Phase structure and confinement properties of noncompact gauge theories I

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Abstract

In the context of reviewing noncompact lattice gauge models at zero and finite temperature we study in detail a contribution of the invariant measure and the time-like plaquette configurations to correlation functions, analyze the problem of the compactness of the potentials in respect to the confinement and indicate the essential features to deal with the Wilson gauge theory in the weak coupling region. A method for calculating an effective confining noncompact model is also proposed.

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1 Introduction

Since K. Wilson proposed [1] twenty years ago to quantize the field theory on a lattice in the Euclidean space-time with an exact gauge invariance in order to make the strong coupling calculations, the lattice approach combined especially with the numerical Monte Carlo simulations has provided a huge progress of the quantum chromodynamics (QCD). However, the problem of a confinement mechanism did not become less intriguing because a conceptually simple mechanism in strong coupling regime could not proceed along the same line to the continuum theory. The principal difficulties here is to define the proper configurations (monopoles, vortices,...) of the compact lattice gauge fields which are the most essential ones for forming the confining forces, and how one may identify them in the continuum theory?

In this paper we make an attempt to advance this question again constructing an effective noncompact model starting from the lattice gauge theory (LGT) and aiming to analyze the weak coupling region. Generally speaking one should presuppose the existence of the only mechanism of confinement both on the lattice and in the continuum, in order to address this problem unambiguously. Because it is not obvious, one must argue the proffered statement. We shall come back to this point later, accepting this as conjecture for now. Besides, we need to develop an ingenious approach to construct an adequate quantum theory of noncompact potentials starting from compact LGT.

Seems, such a possibility does exist. Exploring the lattice models which are analytically solvable in a sense and exhibit the confinement property, we could identify the important configurations and construct their correct noncompact limit (of course, if it exists). This is the principal strategy of the following paper. As the first step, we elaborate the chromoelectric part of the Wilson action (WA) which states in this approach the existence of confining forces in the low temperature phase. However, the quantum noncompact lattice theory based on the naive limit [3] of the chromoelectric part of WA does not possess this property compelling to analyze a nonperturbative limit of the model. (One may worry at this point that dealing with the chromoelectric part of the WA we are trapped by the strong coupling region which could be far away from the continuum limit. At this stage, however, our goal is to find a noncompact *lattice* theory which would belong to the same universality class as the compact LGT. Continuum limit can be accomplished after including the chromomagnetic part of WA in our scheme).

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[3] Throughout this paper we use the following limits of the lattice theory: 1) naive limit means the expansion of lattice gauge field matrices around the unit matrix and leads to the classical (lattice or continuum) Yang-Mills action; 2) perturbative limit of Wilson LGT is taken as a limit of vanishing lattice spacing together with small coupling constant expansion and conventional renormalization procedure done. This limit coincides with the corresponding perturbative expansion of the continuum theory; 3) nonperturbative limit is defined as a limit $a \to 0$ together with a proper renormalization procedure after integrating the lattice partition function over configurations of the gauge fields which are far from the unit matrix and hence are missed in the perturbative expansion; 4) mentioning a noncompact (perturbative or nonperturbative) limit we mean an effective action in terms of noncompact gauge potentials obtained in the limit of a small coupling constant after a partial summation over compact gauge fields.

The strong confirmation that in such a way we may achieve a desirable effective noncompact model (and even to find its continuum quantum limit) one finds in [2, 3], where it was shown that confinement can be obtained in the effective model for A_0 gauge field only and, in the first approximation, we may set space-gauge fields A_n equal to zero. In fact, the effective action given in Refs. [2, 3] shares the same important features as an effective theory for the time-component of gauge field A_0, which is calculated from the chromoelectric lattice action.

Another problem closely connected with forementioned is the deconfinement phase transition, which takes place in compact lattice theory at finite temperature. As is known, the chromoelectric part of the compact action is well indicative again, exhibiting the deconfinement.

Advertising the worthwhile results of our investigations, we would like to mention a construction of a nonperturbative noncompact limit of the Wison model in the weak coupling region and evaluation of the corresponding effective model. What we obtained differs from the naive noncompact generalization of the Yang-Mills theory because it includes $Z(N)$ symmetry of WA and an influence of the invariant measure (as specified below). We demonstrate that the mechanism of confinement in the model developed is essentially the same, as that in the initial compact theory and the new ingredients of noncompact formulation are playing the crucial role to have confinement available. A string tension is evaluated in the model and a generalization to include the chromo-magnetic part of WA is argued. Certainly, it does not solve the confinement problem but permits to have a noncompact formulation on the same footing as compact LGT.

We are going to present these results in two articles. The present paper is organized as follows. In sect. 2 we remind briefly the decisive features of the compact LGT in the strong coupling limit. Sect. 3 is devoted to discussion of the noncompact lattice models and their connection, both with the compact ones and with the continuum Yang-Mills theory. We construct and analyze noncompact model with compact A_0 integration in sect. 4. In sect. 5 we discuss an effective way to include an invariant group measure into noncompact models. We close in sect. 6 with a discussion of compactness problem of potentials and give simple examples how compactification could lead to the linear potential between probe charges. On the other hand we claim it is quite enough in some models with compact variables to perform noncompact Gaussian integration only over dominating configurations to achieve linear potential. The main issue of this discussion is a demonstration of how noncompact theory can confine in the same way that compact theory does.

2 The essentials of strong coupling compact LGT

Let us consider compact $SU(N)$ and $SU(N)/Z(N)$ gauge theories on the lattice. The Wilson formulation of LGT has the following form [1]

$$Z = \int D\mu(U) \exp(\lambda \sum_p \Omega(\partial p)), \qquad (1)$$

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where Ω is a character of the fundamental representation of a compact Lie group G, $D\mu(U)$ is the invariant integration measure and $\lambda = \frac{2N_c}{g^2}$. We would like to recall now some properties of the Wilson LGT which will be essential here. The majority of exact analytical results obtained by studying the theory (1) were achieved by strong coupling expansion. These results in respect to confining properties are usually related to the Wilson criterion of confinement, expressed by the area law for the Wilson loops, which are non-trivial on the $Z(N)$ subgroup [1]: If the Wilson loop $\Omega_\nu(\partial C) = Tr \prod_{l \in C} U_\nu(l)$ obeys the area behaviour

$$< \Omega_\nu(\partial C) > \leq K_0 \exp(-K_1 area(C)), \qquad (2)$$

the static colour charges in representation containing $Z(N)$ will be confined. It was proved in [4], that $< \Omega_\nu(\partial C) >$ shows the area behaviour in a region of a convergence of the strong coupling expansion for the $SU(N)$ gauge group. At the same time, the Wilson loop in the adjoint representation obeys the perimeter law [4]. What are the mechanisms of the such behaviour? Two possibilities mainly dominate through the discussions - monopole condensation and vortex condensation.

Here we are sticking to the opinion that the status of the vortex condensation mechanism is better analytically founded, at least in the strong coupling region thus preferring the special $Z(N)$ configurations contributing to path integral to provide a confinement. In [5] a sufficient condition for confinement by $Z(N)$ vortex condensation was derived. $Z(N)$ vortices there take a special form of $Z(N)$ singular transformations performed over a two-dimensional closed surface, and their condensation means that they must become "fat" in a certain way. A direct calculation up to very high orders in $\lambda \sim g^{-2}$ confirms the expected behaviour of the condensate in pure $SU(2)$ gluodynamics according to the mentioned theorem [5, 6]. It was also proven [6], that a coefficient at the area law for the vortex free energy exactly equals the string tension. Indeed, the string tension (coefficient K_1 in (2)) calculated from the vortex condensate is in accordance with MC data in the region of the strong and the intermediate coupling. The evaluation of the vortex condensate in the weak coupling region can be found in [7]. A crucial feature of this mechanism is the breakdown of the $SU(N)$ local gauge symmetry up to its $Z(N)$ local subgroup [8]. This dynamical Higgs mechanism leads to the long-interacting forces between colour charges, disordered behaviour of the Wilson loop and screening of all the gluonic states.

Believing in this strong coupling confinement picture and taking into account that there is no phase transition at zero temperature in $SU(2)$ and $SU(3)$ gauge theories, one may hope that this mechanism would persist in the continuum theory as well, if $Z(N)$ configurations survive the transition to the weak coupling regime. Certainly, it is not the case at the naive continuum limit and at the continuum limit of perturbative expansion. Therefore, the nonperturbative limit must be studied. It has been demonstrated in [9] that a nonperturbative continuum limit of the $SU(N)$ LGT contains $Z(N)$-vortices already in a bare Lagrangian. It is interesting to note that the monopole configurations do not contribute to such not naive continuum limit [9]. These facts are heuristically important though the theory having been exposed in [9]

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is different from the conventional Yang-Mills one (see sect. 6 for more discussion of this point). Usual objection against this $Z(N)$ confinement mechanism comes from the observation that the Wilson loop in the adjoint representation shows presumably the area law behaviour in the limit $N_c \to \infty$ (N_c is a number of colours) despite this representation does not feel $Z(N)$ variables. This objection has been discussed in [9, 10] and we refer the interested readers to that discussion. It is worth mentioning that, in fact, the monopole mechanism of confinement runs into similar problems as well [11]; we do not know whether there exist any answers to the questions put forth in the paper [11] (this remark concerns only abelian projected monopoles; there exists another mechanism of confinement by $SU(N)/Z(N)$ dynamical monopoles, see [12]). Anyway, we will not specify $Z(N)$ configurations in what follows, so that only their presence is important.

Going to display the phase structure of $SU(N)/Z(N)$ compact LGT, one should remember that Lagrangian ? includes the adjoint characters $\Omega(\partial p)$ [8]. The fundamental Wilson loop equals zero [8], implying the so-called "superconfinement" of the static charges. There is a phase transition at a critical coupling constant presumably of the first order, related to the condensation of the $Z(N)$ monopoles [14] in this theory. The mixed theory $S = \lambda_1 \Omega^{adj} + \lambda_2 \Omega^{fun}$ leads to the picture of two phases existing at zero temperature: the confining phase with the area law behaviour, and the deconfining one. Due to the absence of $Z(N)$ configurations in the bare Lagrangian of $SU(N)/Z(N)$ LGT, we cannot determine vortex potentials and consequently, condensates as well, at least in the same manner. Certainly, they are absent at the level of bare Lagrangian. A similar situation takes place in the positive plaquette model [15] which eliminates all thin $Z(N)$ vortices from the standard $SU(2)$ LGT. MC-simulations indicate that string tension in this model is much less than in the standard $SU(2)$. Since the Lie algebras of $SU(N), SU(N)/Z(N)$ and positive plaquette model are the same, the naive continuum limits of these models are the same as well, and equal the Yang-Mills action. This means that $Z(N)$ configurations disappear from $SU(N)$ in this limit.

Let us, therefore, display the role of the $Z(N)$ subgroup in the phase structure of the compact $SU(N)$ model at a finite temperature. We would also like to pay an attention to the chromoelectric part of the model in this example. The partition function at a finite temperature

$$Z = \int D\mu(U_n)D\mu(U_0) \exp(\lambda \sum_p \Omega(\partial p) + \lambda_0 \sum_{p_0} \Omega(\partial p_0)), \qquad (3)$$

where $p_0, (p)$ are time-like (space-like) plaquettes and

$$\lambda_0 = \xi \frac{2N_c}{g^2}, \ \lambda = \xi^{-1} \frac{2N_c}{g^2}, \ \xi = \frac{a_\sigma}{a_t}, \qquad (4)$$

is calculated at the following boundary conditions:

$$U_\mu(x,t) = U_\mu(x,t+N_t). \qquad (5)$$

These conditions (5) generate new physical degrees of freedom which can be taken as the eigenvalues of the Polyakov loop [16]

$$W_x = P \prod_{t=1}^{N_t} U_0(x, t). \tag{6}$$

The compactness in the temporal direction leads to a $Z(N)$ global symmetry of the model. This means, multiplication of all links in the time direction in the three dimensional x, y, z-torus by a $Z(N)$ element does not change the action, though a single Polyakov loop transforms as

$$W_x \longrightarrow z W_x, \ z \in Z(N). \tag{7}$$

Thus, an expectation value of the Polyakov loop can be used as an order parameter to measure a spontaneous breaking of the $Z(N)$ symmetry. The corresponding phase transition is well-known as the deconfining one [17], and in the high-temperature phase the $Z(N)$ symmetry was spontaneously broken [18] (see, however, [19]).

What is the role of $Z(N)$ configurations at a finite temperature? Let us sketch now some recognized results obtained from the chromoelectric part of the action (3), i.e. the term $\lambda_0 \sum_{p_0} \Omega(\partial p_0)$ (to avoid possible confusion we would like to stress that we are dealing with the Euclidean formulation of LGT; "chromoelectric part" of the action means in this case time-like plaquettes). Fixing the temporal gauge $\partial_0 A_0 = 0$ and performing the integration over the space gauge fields $U_n(x, t)$ we come to the partition function of the form in the limit $a_t \rightarrow 0$

$$Z = \int \prod_x D\mu(W_x) \prod_{x,n} \sum_l \exp(-\gamma C_2(l)) \Omega_l(W(x)) \Omega_l^*(W(x+n)), \tag{8}$$

where $\gamma = g^2 (2 T a_\sigma)^{-1}$, Ω_l is the character of the l-th irreducible representation of $SU(N)$ and $T = N_t a_t$ is the temperature. The partition function (8) has been studied in numerous articles through different approaches including MC-simulations. There is a phase transition of the second order for $SU(2)$ and of the first order for $SU(3)$ at some critical value $N_t \lambda_0^{-1}$. The expectation value of the fundamental character behaves according to

$$\langle \{N^{-1} Sp W_x\} \rangle = \begin{cases} 0, \ T < T_c^D, \ confinement \ phase, \\ z * f(T), \ T > T_c^D, \ z \in Z(N), \ f(T) \leq 1, \\ \qquad deconfinement \ phase. \end{cases} \tag{9}$$

Let us limit ourselves to $Z(N)$ subgroup in (8):

$$\int D\mu(W) \quad \rightarrow \quad \frac{1}{N} \sum_{z \in Z(N)},$$
$$\Omega_l(W) \quad \rightarrow \quad z d_l, \ if \ \Omega_l \rightarrow z \Omega_l,$$
$$\Omega_l(W) \quad \rightarrow \quad d_l, \ if \ \Omega \ is \ invariant \ under \ Z(N), \tag{10}$$

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where d_l is the dimension of l-th representation. The resulting model has the same qualitative features as the initial one in (8). Both deconfinement phase transition and confinement take place. If we consider expansion for W_x around the unit matrix (as it is made at the naive continuum limit), we will lose this phase structure, because the system stays in one of the minima of the $Z(N)$-broken deconfined phase. Let us briefly summarize. $Z(N)$ configurations might play the crucial role in the confinement mechanism. It follows, if one wants to construct a noncompact theory which could display this confinement mechanism it will be necessary to include these configurations in such a theory. In fact, it is the old problem. There were some attempts to implement $Z(N)$ configurations into the Yang-Mills theory (see, for instance, [9] and references therein). We would like to use an other method described in the following text.

3 The essentials of noncompact LGT

In order to show the principal difference between noncompact gauge theories and compact ones, we shall briefly point out some aspects of noncompact Yang-Mills theories on the lattice. They were introduced first in [20] and studied intensively in [21, 22, 23, 24] (see also references in [24]). The basic element is the gauge potential $A_\mu = A_\mu^a t^a$, where t^a are generators of the $SU(N)$ group. The derivatives are represented by the finite-difference form

$$\partial_\mu f \rightarrow \frac{1}{a}[f(x + \mu) - f(x)], \qquad (11)$$

and integrals over the four-dimensional space are changed into sums over all lattice sites. The path integral is defined as the integral over all noncompact gauge fields $A_\mu(x)$ calculated in each lattice site. The partition function is, therefore, defined by the relation:

$$Z = \int \prod_{x,\mu} dA_\mu(x) exp[-a^4 \sum_x \sum_{\mu,\nu} (F_{\mu,\nu})^2]. \qquad (12)$$

The gauge invariance in these models is explicitly broken. Hence, the gauge fixing mechanism represented by the Faddeev-Popov ansatz cannot be directly simulated by the Monte-Carlo process. The method based on simulation of a diffusion equation was proposed [21]. Gauge fixing is assured by introducing of the local gauge fixing force to the diffusion equation, tangent to the gauge orbit. Expectation values of gauge invariant quantities should be independent of such forces. Despite the explicit breakdown of the gauge invariance, in the limit $g^2 \rightarrow 0$ the asymptotic freedom is presented [20, 21]. One may suggest that the breakdown of the gauge invariance on the level of the bare Lagrangian is not of crucial importance due to restoration of the gauge symmetry in the expected region $a \rightarrow 0$ of the quantum theory, since the terms that caused the breakdown are proportional to the lattice spacing. The main contribution to the path integral results from a compact region defined by a gauge condition (local gauge force) [21]. No evidence for confinement was found. The string tension vanished even at very strong values of the coupling constant. The Wilson loop obeyed the perimeter law [20, 21, 22]. The expectation value of the Polyakov loop

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always differs from zero. Similar behaviour was also observed in the theory with the gauge $A_0 = 0$ [20]. From the point of view of these facts, noncompact gauge theories resemble $SU(N)/Z(N)$ compact ones in the weak coupling region, rather than $SU(N)$.

Some attempts to find a proper solution were connected to the fact that the explicit violation of the gauge symmetry is the reason for the absence of the confining forces [23, 24]. However, this opinion does not look to be well motivated. As we pointed out earlier, in the quantum noncompact theory the asymptotic freedom is observed and the contribution to the path integral results from the compact region. Thus, there is, actually, no reason to believe that the gauge invariance is not restored in the quantum theory after taking the limit $a \to 0$. One more argument comes from the consideration of the finite temperature behaviour of the noncompact model (see section 4 of the present paper). If we believe in a common mechanism of the confinement in the compact lattice gauge theory and in the noncompact gauge theory then we should expect similar behaviour of the Wilson (Polyakov) loops in the noncompact gauge model restricted to the chromoelectric part as well. If we calculate the partition function ($SU(2)$ gauge group for simplicity)

$$Z = \int \prod_{x,\mu} dA_\mu(x,t) exp[-a^4 \sum_x (F_{0,\mu})^2] \tag{13}$$

at the periodic boundary conditions $A_\mu(x,t) = A_\mu(x, t + N_t)$ in the temporal gauge, we shall find that $\langle TrW \rangle \neq 0$ at any temperature, perhaps with exception of the case of the infinite coupling $g^2 \to \infty$ [21]. Thus, the model (13) does not display confinement behaviour. Let us emphasize that the final result is the gauge invariant as well as the expression for the partition function after integration out of space gauge fields $A_n(x)$ (see for technical details the next section). This, together with independence of the MC results of the gauge and the restoration of the asymptotic freedom, refutes the usual objection concerning a connection between the vanishing of the string tension and the breakdown of the gauge symmetry in noncompact models.

In fact, some kind of invariant integration over gauge fields is present in all models constructed with the goal of avoiding this explicit violation of the gauge invariance [23, 24]. For instance, the model proposed in [23] is equivalent to the dielectric LGT introduced in [25]. Let us consider the $SU(2)$ Yang-Mills action with potentials [23]

$$A_\mu^{Y-M} = A_\mu^a t^a \to A_\mu^d = A_\mu^{Y-M} + IT_\mu, \tag{14}$$

where T_μ is a new noncompact potential proportional to a unit matrix in colour space. Rewriting the obtained action on the lattice in the finite-difference form we have as result the dielectric theory [25] since we can use the representation $A_\mu^d = \rho_\mu(x)U_\mu(x)$ where $U_\mu(x) \in SU(2)$, $0 \leq \rho < \infty$. We are allowed to choose the potential for the dielectric field ρ in such a form that [23]: 1) naive continuum limit equals the standard Yang-Mills action; 2) Wilson loops and corresponding string tension behave like those in the compact Wilson model; 3) at the weak coupling asymptotic freedom exists. Thus, the theory is noncompact but confinement of static charges takes place. Since, however, the integration measure includes the invariant measure of $SU(2)$ group, it

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can be the reason of the area law. Further, introduce the following restriction for the noncompact field T_μ in (14): $0 \le T_\mu < \infty$. It follows, that we should consider $SU(N)/Z(N)$ as the gauge group since $U \in SU(N)/Z(N)$ in this case. So, as we have discussed earlier we have to use adjoint $SU(2)$ representation for gauge matrix U. It means immediately that the fundamental Wilson loop equals zero whereas the adjoint loop can show critical behaviour.

One more approach starting from a generating functional for noncompact Yang-Mills theory has been discussed in Ref.[24] where noncompact fields are exposed to random compact gauge transformations (instead of the Faddeev-Popov ansatz) at all lattice sites during every Monte-Carlo sweep. Gauge invariance can be restored in this approach and linearly rising potential has been observed. In the meantime we cannot accept as conclusive, the original interpretation of the nature of such behaviour. Indeed, random compact gauge transformations introduce $Z(2)$ variables into the simulated theory, creating a reason for the appearing of linear potential. We believe, in order to comprehend the problem, the supplementary MC-simulations with random gauge transformations $V(2)$ belonging to: 1) $Z(2)$ and 2) $SU(2)/Z(2)$ are highly desirable. Then, what we would expect in the first case (no explicit gauge symmetry restoration) is the confining behaviour in the strong coupling region and the deconfining phase transition in the weak coupling region. In fact, the basis for this is supported by the resemblance of the resulting theory and the $Z(2)$ gauge theory. The Wilson loop behaviour in the full range of coupling is less predictable for the second case. Surely, if we add a summation over $Z(2)$ variables to the noncompact integration, the Wilson loop will equal zero. We do not expect that the adjoint Wilson loop will obey area law, at least in the region of weak coupling.

In our opinion, there are two different explanations of such behaviour of noncompact lattice models:

1) The confinement mechanism in continuum gauge theory is different from the one on the lattice (the so-called "light" confinement mechanism which works only in the presence of the dynamical quarks [3]) and this mechanism could work in noncompact lattice models.

2) Noncompact Yang-Mills theory belongs to the other universality class without quark confinement.

The third possibility, namely that confinement is solely the property of compact gauge theory is not confirmed by an example of the dielectric gauge theory discussed above. The following facts indicate that the second choice could be the right one:

i) There is no phase transition, depending on the coupling constant, at zero temperature in pure $SU(2)$ and $SU(3)$ gauge theories; it follows that a nonperturbative weak coupling limit in a bare constant can define a noncompact model different from the naive lattice Yang-Mills theory.

ii) The expectation value of the Polyakov loop differs from zero in $SU(N)$ gauge theory if the vacuum is not invariant under $Z(N)$ rotations[2]. The standard Yang-Mills theory with a flat integration measure does not possess $Z(N)$ invariance.

iii) $SU(N)$ compact theory has N global minima whereas noncompact Yang-Mills

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theory has alone minimum. According to [24], such a periodicity could be responsible for the confinement but all the minima with the exception of the trivial one are nonphysical. We do not think that the argument of Ref. [24] is correct in this respect.

We believe, in full accordance with the results [20, 21, 22] discussed above, that the compact Wilson theory and the noncompact lattice Yang-Mills theory belong to two different classes of universality: we do not expect the Yang-Mills theory in its naive lattice form to be able to describe confinement. Our method of discovering a proper theory is to construct a noncompact model starting from the compact Wilson model but not from the continuum Yang-Mills theory taking its naive lattice form (12). In order that the readers have a guideline to the manuscript we present here a short description of the main ideas. To secure a confining theory in the weak coupling region we propose to execute the summation over $Z(N)$ variables in the compact formulation and then to take a noncompact limit expanding the resulting $SU(N)/Z(N)$ matrices around all minima of the effective action obtained. Further, as is known from the studies of the strong coupling lattice models, invariant group measure (at least, for A_0 gauge field) can be of great importance for the finite temperature confinement (for the definition of the invariant measure, see section 5). The flat integration measure for the A_0 field fails to respect the $Z(N)$ global symmetry of the vacuum [2]. Thus, the next step should be to include the invariant measure contribution in the noncompact effective model. Of course, the principal questions appearing here are the expansion of $SU(N)/Z(N)$ gauge matrices in the points of the minima, and what form can be used for the invariant measure.

4 Noncompact model with compact A_0 integration

As the first step, we are going to explore the $SU(2)$ noncompact model defined by Eq.(13). We would like to reexamine the continuum limit of the chromoelectric part of the lattice action in order to include $Z(2)$ invariance and compact A_0 integration in the finite-difference gauge theory. We begin by rewriting the chromoelectric part of the action (3) using the following gauge transformations of space gauge matrices $U_n(x)$ [26]

$$U_n(x,t) \longrightarrow (V_x)^t U_n(x,t)(V_{x+n})^{-t},$$
$$U_n^+(x,t) \longrightarrow (V_{x+n})^t U_n^+(x,t)(V_x)^{-t}, \quad n = 1,...,d$$

$$(15)$$

where V is the U_0 gauge matrix in the static gauge. Next, the chromoelectric part becomes

$$\lambda_0 \sum_{p_0} \Omega(\partial p_0) \to \bar{S}(E) = \frac{2a_n}{a_t g^2} \sum_{x,n}$$

$$\left\{ \sum_{t=0}^{N_t-2} Sp[I - U_n(x,t)U_n^+(x,t+1)] + Sp[I - U_n(x,N_t-1)W_x U_n^+(x,0)W_{x+n}^*] \right\}. \quad (16)$$

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Computing the continuum limit of $\bar{S}(E)$ we consider that at the limit $a_t \to 0$ we have $W_x \to \exp(i\beta \ g \ A_0(x))$ in the given gauge. Assuming the smoothness of the A_0 field in the sense that

$$W_x W^*_{x+n} \ \approx \ \exp(-i\beta g a_n \partial_n \ A_0(x)) \ \approx \ I \ - \ i\beta g a_n \partial_n \ A_0(x) \ + \ ... \qquad (17)$$

by help of the definition $\frac{\delta_{t,0}}{a_t} \ \to \ \delta(t)$ one obtains

$$\bar{S}_{con}(E) \ = \ \int d^3x \int_0^\beta dt \{ (\partial_t A_n)^2 + \delta(t) Sp[L_1 + L_2/a_t] \}, \qquad (18)$$

where

$$L_1 \ = \ A_n(\partial_t A_n) \ + \ \beta(\partial_n A_0) \ (\partial_t A_n) - \ A_n W(x)(\partial_t A_n) W^*(x), \qquad (19)$$

$$L_2 \ = \ A_n^2 \ + \ \frac{1}{2}\beta^2 \ (\partial_n A_0)^2 \ - \ A_n W(x) A_n W^*(x). \qquad (20)$$

The first term in Eq.(18) corresponds to the gauge $A_0 = 0$. However, this gauge is incompatible with periodic boundary conditions. A_0 can be set equal to zero everywhere except at one singular point [27]. The second term in Eq.(18) reflects this fact. Using the decomposition

$$W(x) \ \sim \ I \ + \ i\beta g A_0(x) \ + \ ...,$$

we can easily demonstrate that the function at the δ-symbol in Eq.(18) corresponds to the remaining terms in $(F_{0,n})^2$. Due to the preceding discussion we do not use the last decomposition. We are going to study the partition function $\bar{S}_{con}(E)$ in the finite difference formalism. Because the gauge fixing in Eq.(18) is equivalent to the static one (i.e. $\partial_0 A_0 = 0$), the results, calculated in both gauges, are the same. We have in Eq.(18) the singular term L_2/a_t. Hence, a regularization procedure is necessary to define it properly. Applying the finite difference approximation in Eq.(18) we discover through algebra the relation to the partition function

$$Z \ = \ \int \prod_x d\mu(W_x) \prod_{x,n} Z_{1,2}(W) \ Z_3(W) \exp(-S^{(1)}), \qquad (21)$$

where

$$S^{(1)} \ = \ \frac{2a_n}{a_t}\beta^2 \ \sum_x (\partial_n A_0(x))^2, \quad \partial_n A_0(x) \ = \ A_0(x+n) - A_0(x). \qquad (22)$$

Z_i is the path integral over space gauge fields:

$$Z_{1,2} \ = \ (Det \ M^{bc}_{tt'})^{-1/2} \ =$$
$$\int \prod_{t=0}^{N_t-1} dA_n^1(t) \ dA_n^2(t) \exp\{-A_n^b(t) \ M^{bc}_{tt'} \ A_n^c(t') \ \}, \qquad (23)$$

where $b, \ c \ = \ 1, 2$ are the colour indices and

$$Z_3 \ = \ \int \prod_{t=0}^{N_t-1} dA_n^3(t) \exp\{-A_n^3(t) \ \tilde{M}_{tt'} \ A_n^3(t')$$

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$$+\frac{2a_n^2}{a_t}\beta(\partial_n A_0(x))\sum_{t=0}^{N_t-1}(\delta_{t,N_t-1}-\delta_{t,0})\ A_n^3(t)\ \}=$$

$$(Det\ \tilde{M}_{tt'})^{-1/2}\ \exp\left[\frac{2a_n^4}{a_t^2}\beta^2(\partial_n A_0(x))^2[(\tilde{M}_{0,0})^{-1}-(\tilde{M}_{0,N_t-1})^{-1}]\right]. \tag{24}$$

We use the representation:

$$M_{tt'}^{bc}\ =\ \frac{a_n^3}{a_t}\begin{pmatrix} 2I & -I & \cdots & 0 & m^{cb} \\ -I & 2I & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 2I & -I \\ m^{bc} & 0 & \cdots & -I & 2I \end{pmatrix}$$

and

$$\tilde{M}\ =\ M_{tt'}^{bc}(m^{bc}=m^{cb}=I).$$

m^{bc} is the matrix $2\otimes 2$ constructed from the scalar $Sp\ W(x)$ and the octet $Sp\sigma^3 W(x)$ parts of the Polyakov loop. Through parametrization

$$W(x)=\exp(\frac{i\varphi(x)\sigma^3}{2}),\ \varphi(x)=\beta g A_0(x), \tag{25}$$

one finds

$$m^{cb}=\begin{pmatrix} \cos\varphi(x) & -\sin\varphi(x) \\ \sin\varphi(x) & \cos\varphi(x) \end{pmatrix}. \tag{26}$$

Introducing the notation

$$S_{eff}(A_0(x))\ =\ S^{(1)}-\frac{2a_n^4}{a_t^2}\beta^2\sum_{x,n}(\partial_n A_0(x))^2\left[(\tilde{M}_{0,0})^{-1}-(\tilde{M}_{0,N_t-1})^{-1}\right], \tag{27}$$

we can represent the partition function in the form

$$Z=[(Det\ \tilde{M}_{tt'})^{-1/2}]^{N_\sigma^3}\int e^{-S_{eff}(A_0(x))}\prod_{x,n}(Det\ M_{tt'}^{cb})^{-1/2}\ \prod_x d\mu(W_x). \tag{28}$$

It follows from (24 - 28) that the third color component of the gauge field does not interact with the first and the second components (because of chosen gauge). However, only the third component leads to the second term in (27). Since $\tilde{M}_{tt'}$ does not depend on $W(x)$ this contribution renormalizes a free theory $S^{(1)}$. At $N_t\to\infty$ we have precisely

$$S_{eff}=\alpha\sum_{x,n}(\partial_n A_0(x))^2,\ \alpha=const\frac{a_n}{a_t}\beta^2. \tag{29}$$

The determinant can be calculated to produce

$$Det M_{tt'}^{cb}=(\frac{2a_n^3}{a_t})^{N_t}\sin^4\frac{\varphi(x)}{2}. \tag{30}$$

Since $d\mu(W_x) = \sin^2\frac{\varphi(x)}{2}d\varphi(x)$ we finally obtain the partition function to be of the form (up to an irrelevant constant)

$$Z \sim \int e^{-S_{eff}(A_0(x))} \prod_x \left[\frac{\sin^2\frac{\varphi(x)}{2}d\varphi(x)}{(\sin^2\frac{\varphi(x)}{2})^d} \right], \tag{31}$$

where d is the space dimension. It is clear from (31) that the singular contribution to (18) is well-controlled in the present regularization (since there are no time derivatives at the singular term and all singular terms are proportional to a_t^{-1}). Obviously, the constructed theory will be equivalent to (3) (restricted to the chromoelectric part) when the matrices U_n are expanded around unit matrices and U_0 are being kept in their lattice form (it is also the proof that these calculations do not depend on chosen gauge). To verify this we note that all equations (21-24) are valid in this case and matrix $M_{tt'}^{cb}$ becomes

$$M_{tt'}^{cb} \rightarrow \overline{M}_{tt'}^{cb} = \begin{pmatrix} 2I & -m^{bc} & \cdots & 0 & m^{cb} \\ -m^{bc} & 2I & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 2I & -m^{bc} \\ m^{bc} & 0 & \cdots & -m^{bc} & 2I \end{pmatrix}.$$

Since $Det M_{tt'}^{cb} = Det \overline{M}_{tt'}^{cb}$ we obtain the same result (31).

Let us now discuss this example comparing (31) with (8). In parametrization (25) the character expansion in (8) can be expressed as

$$\prod_{x,n} \Omega_l(W_x)\Omega_l(W_{x+n}) = \prod_x (\sin^2\frac{\varphi_x}{2})^{-d} \prod_{x,n} \sin\frac{\varphi_x}{2}(2l+1) \sin\frac{\varphi_{x+n}}{2}(2l+1). \tag{32}$$

We conclude from this that (8) and (31) are different theories. If we integrate over $d\mu(W_x)$ in (8) we will find that only closed loops contribute to the partition function whereas there is no such property in (31). However, just this property is the cause of linear potential at a low temperature. Further, using the Poisson summation formula applied to calculate the sum over characters in (32) we can easily show that at a high temperature (lattice spacing a is fixed)

$$\prod_{x,n}\sum_l \exp(-\gamma C_2(l))\Omega_l(W_x)\Omega_l(W_{x+n}) \longrightarrow_{T\rightarrow\infty} \exp(-S_{eff} + S_{loc} + O(a)) \tag{33}$$

with S_{eff} to be of the form as in (29) [17, 28], S_{loc} is a local function of A_0, and the measure in (31) should be treated as smooth function. The correlation function of the Polyakov loops has a form corresponding to a screening potential between probe quarks in this region. Hence, the model presented in (31) is capable of describing only the high temperature deconfined phase. It follows immediately from this consideration that the pure Yang-Mills theory (13) does not describe the confinement phase since we

have taken into account a larger number of gauge configurations (we did not expand the Polyakov loops around unit matrix) generated by compact fields on time-like plaquettes. We are convinced from this example that (3) and (12) can belong to different universality classes. We could obtain slightly more information if we did not presuppose the smoothness of A_0 field in the sense (17). The smoothness of A_0 means that neighboring "spins" W_x and W_{x+n} are oriented approximately in one direction, while in the confinement phase the configurations being essential for confinement should be strongly disordered (for example, $W_x \sim I$, $W_{x+n} \sim -I$). In this case the expansion (17) will be obviously invalid. If we had constructed the continuum limit in the time direction only, we would obtain the effective action of the form

$$S_{eff} = \gamma \sum_{x,n} \cos(\varphi_x - \varphi_{x+n}). \tag{34}$$

This action coincides with the effective action for the $U(1)$ lattice compact theory appearing in the Hamiltonian formulation at a finite temperature. There is a phase transition from the low temperature confining phase to the deconfining one in this theory. However, this method of calculation is not mathematically well-founded for the $SU(2)$ gauge group. If $W_x \sim I$ and W_{x+n} is far from this configuration, then the expansion of $U_n(x,t)$ around the unit matrix will not lead to the true minima of the action. Two possible avenues to promote our calculations are available in principle. The first one consists of applying some conjectures proposed in [2] and, then, in transition from theory (31) to some effective model which should be calculated in the framework of the renormalization group scheme. There exists a concern that not all the important configurations have been taken into account. The last example (34) demonstrates that space gauge field configurations $Z \in U_n(x)$ could be essential for obtaining the true minima of the quantum theory. In the next section we analyze the first of these possibilities.

5 JLP - model and simulation of the invariant measure contribution

The next model we would like to examine was proposed in [2] (see also [3]) (in what follows we will call the model: the JLP-model). The question is, how could one simulate the contribution of the invariant group measure in a noncompact (either lattice or continuum) theory. A flat integration measure fails to respect the $Z(N)$ global symmetry of the lattice action. In this case the expectation value of the Polyakov loop differs from zero. Usually, the definition of the invariant measure on $SU(N)$ group includes a compact region of integration and weight function in the corresponding integrand. In this section speaking about invariant measure we mean only this weight function which is local contribution to the action of LGT. The basic idea of the JLP-model consists in the assumption that one should simulate this contribution making use of a local $Z(N)$ invariant potential for A_0 gauge field. Then, a symmetry argument suggests that the

action of the confining $SU(2)$ noncompact model involves a non-polynomial periodic term depending on the A_0 gauge field:

$$S_{Y.-M.} \rightarrow \frac{1}{g^2} \int Sp(F_{\mu\nu})^2 d^4x - \frac{1}{a^4} \int d^4x \ln(\sin^2 A_0(x,t)ag), \tag{35}$$

where, following [2], we did not fix the static gauge but chose the diagonal form for A_0. The basic assumption is that the cutoff (lattice spacing in our case) of the theory is renormalizable and is left finite in the continuum. If this is the case, the renormalized Lagrangian is of the sine-Gordon form. Thus, we have for the action

$$S_{eff} = S_{Y.-M.} + \mu \sum_{m=1}^{\infty} \nu_m \cos mTgA_0 = S_{Y.-M.} + V(A_0), \tag{36}$$

where ν_m is an arbitrary coefficient in the model. The potential $V(A_0)$ is a sum over the characters of the $SU(2)$ gauge group which are trivial on $Z(2)$. The new constant μ can be interpreted as the so-called hidden coupling constant [29] and should be calculated in the course of the renormalization procedure. If one now takes another assumption, namely that the dynamics of the space gauge fields is not essential for the confinement, we will get the following effective theory

$$S_{eff} = \int d^4x \left(\frac{1}{g^2} Sp(\partial_\mu A_0(x))^2 - V(A_0)\right). \tag{37}$$

It has been claimed in [2] that the Wilson loop obeys the area law and this leads to the linear potential between probe quarks. The string tension appears to be proportional to μ. The model possesses global $Z(2)$ symmetry which, however, appeared to be broken at any values of the coupling constants μ and λ [30]. This seems to be in contradiction with the main idea of [2], since the invariant measure was introduced to preserve the $Z(2)$ symmetry of the vacuum. In ref.[30] it has been rigorously shown that the correlation functions of the kind $< \sin A_0(0)/2 \sin A_0(R)/2 >$ behave like those in the free scalar model. This leads to the nonzero string tension $\alpha = \mu/\lambda$ if we choose the appropriate sign in μ (the effective action (37) should have a maximum at $A_0 = 0$).

We studied a simplified version of the model (37) with $\nu_m = \delta_{m,1}$. This approach is sufficient for our arguments since the string tension in [2] is non-zero at this level. Thus, we start from the partition function

$$Z = \int_{-\infty}^{\infty} \prod_x du_x e^{-S(u)}, \tag{38}$$

where the action is of the sine-Gordon type

$$S(u) = \lambda \sum_{x,x'} u_x M_{x,x'} u_{x'} - \mu \sum_x \cos u_x \tag{39}$$

with

$$uMu = du_x^2 - \sum_n u_x u_{x+n}, \tag{40}$$

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and d is the space dimension. We are going to calculate the following correlation function

$$\Gamma(R) = < e^{iu(-R/2)/2} e^{-iu(R/2)/2} >, \qquad (41)$$

which can be interpreted as a correlation function of two Polyakov loops in the static diagonal gauge (after we finished all calculations without gauge fixing, we found that all results were essentially the same). Introducing external sources $\eta_x = \frac{1}{2}(\delta_{x,-R/2} - \delta_{x,R/2})$ into the partition function (38), we define $\Gamma(R)$ as

$$\Gamma(R) = Z_\eta / Z, \qquad (42)$$

where

$$Z_\eta = \int_{-\infty}^{\infty} \prod_x du_x e^{-S(u) + i \sum_x \eta_x u_x}. \qquad (43)$$

The corresponding potential between probe quarks is then

$$V(R) = -\ln \Gamma(R) - V_0, \qquad (44)$$

where V_0 is the self energy of two static charges. We want to investigate two asymptotic regions on the plane (μ, λ): 1) $\mu \gg 1, \lambda \gg 1$ and 2) $\mu \ll 1$. Let us begin with the first asymptotic. Integrating the u_x field, one obtains for Z_η up to an irrelevant constant

$$Z_\eta = (det\lambda M)^{-1/2} \sum_{l_x} \prod_x I_{l_x}(\mu) \exp[-\frac{1}{\lambda} \sum_{x,x'} (l_x + \eta_x) M_{x,x'}^{-1} (l_{x'} + \eta_{x'})]. \qquad (45)$$

Here, I_l is the modified Bessel function. Taking its asymptotic behaviour at $\mu \gg 1$ we can make use of the Poisson summation formula to calculate the sum over l_x in (45). After this procedure one arrives at the equation for the potential, to be of the form in the limit $N \to \infty$, where N is the number of lattice sites

$$V(R) = q(R) + \frac{1}{\lambda} \int d^3k \sin^2 \frac{k_\sigma R_\sigma}{2} (\frac{1}{M_k} - \frac{1}{M_k + \frac{2\mu}{\lambda}}) - V_0, \qquad (46)$$

where we have denoted

$$q(R) = \frac{1}{\lambda} \sum_{x,x'} \eta_x M_{x,x'}^{-1} \eta_{x'} \qquad (47)$$

and

$$M_k = d - \sum_{\sigma=1}^{d} \cos k_\sigma. \qquad (48)$$

Calculating the right-hand side of eq.(46) we find the potential of the general form for $R \to \infty$

$$V(R) \sim \frac{a}{R} - \frac{be^{-mR}}{R} \qquad (49)$$

with $m = \frac{2\mu}{\lambda}$ and a, b are R-independent constants.

Considering asymptotic $\mu \ll 1$ (just this case corresponds to the regime of [2]), it is convenient to rewrite Z_η in an equivalent form as

$$Z_\eta = e^{-q(R)} \int_{-\infty}^{\infty} \prod_x du_x \exp[-\lambda \sum_{x,x'} u_x M_{x,x'} u_{x'} - \mu \sum_x \cos(u_x + iQ_x)], \qquad (50)$$

where $Q_x = \frac{2}{\lambda} \sum_{x'} M_{x,x'}^{-1} \eta_{x'}$. In the first order in μ we find the potential $V(R)$ to be (taking into account contribution of V_0):

$$V(R) = -\frac{1}{2} M_{R/2,-R/2}^{-1} + \mu \sum_x W_x(R). \qquad (51)$$

We introduced here

$$W_x(R) = 4 e^{-\frac{1}{\lambda} M_{x,0}^{-1}} \sinh \frac{M_{x,-R/2}^{-1}}{2\lambda} \sinh \frac{M_{x,R/2}^{-1}}{2\lambda} \cosh \frac{M_{x,R/2}^{-1} - M_{x,-R/2}^{-1}}{2\lambda}. \qquad (52)$$

If we consider asymptotic $\lambda \gg 1$ we can approximately represent

$$W_x(R) \approx \frac{M_{x,-R/2}^{-1} M_{x,R/2}^{-1}}{\lambda^2}. \qquad (53)$$

Eq.(52) with asymptotic behaviour (53) corresponds, in our lattice notations, one-to-one to the result of [2] where the confining potential has emerged from the term $\sum_x W_x(R)$. We performed both analytical and numerical evaluations of the sum (51) in the approach

$$W_x(R) \approx e^{-\frac{1}{\lambda} M_{x,0}^{-1}} \left[\frac{M_{x,-R/2}^{-1} M_{x,R/2}^{-1}}{\lambda^2} \right]. \qquad (54)$$

Actually this sum is divergent, but it is decreasing as a function of R for any finite number of lattice sites. We did the computations for various values of $\lambda \geq 1$ and for $N = 20^3, 30^3, 40^3$. Certainly, if we took at this stage the continuum limit in (53) and followed the procedure of [2] we would get the linear potential. But we do not think that this procedure is well founded. We considered the limit $N = \infty$ and introduced a different regularization to compute $V(R)$. $\sum_x W_x(R)$ can be represented in the approach (53) as

$$W_x(R) \approx lim_{\epsilon \to 0}(-\frac{\partial}{\partial \epsilon} \sum_{k_\sigma} \frac{e^{ik_\sigma R_\sigma}}{d + \epsilon - \sum_{\sigma=1}^{d} \cos k_\sigma}). \qquad (55)$$

The potential can be found after subtraction of the R-independent divergences from the last equation. Indeed, increasing with R potential appears to have a negative sign. To achieve a confining potential we have to choose $\mu < 0$. In [2] μ enters the effective action just with this sign. We were not able to prove that this result is independent of the regularization scheme for evaluations of these divergent sums. Besides, there is an additional term $e^{-\frac{1}{\lambda} M_{x,0}^{-1}}$ in $W_x(R)$ which was missed in [2]. This term improves the

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convergence of the whole sum but the renormalization procedure becomes even more complicated.

A more reliable way to calculate (50), in our opinion, is to use the saddle point method. This method leads, in the asymptotic under consideration, to the potential which has the form (49) (we omitted all calculations since they are quite transparent). This result is in agreement with rigorous results of [30]. If the correlation function $< \sin A_0(0)/2 \sin A_0(R)/2 >$ is exponentially decreasing and $Z(2)$ global symmetry is broken then the correlation function $< \cos A_0(0)/2 \cos A_0(R)/2 >$ is close to unity. Consequently, the correlation function (41) is close to unity as well, which implies a nonconfining potential.

We would like to summarize the main consequences and to provide some comments on the reliability of this approach to the confinement in noncompact models. Taking suitable correlation functions (see theorem 4 in [30]), the nonzero string tension can be found even in the continuum limit after proper renormalization procedure. If we may interpret these correlation functions as those of the Polyakov loops in the effective model, then we have confinement of static charges. Nevertheless, $Z(2)$ global symmetry is broken at all couplings. This type of confinement resembles that of the $U(1)$ Villain lattice model investigated in [30]. The sine-Gordon model is there an effective model of the lattice abelian theory with Villain action. Thus, the first question is whether this mechanism can reproduce the specific features of confinement of the $SU(2)$ Wilson model. In this model $Z(2)$ global symmetry is unbroken at zero and at low temperatures. A broken $Z(2)$ implies a screening potential between probe quarks, and, thus a deconfinement phase. Thereby, we have to use (41) as correlations of the Polyakov loops in this model since, in any other case, it is unclear how to deal with the deconfinement transition if we have the linear potential in the $Z(2)$ broken phase. In our opinion, to reproduce the specific features of the $SU(2)$ Wilson theory we have to preserve not only the global center symmetry, but also the local center symmetry. In principle, the global symmetry can be spontaneously broken as it happens in the standard sine-Gordon model, whereas there should be no such breakdown at zero temperature $SU(N)$ models. To achieve the above stated goal, it is not sufficient to introduce the invariant measure into effective action. We present a modification of the JLP model which respects *local $Z(N)$* symmetry in [13].

The second question concerns the assumption that the dynamics of space gauge potentials is not essential for the confinement. This may not be the case, and we have discussed in the previous chapter that $Z(2)$ configurations contained in the compact lattice field $U_n(x)$ can be of great importance. Let us consider the compact formulation at zero temperature. If we set $U_n(x) = I$ everywhere, we get as a result an XY model for the $U_0(x)$ gauge field with an integration over the compact $SU(2)$ measure. There is a phase transition in this model implying deconfinement of static quarks. However, no phase transition should take place in $SU(2)$ at zero temperature. Hence, we are not allowed to neglect dynamics of space gauge potential, at least in this naive form. Further, it is obvious from (31) that the noncompact integration over space potentials can significantly change the effective integration measure for the A_0 gauge field because

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the determinant (30) generates a local contribution to the measure. Moreover, we can see from (28), (30) that the noncompact integration over space gauge potentials generates just the invariant measure. On the other hand, the contribution of the sine-Gordon type can appear not only from the invariant measure but also from the effective action (33). The calculation of S_{loc} in (33) shows that this term is proportional to the $\cos agA_0$ up to the corrections $O(a)$. The situation becomes even more complicated when we consider the chromomagnetic part of the action. The invariant measure can be cancelled completely in this case as has been shown in [31] (in fact, there is currently no common opinion on the cancellation of the invariant measure - see for discussion [32]). Hopefully there should be no such cancellation at zero temperature. Here, the invariant measure can be included into the effective action together with a compact measure for $Z(N)$ space gauge configurations.

6 Compactness and noncompactness in confinement: discussion of simple models

In this section we discuss the problem of compactness and its importance regarding confinement. In the broad class of lattice models, the compactness of the potentials entering the original action, is an essential condition for confinement. On the other hand, the compactness itself does not lead to the linear potential. Indeed, the compactification performed following the scheme

$$\int_{-\infty}^{\infty} d\phi f(\phi) = \int_0^{2\pi} d\phi F(\phi), \ F(\phi) = \sum_{j=-\infty}^{\infty} f(2\pi j + \phi)$$

can change nothing in the correlation functions, so we adduce the following example when the transition to the compact theory provides confinement. We start from the theory of a scalar noncompact field in continuum with the action

$$S = J_0 \frac{1}{2} \int d^4x \sum_n (\partial_n \phi(x))^2 + J_1 \int d^4x \cos \phi(x), \tag{56}$$

where $n = 1, ..., d$ and represent it on the lattice as

$$S^{lat} = J_0^l \frac{1}{2} \sum_{x,n} (\Delta_n \phi(x))^2 + J_1^l \sum_x \cos \phi(x), \tag{57}$$

where J_0^l and J_1^l is connected with J_0 and J_1. The compactified version of (57) has the form (up to an irrelevant constant and up to a^2 in lattice spacing a)

$$S^{lat}_{compt} = \sum_{x,n} (J_- \cos \frac{\phi_{x+n} - \phi_x}{2} + J_+ \cos \frac{\phi_{x+n} + \phi_x}{2}). \tag{58}$$

This expression coincides with the effective three-dimensional action for $SU(2)$ gluody-namics at a finite temperature in the strong coupling limit provided that $J_- - J_+ \sim 0$

(see, for instance, [33]). One can find linear potential in this model at small J_- ($d = 3$) and deconfinement transition to a phase with screening potential when J_- is increasing. What lesson may we extract from this? The model (58) is a version of the well-known three-dimensional XY model which displays a phase transition from strong to weak coupling behaviour. At the small value of J_- (low temperature strong coupling region of finite temperature $SU(2)$ theory or high temperature region of the spin system) correlation functions fall exponentially and system is in the disordered phase. Regarding $SU(2)$ language, it means the area law for the Wilson loops. This behaviour is due to the vortex loops which percolate through the lattice [34] (or become fat because of condensation, in other terminology). In the weak coupling phase only short noninteracting vortex loops are allowed, thus the behaviour of the system is mainly defined by the contribution of the spin waves. Neglecting entirely the vortex contribution in this region, we come to the free theory of the scalar field on the lattice. The theories (56),(57) are in the same universality class while the theory (58) belongs to another class, although it originates from (56) and has only this naive continuum limit. Thereby, the question is whether it is possible that the theory (58) could define a new continuum theory with a disordered phase caused by vortex condensation? In other words, can we construct a continuum limit of the model (58) at the small value of J_- (strong coupling region)? We investigate this problem in the next paper [13].

Our next example is related to the fact that a form of the effective variables in the action can also be essential for confinement. To illustrate this suggestion by way of a solvable model we need to restrict ourselves to models which can be effectively reduced (for dominating configurations) to the Gaussian integrals after changing variables $\phi_x \to u(\phi_x) = u_x$ and expanding the action around the main minimum. Thus, we consider the action

$$S_{eff} = -\frac{\beta}{2} \sum_{x,x'} u_x M_{x-x'} u_{x'} + \sum_x V(u_x) + i \sum_x \eta_x \phi_x(u_x), \qquad (59)$$

supposing that the potential $V(u_x)$ includes the contributions both of the invariant measure and of the Jacobian of the substitution $\phi_x \to u_x$. The integration in the partition function is performed with the measure $\prod_x du_x$ over the entire noncompact region. When $u_x = \phi_x$ we have a model which is close to the JLP model if V represents the periodic potential. Let us suppose that a proper effective variable is $u_x = e^{i\phi_x}$. Then we immediately obtain

$$e^{-F(R)} = < u_0 u_R^* > \approx \frac{e^{-mR}}{R}, \qquad (60)$$

where

$$m = \frac{1}{N} \sum_x (\frac{\beta}{2} M_x + \frac{V''(0)}{2}) \qquad (61)$$

is the string tension in this model. Formally, the principal point in this calculation was the transition from $\Gamma(R) = < \phi_0 \phi_R >$ to $\Gamma(R) = < u_0 u_R^* >$ when $u_x = e^{i\phi_x}$. Let us imagine that the original variables ϕ_x were compact variables. Then, this rather trivial example demonstrates that in some models with compact variables it is sufficient to

perform only noncompact Gaussian integration over the dominating configurations to achieve the linear potential.

Our next example concerns the noncompact model presented in [9, 10]. This model is of the same spirit like [5], where a sufficient condition for the confinement was derived (see our discussion in the section 2). It has been proven that if probability distribution of the vortices in the compact model obeys the area law it will lead to the disordering behaviour of the Wilson loop and, consequently, to the confining potential. This statement can be generalized for the noncompact model calculated in [9] (see [35]). Unfortunately, both the theory [9] and this generalization of the $Z(N)$ vortex confinement mechanism to the noncompact models [35] appear very formal. As we pointed out in section 2, the theory of [9] is noncompact and not a naive limit of the Wilson model. It includes, in addition to Yang-Mills potentials, singular $Z(N)$ transformations performed over two-dimensional closed surfaces. The corresponding path integral contains a summation over all possible two dimensional surfaces and determinants in the external singular fields. It is unclear at the moment whether it is possible to execute all these summations and to calculate the corresponding determinants (it is the main reason for not adducing any calculations here with discussed theory). As such, we think this model demonstrates that $Z(N)$ variables can be included in the noncompact limit of the compact theory.

The last example concerns a mathematical origin of confinement in the Wilson theory and how its origin can be reproduced in noncompact models [36]. We start from the finite temperature partition function for $SU(N)$ gluodynamics, obtained within the approach of time-like plaquettes, as

$$Z = \int \prod_x D\mu_x \prod_{x,n} \sum_l K_l(\gamma)\Omega_l(x)\Omega_l^*(x+n). \qquad (62)$$

Here, $D\mu_x$ is the invariant integratiom measure and $\Omega_l(x)$ is the character of the l-th irreducible representation. Performing the invariant integration in the partition function we can easily check that closed loops only contribute to the partition function. For the correlation function $< \Omega_f(0)\Omega_f^*(R) >$ (f marks fundamental representation) we find out that the first nontrivial term, surviving the invariant integration in the region $K_l(\gamma) \to 0$ (low temperature), is the shortest path between points 0 and R on the lattice:

$$< \Omega_f(0)\Omega_f^*(R) > \approx (K_f(\gamma))^R + O(K(\gamma)). \qquad (63)$$

Could this picture be reproduced for noncompact fields? The positive answer becomes straightforward supposing that we have the following partition function for the noncompact field u_x

$$Z = \int_{-\infty}^{\infty} \prod_x [du_x e^{V(u_x)}] e^{\check{S}(u)} \qquad (64)$$

and $e^{\check{S}(u)}$ can be expanded as

$$e^{\check{S}(u)} = \prod_{x,n} \sum_l C_l(\gamma)L_l(u_x)L_l(u_{x+n}), \qquad (65)$$

where functions L_l form the complete orthonormal basis in the space of quadratically integrable functions with the weight $e^{V(u_x)}$

$$\int_{-\infty}^{\infty} du e^{V(u)} L_l(u) L_k(u) = \delta_{l,k}. \tag{66}$$

Mathematically it indicates the same property as above: closed loops only contribute to the partition function (64). Calculating the correlation function $< L_k(0)L_k(R) >$ we find the linear potential in a similar manner as in the model with compact invariant integration:

$$< L_k(0)L_k(R) > \approx (C_k(\gamma))^R + O(C(\gamma)), \tag{67}$$

if the following equation is fulfilled

$$\int_{-\infty}^{\infty} du e^{V(u)} L_l(u) = 0. \tag{68}$$

Two points should be stressed here. The weight $e^{V(u)}$ plays a role of the invariant measure of the compact model. It enters the action $S = V(u) + \tilde{S}$ as the local potential. The role of the $Z(N)$ symmetry is to pick up those functions L_k within the complete basis $\{L\}$ which satisfy eq.(68). It is clear from the procedure above that this method could be directly applied to the full Wilson action with plaquette interaction. We should take a product of the L_l functions in this case along the perimeter of the minimal plaquette with the same weight, and to sum over l in (65) obeying (68).

Finally, the last question has to be answered: whether the partition function (64) may correspond to any quantum field theory with acceptable properties besides confining ones? In the paper [13] we try to synthesize all the essential results of this paper into a general picture and present an investigation of the $SU(2)$ compact Wilson model in the region of weak coupling. We shall demonstrate that models of such types (64) can be defined as a noncompact limit of the Wilson theory in the weak coupling region if we execute summation over $Z(N)$ variables. We shall calculate an effective noncompact model and prove its confining behaviour.

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