INHOMOGENEOUS NUCLEATION OF QUARK–GLUON PLASMA IN HIGH ENERGY NUCLEAR COLLISIONS

Joseph I. Kapusta and **Axel P. Vischer**[†]

School of Physics and Astronomy
University of Minnesota
Minneapolis, MN 55455

Abstract

We estimate the probability that a hard nucleon-nucleon collision is able to nucleate a seed of quark–gluon plasma in the surrounding hot and dense hadronic matter formed during a central collision of two large nuclei at AGS energies. The probability of producing at least one such seed is on the order of 1-100%. We investigate the influence of quark–gluon plasma formation on the observed multiplicity distribution and find that it may lead to noticeable structure in the form of a bump or shoulder.

1

1 Introduction

One of the mysteries of heavy ion physics at Brookhaven National Laboratory's AGS is: *If hadronic cascade event simulators like RQMD [1] and ARC [2] produce energy densities approaching 2 GeV/fm³, yet agree with experiment, where is the quark–gluon plasma?* After all, numerous estimates of the onset of quark–gluon plasma agree that it should occur at about that energy density, and if there is a first order phase transition, then the onset of the mixed phase would occur at an even lower density. One possibility is that no phase transition occurs even at these high densities, but it is difficult to understand how composite objects like hadrons can overlap so strongly in position space without the matter undergoing some qualitative change in character. A second possibility is that the distribution of observed hadrons in the final state is insensitive to the dynamics of the matter when it is most hot and dense. (Unfortunately there are no measurements of direct photons or dileptons at the AGS which might probe this stage of the collision.) There is some evidence for this which comes from artificially modifying hadronic cross sections at high density [3]. It may be understood by recognizing that once a system reaches local thermal equilibrium it is basically irrelevant how it got there.

Recently we proposed a third possibility [4]: *Most collisions at AGS energies produce superheated hadronic matter and are describable with hadronic cascade simulators, but in rare events a droplet of quark–gluon plasma is nucleated which converts most of the matter to plasma.* We estimated the probability of this to occur, using homogeneous nucleation theory, to be on the order of once every 100 to 1000 central collisions of large nuclei. Our estimate was based on the probability that thermal fluctuations in a homogeneous superheated hadronic gas would produce a plasma droplet, and that this droplet was large enough to overcome its surface free energy to grow. In this paper we consider another source of plasma droplet production which is essentially one of nonthermal origin. Specifically, we estimate the probability that a collision occurs between two highly energetic incoming nucleons, one from the projectile and one from the target, that this collision would have produced many pions if it had occurred in vacuum, but because it occurs in the hot and dense medium its collision products are quark and gluon fields which make a small droplet of plasma. Although there is a large uncertainty in our estimates, we find that this inhomogeneous nucleation of plasma may be more probable than homogeneous nucleation by one to two orders of magnitude.

In this paper we also consider the problem of observation of the effects of nucleation of plasma in rare events. We are guided by observations of multiplicity distributions in $p\bar{p}$ collisions at the CERN and Fermilab colliders. In those distribution, one sees a shoulder developing at high multiplicity at an energy of 540 GeV, which turns into a noticeable bump at higher energies. The real cause of this struc-

2

ture is not known, but may be due to minijet production. If plasma is nucleated in some fraction of central nucleus–nucleus collisions at the AGS, a similar structure may develop.

2 Kinetic Model of Hard Nucleon-Nucleon Collisions

In this section we develop a simple kinetic model which allows us to estimate the number of high energy nucleon-nucleon scatterings occurring in the high density medium formed during a collision between heavy nuclei. These scatterings occur when a projectile nucleon penetrates the hot and dense matter to collide with a target nucleon which has also penetrated the hot and dense matter. The energy loss of the colliding nucleons must be taken into account to obtain a reasonable estimate of the energy available for meson production in the nucleon-nucleon collision.

To first approximation we can visualize the initial stage of a heavy ion collision at the AGS in the nucleus-nucleus center-of- momentum frame as two colliding Lorentz contracted disks. See Figure 1. At time $t = 0$ they touch; subsequently they interpenetrate, forming hot and dense matter in the region of overlap. During this stage, additional matter streams into the hot zone even as this zone is expanding along the beam axis. The nucleons streaming in undergo scatterings with the hot matter already present, degrading their longitudinal momentum and producing baryonic isobars and/or mesons. Finally, at time $t_0 = L/2v\gamma$, all the cold nuclear matter has streamed into the region of overlap, and expansion and cooling begins. Here, L is the nuclear thickness, v is the velocity in the center-of-momentum frame, and γ is the associated Lorentz contraction factor. This is a very simplified picture of the early stage of the collision, but it seems to semi-quantitatively represent the outcome of both the ARC and RQMD simulations [1, 2, 4].

We are interested in the possibility that an incoming projectile nucleon suffers little or no energy loss during its passage to the longitudinal point z inside the hot and dense zone where it encounters a target nucleon which also has suffered little or no energy loss. The energy available in the ensuing nucleon-nucleon collision, $\sqrt{s}$, can go into meson production. Suppose that a large number of pions would be produced if the collision had happened in free space. Clearly, the outgoing quark and gluon fields cannot be represented as asymptotic pion and nucleon states immediately. The fields must expand and become dilute enough to be called real hadrons. If this collision occurs in a high energy density medium, the outgoing quark-gluon fields will encounter other hadrons before they can hadronize. It is reasonable to suppose that this "star burst" will actually be a seed for quark-gluon plasma formation if the surrounding matter is superheated hadronic matter. We

need a semi-quantitative model of this physics.

A fundamental result from kinetic theory is that the number of scattering processes of the type $1 + 2 \to X$ is given by

$$N_{1+2 \to X} = \int dt \int d^3x \int \frac{d^3p_1}{(2\pi)^3} f_1(\mathbf{x}, \mathbf{p}_1, t) \int \frac{d^3p_2}{(2\pi)^3} f_2(\mathbf{x}, \mathbf{p}_2, t) \, v_{12} \, \sigma_{1+2 \to X}(s_{12}) \,. \quad (1)$$

Here v_{12} is a relative velocity,

$$v_{12} = \frac{\sqrt{(p_1 \cdot p_2)^2 - m_N^4}}{E_1 \, E_2} \,, \quad (2)$$

where p_i denotes the four-momentum of nucleon i and $E_i = \sqrt{\mathbf{p}_i^2 + m_N^2}$ its energy. The f_i are phase space densities normalized such that the total number of nucleons of type i is

$$N_i^{\text{tot}} = \int \frac{d^3x d^3p}{(2\pi)^3} f_i(\mathbf{x}, \mathbf{p}, t) \,. \quad (3)$$

A differential distribution in the variable Y is obtained by replacing σ with $d\sigma/dY$.

For our purpose it is reasonable to represent the colliding nuclei as cylinders with radius R and thickness L. All the action is along the beam axis. We assume that the phase space distributions are independent of transverse coordinates x and y and of transverse momentum. Integrating over the cross sectional area of the nuclei, and counting only those collisions that occur within the hot zone, yields

$$N_{1+2 \to X} = \pi R^2 \int_0^{t_0} dt \int_{-vt}^{vt} dz \int \frac{dp_{1z} \, dp_{2z}}{(2\pi)^2} f_1(z, p_{1z}, t) \, f_2(z, p_{2z}, t) \, v_{12} \, \sigma_{1+2 \to X}(s_{12}) \,.$$
$$(4)$$

Here there is a change in notation: $f_i(z, p_{iz}, t)/2\pi$ is the probability per unit volume to find a nucleon i with longitudinal momentum p_{iz} at longitudinal position z at time t. The integration limits on z ensure that the collisions under consideration really occur in the hot zone; see Figure 1. The integration limits on t mean that we only count those collisions which occur before the system begins its cooling stage. The depth in the hot zone to which nucleon 1 has penetrated is $d_1 = (vt + z)/2$, and the depth to which nucleon 2 has penetrated is $d_2 = (vt - z)/2$. We neglect the decrease in velocity of the nucleons as they travel through the hot zone. This is an acceptable approximation because in the end we are interested only in those nucleons which suffer a small energy loss in traversing the hot matter.

We construct the phase space distribution as follows:

$H(x, N) =$ probability that the nucleon has momentum fraction x after
making N collisions;

$S(N, d)$ = probability that the nucleon has made N collisions after penetrating to a depth d;

$\sum_{N=0}^{\infty} H(x, N) S(N, d)$ = probability that the nucleon has momentum fraction x after penetrating to a depth d.

The distribution functions are normalized to unity.

$$\int_0^1 \frac{dx}{x} H(x, N) = 1 \tag{5}$$

$$\sum_{N=0}^{\infty} S(N, d) = 1 \tag{6}$$

The phase space density of nucleon i is then taken to be

$$\frac{dp_{zi}}{2\pi} f_i(z, p_{iz}, t) = \gamma\, n_0\, \frac{dx_i}{x_i} \sum_{N_i=0}^{\infty} H(x_i, N_i)\, S(N_i, d_i)\,, \tag{7}$$

where n_0 is the average baryon density in a nucleus, about 0.145 nucleons/fm^3. As a check, we can compute the number of nucleons which have entered the hot zone as a function of time.

$$N_i^{\mathrm{part}}(t) = \int \frac{d^3x\, dp_{iz}}{2\pi} f(z, p_{1z}, t)\, \Theta(d_i) = 2\pi R^2 \gamma n_0 v t \tag{8}$$

The step function fixes the limits on the z integration. The number of participating nucleons grows linearly with time, and at time t_0 we get $N_i^{\mathrm{part}}(t_0) = \pi R^2 L n_0$, which is the total number of nucleons in the nucleus.

The number of elementary nucleon-nucleon collisions can now be expressed as

$$\begin{aligned}
N_{1+2\to X} &= \pi R^2 \gamma^2 n_0^2 \int_0^{t_0} dt \int_{-vt}^{vt} dz \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2}\, v_{12}\, \sigma_{1+2\to X}(s_{12}) \\
&\quad \sum_{N_1=0}^{\infty} \sum_{N_2=0}^{\infty} H(x_1, N_1)\, S(N_1, d_1)\, H(x_2, N_2)\, S(N_2, d_2)\,.
\end{aligned} \tag{9}$$

Since the nucleons' velocities are antiparallel the velocity factor is

$$v_{12} = \frac{x_1 p_0}{\sqrt{x_1^2 p_0^2 + m_N^2}} + \frac{x_2 p_0}{\sqrt{x_2^2 p_0^2 + m_N^2}}\,, \tag{10}$$

where p_0 is the beam momentum in the center-of-momentum frame.

The survival function $S(N, d)$ is characterized by the mean free path λ of nucleons in the hot and dense hadronic matter. For a dilute gas the inverse of the mean

5

free path is the sum of products of the cross section of the nucleon with the density of objects it can collide with.

$$\lambda^{-1} = \sum_i n_i \sigma_i \tag{11}$$

Average particle densities, including baryons and mesons, were computed in ref. [4] for the hot and dense matter under consideration. A plot of the density as a function of beam energy is shown in Figure 2. Assuming an average hadron-nucleon cross section of 25 mb, we find $\lambda = 0.4$ fm at a laboratory beam energy of 11.6 GeV/nucleon. This is very short, and just emphasizes the physics we discussed in the introduction concerning hadronic matter versus quark-gluon plasma.

We assume that the collisions suffered by the nucleons are independent and can be characterized by a Poisson distribution.

$$S(N, d) = \frac{1}{N!} \left(\frac{d}{\lambda} \right)^N \exp \left(-\frac{d}{\lambda} \right) \tag{12}$$

Here d/λ is the average number of scatterings in a distance d.

The invariant distribution function $H(x, N)$ describes the momentum degradation of a nucleon propagating through the hot zone. This distribution function was introduced in the evolution model of Hwa [5]. In this model the nucleon propagates on a straight line trajectory and interacts with target particles contained within a tube with area given by the elementary nucleon–nucleon cross section σ_{NN}. Csernai and Kapusta [6] solved the resulting evolution equations and found that the invariant distribution function in this model is given by

$$H(x, N) = x \sum_{n=1}^{N} \left(\begin{array}{c} N \\ n \end{array} \right) w^n (1 - w)^{N-n} \frac{(-\ln x)^{n-1}}{(n - 1)!} + (1 - w)^N \delta(x - 1). \tag{13}$$

The δ-function represents elastic and soft inelastic contributions to the evolution of the nucleon through the matter. The probability w is the ratio of inelastic to total nucleon– nucleon cross section. It corresponds to the probability that the nucleon scatters inelastically and therefore drops out of the evolution described by H; it is approximately 0.8 in free space. Csernai and Kapusta found that it reduces to about 0.5 for nucleons propagating through a nucleus. This value allowed them to obtain a good representation of data with beam energies in the range of 6-405 GeV. In our case the nucleon is propagating through hot and dense hadronic matter. We keep w as a free parameter since we don't know how the value of w changes due to the thermal excitations and the increased density.

We are interested in the number of pion-producing nucleon-nucleon collisions with a relatively high center-of-momentum energy squared s. Our basic result from

6

this section is

$$\frac{dN_{\rm in}^{\rm hard}}{ds} = \pi R^2 \gamma^2 n_0^2 \, \sigma_{\rm in}(s) \int_0^{t_0} dt \int_{-vt}^{vt} dz \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \, v_{12} \, \delta(s - s_{12})$$

$$\sum_{N_1=0}^{\infty} \sum_{N_2=0}^{\infty} H(x_1, N_1) \, S(N_1, d_1(z,t)) \, H(x_2, N_2) \, S(N_2, d_2(z,t)) \,. \quad (14)$$

Here $\sigma_{\rm in}$ is the inelastic nucleon-nucleon cross section, and $\sqrt{s_{12}}$ is the total energy in the nucleon-nucleon collision where the nucleons have momentum fractions x_1 and x_2.

3 Meson Production Cross Sections

A phase transition to quark–gluon plasma will become thermodynamically favorable if the energy density is large enough. The corresponding phase boundary in the temperature/chemical potential plane was explored in [4]. Until now we have only selected nucleon-nucleon scatterings in which the total available energy $\sqrt{s}$ is large. In addition, we need to specify what fraction of this energy goes into meson production. In this section we estimate the pion number distribution function $P_n(s)$, which is the probability of producing n pions in a nucleon–nucleon collision in free space. The pion number distribution function is linked to the cross section σ_n for producing n pions by

$$P_n(s) = \sigma_n(s)/\sigma_{\rm in}(s) \,. \quad (15)$$

Given $P_n(s)$ we can estimate the number of nucleon-nucleon collisions that would lead to the production of n pions as

$$N_n = \int_{s_{\rm min}}^{4E_0^2} ds \, P_n(s) \, \frac{dN_{\rm in}^{\rm hard}}{ds} \,. \quad (16)$$

The lower limit of integration is fixed by kinematics and the upper limit is determined by the beam energy.

We shall approximate the pion number distribution function $P_n(s)$ with a binomial [7] and choose the parameters of this binomial such that we have some rough agreement with experiment [8].

$$P_n(s) = \left(\begin{array}{c} n_{\rm max} \\ n \end{array} \right) \xi^n (1 - \xi)^{n_{\rm max} - n} \quad (17)$$

The maximum number of pions produced in a nucleon-nucleon collision is determined by kinematics.

$$n_{\rm max}(s) = {\rm Integer} \left(\frac{\sqrt{s} - 2m_N)}{m_\pi} \right) \quad (18)$$

7

The parameter ξ is related to the mean multiplicity by

$$\xi(s) = \frac{\langle n \rangle}{n_{\max}} = \frac{3}{n_{\max}} \left(\frac{1}{4} \langle n_{pp}^- \rangle + \frac{1}{2} \langle n_{pn}^- \rangle + \frac{1}{4} \langle n_{nn}^- \rangle \right) . \qquad (19)$$

Here $\langle n \rangle$ is the average pion multiplicity averaged over pp, pn and nn collisions while $\langle n_{pp}^- \rangle$, $\langle n_{pn}^- \rangle$ and $\langle n_{nn}^- \rangle$ represent the average negative pion multiplicity in those collisions. All average multiplicities are functions of s, of course. The factor of 3 is due to isospin averaging.

Experimental data were compiled and parametrized in [9] as

$$\begin{aligned}
\langle n_{pp}^- \rangle &= -0.41 + 0.79 F(s) \\
\langle n_{pn}^- \rangle &= -0.14 + 0.81 F(s) \\
\langle n_{nn}^- \rangle &= +0.35 + 0.77 F(s) .
\end{aligned} \qquad (20)$$

The function F was introduced by Fermi [10],

$$F(s) = \frac{\left(\sqrt{s} - 2m_N \right)^{3/4}}{s^{1/8}} , \qquad (21)$$

with s measured in GeV^2. The parametrizations in (20) describe the data rather well except in the threshold region. We approximate the inelastic nucleon–nucleon cross section $\sigma_{\rm in}$ by the inelastic proton–proton cross section. A convenient parametrization is given in [8],

$$\sigma_{\rm in} = 30.9 - 28.9 \, p_L^{-2.46} - 0.835 \ln p_L + 0.192 \ln^2 p_L , \qquad (22)$$

where p_L is the laboratory momentum in GeV/c and the cross section is in mb. This parametrization is good for $p_L > 0.968$ GeV/c.

The pion production cross sections, as described above, are displayed in Figure 3. They have the right shapes and the right orders of magnitude compared to data [8]. However, direct comparison is not possible. First of all, data generally does not exist for final states with π^+, π^-, and π^0. Usually, exclusive experiments can only measure charged mesons or neutral mesons, not both. Secondly, we have not been so sophisticated as to include vector mesons, the η meson, and kaons. For our purpose such sophistication is probably not necessary. We care only about the probability that a nucleon-nucleon collision leads to a significant amount of energy release in the sense of conversion of initial kinetic energy to meson mass. We are essentially basing our results on the total inelastic cross section, the average meson multiplicity, kinematics, and entropy. Our analysis would be better if we had a handle on the width of the multiplicity distribution, averaged over the initial state isospin and summed over the final state isospin.

4 Star Burst Probabilities

In this section we put together the ingredients developed in the last two and compute the number of star bursts which may become nucleation sites or seeds for plasma formation and growth.

The nucleon-nucleon collisions may be referred to as primary-primary, primary-secondary, and secondary-secondary, depending on whether the nucleons have scattered from thermalized particles in the hot zone (secondary) or not (primary). The easiest contribution to obtain is the primary-primary. All integrations and summations can be done analytically with the result

$$\frac{dN_{\text{in}}^{\text{prim}-\text{prim}}}{ds} = 4\pi R^2 \sigma_{\text{in}}(s) \left(\frac{\lambda \gamma n_0}{w}\right)^2 \left[1 - \left(1 + w\,\frac{vt_0}{\lambda}\right) \exp\left(-\frac{vt_0}{\lambda}\right)\right] \delta(s - 4E_0^2).$$

(23)

The formulas for the primary-secondary and secondary-secondary contributions can be simplified to some extent but in the end some summations remain which must be done numerically.

The number of nucleon-nucleon collisions as a function of s are plotted in Figure 4. Both $w = 0.5$ and 0.8 are shown; there is little difference. The laboratory beam energy is 11.6 GeV per nucleon and the nuclei are gold. The spike represents the delta function from primary-primary collisions. The contribution from primary-secondary collisions falls from about 11 to 7 GeV^{-2} as s goes from 9 to 25 GeV. The contribution from secondary-secondary collisions is almost negligible.

The pion multiplicity distribution arising from these hard collisions is shown in Figure 5. It drops by more than nine orders of magnitude in going from 6 pion production to 18 pion production. Typically there is only one hard nucleon-nucleon collision leading to the production of seven pions in a central gold-gold collision at this energy.

We are interested in the possibility that one of these star bursts nucleates quark-gluon plasma. The precise criterion for this to happen is not known. However, we can make some reasonable estimates. In [4] we estimated that a critical size plasma droplet at these temperatures and baryon densities would have a mass of about 4 GeV. Any local fluctuation more massive than this would grow rapidly, converting the surrounding superheated hadronic matter to quark-gluon plasma. A similar estimate, based on the MIT bag model, a simpler hadronic equation of state (free pion gas), and with zero baryon density, was obtained much earlier [11]. Another estimate is obtained by the argument that at these relatively modest beam energies most meson production occurs through the formation and decay of baryon resonances: Δ, N^*, etc. The most massive observed resonances are in the range of 2 to 2.5 GeV. Putting two of these in close physical proximity leads to a mass of 4 to 5 GeV. We now need an estimate of the number of pions this critical mass

9

corresponds to. Let us assume that each particle, nucleon and meson, carries away a kinetic energy equal to one half its rest mass. If a particle would have too great a kinetic energy then it might escape from the nucleon-nucleon collision volume long before its neighbors and so would not be counted in the rest mass of the local fluctuation. Taking 4 GeV, dividing by 1.5, and subtracting twice the nucleon mass leaves about 6 pion rest masses. So our most optimistic estimate is that one needs a nucleon-nucleon collision which would have led to 6 pions if it had occurred in free space. One might be less optimistic and require the production of 8 or 10 pions instead.

In Figure 6 we show the total number $N_>$ of nucleon-nucleon collisions which would lead to the production of at least n_{crit} pions. We may view n_{crit} as the minimum number necessary to form a nucleation site or plasma seed. If $n_{\mathrm{crit}} = 6$ is the relevant number then there are on average 7 such nucleon-nucleon collisions per central gold-gold collision. If 8 or 10 are the relevant multiplicities then there is only one such critical star burst every 1 or every 25 central gold-gold collisions, respectively. These numbers vary somewhat with w; the numbers quoted are averages. Conservatively, we may conclude that the probability of at least one plasma seed appearing via this mechanism is in the range of 1 to 100% per central gold-gold collision at the highest energy attainable at the AGS. These probabilities are about one to two orders of magnitude greater than those estimated in [4] on the basis of thermal homogeneous nucleation theory.

5   Consequences for the Multiplicity Distribution

The results of the last section confirm the possibility of producing quark–gluon plasma droplets in rare events at AGS. Once formed the droplets grow rapidly due to the significant superheating of the hadronic matter. This process was explored in [4] where it was found that the radii of such droplets can reach $3 - 5$ fm. Since the phase transition is occurring so far out of equilibrium we would expect a significant increase in the entropy of the final state. This could be seen in the ratio of pions to baryons, for example, or in the ratio of deuterons to protons [12]. Along with the increased entropy should come a slowing down of the radial expansion due to a softening in the matter, that is, a reduction in pressure for the same energy density. Together, these would imply a larger source size and a longer lifetime as seen by hadron interferometry [13].

In this section we study one of the experimental ramifications in detail. Specifically, we look at the charged particle multiplicity distributions and investigate under what conditions one might be able to detect the rare events from the structure of this distribution.

In Figure 7 we plot the ratio of entropy to total baryon number S/B for the

hadronic and quark–gluon plasma phase for fixed beam energies. Fixed beam energy means that initially both the energy density and the baryon number density of the system is given which then determine the corresponding entropies via the equation of state. We use the equation of state discussed in [4] for all further calculations. It is helpful to consider two extreme and opposite scenarios. Either the matter stays all the time in the hadronic phase, or the matter has been completely converted to quark–gluon plasma by the time t_0 and only hadronizes later. The difference of the entropies produced in these two scenarios is given by the difference of the two curves in Figure 7. It represents an upper limit on the additional number of pions produced. Since the temperature is comparable to or larger than the pion mass the excess entropy is proportional to the maximum number of excess pions

$$3\frac{\Delta N_-}{B} = \frac{1}{3.6}\frac{\Delta S}{B}\,.\tag{24}$$

The number of additional negatively charged pions per baryon $\Delta N_-/B$ is linearly related to the entropy difference ΔS determined from Figure 7. The result is shown in Figure 8 for central Au + Au collisions. At beam energies of 11.6 GeV/A we produce 0.33 additional negatively charged pions per participating baryon. This is an upper limit, and in reality we would expect less.

These additional mesons might be visible in the charged particle multiplicity distribution which would have the form

$$P_n = (1 - q)\, P_n^{\mathrm{had}}(N_{\mathrm{had}}) + q\, P_n^{\mathrm{qg}}(N_{\mathrm{qg}})\,.\tag{25}$$

Here q is the probability of finding a central event in which plasma is formed, P_n^{had} is the multiplicity distribution for purely hadronic events with mean N_{had}, and P_n^{qg} is the multiplicity distribution for events in which a plasma was formed with mean N_{qg}.

Experimentally one would expect to see a bump in P_n at larger values of n. A structure like that was found in charged particle multiplicity distributions in $p\bar{p}$ collisions at the CERN [14, 15] and Fermilab [16, 17] colliders. For energies larger then 540 GeV a shoulder develops in the multiplicity distribution, becoming more pronounced as the beam energy increases. It is assumed that this structure is due to the onset of minijets. It is definitely an indication of new physics.

In Figure 9 we plot the charged particle multiplicity distribution for $p\bar{p}$ collisions at $\sqrt{s} = 900$ GeV from the UA5 collaboration [14]. For energies less then 500 GeV it was found that the distribution could be well described by a negative binomial distribution of the form

$$P_n(\bar{n}, k) = \left(\begin{array}{c} n + k - 1 \\ k - 1 \end{array}\right)\left[\frac{\bar{n}/k}{1 + (\bar{n}/k)}\right]^n \frac{1}{[1 + (\bar{n}/k)]^k}\,.\tag{26}$$

11

The parameter k characterizes the width of the distribution. For $k \to \infty$ we recover a Poisson distribution, the distribution with the smallest width. One can see from the figure that at 900 GeV a single negative binomial (NBD) cannot describe the data anymore. A double negative binomial (DNBD) of the form discussed in eq. (25) on the other hand describes it very well. The question remains to what extent a similar analysis might be able to reveal rare events of quark–gluon plasma production at AGS.

A rough criteria for the observability of such structure in distributions of the form (25) is

$$\frac{2}{\sqrt{N_{\mathrm{bin}}}} P^{\mathrm{had}}_{N_{\mathrm{qg}}} = q P^{\mathrm{qg}}_{N_{\mathrm{qg}}} \, . \tag{27}$$

Here N_{bin} is the number of observed central Au + Au collisions for which the central multiplicity of the bin is N_{qg}. The right–hand side of eq. (27) is the magnitude of the rare events to the overall multiplicity, while the left hand side gives the statistical resolution. The assumption here is that q is small, so that at N_{qg} we can use $P_n \sim P^{\mathrm{had}}_n$ for the left–hand side.

To obtain a feeling for the shape and applicability of eqs. (26) and (27) we plot in Figures 10 and 11 different negatively charged particle multiplicity distributions as might be expected for central Au + Au collisions at AGS with $E_{\mathrm{beam}} = 11.6$ GeV/A. From [9] we obtained the mean for purely hadronic events to be $N_{\mathrm{had}} = 145$. This is slightly larger than the value $N_{\mathrm{had}} = 131 \pm 21$ cited in [9] for 355 ± 7 participating nucleons since we are assuming that all $2A$ nucleons are participating in the collision. The result depicted in Figure 8 for the upper limit on the additional number of negatively charged pions produced per participating baryon allows us to deduce an upper limit of $N_{\mathrm{qg}} = 193$ on the mean for the events with quark–gluon plasma production. In Figure 10 we plot the negatively charged particle multiplicity distribution defined in eq. (25) for different values of the probability q. We use Poisson distributions for P^{had} and P^{qg} and take the upper limit for rare events $N_{\mathrm{qg}} = 193$ as the mean for P^{qg}. A shoulder develops for small q and becomes more pronounced the larger q is. In Figure 11 we fix $q = 0.1$ and investigate the effect of different values of the mean N_{qg} of the distribution for events with some quark–gluon plasma production. If this mean is close to the mean of purely hadronic events we will only find some broadening of the overall distribution. This would be the case if the phase transition is weakly first order or second order. For larger N_{qg} we begin to see a well established shoulder develop. For large N_{qg} a second maximum appears.

It is clear that the exact values of the probability q and of the mean N_{qg} of rare events will be crucial for the experimental observation of a phase transition. We have provided a first glimpse into this problem, but in the end it is up to experiment to discover new physics in multiplicity distributions at the AGS.

12

6 Summary and Conclusion

We have estimated the probability that hard nucleon-nucleon collisions initiate the formation of seeds of quark-gluon plasma at AGS energies. Based on our previous studies we know that these will grow rapidly to convert most of the superheated hadronic matter to quark-gluon plasma. Our estimates are based on reasonable assumptions and approximations to the kinetic theory of hadronic physics. Better estimates could be made using event simulators like RQMD and ARC together with more detailed knowledge of multi-particle production in nucleon-nucleon collisions. We find that anywhere from 1% to 100% of central Au + Au collisions should lead to significant quark-gluon plasma formation. A major assumption is that there is a phase transition and that it is first order.

We have already proposed that the formation of plasma in rare events should have an observable consequence for hadron interferometry, deuteron production, and the meson multiplicity distribution. In this paper we have studied the effect on the multiplicity distribution. It would be observable as a shoulder or second maximum at some multiplicity higher than the most probable one. If there is a phase transition but it is second order or weakly first order then the effect will be much more difficult to see. We eagerly await the results of experiments.

Acknowledgements

References

[1] H. Sorge, H. Stöcker and W. Greiner, Annals Phys. (NY) **192**, 266 (1989); R. Mattiello, H. Sorge, H. Stöcker and W. Greiner, Phys. Rev. Lett. **63**, 1459 (1989); H. Sorge, A. V. Keitz, R. Mattiello, H. Stöcker and W. Greiner, Phys. Lett. **243B**, 7 (1990); A. V. Kreitz, L. Winckelmann, A. Jahns, H. Sorge, H. Stöcker and W. Greiner, Phys. Lett. **263B**, 353 (1991).

[2] Y. Pang, T. J. Schlagel and S. H. Kahana, Phys. Rev. Lett. **68**, 2743 (1992); T. J. Schlagel, Y. Pang and S. H. Kahana, Phys. Rev. Lett. **69**, 3290 (1992); S. H. Kahana, Y. Pang, T. J. Schlagel and C. Dover, Phys. Rev. C **47**, R1356 (1993).

[3] Y. Pang, proceedings of *Quark Matter '95* (Monterey, CA, Jan. 1995) to appear.

[4] J. I. Kapusta, A. P. Vischer and R. Venugopalan, Phys. Rev. C **51**, 901 (1995).

[5] R. C. Hwa, Phys. Rev. Lett. **52**, 492 (1984).

[6] L. P. Csernai and J. I. Kapusta Phys. Rev. D **29**, 2664 (1984); ibid. **31**, 2795 (1985).

[7] There is always some uncertainty in how much of the $n = 0$ term in the binomial is to be associated with diffractive and elastic events; this affects the exact measure of σ_{in} as used in eq. (15). These uncertainties are relatively minor for us.

[8] A. Baldini, V. Flaminio, W. G. Moorhead and D. R. O. Morrison, in *Landolt–Börnstein*, New Series I/12b, Springer, 1988, pp. 149–180.

[9] M. Gaździcki and D. Röhrich, Z. Phys. C **65**, 215 (1995).

[10] E. Fermi, Prog. Theor. Phys. **5**, 570 (1950).

[11] J. I. Kapusta, Phys. Lett. **143B**, 233 (1984).

[12] P. J. Siemens and J. I. Kapusta, Phys. Rev. Lett. **43**, 1486 (1979).

[13] S. Pratt, Phys. Rev. D **33**, 1314 (1986).

[14] UA5 Collaboration, Z. Phys. C **43**, 357 (1989).

[15] C. Fuglesang, for the UA5 Collaboration: "UA5 Multiplicity Distributions and Fits of Various Functions", proceedings of *Multiparticle Dynamics: Festschrift for Leon van Hove* (La Thuile, Valle d'Aosta, Italy, March 1989) eds. A Giovannini and W. Kittel, World Scientific, 1990.

[16] F. Rimondi, for the CDF Collaboration: "Multiplicity Distributions in $\bar{p}p$ Interactions at $\sqrt{s} = 1800$ GeV", proceedings of the *XXIII International Symposium on Multiparticle Dynamics* (Aspen, CO, Sept. 1993) eds. M. M. Block and A. R. White, World Scientific, 1994.

[17] C. S. Lindsey, for the E735 Collaboration, Nuc. Phys. **A544**, 343c (1992).

Figure Captions

Figure 1: Schematic of a central collision between two nuclei. Nucleus 1 is incident from the left and nucleus 2 is incident from the right. The shaded area is the hot and dense overlap zone which is expanding along the beam axis with the original beam velocity v. A hard nucleon-nucleon collision leading to a large energy release, or star burst, is indicated at the longitudinal position z.

Figure 2: The density of hadrons n_{tot} in the hot and dense overlap zone as a function of laboratory beam energy E_{beam}.

Figure 3: The pion production cross sections σ_n versus energy s as computed according to the text. Note that they are averaged over initial state isospin.

Figure 4: Distribution dN/ds in s of hard nucleon-nucleon collisions taking place in the hot zone. The value of w is 0.5 (4a) and 0.8 (4b).

Figure 5: Number of hard nucleon-nucleon collisions N_n leading to a particular final state pion multiplicity n.

Figure 6: Number of hard nucleon-nucleon collisions $N_>$ with at least n_{crit} pions produced.

Figure 7: Ratio of entropy to baryon number S/B for fixed beam energy.

Figure 8: Upper limit on the additional number of negative pions produced per participating baryon $\Delta N_-/B$ in central Au + Au collisions as a function of beam energy.

Figure 9: Charged particle multiplicity distribution for $\bar{p}p$ collisions at $\sqrt{s} = 900$ GeV. The parameters for the fits are taken from [15] with a probability of the second, high multiplicity, component being 0.35.

Figure 10: Negatively charged particle multiplicity distribution for central Au+ Au collisions at $E_{beam} = 11.6$ GeV/A for different values of the probability q. The mean for purely hadronic events is taken to be $N_{had} = 145$ while the mean for events with quark–gluon plasma production is taken to be $N_{qg} = 193$.

Figure 11: Negatively charged particle multiplicity distribution for central Au + Au collisions at $E_{\mathrm{beam}} = 11.6$ GeV/A for different values of the mean N_{qg} for rare events. The probability is fixed at $q = 0.1$ and the hadronic mean multiplicity is fixed at $N_{\mathrm{had}} = 145$.