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Gravitating, Gravitational and Dilatonic Sphalerons

George Lavrelashvili

Institut für Theoretische Physik
Universität Zürich
Winterthurerstrasse 190
CH-8057 Zürich, Switzerland

Abstract. In the present contribution we shall give a brief review of the main properties of sphalerons in various theories with a Yang–Mills field.

1. Introduction

As is well known, the vacuum in gauge theories has a complicated structure [1]. One can label vacua with a topological (Chern–Simons) number. Vacua with different topological numbers are separated by a potential barrier, whose height is set by the sphaleron [2], [3].

Topologically nontrivial fluctuations of the gauge field lead to fermion number nonconservation [4] via the anomaly [5]. The fermion number nonconservation in topologically nontrivial gauge field backgrounds can be described by the generalized Bogolyubov transformation technique [6], [7] or in terms of a level-crossing picture [8]. The rate of the fermion number nonconservation depends on the energy of the process. The sphaleron solution determines the energy scale for processes with a strong nonconservation of fermion number.

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Recently a discrete series of static, spherically symmetrical solutions of Einstein–Yang–Mills (EYM) theory was found was found by Bartnik and McKinnon [9]. It turns out that Bartnik–McKinnon (BMK) solutions are gravitational analogues of the electroweak sphaleron [10], [11]. Solutions of the same nature were found in Yang–Mills-dilaton (YMD) theory [12], [13]. It was understood that the existence of all of these solutions is related to topological properties of the YM field configuration space.

In the present contribution we give a brief review of different YM sphalerons.

In the next section we describe sphaleron solutions in YMH, EYM and YMD theories. In section 3 we discuss the main properties of these solutions and their interpretation. In section 4 we discuss similarity and difference in various sphalerons. Section 5 contains concluding remarks.

2. Sphaleron solutions in different theories

In the present section we shall discuss several theories with the YM field in which sphaleron solutions exist, namely YMH with the Higgs doublet, EYM, and YMD, and describe the main properties of sphalerons.

2.1. Electroweak sphaleron

The electroweak sphaleron has a relatively long history. It was found by R.F. Dashen, B. Hasslacher, and A. Neveu (DHN) [14] in 1974 in relation with hadron physics and rediscovered later in the context of nuclear physics by Boguta [15]. In 1983 an analysis of the properties of the configuration space of an $SU(2)$ YM field [2] led to the claim of existence of saddle point solutions in YMH theory. It was realized [3] that they coincide with the DHN solutions.

The argument for the existence of sphalerons runs as follows [2]. Let us consider a one-parameter family of configurations interpolating between the vacua with different topological (Chern–Simons) numbers. The asymptotic behaviour of the gauge field defines a map $S^2 \to SU(2) \simeq S^3$. A suitable one-parameter family of maps is topologically equivalent to a single nontrivial map $S^3 \to S^3$. Denoting the maximum of the (static) energy along each path l by $E(l)$, we may take the minimum of $E(l)$ running through all nontrivial paths. This minimum corresponds to a saddle point of the energy functional.

In pure YM theory there is no scale and there are no static solutions [16] in $(1+3)$ dimensions. One way out is to introduce a Higgs field.

The action for the YMH theory has the form

$$S_{\mathrm{YMH}} = \int \left(-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\ \mu\nu} + (D^\mu \Phi)^\dagger D_\mu \Phi - \lambda(\Phi^\dagger \Phi - \frac{v^2}{2})^2 \right) d^4x \tag{1}$$

where $F^a_{\mu\nu}$ is the $SU(2)$ gauge field strength, $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \epsilon^{abc} W^b_\mu W^c_\nu$, and $a = 1, 2, 3$ is the $SU(2)$ group index, $\mu, \nu = 0, 1, 2, 3$ are space-time indices. Covariant derivatives are defined by $D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2} \tau^a W^a_\mu \Phi$.

We are interested in spherically symmetric solutions. The most general spherically symmetric ansatz for the *SU(2)* Yang–Mills field W^a_μ can be written (in the Abelian gauge) as [17]

$$\begin{aligned} W^a_t &= (0, 0, A_0)\,, \qquad & W^a_\theta &= (\phi_1, \phi_2, 0)\,, \\ W^a_r &= (0, 0, A_1)\,, \qquad & W^a_\varphi &= (-\phi_2 \sin\theta, \phi_1 \sin\theta, \cos\theta)\,. \end{aligned} \tag{2}$$

For the Higgs field we take

$$\Phi = \frac{v}{\sqrt{2}} \left[H + iK(\vec{n} \cdot \vec{\tau})\right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{3}$$

where $\vec{\tau}$ are the usual Pauli isospin matrices and $\vec{n} = \vec{x}/r$.

The ansatz (2) is form-invariant under gauge transformations around the third isoaxis, with A_α transforming as a $U(1)$ gauge field on the reduced space-time (t, r), whereas $\phi = \phi_1 + i\phi_2$ is a scalar field of charge one with respect to the $U(1)$. Introducing $\chi = H + iK$ we find

$$A_\alpha \to A_\alpha + \partial_\alpha \Omega, \qquad \phi \to \mathrm{e}^{i\Omega}\phi, \qquad \chi \to \mathrm{e}^{i\Omega/2}\chi. \tag{4}$$

With respect to this $U(1)$ one may define the 'charge conjugation'

$$A_\alpha \to -A_\alpha, \qquad \phi \to \overline{\phi}, \qquad \chi \to \overline{\chi}. \tag{5}$$

The even sector with respect to this charge conjugation is given by

$$A_0 = 0,\ A_1 = 0,\ \phi_1 \equiv W(r),\ \phi_2 = 0, K = 0. \tag{6}$$

In this sector the ansatz (2) is equivalent to the usual "monopole" ansatz

$$W^a_0 = 0, \quad W^a_i = \epsilon_{aij} \frac{n_j}{r} (1 - W(r)) \tag{7}$$

with $n_j = x_j/r$. And the Higgs field is

$$\Phi = \frac{v}{\sqrt{2}}\, H(r) \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{8}$$

The reduced action in this sector has the form

$$S^{\rm red}_{\rm YMH} = -\frac{4\pi v}{g}\int\left(\left(\frac{dW}{d\xi}\right)^2 + \frac{(W^2-1)^2}{2\xi^2}\right.$$
$$\left. + \frac{\xi^2}{2}\left(\frac{dH}{d\xi}\right)^2 + \frac{H^2(1+W)^2}{4} + \frac{1}{4}\frac{\lambda}{g^2}\xi^2(H^2-1)^2\right)d\xi \tag{9}$$

where $\xi = gvr$.

The corresponding equations of motion are

$$\frac{d^2W}{d\xi^2} = \frac{W(W^2-1)}{\xi^2} + \frac{H^2}{4}(1+W)\,,$$
$$\frac{d}{d\xi}\left(\xi^2\frac{dH}{d\xi}\right) = \frac{H(1+W)^2}{2} + \frac{\lambda}{g^2}\xi^2(H^2-1)H\,. \tag{10}$$

The solution has to interpolate between

$$W = 1\,, \qquad H = 0\,, \tag{11}$$

at $\xi \to 0$ and

$$W = -1\,, \qquad H = 1 \tag{12}$$

for $\xi \to \infty$.

It was found [14], [3] that equations (10) indeed have a sphaleron solution $\{W(r), H(r)\}$ which satisfies boundary conditions (11), (12).

2.2. Gravitational sphaleron

In 1988 Bartnik and McKinnon unexpectedly found a discrete sequence of globally regular solutions of the EYM theory.

We say unexpectedly, because neither vacuum Einstein nor pure YM theory has nontrivial globally regular, static, finite energy solutions [16]. There are also no such solutions in the EYM theory in $(2+1)$ dimensions [18].

The action for the EYM theory has the form

$$S_{\rm EYM} = \frac{1}{4\pi}\int\left(-\frac{1}{4G}R - \frac{1}{4g^2}F^a_{\mu\nu}F^{a\ \mu\nu}\right)\sqrt{-g}\,d^4x \tag{13}$$

where g denotes the gauge coupling constant and G is Newton's constant.

A convenient parametrization for the metric turns out to be

$$ds^2 = S^2(r)N(r)dt^2 - \frac{dr^2}{N(r)} - r^2 d\Omega^2 \ , \tag{14}$$

where $d\Omega^2 = d\theta^2 + \sin^2(\theta)d\varphi^2$ is the line element of the unit sphere.

For the $SU(2)$ YM potential we make the usual ('magnetic') spherically symmetric ansatz (7). Substituting this ansatz into the action we obtain the reduced action

$$S^{\rm red}_{\rm EYM} = -\int S\left[\frac{1}{2G}(N + rN' - 1) + \frac{1}{g^2}\left(NW'^2 + \frac{(1-W^2)^2}{2r^2}\right)\right] dr \ , \tag{15}$$

where a prime denotes $\frac{d}{dr}$.

The resulting field equations are

$$\begin{aligned}
(NSW')' &= S\frac{W(W^2-1)}{r^2} \ , \\
N' &= \frac{1}{r}\left(1 - N - 2\left(NW'^2 + \frac{(1-W^2)^2}{2r^2}\right)\right) \ , \\
S^{-1}S' &= \frac{2W'^2}{r} \ .
\end{aligned} \tag{16}$$

The field equations (16) have singular points at $r = 0$ and $r = \infty$ as well as points where $N(r)$ vanishes. Regularity at $r = 0$ of a configuration requires $N(r) = 1 + O(r^2)$, $W(r) = \pm 1 + O(r^2)$ and $S(r) = S(0) + O(r^2)$. Since W and $-W$ are gauge equivalent we may choose $W(0) = 1$. Similarly we can assume $S(0) = 1$ since a rescaling of S corresponds to a trivial rescaling of the time coordinate. Inserting a power series expansion into (16) one finds

$$\begin{aligned}
W(r) &= 1 - br^2 + O(r^4) \ , \\
N(r) &= 1 - 4b^2r^2 + O(r^4) \ , \\
S(r) &= 1 + 4b^2r^2 + O(r^4) \ ,
\end{aligned} \tag{17}$$

where b is an arbitrary parameter.

Similarly assuming a power series expansion in $\frac{1}{r}$ at $r = \infty$ for asymptotically flat solutions, one finds $\lim\limits_{r\to\infty} W(r) = \{\pm 1, 0\}$. It turns out that $W(\infty) = 0$ cannot occur for globally regular solutions, so we concentrate on the remaining cases. One finds

$$W(r) = \pm\left(1 - \frac{c}{r} + O\left(\frac{1}{r^2}\right)\right) \,,$$
$$N(r) = 1 - \frac{2M}{r} + O\left(\frac{1}{r^4}\right) \,,$$
$$S(r) = S_\infty\left(1 + O\left(\frac{1}{r^4}\right)\right) \,, \tag{18}$$

where again c, M and S_∞ are arbitrary parameters and have to be determined from numerical calculations.

It was found [9] that equations (16) admit a discrete sequence of finite-energy solutions $\{W_n, N_n, S_n\}$ which interpolate between the asymptotic behaviours (17) for $r \to 0$ and (18) for $r \to \infty$.

2.3. Dilatonic sphaleron

As mentioned earlier, there are no static solutions in the pure YM theory in $(3+1)$ dimensions. The reason is that pure YM theory is repulsive. In order to have solutions with finite energy one needs some extra field providing attraction that compensates YM repulsion. In the case of the electroweak sphaleron this attraction is provided by a Higgs field. It was realized [12], [13] that the role of a binding force can be provided by a dilaton field as well.

Introducing a dilaton field we naturally obtain a EYMD theory with the action

$$S_{\mathrm{EYMD}} = \frac{1}{4\pi}\int\left(-\frac{1}{4G}R + \frac{1}{2}(\partial\varphi)^2 - \frac{\mathrm{e}^{2\kappa\varphi}}{4g^2}F^2\right)\sqrt{-g}\,d^4x \tag{19}$$

where κ and g respectively denote the dilatonic and gauge coupling constant and G is Newton's constant.

This theory depends on a dimensionless parameter $\gamma = \frac{\kappa}{g\sqrt{G}}$. In the limit $\gamma \to 0$ one gets the EYM theory studied in [9]. The value $\gamma = 1$ corresponds to a model obtained from heterotic string theory [19]. We found strong indications that the lowest-lying regular solution for this value of γ may be obtained in closed form [20], [21].

We will not discuss here the case of general γ [20], [22], but rather concentrate on the limiting case $\gamma \to \infty$ where one obtains the YM-dilaton theory in flat space [12], [13].

In the flat case when gravity decouples we get a YMD theory defined by the action

$$S_{\rm YMD} = \frac{1}{4\pi}\int\left(\frac{1}{2}(\partial\varphi)^2 - \frac{e^{2\varphi}}{4g^2}F^2\right)d^4x. \tag{20}$$

The corresponding reduced action is

$$S^{\rm red}_{\rm YMD} = -\int dr\left[\frac{r^2}{2}\varphi'^2 + e^{2\varphi}\left(W'^2 + \frac{(W^2-1)^2}{2r^2}\right)\right] \tag{21}$$

with resulting field equations

$$\begin{aligned} W'' &= \frac{W(W^2-1)}{r^2} - 2\varphi' W' \, , \\ (r^2\varphi')' &= 2e^{2\varphi}\left(W'^2 + \frac{(1-W^2)^2}{2r^2}\right) \, . \end{aligned} \tag{22}$$

These equations are invariant under a shift $\varphi \to \varphi + \varphi_0$ accompanied by a simultaneous rescaling $r \to r e^{\varphi_0}$. Hence globally regular solutions can be normalized to $\varphi(\infty) = 0$.

It was found [12], [13] that equations (22) have a discrete sequence of finite energy solutions $\{W_n, \varphi_n\}$, where $n = 1, 2, 3, ...$ labels the number of zeros of the gauge function $W_n(r)$.

The mass of this solution varies from ≈ 0.8 for $n = 1$ to 1.0 for $n \to \infty$ in natural units κg.

The asymptotic behavior of these solutions for $r \to 0$ is

$$\begin{aligned} W_n(r) &= 1 - b_n r^2 + O(r^4) \, , \\ \varphi_n(r) &= \varphi_n(0) + 2e^{2\varphi_n(0)} b_n^2 r^2 + O(r^4) \, , \end{aligned} \tag{23}$$

and for $r \to \infty$ is

$$\begin{aligned} W_n(r) &= (-1)^n\left(1 - \frac{c_n}{r} + O\left(\frac{1}{r^2}\right)\right) \, , \\ \varphi_n(r) &= -\frac{d_n}{r} + O\left(\frac{1}{r^4}\right) \, . \end{aligned} \tag{24}$$

The parameters $b_n, \varphi_n(0), c_n$ and d_n have to be determined by numerical calculations.

2.4. Gravitating sphaleron

A natural generalization of the YMH theory is obtained taking gravity into account. In this way one gets EYMH theory, which has gravitating sphaleron [23], [24] solutions.

This theory contains two scales, gravitational and electroweak. As a result one gets two kinds of excitation modes: gravitational (analogous to the BMK mode) and electroweak. The lowest solution is with one sphaleron node. The next one, with two nodes, contains one sphaleron node and one gravitational node.

3. Properties and interpretation

The main properties of the sphalerons are as follows:

(i) they have finite energy
(ii) they have fractional topological charge
(iii) there are fermion zero modes in the background of these solutions
(iv) they are saddle points of the action.

We call solutions of EYM and YMD sphalerons since they possess all the properties (i)–(iv). More precisely, solutions with odd n are sphalerons, while solutions with even n correspond to trivial loops in configuration space.

(i) The mass of the sphaleron in the standard model of electroweak interactions is of order of a few TeV. The EYM sphalerons have typical masses of order $1/g\sqrt{G}$. If we assume that our model is part of the string theory, or in other words if we are granted the mass scale parameter $M_{\rm Pl}$, the mass of the solutions of the Einstein–Yang–Mills theory is of the order of unity in Planckian units. In the EYMD theory the mass of the solutions decreases with increasing dilatonic coupling constant γ and for large γ goes to zero like $M_{\rm EYMD} \sim \frac{M_{\rm Pl}}{\gamma^2}$.

(ii) It was shown [2], [3] how to assign a topological (baryon) number to the sphaleron. It turns out to be $\frac{1}{2}$. In the gauge we use here one can read this off from the asymptotic behaviour of the gauge field.

(iii) It was found that there are fermion zero modes in the background of the electroweak [25], gravitational [26] and dilatonic [27] sphalerons. This is in perfect agreement with the picture of level-crossing phenomena [8].

(iv) The electroweak sphaleron has just one negative mode [2], [3], [28].

Various aspects of the stability of BMK solutions have been analyzed [29], [30], [31], [32], [33], [34], [35]. It is natural [34] to distinguish two different kinds of instabilities, which we call ‘sphaleron’ and ‘gravitational’ instabilities. Gravitational instabilities have no analogue for the flat-space sphaleron, whereas instabilities of the former type have same nature as for the YMH sphaleron. It was found [29], [30] that the BMK solutions are unstable and the numerical results for the few lowest solutions led to the conclusion that the n^{th} Bartnik–McKinnon solution has exactly n unstable gravitational modes. It was shown analytically [31], [32] that there exists at least one sphaleron-like unstable mode for each member of the BMK family.

Numerical studies [34] led to the claim that the n^{th} BMK solution has exactly n sphaleron-like instabilities, so that altogether the n^{th} BMK solution has $2n$ unstable modes, n of either type. It is quite remarkable that the conjecture about the number of sphaleron-like instabilities can be proven [35] in spite of the fact that the BMK solutions are not known analytically.

Numerical studies indicates that the same conjecture is true for dilatonic sphalerons, namely the n^{th} solution of the YMD theory has exactly $2n$ unstable modes [27].

4. Comparison of different sphalerons

It is an interesting question why such different theories as YMH, EYM and YMD posses similar solutions. The "explanation" lies in the presence of a YM field in all of these cases. Introducing a proper "time" coordinate τ (different in each case) one arrives at an equation of the following type:

$$\frac{d^2W}{d\tau^2} = -\frac{dU}{dW} + \lambda(\tau)\dot{W} + h(\tau), \tag{25}$$

where $U(W) = -(W^2-1)^2/4$ is an inverted double well potential, and the coefficients $\lambda(\tau)$ and $h(\tau)$ for each theory are shown in the Table.

Theory	$\lambda(\tau)$	$h(\tau)$
YM	1	0
YMH	1	$\frac{e^{2\tau}H^2}{4}(1+W)$
EYM	$K - \frac{\dot{K}}{K} - \frac{2\dot{W}^2}{Kr}$	0
YMD	$1-2\dot{\varphi}$	0

Table. Coefficients in equation (25) for different theories. $K \equiv \sqrt{N}$.

Without the last term $h(\tau)$ the equation (25) has a simple mechanical analogue, the motion of a "particle" in the potential $U(W)$ under the influence of friction, with the coefficient $\lambda(\tau)$. The last term, which is present only in the YMH case, plays the role of a "time dependent" potential.

In order to have a finite-energy field-theoretical solution the "particle" in our mechanical analogy should start at $W = 1$ for $\tau \to -\infty$ with $\dot{W} = 0$ and end up at $W = \pm 1$ for $\tau = +\infty$.

It is obvious that there are no static solutions in the pure YM theory, because the constant friction term prevents the "particle" from stopping at the top, $W = -1$.

In the YMH case the friction term is the same but the potential is deformed with increasing τ.

In EYM and YMD cases the friction coefficient depends on the derivatives of other fields. Thus one can have positive as well as negative friction. This allows for excited solutions in the cases with gravity and dilaton, corresponding to oscillations of the "particle".

Although all the solutions we discuss are similar, there is an important difference. In contrast to the electroweak sphaleron the gravitational and dilatonic sphalerons have an even number of negative modes [34], [27]. ((This fact should be considered in view of the recent paper of Rubakov and Shvedov [36].))

5. Concluding remarks

We have shown that there are saddle point solutions of similar nature in YMH, EYM and YMD theories. We emphasized the similarities and differences between these solutions.

In addition to the globally regular solutions considered above there are also black hole solutions [37], [20], [23] of corresponding theories. They may be considered as black holes sitting inside sphalerons. These solutions are of interest as counterexamples for a "no-hair" conjecture.

The "zoo" of solutions described is an interesting problem of mathematical physics. The sphaleron is important in electroweak baryogenesis. The role and importance of the gravitational and dilatonic analogues is not yet clear.

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