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Pure electroweak mechanism for the electric dipole moment of neutron in the Kobayashi-Maskawa model.

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Abstract

The pure electroweak three-loop mechanism for the induced electric and chromo-electric dipole moments of quarks is studied in the Kobayashi-Maskawa model with three and four generations. In the standard three generation case, this mechanism is found to produce a negligible contribution to the electric dipole moment of neutron. In the presence of the fourth heavy generation, however, pure electroweak corrections are important and might be several times larger than the corresponding QCD contribution for the masses of heaviest quarks $\sim$ 500-600 GeV. The resulting electric dipole moment of neutron naturally arises at the level of $10^{-29}\, e \cdot cm$. The effects of the fourth generation physics are parametrized at standard electroweak scale by the presence of the effective charged right-handed currents.

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1 Introduction

In this letter, we consider the pure electroweak three loop contribution to the neutron electric dipole moment in the Kobayashi-Maskawa (KM) model with four generations. Besides its interesting predictions for B-meson physics [1], the enlarged variant of the KM model leads to the enhancement of the neutron electric dipole moment (EDM) in comparison with Standard Model case [2]. This enhancement is linked with the short distance contribution to the electric and chromoelectric dipole moments (CEDM) of quarks.

The origin of CP-violation in the Kobayashi-Maskawa (KM) model resides in the complexity of the quark mixing matrix. This requires four semi-weak vertices to generate a flavour-diagonal CP-odd amplitude. It was shown that the EDMs and CEDMs of quarks cannot be generated at the lowest possible two-loop order [3]. Thus, the QCD corrections are brought in to prevent these quantities from the identical cancellation. The detailed study of these operators at three loop order (two electroweak plus one gluonic) was done in SM by Khriplovich [4] and by us for its four generation modification [2].

Here, we replace the hard gluon loop by another one of electroweak in the EDM inducing graphs. Specifically, we concentrate on the large renormalization factor of the axial coupling of Z-boson with fermion proportional to m_t^2 in SM and its relevance for the induced EDM in the presence of an extra heavy generation of quarks [5].

The purpose of this work is to compute EDMs and CEDMs at three loop electroweak order and then compare our results with corresponding QCD induced values [2, 4].

2 Electroweak corrections to EDM

At first glance the problem of this calculation appears to be very complicated. However, taking into account the explicit mass hierarchy in this problem, we reduce the three-loop calculation to one-loop integrals. We assume that:

$$m_h^2;\ m_g^2 \gg m_t^2 \gg m_w^2 \gg m_i^2, \tag{1}$$

where we have denoted the heaviest quarks as h and g; i represents the standard set of "light" flavours: u, d, s, c and b. The first inequality is assumed in order to single out parametrically the contributions proportional to m_h^2 or m_g^2.

The typical representatives of the diagrams to be calculated are depicted in Fig. 1. The solid line represents the fermions; wavy lines are charged electroweak bosons and the dashed line are the neutral ones. The position of an external photon or gluon is not indicated here.

In the following calculation, we consider the EDM and CEDM operators of the strange quark. Then the arrangement of flavours along the fermion line is determined in SM uniquely by [4]:

$$i\mathrm{Im}(V_{ts}^* V_{td} V_{cd}^* V_{cs}) s[t(b-d)u - u(b-d)t + u(b-d)c - c(b-d)u + c(b-d)t - t(b-d)c]s. \tag{2}$$

The enlarged KM matrix possesses in general three independent CP-odd invariants, one of which being distinguished by the dynamical enhancement [2]:

$$i\mathrm{Im}(V_{ts}^{*}V_{tb}V_{hb}^{*}V_{hs})s[t(b-g)h-h(b-g)t+U(b-g)t-t(b-g)U+h(b-g)U-U(b-g)h]s. \quad (3)$$

The capital U here denotes the propagation of u and c quarks which we are free to regard massless and degenerate inside the loops. This degeneracy is the factor which leads to the identical cancellation of the amplitude (2) in the SM. Therefore (2) is forced to be proportional to m_c^2 whereas (3) is determined by heavy mass scale like m_t^2 [2]. This is the main source of the EDM enhancement in the KM model with four generations. The additional source of the enhancement probably lies in the numerical significance of the CP-odd combination $\mathrm{Im}(V_{ts}^{*}V_{tb}V_{hb}^{*}V_{hs})$ which could naturally reach the level of λ^5, where λ is the Wolfenstein parameter [1, 2].

We begin from the shortest distances and calculate first the effective one-loop induced flavour changing neutral currents "electroweak penguin", which is well known from the kaon physics:

$$\mathcal{L}^{(1)}=\sum_{i,j,f}a_jV_{ij}V_{fj}^{*}\bar{q}_i\gamma_\mu(1-\gamma_5)q_fZ_\mu \quad (4)$$

To a good accuracy, it is sufficient to take only leading contribution to the coefficients a_j which are proportional to the square of the heaviest masses:

$$\mathcal{L}^{(1)}=\frac{g_w}{\cos\theta_W}\frac{\alpha_w}{16\pi}Z_\mu\left[\frac{m_h^2}{m_w^2}\sum_{i,f}V_{hi}V_{hf}^{*}\bar{q}_f\gamma_\mu\frac{(1-\gamma_5)}{2}q_i-\frac{m_g^2}{m_w^2}\sum_{i',f'}V_{i'g}V_{f'g}^{*}\bar{q}_{f'}\gamma_\mu\frac{(1-\gamma_5)}{2}q_{i'}\right], \quad (5)$$

in analogy to the SM expression with dependence on m_t^2. The heavy mass dependence here originates from the longitudinal part of W-boson propagator or from the equivalent graph with charged non-physical higgs boson [5]. Taking the new effective vertex generated by (5), we reduce the remaining computation to the one presented in Fig. 2.

The second step is to integrate out the neutral boson, which could be done along the same line. At this point, however, it is useful to treat the three and four generation models separately.

In the four generation case, the characteristic loop momenta are large, which allow us again to take only the longitudinal part of the Z-boson propagator. This leads us to obtain effective charged right handed currents in an easy way. Taking into account the flavour structure in (3), we obtain the effective lagrangian for the s - t transition:

$$\mathcal{L}^{(2)}\simeq-\frac{g_w}{\sqrt{2}}V_{tb}V_{hb}^{*}V_{hs}(\frac{\alpha_w}{16\pi})^2\frac{m_tm_s}{m_w^2}\frac{(m_h^2+m_g^2)}{m_w^2}\log(\frac{m_{h(g)}^2}{m_t^2})\bar{q}_t\gamma_\mu\frac{(1+\gamma_5)}{2}q_sW_\mu^{+}+(h.c.), \quad (6)$$

where we have omitted all constants in comparison with "large" logarithmic factors. This logarithmic accuracy is motivated by theoretical considerations. However, in the final numerical result, we set all logarithms to unity. This allows us to avoid the problem of a true two-loop calculation which is not reducible to two independent integrations. It is important to remember that the vertices (5) and (6) are indeed effective and do not survive if the incoming momenta are larger than all masses of particles flowing inside the loops. For this reason, the upper limit for logarithmic integral over the loop momentum coincides with m_h if $m_h < m_g$ and with m_g if $m_g < m_h$. The analysis of the precision electroweak data suggests that h and g quarks must be sufficiently degenerate in masses. From here to below, we put $m_g \simeq m_h$. It is worth to note also the constructive interference between the two terms in (6) proportional to m_g^2 and m_h^2, in contrary to the mass dependence of the electroweak parameter ρ [5]. The contribution of the fourth generation to ρ vanishes at $m_h = m_g$.

The situation is quite different in the SM. Due to the presence of right-handed currents in the s-c or d-c transition, those transition amplitudes are suppressed not only by $m_{s(d)}$ and m_c but also by the factor m_b^2/m_Z^2 reflecting the GIM property of (2). Taking into account that the result for EDM of s or d quark must be proportional to m_c^2, although the interchange of a gluon loop by an electroweak one replaces $\alpha_s(q^2 \simeq m_b^2)$ by $\alpha_w \frac{m_t^2 m_b^2}{m_w^4}$, we deduce that the total effect is negligibly small.

The presence of right-handed currents itself does not necessarily imply CP-violation. The latter arises through the complex phase of $V_L V_R^\dagger$, the product of right- and left-handed KM matrices,

$$\mathcal{L} = \frac{g_w}{2\sqrt{2}} W_\mu^+ \sum_{f,i} [\bar{q}_f (V_L)_{fi} \gamma_\mu (1-\gamma_5) q_i + \bar{q}_f (V_R)_{fi} \gamma_\mu (1+\gamma_5) q_i] + (h.c.), \tag{7}$$

and is known to exist even in two generations. This is exactly what happens at the last stage of our calculation. EDM and CEDM of s-quark results from the mixing between second and third generations in the presence of right-handed currents given by (6) depicted in Fig. 3. The results are:

$$d_s = -\frac{5e}{3} \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) (\frac{\alpha_w}{4\pi})^2 \frac{1}{16\pi^2} \frac{G_F}{\sqrt{2}} m_s \frac{m_t^2 m_h^2}{4 m_w^4} \log(\frac{m_h^2}{m_t^2}) \tag{8}$$

$$\tilde{d}_s = -g_s \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) (\frac{\alpha_w}{4\pi})^2 \frac{1}{16\pi^2} \frac{G_F}{\sqrt{2}} m_s \frac{m_t^2 m_h^2}{4 m_w^4} \log(\frac{m_h^2}{m_t^2}), \tag{9}$$

where d_s and $\tilde{d}_s$ are determined as the coefficients in front of $\frac{1}{2}\bar{q}(F\sigma)\gamma_5 q$ and $\frac{1}{2}\bar{q}t^a(G^a\sigma)\gamma_5 q$ respectively. Due to the inequality (1), we have taken into account only the longitudinal part of W-propagator. The use of longitudinal parts of gauge boson propagators throughout the calculation of EDM in four generation case maximizes the size of CP-violation. This also means that the obtained result arises entirely from the Higgs sector phenomenon. Then, the whole calculation could be performed in the t'Hooft-Feynman gauge and therefore only scalar bosons should be taken into account. Thus, we can rewrite the results given in (9) and (8) in terms of Yukawa couplings $f_i = m_i/v$ of heavy quarks:

$$\frac{\tilde{d}_s}{g_s} = \frac{3 d_s}{5e} = -\mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) \frac{1}{1024\pi^6} \frac{G_F}{\sqrt{2}} m_s f_t^2 f_h^2 \log(\frac{f_h^2}{f_t^2}), \tag{10}$$

where $v = 246\text{GeV}$ is the vacuum expectation value of the scalar field.

The extraction of the EDM of neutron resulting from the effective interaction (9) depends on our understanding of low-energy hadronic physics. Based on the chiral perturbation estimation proposed in [6], we find the EDM of neutron at the level:

$$d_N \sim e \mathrm{Im}(V_{ts}^* V_{tb} V_{hb}^* V_{hs}) f_t^2 f_h^2 \cdot 2 \cdot 10^{-26}\ cm \tag{11}$$

The comparison of (11) with the corresponding QCD-induced contribution to EDM obtained earlier in Ref.[2] shows, in principle, the same order of magnitude for both results. In the most optimistic scenario about the values of CP-odd phase invariant, combined with the masses of heavy quarks around 500GeV, we obtain the neutron EDM to arise at the level:

$$d_N \sim 10^{-29}\ e \cdot cm. \tag{12}$$

The main source of the numerical smallness is connected with the tiny numerical coefficient in expression (10).

3 Discussions and Conclusions

We would like to point out that there exists another contribution to the EDM of neutron from the physical Higgs boson loop which should be taken into account as well. One may undertake that calculation along the same line and obtain the effective right-handed currents as well. This means that the value of EDM depends not only on unknown m_g and m_h but also on the mass of real Higgs boson. To our logarithmic accuracy, these corrections are unimportant if we believe that m_{Higgs} is also very large, somewhere around $m_{h(g)}$.

The last diagram which could contribute to EDM at this order is the "rainbow" graph, Fig. 4, where all wavy lines are W-bosons. It could be checked, however, that even if this diagram provides any nonvanishing contribution, it cannot generate an $m^2_{h(g)}$-dependence in the result.

Our estimate shows that the electroweak contributions to the EDM of neutron are comparable with QCD ones in the case of four generations and negligibly small in SM. The QCD effects dominate over pure electroweak contribution at $m_{h(g)} \sim m_t$, whereas at $m_{h(g)} \sim 500 - 600$GeV the latter dominates. The numerical result (12) then could be regarded as the maximal value of EDM which can be obtained through the Kobayashi-Maskawa type of CP-violation in the presence of an additional heavy generation. Despite the significant enhancement in comparison with SM case, it is too low to be detected even at the future generation of experiments aiming at the search of EDM. This implies also that the reliable information and limits on the parameter of the model with four generations may come only from the analysis of the electroweak precision data and K, B-meson physics [1]. The latter case requires a further analysis on the role of large electroweak radiative corrections.

As a concluding remark, we have emphasized the main mechanism for the induced EDM of neutron in the hypothetical case of two or more additional heavy generations, (h, g); (h', g');..., with large mixing between them. Then the maximum of CP-violation at low energies comes from Weinberg operator [7] which is known to exist in KM model at the lowest possible three-loop order [8]. In the four generation case, this operator is suppressed by the factor m_b^2/m_w^2 [2], which disappears at five or more generations. At the same time, there is no severe limits on the mixing between heavy generations from the low energy phenomenological data and one may expect the corresponding CP-odd invariant, $\mathrm{Im}(V^*_{tg'}V_{tb}V^*_{hb}V_{hg'})$ to be large.

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