BEYOND LEADING LOGARITHM PARTON DISTRIBUTIONS IN THE PHOTON

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Abstract

I discuss Beyond Leading Logarithm parton distributions in the photon and study the constraints put on the latter by data on $F_2^\gamma(x, Q^2)$, and on the jet production in photo-proton and photon-photon collisions.

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1. Introduction

For the past fifteen years, the photon structure function $F_2^\gamma(x,Q^2)$, measured in deep inelastic lepton-photon scattering (Fig. 1), has generated considerable theoretical and experimental work [1], and, more recently, the beginning of HERA has reactivated the interest in the quark and gluon distributions in real photons. It is indeed expected that photoproduction experiments [2, 3] will allow to measure these distributions with a good precision. Another interest of photoproduction reactions consists in the coupling of the gluon (from the photon) to the hard subprocesses (Fig. 2) which offers a direct determination of its distribution [4] : in $\gamma\gamma^*$ DIS experiments it is only through the evolution equations for F_2^γ that the gluon distribution may be determined.

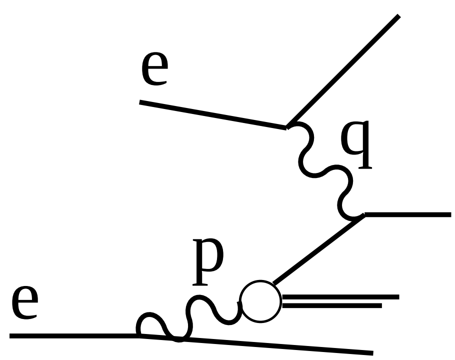

Fig. 1 : Deep inelastic scattering on a real photon ($p^2 \simeq 0$).

Jet production in $\gamma\gamma$ collisions also offers the possibility to measure the quark and gluon distributions in real photons, and data from TRISTAN [5, 6] are very interesting. In this reaction the parton distributions may doubly intervene and the cross-section is very sensitive to the latter. The x-region probed at HERA ($x \sim .1$) and TRISTAN ($x \sim .4$) are complementary whereas the Q^2-regions are very similar ($Q^2 \simeq p_\perp^2 = 25$ GeV^2). Therefore the joint study of HERA and

TRISTAN [7, 8] results should constrain rather tightly these parton distributions.

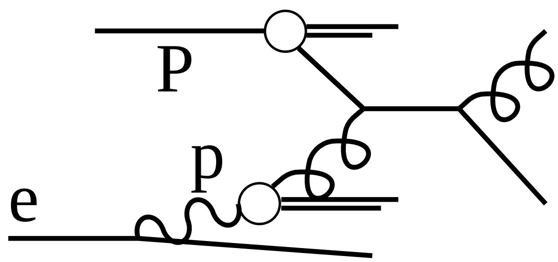

Fig. 2 : Jet photoproduction through the subprocess gluon + quark → gluon + quark.

In this talk, I would like to discuss the photon structure function, stressing the importance of the Higher Order QCD corrections, of the factorization scheme and of the problem of the non perturbative input to F_2^γ. In particular I show how the non peturbative component must be modified in order to take into account the specificity of the $\overline{MS}$ factorization scheme.

Then I will compare the theoretical predictions with experimental results on F_2^γ, and discuss the possibility to obtain indications on the non perturbative input from data. Finally I will consider jet production in $\gamma\gamma$ collisions and in photoproduction and the constraints put by the data on the quark and gluon distributions.

2. The parton distributions in the photon

The parton contents of the photon can be measured in deep inelastic scatte ring experiments in which the virtual photon γ^* of momentum q ($Q^2 = -q^2 >> \Lambda^2$) probes the short distance behavior of the real photon γ of momentum p (Fig. 1). The structure function F_2^γ of this reaction is proportional, in the Leading Logarithm (LL) approximation, to the quark distributions in the real photon

$$F_2^\gamma(x,Q^2) = x\sum_{f=1}^{n_f} e_f^2 \left(q_\gamma^f(x,Q^2) + \bar{q}_\gamma^f(x,Q^2)\right) \quad . \tag{1}$$

The sum in (1) runs over the quark flavors and $x = Q^2/2p.q$.

It is instructive to consider the contribution to F_2^γ of the lowest order diagrams of Fig. 3. Contrarily to the case of a hadronic target, the lower part of the diagram is known : it is given by the coupling of photon to quark.

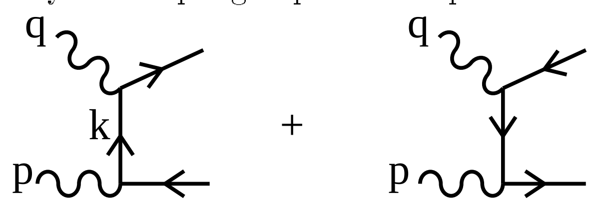

Fig.3

This contribution is therefore exactly calculable, with the following result for a quark of charge e_f :

$$F_2^\gamma(x,Q^2)/x = 3e_f^4\frac{\alpha}{\pi}\left\{\left(x^2+(1-x)^2\right)\ell n\frac{Q^2}{m_f^2}\right.$$
$$\left.+\left(x^2+(1-x)^2\right)\ell n\frac{1-x}{x}+8x(1-x)-1\right\}. \qquad (2)$$

However our result (2) is not directly related to a physical process, because it depends on the unknown quark mass m_f, used as a cut-off to regularize a logarithmic divergence. Actually this perturbative approach is certainly not valid when the virtuality $|k|^2$ of the exchanged quark becomes small. We then go into a non perturbative domain where we lack theoretical tools and we must resort to models to describe non perturbative (NP) contributions to F_2^γ. A popular model is the "Vector Meson Dominance Model" (VDM) which consider that the real photon couple to vector mesons. Therefore the real photon, besides a direct coupling to a $q\bar{q}$ pair, has a VDM component which is also probed by the virtual photon.

The latter component contributes to F_2^γ and must be added to expression (2). Keeping only the term in (2) proportional to $\ell n\ Q^2/m_f^2$ (LL approximation), we write

$$F_2^\gamma(x,Q^2) = 3e_f^4\frac{\alpha}{\pi}x\left(x^2+(1-x)^2\right)\ell n\frac{Q^2}{Q_0^2} + x\sum_{V=\rho,\omega,\phi} e_f^2\left(q_f^V(x)+\bar{q}_f^V(x)\right) \quad . \qquad (3)$$

The scale Q_0^2 is the value of Q^2 at which the perturbative approach is no more valid. The perturbative contribution vanishes at $Q^2 = Q_0^2$ and F_2^γ is described only by the non perturbative contribution $q_f^{NP}(x) = q_f^V(x)$ which describes the quark contents of vector mesons.

We have to keep in mind that this way of treating the non perturbative part of F_2^γ is due to our lack of theoretical understanding of this contribution. There are other approaches [9], especially that of Ref. [10] which takes into account the interaction between the quarks and the gluon condensate. These different approaches must ultimately be compared with experiment.

QCD corrections to the diagrams of Fig. 3 do not change the basic structure of expression (3). In the LL approximation [11, 12], the perturbative quark distribution is given by the sum of ladder diagrams (Fig. 4) (for simplicity we consider only one flavor),

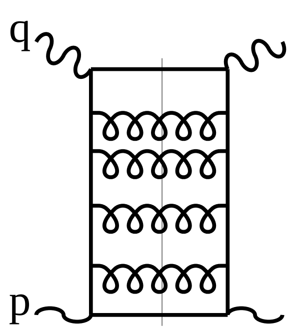

Fig. 4 : Ladder diagram contribution to F_2^γ (the thin line cuts final partons).

whereas the non perturbative part acquires a Q^2-dependence which is identical to that of a quark distribution in a hadron. Thus the total quark and gluon distributions are given by (AN is for anomalous, a designation introduced in Ref. 11 for the perturbative distribution)

$$q_\gamma(n, Q^2) = q_\gamma^{AN}(n, Q^2) + q_\gamma^{NP}(n, Q^2) \equiv d_q(n, Q^2)$$
$$g_\gamma(n, Q^2) = g_\gamma^{AN}(n, Q^2) + g_\gamma^{NP}(n, Q^2) \equiv d_g(n, Q^2) \qquad (4)$$

which verify the inhomogeneous equation ($i, j = q, \bar{q}, g$)

$$Q^2 \frac{\partial d_i(n,Q^2)}{\partial Q^2} = \frac{\alpha}{2\pi} k_i^{(0)}(n) + \frac{\alpha_s(Q^2)}{2\pi} P_{ij}^{(0)}(n) d_j(n,Q^2) \quad . \tag{5}$$

As in (3) we introduce the boundary condition [13] Q_0^2 in (4) so that $q_\gamma^{AN}(n,Q_0^2)$ $g_\gamma^{AN}(n,Q_0^2)$ vanish when $Q^2 = Q_0^2$. $k_q^{(0)}(n) = \int_s^1 dx \; x^{n-1} k_q^{(0)}(x)$ is the moment of the Altarelli-Parisi kernel describing the splitting of a photon into a $q\bar{q}$ pair (the bottom rung of the ladder) whereas $P_{ij}^{(0)}$ are the usual AP kernels.

The modifications of these LL results due to Higher Order (HO) QCD corrections [14, 15, 16] are obtained by replacing the LL kernels of (5) by kernels involving HO contributions

$$\begin{aligned} k_i(n) &= \frac{\alpha}{2\pi} k_i^{(0)}(n) + \frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi} k_i^{(1)}(n) + \cdots \\ P_{ij}(n) &= \frac{\alpha_s}{2\pi} P_{ij}^{(0)}(n) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(1)}(n) + \cdots \quad , \end{aligned} \tag{6}$$

and by a modification of the expression of F_2^γ in terms of parton distributions (the gluon contribution is now explicitly written)

$$F_2^\gamma(n-1,Q^2) = e_f^2 C_q(n,Q^2)\left(q_\gamma(n,Q^2) + \bar{q}_\gamma(n,Q^2)\right) + C_g(n,Q^2) g_\gamma(n,Q^2) + C_\gamma(n) \tag{7}$$

where C_γ is the "direct term", given by the part of (2) not proportional to $\ell n \frac{Q^2}{m_f^2}$. C_q and C_g are the well-known Wilson coefficients which are identical to those found in the case of a hadronic target.

A delicate point when working beyond the LL approximation is that of the factorization scheme. A change in the factorization scheme is translated into a change in $k^{(1)}$ and C_γ but in such a way that the physical quantity F_2^γ remains unmodified (at order α_s^0). On the other hand q_γ is not an invariant with respect to the factorization scheme and a change in $k_i^{(1)}$ causes modifications in q_γ^{AN} and q_γ^{NP}. Therefore the separation (4) in a perturbative and a non perturbative part is not scheme invariant and the statement that q_γ^{NP} can be described by VDM has no meaning, unless one specifies in which factorization scheme it is valid.

Within the $\overline{MS}$ scheme we obtain for C_γ the expression given by the part of Eq. (2) not proportional to $\ell n Q^2/m_f^2$. We observe that the $\ell n(1-x)$ factor becomes very large near the boundary of phase space and then it does not appear as a correction in Eq. (7) since it may become numerically larger that the leading terms q_γ which are enhanced by a factor $1/\alpha_s$. In order to keep the concept of a perturbative expansion useful, it is proposed in [17] to introduce the DIS_γ factorization scheme where the choice

$$C_\gamma(x)|_{DIS_\gamma} = 0 \tag{8}$$

is made. The parton distributions thus defined satisfy an evolution equation of type (5, 6) with the inhomogeneous terms $k^{(1)}$ replaced by

$$\begin{aligned} \frac{\alpha}{2\pi} k_q^{(1)}\Big|_{DIS_\gamma} &= \frac{\alpha}{2\pi} k_q^{(1)} - P_{qq}^{(0)}\ C_\gamma/2e_f^2 \\ \frac{\alpha}{2\pi} k_g^{(1)}\Big|_{DIS_\gamma} &= \frac{\alpha}{2\pi} k_g^{(1)} - P_{gq}^{(0)}\ C_\gamma/e_f^2 \end{aligned} \tag{9}$$

as can be immediately derived by expressing the $\overline{MS}$ parton distributions q_γ in terms of their DIS_γ counterparts

$$\begin{aligned} q_\gamma &= q_\gamma|_{DIS_\gamma} - C_\gamma/2e_f^2 \\ g_\gamma &= g_\gamma|_{DIS_\gamma} \end{aligned} \tag{10}$$

in Eq. (5, 6). Note that the homogeneous terms are not affected by this transformation. It is shown in [17] that, in this convention, the leading logarithmic and the beyond leading logarithmic parton distributions remain very close to each other over the all range in x.

We adopt here a different approach which also absorbs the troublesome "large" $\ell n(1-x)$ terms with the added advantages that the parton distributions we define are universal (i.e. independent of the reference process) and obey the $\overline{MS}$ evolution equations, as all hadron structure functions in practical use today. A careful analysis [18] of the box diagram (Fig. 3) indeed shows that it is possible to define a factorization-scheme-invariant non perturbative input $\bar{q}_\gamma^{NP}(n, Q^2)$, related to the non perturbative input defined in (4) by the relation

$$q_\gamma^{NP}(n, Q_0^2) = \bar{q}_\gamma^{NP}(n, Q_0^2) - C_0(n)/2e_f^2 \tag{11}$$

with $C_0(x)$ given in the $\overline{MS}$ scheme by

$$C_0(x) = 3e_f^4 \frac{\alpha}{\pi} \left\{ \left(x^2 + (1-x)^2 \right) \ell n(1-x) + 2x(1-x) \right\} \quad . \tag{12}$$

Actually all the scheme dependence of q_γ^{NP} (Eq. 4) is contained in $C_0(x)$; $\bar{q}_\gamma^{NP}$ is factorization scheme independent and we use the Vector Meson Dominance Model to describe it.

Fig. 5 : Predictions for F_2^γ compared with AMY data [19]. Non perturbative input of Eq. 11 with $\bar{q}_\gamma^{NP} = q_\gamma^{VDM}$ and $Q_0^2 = .25$ GeV^2 (full curve) ; same input and $Q_0^2 = 1.$ GeV^2 (dotted curve) ; input of Eq. 11 with $C_0(n) = 0$ and $Q_0^2 = .25$ GeV^2 (dashed curve).

Let us now turn to a comparison between the theoretical calculations and the experimental results. Details on the VDM input and on the treatment of the massive charm quark can be found in Ref. [18]. In Fig. 5, we see a prediction obtained with $Q_0^2 = .25$ GeV^2 ($\Lambda_{\overline{MS}} = 200$ MeV) compared with AMY data [19]. The sensitivity to the $C_0(x)$ term of Eq. (11) is also shown, as well as the sensitivity to the value of Q_0^2. A similar comparison with JADE data [20] is displayed in Fig. 6 on which we can observe the effect of the non perturbative input described by VDM.

Fig. 6 : JADE data [20]. Non perturbative input of Eq. 11 with $\bar{q}_\gamma^{NP} = q_\gamma^{VDM}$ and $Q_0^2 = .5$ GeV^2 (full curve). With $\bar{q}_\gamma^{NP} = 0$ (dashed curve).

We clearly see from these comparisons that a satisfactory agreement is obtained between theory and experiment but the accuracy of the data are not good enough to put constraints on the non perturbative input and on the value of Q_0^2. Let us however notice that we compared with large-Q^2 data corresponding to an important contribution of the anomalous distribution (Eq. 4) which has a term proportional to $\ell n Q^2/\Lambda^2$. The low-Q^2 region emphasizes the role of the non perturbative input as it can been observed in Fig. 7 which displays PLUTO [21] and TOPAZ [22] data. The nice agreement of the theoretical prediction with these data shows that the non perturbative input is reasonably described by VDM and a value $Q_0^2 = .5$ GeV^2.

Fig. 7 : PLUTO data [21] and TOPAZ data [22]. Same as fig. 6.

In this talk I only give comparisons between the distributions functions of Ref. [18] and some data. Two other Beyond Leading Logarithm (BLL) parametrizations of the parton distributions in the photon exist [23, 24], similar to the one presented here. The interested reader can find a discussion of these parametrizations and more comparisons with data in Ref. [25].

3. Jet production in $\gamma\gamma$ collisions and in photoproduction

The jet production in $\gamma\gamma$ collisions is a powerful tool for measuring parton distributions in the photon ; the cross-section is indeed very sensitive to these distributions since they may intervene twice in calculations. In this reaction, as in the photoproduction case, it is important to compare the experimental results with theoretical expressions calculated beyond the Leading Logarithm approximation.

It is only under these conditions that precise parton distributions can be extracted from data. A detailed discussion of the jet production at TRISTAN has been given in Ref. 7 that I summarize here.

When an inclusive jet cross-section is calculated beyond the LL aproximation, both the parton distributions and the subprocesses must be corrected. For instance, let us consider the reaction of Fig. 2 which can be symbolically written

$$\frac{d\sigma^{jet}}{d\vec{p}_\perp dy} = \sum_{ij} \mathcal{P}^i_\gamma \otimes \widehat{\sigma}\,(ij \to jet\ X) \otimes \mathcal{P}^j_P \tag{13}$$

where $\mathcal{P}^i_\gamma$ and $\mathcal{P}^j_P$ are the parton distributions in the photon and in the proton, and where $\widehat{\sigma}$ is the subprocess cross-section. Expanding $\mathcal{P}^i_\gamma$ and $\widehat{\sigma}$ in power of α_s, we obtain an expression

$$\frac{d\sigma^{jet}}{d\vec{p}_\perp\ d\eta} = \sum_{i,j} \left(\frac{4\pi}{\alpha_s(p^2_\perp)} a_i + b_i \right) \otimes \left(\alpha\ \alpha_s(p^2_\perp) \widehat{\sigma}^{BORN}_{ij} + \alpha\ \alpha^2_s(p^2_\perp) K_{ij}\right) \otimes \mathcal{P}^j_P \tag{14}$$

which shows the Leading Logarithm contributions to the jet cross-section (associated with a_i and $\widehat{\sigma}^{BORN}_i$ which describe the $2 \to 2$ subprocesses), and the BLL QCD corrections coming from b_i and K_{ij}. I do not discuss the well-known parton distributions in the proton and concentrate on the incident photon.

The term b_i describes the effects of the HO corrections to the evolution equation (5) and it is proportional to $k^{(1)}$, $P^{(1)}$ (cf. eq. (6)) and β_1 (the two-loop coefficients of the function $\beta(\alpha_s)$). These parton distributions have been discussed in section 2.

The HO terms K_{ij} correspond to $2 \to 3$ subprocesses and to virtual corrections to the $2 \to 2$ Born subprocesses. They contain terms which compensate the scale-dependence of the LL expressions and make the corrected inclusive cross-section more stable with respect to variations of the renormalization and factorization scales μ and M [7].

The theoretical results compared below with TRISTAN data are obtained with the distribution functions described in section 2 ; they include BLL QCD corrections and the scales have been set equal to $p_\perp$: $\mu = M = p_\perp$. The effects of the convolution with the Weizsäcker-Williams photon spectrum in the electron is also carefully discussed in Ref. [7].

Fig. 8 : TOPAZ data [5] on inclusive jet production and theoretical predictions for $\int_{-.7}^{.7} d\eta \frac{d\sigma^{e^+e^- \to jet}}{dp_T \; d\eta}$. The top curve is the theoretical prediction based on the standard photon structure functions, the middle one is based on structure functions with half the VDM input, and the lower one is based on the perturbative component only. The dash-dotted curve is the “direct contribution”.

Fig. 9 : AMY data [6] on inclusive jet production and theoretical predictions for $\int_{-.1}^{.1} d\eta \ d\sigma/dp_\perp \ d\eta$. The top curve is the theoretical prediction based on the standard photon structure functions.

We observe a good agreement between theory and data in which the contribution due to the parton distributions in the photon play an essential role. Without the latter, the theoretical prediction would be given by the dash-dotted curve of Fig. 8. We also notice that the theoretical cross-section is not very sensitive to the non-perturbative inputs of the parton distributions : so that we can say that the good agreement found in Fig. 8 and 9 is a great success of perturbation QCD.

The observed disagreements for $p_\perp$ smaller than 5 GeV/c may be, at least partly, attributed to the fact that we neglected the charm quark mass in our calculations, thus enhancing by some 15 % the cross section at $p_\perp = 3$ GeV/c [7].

Let us conclude this study of $\gamma\gamma$ collisions by saying that more precise data should certainly improve the quality of the test of QCD and shall allow us to constrain the non-perturbative inputs.

The photoproduction of large-$p_\perp$ jet at HERA is another source of novel indications concerning the parton distributions in the photon (Fig. 2). In this case as well BLL calculations of the jet cross-section are available [4, 26] and may be compared with recent experimental results [27, 28]. In Fig. 10 a comparison between a theoretical prediction [8] and data [27] is displayed.

Fig. 10 : Jet cross-section as a function of the pseudo-rapidity η : full line. (The cross-section is integrated over $p_\perp$ from $p_\perp$=7GeV/c). Gluon distribution in the photon = 0 : dashed line. Gluon in the proton = 0 : dots.

We notice a reasonable agreement between theory and data. Dashed and dotted lines in Fig. 10 show the sensitivity of this jet cross-section to the gluon contents of the photon and of the proton, and demonstrate that the rapidity dependence is a very good tool to explore and measure these distributions.

4. Conclusion

In this talk I have presented a set of parton distributions in real photons which take into account Beyond Leading Logarithm QCD corrections. I also showed how to modify the non perturbative input to make it factorization scheme independent.

The agreement between theory and data is satisfactory as regards the photon structure function $F_2^\gamma(x, Q^2)$. However large error bars do not allow us to draw any definite conclusion and make necessary the comparison with other experimental results.

Particularly promising is the jet production in photon-hadron and photon-photon reactions. The sensitivity of the cross-sections to the parton distributions is large and should allow a good determination of the gluon contents of the photon. The good agreement between theory and data in what concerns the jet cross-section $d\sigma(\gamma\gamma \to jet + X)/d\vec{p}_\perp \, d\eta$ is a success for perturbative QCD.

Acknowledgements

I would like to thank Dr. T. Tauchi and his colleagues for the organization of this interesting meeting, and Dr. Y. Shimizu and the "Minami Tateya" group for their warm hospitality and for many interesting discussions.

References

[1] Reviews on the photon structure function may be found in C. Berger and W. Wagner, Phys. Rep. 146 (1987) 1 ; H. Kolanoski and P. Zerwas, in High Energy e^+-e^- physics, World Scientific, Singapore, 1988, Eds. A. Ali and P. Söding. J. H. Da Luz Vieira and J. K. Storrow, Z. Phys. *C*51 (1991) 241.

[2] H1 Collaboration, T. Ahmed et al., Phys. Lett. *B*297 (1992) 205.

[3] Zeus Collaboration, M. Derrick et al., Phys. Lett. *B*297 (1992) 404.

[4] For a review on the photoproduction at HERA, see M. Fontannaz, "The Photon Structure Function at HERA", Orsay preprint LPTHE 93-22, talk given at the XXI International Meeting on Fundamental Physics, Miraflores de la Sierra, Spain (May 1993), and "Hard Processes and Polarization Phenomena in QCD", talk at the "Workshop on Two-Photon Physics at LEP and HERA", Lund 1994, edited by G. Jarlskog and L. Jönsson, Lund University.

[5] TOPAZ collaboration, H. Hayashi et al., Phys. Lett. B314 (1993) 149.

[6] AMY collaboration, R. Tomaka et al., Phys. Lett. B325 (1994) 248.

[7] P. Aurenche, M. Fontannaz, J. Ph. Guillet, Y. Shimizu, J. Fujimoto and K. Kato, Prog. Theor. Physics 92 (1994) 175 ;
P. Aurenche et al., talk at the "Workshop on Two-Photon Physics at LEP and HERA", Lund 1994, edited by G. Jarlskog and L. Jönsson, Lund University.

[8] P. Aurenche, M. Fontannaz and J. Ph. Guillet, Phys. Lett. B338 (1994) 98.

[9] I. Antoniadis and G. Grunberg, Nucl. Phys. B213 (1983) 455.

[10] A. S. Gorski, B. L. Ioffe, A. Yu Khodjaminian, A. Oganesian, Z. Phys. C44 (1989) 523.

[11] E. Witten, Nucl. Phys. B210 (1977) 189.

[12] A review on QCD corrections, and previous references on the subject can be found in : F. M. Borzumati and G. A. Schuler, Z. Phys. C58 (1993) 139.

[13] M. Glück and E. Reya, Phys. Rev. D28 (1983) 2749.

[14] W. A. Bardeen and A. J. Buras, Phys. Rev. D20 (1979) 166 ; 21 (1980) 2041 (E).

[15] M. Fontannaz and E. Pilon, Phys. Rev. D45 (1992) 382.

[16] E. Laenen, S. Riemersma, J. Smith and W. L. van Neerven, Phys. Rev. D49 (1994) 5753.

[17] M. Glück, E. Reya and A. Vogt, Phys. Rev. D45 (1992) 3986.

[18] P. Aurenche, M. Fontannaz and J.-Ph. Guillet, Z. Phys. C. (1994).

[19] T. Sasaki et al., Phys. Lett. B252 (1990) 491.

[20] JADE collaboration, W. Bartel et al., Z. Phys. C24 (1984) 231.

[21] PLUTO collaboration, Ch. Berger et al., Nucl. Phys. B281 (1987) 365 ; Phys. Lett. 142B (1984) 11.

[22] TOPAZ collaboration, K. Muramatsu et al., Phys. Lett. B332 (1994) 447.

[23] M. Glück, E. Reya and A. Vogt, Phys. Rev. D46 (1992) 1973.

[24] L. E. Gordon and J. K. Storrow, Z. Phys. C56 (1992) 307.

[25] A. Vogt, Proceedings of the Workshop "Two-Photon Physics at LEP and HERA", Lund, May 1994.

[26] J. Ph. Guillet, talk at the "Workshop on Two-Photon Physics at LEP and HERA", Lund 1994.

[27] H1 collaboration, "27th International Conference on HEP", Glasgow 1994, presented by H. Hufnagel.

[28] ZEUS collaboration, M. Derrick et al., DESY 94-176.