likelihood function is peaked at $N_\nu = 2$ and is approximately gaussian with $\sigma_{N_\nu} = 0.3$. On this basis it was claimed that the standard theory of nucleosynthesis is ruled out with 99.7% confidence. By so doing equal weight was implicitly given to all values of N_ν (uniform priors). The prior for $N_\nu = 3$ (the standard model of particle physics) is certainly orders of magnitude greater than that for $N_\nu = 2$ (for which no well developed model exists). The likelihood function $\mathcal{L}(N_\nu)$ which properly included prior information would certainly not be peaked at $N_\nu = 2.0$.

In Figs. 5 and 6 we show the 95% contours of the likelihood functions $\mathcal{L}(\eta, \sigma_Y)$ and $\mathcal{L}(\eta, \Delta Y)$ for Models 0, 1, and 2 and both values of the central ^{7}Li abundance. Both figures suggest the same thing: the uncertainty in the primordial ^{4}He abundance has been underestimated. In the $\sigma_Y - \eta$ plane $\sigma_Y = 0.003$ does not intersect the 95% credibility contour, and in the $\Delta Y - \eta$ plane $\Delta Y = 0$ does not intersect the 95% credibility region (except for Model 1, where they barely do). The 95% credibility contour in the $\sigma_Y - \eta$ plane becomes independent of σ_Y for $\sigma_Y \gtrsim 0.008$, with 95% credibility interval $\eta \simeq (3 - 6.5) \times 10^{-10}$ (allowing both for the uncertainty in the astration of ^{7}Li and in the chemical evolution of D and ^{3}He).

The 95% credibility contours in the $\Delta Y - \eta$ plane suggest that the primeval ^{4}He abundance has been systematically underestimated, by an amount $\Delta Y \approx +0.01$. (Though it should be noted that Model 1 and the lower ^{7}Li abundance are just consistent with $\Delta Y = 0$ at 95% credibility.) Put another way, D, ^{3}He, and ^{7}Li are concordant and ^{4}He is the outlayer. (This can also be seen in Fig. 2.) When the likelihood function is marginalized with respect to ΔY, the 95% credibility interval is $\eta \simeq (2 - 6.5) \times 10^{-10}$ (allowing again for the uncertainty both in astration of ^{7}Li and in the chemical evolution of D and ^{3}He).

To conclude, the fact that systematic uncertainties dominate precludes crisp statistical statements. The lack of a viable alternative to the standard theory of nucleosynthesis complicates matters further as the most powerful statistical techniques assess relative viability. However, the rigorous techniques that we have applied point to several conclusions that are insensitive to assumptions made about systematic uncertainty:

- The predictions of the standard theory of primordial nucleosynthesis are only consistent with the extant observations with 95% credibility provided $\eta \simeq (2 - 6.5) \times 10^{-10}$.
- Our analysis suggests that the primordial ^{4}He abundance has been systematically underestimated ($\Delta Y \approx +0.01$) or that the random errors have been underestimated ($\sigma_Y \approx 0.01$). Only for Model 1 (extreme destruction of ^{3}He) are $\Delta Y = 0$ and $\sigma_Y = 0.003$ in the 95% credibility region (cf., Figs. 5 and 6).
- The limit to N_ν depends upon the systematic uncertainties in the ^{4}He abundance (cf., Table 1); taking $|\Delta Y| \leq 0.02$, which is four times the estimated systematic error and also encompasses the 95% likelihood contour in the $\Delta Y - \eta$ plane, leads to the 95% credible limit $N_\nu < 3.9$.

This more rigorous analysis provides additional support for the conclusions reached previously about the concordance interval for η [2]. The limit to the number of neutrino species is less stringent than previously quoted bounds [2, 3] because we allowed a chemical evolution model with the most extreme destruction of ^{3}He (which permits low values of η where ^{4}He production is lower) as well as a large systematic offset in the ^{4}He abundance.

Finally, there are two measurements that should reduce the systematic uncertainties significantly, permitting a sharper test of big-bang nucleosynthesis. The first is a determination of the primeval D abundance by measuring D-Lyα absorption due to high-redshift hydrogen clouds. The second is a determination of the primeval ^{7}Li abundance by studying short period, tidally locked pop II halo binaries; astration is believed to involve rotation-driven mixing astration and is minimized in these stars because they rotate slowly [13]. At the moment, there are conflicting measurements and upper limits for the primeval D abundance seen in high-redshift hydrogen clouds [14], and there is one study which indicates that the ^{7}Li abundance in short-period binaries is no higher (evidence against significant astration) and another that finds weak evidence that the ^{7}Li abundance is higher [15].

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Figure Captions

Figure 1: The predicted light-element abundances (with 2σ theoretical errors); rectangles indicate consistency intervals, which all overlap for $\eta = (2.5 - 6) \times 10^{-10}$.

Figure 2: Likelihood functions for D and ^{3}He (lower solid curves, from left to right: Models 1, 0, and 2), ^{4}He (dotted curves, from left to right: $\sigma_Y = 0.01$, $\sigma_Y = 0.003$, and $\Delta Y = 0.01$), and ^{7}Li (broken = high ^{7}Li, solid = low ^{7}Li).

Figure 3: Reduced χ^2 as function of η for eight different sets of assumptions about the systematic uncertainties (see text for details).

Figure 4: The 5% of maximum likelihood contours for $\mathcal{L}(N_\nu, \Delta Y, \sigma_Y = 0.003)$ (solid curves = low ^{7}Li, broken curves = high ^{7}Li). Because N_ν and ΔY are not independent parameters, the contours of likelihood are diagonal lines and the likelihood function is not compact.

Figure 5: The likelihood function $\mathcal{L}(\sigma_Y, \eta, \Delta Y = 0)$ (solid curves = low ^{7}Li, broken curves = high ^{7}Li).

Figure 6: Same as Fig. 5 for $\mathcal{L}(\Delta Y, \eta, \sigma_Y = 0.003)$.