BU–TH–95/2
MADPH–95–898
July 1995

Resolved Photon Processes

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Abstract

We review the present level of knowledge of the hadronic structure of the photon, as revealed in interactions involving quarks and gluons “in” the photon. The concept of photon structure functions is introduced in the description of deep–inelastic $e\gamma$ scattering, and existing parametrizations of the parton densities in the photon are reviewed. We then turn to hard γp and $\gamma\gamma$ collisions, where we treat the production of jets, heavy quarks, hard (direct) photons, J/ψ mesons, and lepton pairs. We also comment on issues that go beyond perturbation theory, including recent attempts at a comprehensive description of both hard and soft γp and $\gamma\gamma$ interactions. We conclude with a list of open problems.

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1) Introduction

The photon is the simplest of all bosons. It is the gauge boson of QED, which implies that it is massless and structureless (i.e., pointlike). Predictions for γe interactions can be made with impressive accuracy, in some cases (e.g., $g-2$ [1]) to better than one part in 10^8. Indeed, for many physicists the great quantitative success of QED is one of the strongest arguments in favour of Quantum Field Theories in general.

At first glance it might therefore be surprising, perhaps even a bit embarassing, that many reactions involving (quasi–)real photons are much less well understood, both theoretically and experimentally. However, a moment's reflection will show that this is really not astonishing at all. The uncertainty principle tells us that for a short period of time a photon can fluctuate into a pair of charged particles. Fluctuations into virtual two–lepton states are well understood, and are in fact a crucial ingredient of the quantitative success of QED. Fluctuations into quark–antiquark pairs are much more problematic, however. Whenever the lifetime of the virtual state exceeds about 10^{-25} sec, corresponding to a characteristic energy or momentum scale of about 1 GeV or less, the (virtual) $q\bar{q}$ pair has sufficient time to evolve into a complicated hadronic state that cannot be described by perturbative methods only. Even if the lifetime is shorter, i.e. the energy–momentum scale is larger, hard gluon emission and related processes complicate the picture substantially.

The understanding of such virtual hadronic states becomes particularly important when they are "kicked on the mass shell" by an interaction of the photon. The most thoroughly studied reactions of this type involve interactions of a real and a virtual photon (e.g. in $e\gamma$ scattering); of two real photons ($\gamma\gamma$ scattering at e^+e^- colliders); and of a real photon with a hadron (e.g. γp scattering). All these reactions allow to probe some aspects of the hadronic nature of the photon, and we will discuss them in turn.

Why is it important to study such reactions? There are at least two answers. First, we can use data taken at present colliders to sharpen our predictions for (background) processes at future colliders. We will see that in some cases this may contribute significantly to the assessment of the potential of future colliders; this is true for high–energy linear e^+e^- colliders that are now being discussed, and especially for the so–called $\gamma\gamma$ colliders. Secondly, while the hadronic structure of the photon certainly has nonperturbative aspects, we expect the photon to be a simpler system than any real hadron, like the proton. After all, no matter how complicated it is, the hadronic structure of the photon certainly originated from the $\gamma q\bar{q}$ vertex; if this vertex vanished, the photon would not have any "hadronic nature" to speak of. No such (in principle) simple starting point can be defined for protons, constituent quarks themselves being highly complicated objects. One can therefore expect photonic reactions to be particularly well suited to study both perturbative and nonperturbative aspects of strong interactions, and everything in between. We should mention right away that the present theoretical description of, say, γp scattering is, if anything, even more complex than that of $\bar{p}p$ scattering; however, we feel that this does not refute our argument that γp collisions are simpler at some fundamental level. At the very least reactions probing the hadronic nature of the photon give us an additional handle on most aspects of perturbative and nonperturbative QCD that are of relevance for collider physics.

The remainder of this article is organized as follows. We start in Sec. 2 with a discussion of the scattering of a highly virtual (probing) photon off an almost real (target) photon, $\gamma^*\gamma$

or $e\gamma$ scattering. These were the first photonic reactions for which predictions were made in the framework of the quark parton model (QPM) [2] and within QCD [3]. $e\gamma$ scattering was also among the first of the "hard" photonic reactions, which can at least partly be described by perturbation theory, to be studied experimentally [4]. Finally, the conceptual simplicity of these processes makes them ideal for introducing the notion of photon structure functions and the parton content of the photon.

The commissioning of the ep collider HERA has opened a new era in the experimental study of hard γp collisions, increasing the available γp centre–of–mass energy by about a factor of ten compared to previous experiments. In Sec. 3 we attempt to cover both the recent intense theoretical activity [5] that has been triggered by this prospect, and the relevant experimental data.

Lately there has also been much progress in the field of hard scattering of two quasi–real photons. An important milestone was set [6] by the AMY collaboration at the TRISTAN collider, who for the first time analyzed their data using an essentially complete leading order QCD event generator. Previous generators had omitted important classes of diagrams, and had therefore not been able to reproduce data taken at the older PEP and PETRA colliders. Recent developments in this area are summarized in Sec. 4.

The emphasis in Secs. 2 to 4 is on hard processes, which are amenable to a perturbative treatment. In Sec. 5 we loosen this restriction and comment on issues that go beyond standard perturbative QCD. In particular, we discuss γp and $\gamma\gamma$ cross–sections, and related quantities, in the minijet model [7]. This model has been introduced to describe purely hadronic (pp, $\bar{p}p$) reactions; some modifications are necessary [8] before it can be used for γp and $\gamma\gamma$ scattering. Finally, Sec. 6 contains some concluding remarks, as well as a list of open problems and experimental challenges.

This field is still very much in flux. Hardly a week goes by without a new experimental study of some reaction that probes the hadronic structure of the photon, a new or refined calculation of a relevant cross–section for given photon structure, and/or a new model for or parametrization of this structure. We nevertheless think it worthwhile to summarize the present status, partly to celebrate what has already been achieved, but mostly to highlight open questions and how to address them. We hope that this review, which updates ref.[9], will be of some use both for those who have already worked on some aspects of the hadronic nature of the photon, and for those who contemplate starting such work.

2) Photon Structure Functions

In this section we introduce the concept of photon structure functions. To this end we first discuss in Sec. 2a deep–inelastic $e\gamma$ scattering, which is theoretically very clean, being fully inclusive; it is thus well suited to serve as the defining process for photon structure functions and the parton content of the photon. For reasons of space we have to be rather brief here; we refer the reader to ref.[10] for a more pedagogical introduction to photon structure functions. In Sec. 2b we then describe existing parametrizations of the photonic parton distribution functions.

2a) Deep–inelastic $e\gamma$ Scattering

Studies of deep–inelastic electron nucleon scattering,

$$eN \to eX, \tag{1}$$

where X is any hadronic system and the squared four momentum transfer $Q^2 \equiv -q^2 \geq 1$ GeV^2, have contributed much to our understanding of strong interactions. The pioneering SLAC experiments [11] played a key role in the development of both the quark parton model (QPM) [12], and, due to the observation of scaling violations and the concept of asymptotic freedom [13], of QCD. Since then our understanding of QCD in general and the structure of the proton in particular has improved greatly. The most recent progress in this area has come from the analysis of data taken at the *ep* collider HERA; see ref.[14] for a study of the impact of these data on the determination of the parton content of the proton.

Formally deep–inelastic $e\gamma$ scattering is quite similar to *ep* scattering. The basic kinematics is explained in Fig. 1. The differential cross section can be written in terms of the scaling variables $x \equiv Q^2/(2p \cdot q)$ and $y \equiv Q^2/(sx)$, where $\sqrt{s}$ is the total available centre–of–mass (cms) energy:

$$\frac{d^2\sigma(e\gamma \to eX)}{dxdy} = \frac{2\pi\alpha_{\rm em}^2 s}{Q^4} \left\{ \left[1 + (1-y)^2 \right] F_2^\gamma(x, Q^2) - y^2 F_L^\gamma(x, Q^2) \right\}; \tag{2}$$

this expression is completely analogous to the equation defining the protonic structure functions F_2 and F_L in terms of the differential cross–section for *ep* scattering via the exchange of a virtual photon.* The special significance [2] of $e\gamma$ scattering lies in the fact that, while (at present) the $x-$dependence of the nucleonic structure functions can only be parametrized from data, the structure functions appearing in eq.(2) can be *computed* in the QPM from the diagram shown in Fig. 2a:

$$F_2^{\gamma,{\rm QPM}}(x, Q^2) = \frac{6\alpha_{\rm em}}{\pi} x \sum_q e_q^4 \left\{ \left[x^2 + (1-x)^2 \right] \log \frac{W^2}{m_q^2} + 8x\,(1-x) - 1 \right\}, \tag{3}$$

where we have introduced the squared cms energy of the $\gamma^*\gamma$ system

$$W^2 = Q^2 \left(\frac{1}{x} - 1 \right). \tag{4}$$

The sum in eq.(3) runs over all quark flavours, and e_q is the electric charge of quark q in units of the proton charge. An experimental test of this equation was thought to not only allow to confirm the existence of pointlike quarks, but also to measure their charges through the e_q^4 factor; both were topics of interest in the early 1970's when the study of $e\gamma$ scattering was first proposed [2].

Unfortunately, eq.(3) depends on the quark masses m_q. If this ansatz is to describe data [15] even approximately, one has to use constituent quark masses of a few hundred MeV here; constituent quarks are not very well defined in field theory. Moreover, we now know that

*The Z exchange contribution to eq.(2) is negligible for Q^2 values that can be achieved in the foreseeable future.

QPM predictions can be modified substantially by QCD effects. In case of proton structure functions these lead to, among other things, scaling violations and a nonzero F_L. In case of $e\gamma$ scattering, QCD corrections are described by the kind of diagrams shown in Figs. 2b,c. Diagrams of the type 2b leave the flavour structure unchanged and are therefore part of the (flavour) nonsinglet contribution to F_2^γ, while diagrams with several disconnected quark lines, as in Fig. 2c, contribute to the (flavour) singlet part of F_2^γ.

The interest in photon structure functions received a boost in 1977, when Witten showed [3] that such diagrams can be computed exactly, at least in the so–called "asymptotic" limit of infinite Q^2. Including next–to–leading order (NLO) corrections [16], the result can be written as

$$F_2^{\gamma,\text{asymp}}(x,Q^2) = \alpha_{\text{em}} \left[\frac{1}{\alpha_s(Q^2)} a(x) + b(x) \right], \tag{5}$$

where a and b are *calculable* functions of x. The absolute normalization of this "aymptotic" solution is therefore given uniquely by $\alpha_s(Q^2)$, i.e. by the value of the QCD scale parameter Λ_{QCD}. It was therefore hoped that eq.(5) might be exploited for a very precise measurement of Λ_{QCD}.

Unfortunately this no longer appears feasible. One problem is that, in order to derive eq.(5), one has to neglect terms of the form $\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^P$, where Q_0^2 is some input scale (see below). Neglecting such terms is formally justified *if* $\alpha_s(Q^2) \ll \alpha_s(Q_0^2)$ and P is positive. Unfortunately the first inequality is usually not satisfied at experimentally accessible values of Q^2, assuming Q_0^2 is chosen in the region of applicability of perturbative QCD, i.e. $\alpha_s(Q_0^2)/\pi \ll 1$. Worse yet, P can be zero or even negative! In this case ignoring such terms is obviously a bad approximation. Indeed, one finds that eq.(5) contains divergencies as $x \to 0$ [3, 16]:

$$a(x) \sim x^{-0.59}, \qquad b(x) \sim x^{-1}. \tag{6}$$

The coefficient of the $1/x$ pole in b is *negative*; eq.(5) therefore predicts negative counting rates at small x. Notice that the divergence is worse in the NLO contribution b than in the LO term a. It can be shown [17] that this trend continues in yet higher orders, i.e. the "asymptotic" prediction for F_2^γ rapidly becomes more and more divergent for $x \to 0$ as more higher order corrections are included: $F_2^\gamma \sim x^{-4.3}$ in 3rd order, $\sim x^{-25.6}$ in 4th order, and so on. Clearly the "asymptotic" solution is not a very useful concept, having a violently divergent perturbative expansion.†

The worst divergencies in $F_2^{\gamma,\text{asymp}}$ occur in the singlet sector, i.e. originate from diagrams of the type shown in Fig. 2c. It has been speculated [18] that this hints at a nonperturbative solution. For example, if the invariant mass of the lowest $q\bar{q}$ pair in Fig. 2c is small, they might form a bound state. Traditionally, however, nonperturbative contributions have been estimated using the vector dominance model (VDM) [19], from the diagrams shown in Fig. 2d: The target photon undergoes a transition into a nearly on–shell vector meson ($\rho,\ \omega,\ \phi, \ldots$), so that $e\gamma$ scattering is "really" $e\rho,\ e\omega, \ldots$ scattering, which should look qualitatively like ep scattering. In particular, the contribution of Fig. 2d should itself be

†Notice that in the same "asymptotic" limit, nucleonic structure functions collapse to a $\delta-$function at $x = 0$. While this is formally correct for infinite Q^2, it gives obviously a poor description of the true proton structure at *any* finite Q^2.

well–behaved, i.e. non–singular; it *cannot* cancel the divergencies of the "asymptotic" solution.[‡]

Indeed, the existence of the contribution shown in Fig. 2d demonstrates that we cannot hope to compute $F_2^\gamma(x,Q^2)$ from perturbation theory alone. Moreover, even if we assume that the VDM correctly describes *all* nonperturbative contributions to F_2^γ, it seems essentially impossible to estimate them without making further assumptions. The problem is that the vector mesons $\rho,\ \omega,\ \phi,\ldots$ are much too short–lived to allow an independent measurement of their parton distribution functions.[§] It therefore seems to us that the only meaningful approach is that suggested by Glück and Reya [20]. That is, one *formally* sums the contributions from Figs. 2a–d into the single diagram of Fig. 2e, where we have introduced quark densities in the photon $q_i^\gamma(x,Q^2)$ such that (in LO)

$$F_2^\gamma(x,Q^2) = 2x \sum_i e_{q_i}^2 q_i^\gamma(x,Q^2), \tag{7}$$

where the sum runs over flavours, e_{q_i} is the electric charge of quark q_i in units of the proton charge, and the factor of 2 takes care or antiquarks. This is merely a definition. In the approach of ref.[20] one does not attempt to compute the absolute size of the quark densities inside the photon. Rather, one introduces input distribution functions $q_{i,0}^\gamma(x) \equiv q_i^\gamma(x,Q_0^2)$ at some scale Q_0^2. This scale is in principle arbitrary, as long as $\alpha_s(Q_0^2)$ is sufficiently small to allow for a meaningful perturbative expansion. In practice, Q_0^2 is usually chosen as the *smallest* value where this criterion is assumed to be satisfied. We will come back to this point in the following subsection.

Given these input distributions, the photonic parton densities, and thus F_2^γ, at different values of Q^2 can be computed using the inhomogeneous evolution equations. In LO, they read [3, 21]:

$$\frac{dq_{\rm NS}^\gamma(x,Q^2)}{d\log Q^2} = \frac{\alpha_{\rm em}}{2\pi} k_{\rm NS}^\gamma(x) + \frac{\alpha_s(Q^2)}{2\pi}\left(P_{qq}^0 \otimes q_{\rm NS}^\gamma\right)(x,Q^2); \tag{8a}$$

$$\frac{d\Sigma^\gamma(x,Q^2)}{d\log Q^2} = \frac{\alpha_{\rm em}}{2\pi} k_{\Sigma}^\gamma(x) + \frac{\alpha_s(Q^2)}{2\pi}\left[\left(P_{qq}^0 \otimes \Sigma^\gamma\right)(x,Q^2) + \left(P_{qG}^0 \otimes G^\gamma\right)(x,Q^2)\right]; \tag{8b}$$

$$\frac{dG^\gamma(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[\left(P_{Gq}^0 \otimes \Sigma^\gamma\right)(x,Q^2) + \left(P_{GG}^0 \otimes G^\gamma\right)(x,Q^2)\right], \tag{8c}$$

where we have used the notation

$$\left(P \otimes q\right)(x,Q^2) \equiv \int_x^1 \frac{dy}{y} P(y) q(\frac{x}{y},Q^2). \tag{9}$$

The P_{ij}^0 are the usual (LO) $j \to i$ splitting functions [22]. The inhomogeneous terms k_i^γ describe $\gamma \to q\bar{q}$ splitting, i.e. the diagram of Fig. 2a; for one quark flavour, one has

$$k_{q_i}^\gamma(x) = 3e_{q_i}^2\left[x^2 + (1-x)^2\right]. \tag{10}$$

[‡]Of course, one can always *define* F_2^γ to be the sum of the (divergent) "asymptotic" solution and a (divergent) "nonperturbative contribution". However, nothing is gained by this as long as one cannot at least estimate the nonperturbative contribution. Our argument shows that the VDM is of no help here.

[§]Assuming that such distributions can even be defined for resonance states; e.g., should a ρ be treated as a $q\bar{q}$ resonance with two valence quarks, or a $\pi\pi$ resonance with four valence quarks?

The k_i^γ of eqs.(8) follow from eq.(10) by taking appropriate sums or differences of quark flavours. Eq.(8a) describes the evolution of the nonsinglet distributions (differences of quark densities), i.e. resums only diagrams of the type shown in Fig. 2b, while eqs.(8b,8c) describe the evolution of the singlet sector ($\Sigma^\gamma \equiv \sum_i q_i^\gamma + \bar{q}_i^\gamma$), which includes diagrams of the kind shown in Fig. 2c. Notice that this necessitates the introduction of a gluon density inside the photon $G^\gamma(x, Q^2)$, with its corresponding input distribution $G_0^\gamma(x) \equiv G^\gamma(x, Q_0^2)$.

It is crucial to note that, given non–singular input distributions, the solutions of eqs.(8) will also remain [20] well–behaved at all finite values of Q^2. This is true both in LO and in NLO [23]. On the other hand, by introducing a priori unknown input distributions, one clearly abandons the hope to make an absolute prediction of $F_2^\gamma(x, Q^2)$ in terms of $\Lambda_{\rm QCD}$ alone. The solutions of eqs.(8) still show an approximately linear growth with $\log Q^2$; in this sense eq.(5) remains approximately correct, but the functions a and b now do depend weakly on Q^2 (approximately like $\log \log Q^2$), and the $x-$dependence of b is *not* computable. In fact, only the nonleading Q^2 dependence (contained in the functions a and b), which corresponds formally to the scaling violations in the case of ep scattering¶, can in this approach be used to determine $\Lambda_{\rm QCD}$; a change of the normalization $1/\alpha_s$ multiplying the first term can always be compensated by adding a constant to the input distributions. Notice that *no* momentum sum rule applies for the parton densities in the photon as defined here. The reason is that these densities are all of first order in the fine structure constant $\alpha_{\rm em}$. Even a relatively large change in these densities can therefore always be compensated by a small change of the $\mathcal{O}(\alpha_{\rm em}^0)$ term in the decomposition of the physical photon, which is simply the "bare" photon [with distribution function $\delta(1-x)$]. Formally this would manifest itself by the addition of $\mathcal{O}(\alpha_{\rm em}^2)$ contributions to the inhomogeneous terms in eqs.(8), which are numerically negligible.‖

Before discussing our present knowledge of and parametrizations for the parton densities in the photon, we briefly address a few issues related to the calculation of F_2^γ. As mentioned above, eqs.(7),(8) have been extended to NLO quite early, although a mistake in the two–loop $\gamma \to G$ splitting function was found [26] only fairly recently. A full NLO treatment of massive quarks is now also available [27] for both F_2^γ and F_L^γ. A first treatment of small$-x$ effects in the photon structure functions, i.e. $\log 1/x$ resummation and parton recombination, has been presented in ref.[28]; however, the predicted steep increase of F_2^γ at small x has not been observed experimentally [29]. Finally, nonperturbative contributions to F_2^γ are expected to be greatly suppressed if the target photon is also far off–shell. One can therefore derive unambiguous QCD predictions [30] in the region $Q^2 \gg P^2 \gg \Lambda^2$, where the first strong inequality has been imposed to allow for a meaningful definition of structure functions.**

¶Recall that in case of $e\gamma$ scattering, the QPM also predicts a logarithmic growth of F_2^γ with Q^2, see eq.(3).

‖It is sometimes argued that one can still use the "asymptotic" NLO prediction for F_2^γ to determine $\Lambda_{\rm QCD}$, if one sticks to the region of large x where the influence of the $x \to 0$ pole is supposed to be weak. However, this rests on the *assumption* that the nonperturbative contributions to F_2^γ are small at large x, and the *second assumption* that the terms that regularize the pole [24] vanish as $x \to 1$. The only way to test these assumptions seems to be to compare the value of $\Lambda_{\rm QCD}$ extracted in this way with other measurements [25] of α_s. In our opinion this shows that a measurement of F_2^γ at fixed Q^2 *cannot* be used to determine $\Lambda_{\rm QCD}$ unambiguously. Recall also that the perturbative expansion of the "asymptotic" prediction in yet higher orders in QCD is extremely problematic.

**If $Q^2 \simeq P^2$, the use of fixed–order perturbation theory is more appropriate, since it includes terms

However, it has recently been pointed out [31] that nonperturbative effects might survive longer than previously expected; an unambiguous prediction would then only be possible for very large P^2, and even larger Q^2, where the cross–section is very small.

2b) Parametrizations of Photonic Parton Densities

As discussed in the previous subsection, the Q^2 evolution of the photonic parton densities $\vec{q}^{\gamma}(x,Q^2) \equiv (q_i^{\gamma}, G^{\gamma})(x,Q^2)$ is uniquely determined by perturbative QCD, eqs.(8) and their NLO extension. However, given the flaws of the "asymptotic" prediction for $\vec{q}^{\gamma}$, one has to specify input distributions $\vec{q}_0^{\gamma}$ at a fixed $Q^2 = Q_0^2$. The situation is therefore in principle quite similar to the case of nucleonic parton densities. In practice, however, the determination of these input distributions is much more difficult in case of the photon, for a variety of reasons.

To begin with, no momentum sum rule applies for $\vec{q}_0^{\gamma}$, as discussed above. This means that it will be difficult to derive reliable information on G_0^{γ} from measurements of F_2^{γ} alone: in LO, the gluon density only enters via the (subleading) Q^2 evolution in F_2^{γ}. In contrast, analyses of eN scattering revealed very early that gluons must carry about 50% of the proton's momentum, thereby fixing the overall scale of the gluon density in the nucleon. We will see below that existing parametrizations for G^{γ} still differ by sizable factors over the entire x range unless Q^2 is very large.

Secondly, so far deep–inelastic $e\gamma$ scattering could only be studied at e^+e^- colliders, where the target photon is itself radiated off one of the incoming leptons. Its spectrum is given by the well–known Weizsäcker–Williams function [32]

$$f_{\gamma|e}(z) = \frac{\alpha_{\rm em}}{2\pi z} \left\{ \left[1 + (1-z)^2 \right] \log \frac{2E^2 \, (1-z)^2 \, (1-\cos\theta_{\rm max})}{m_e^2 z^2} - 2\,(1-z) \right\}, \qquad (11)$$

where E is the electron beam energy and zE the energy of the target photon. In eq.(11) we have assumed that there is no experimentally imposed lower bound on the virtuality $p^2 \equiv -P^2$ of the target photon, so that $P^2_{\rm min} = m_e^2 z^2/(1-z)$; however, we have introduced an upper limit $\theta_{\rm max}$ on the scattering angle of the electron emitting the target photon, in order to allow for antitagging.

Eq.(11) implies that the cross section from the measurement of which F_2^{γ} is to be determined is of order $d\sigma/dQ^2 \sim \alpha_{\rm em}^4/(\pi Q^4) \log(E/m_e)$, see eq.(2). [Recall that F_2^{γ} is itself $\mathcal{O}(\alpha_{\rm em})$.] The event rate is therefore quite small; the most recent measurements [15, 29, 33] typically have around 1,000 events at $Q^2 \simeq 5$ GeV^2, and the statistics rapidly gets worse at higher Q^2. This is to be compared with millions of events in, for example, deep–inelastic νN scattering [34].

Another problem is that the $e^{\pm}$ emitting the target photon is usually not detected, since it emerges at too small an angle. This means that the energy of the target photon, and hence the Bjorken variable x, can *only* be determined from the hadronic system. All existing analyses try to determine x from the invariant mass W, using eq.(4). This is problematic, since at least some of the produced hadrons usually also escape undetected, e.g. in the beam pipes; the measured value of W, usually denoted by $W_{\rm vis}$, is therefore generally *smaller* than the true value. (It can exceed the true W due to the finite energy resolution of real–world

$\propto (P^2/Q^2)^n$, which are not included in the usual structure functions.

detectors.) One has to correct for this by "unfolding" the measured $W_{\rm vis}$ distribution to arrive at the true W (or x) distribution. However, in order to do this one has to model the hadronic system X. In other words, a "measurement" of F_2^γ only seems possible if we already know many details about multi–hadron production in $\gamma^*\gamma$ scattering!

In practice the situation is not quite as bad, since one can check the assumptions made by comparing various distributions predicted by the Monte Carlo with real data. An iterative procedure can then be used to arrive at a model for X that should allow to do the unfolding reliably. However, one should be aware that this might lead to large uncertainties at the boundaries of the accessible range of x values. The reason is that in the event selection one usually imposes upper and lower cuts on $W_{\rm vis}$, corresponding to small and large x, respectively. Relatively minor changes of the model for X can therefore significantly change the predicted efficiency for accepting events with x close to one of the kinematic boundaries. The region of large x (small W) is in any case plagued by higher twist uncertainties (e.g., production of resonances); however, the region of small x is very interesting, since here F_2^γ is dominated by sea quarks whose density is closely related to the gluon distribution. It has been shown explicitly [35] that different ansätze for X can lead to quite different "measurements" of F_2^γ at small x.

Most recent analyses use the "FKP model" of refs.[36] as starting point of the unfolding procedure. Here F_2^γ is split into "soft" and "hard" components, depending on the virtuality of the quark exchanged in the $t-$ or $u-$channel, see figs. 2a–c. The cut–off parameter t_0 separating these two regions is to be fitted from the data. Contributions with $|t| \leq t_0$ are modelled using VDM ideas; in practice this means that a scaling (Q^2-independent) ansatz is used, fitted from low$-Q^2$ data [15]. Contributions with $|t| > t_0$ can be computed from an evolution equation in $\log|t|$, with the boundary condition that this "pointlike" part vanish at $t = t_0$. This procedure is formally equivalent to the one suggested by Glück and Reya [20], *if* we identify $Q_0^2 = t_0$ and impose the upper limit $|t|_{\rm max} = Q^2$ for the $t-$evolution. However, as presently used [37, 29, 33], the unfolding procedure has several weaknesses:

- It uses a parametrization [37] of the "pointlike" part of F_2^γ which includes some terms which are of next–to–leading order in a formal operator product expansion; however, not all such terms are included. In particular, it uses the actual kinematical maximum for the $t-$evolution, $|t|_{\rm max} \simeq Q^2/x$; as a result, the predicted F_2^γ tends to be larger at small and median x than what one would get from the usual Q^2-evolution.* On the other hand, this expression ignores sea quarks, i.e. $g \to q\bar{q}$ splitting; it therefore underestimates the true result for very small x.

- The Q^2 (or t) evolution of the "hadronic part" is ignored; at high Q^2, this over–estimates the soft contribution to F_2^γ at median and large x, and underestimates it at small x.

- While this procedure treats the effect of gluon radiation on the shape of F_2^γ (approximately) correctly, it does *not* include any parton showering in the MC event generator. Rather, the generator produces $q\bar{q}$ pairs whose p_T distribution in the $\gamma^*\gamma$ cms is either exponential (for the "hadronic part") or follows the QPM prediction (for the "pointlike part"). This $q\bar{q}$ pair is then fed into a string–based fragmentation program. While

*Very roughly, one replaces the overall growth $\propto \log Q^2$ by $\log\left(Q^2/x\right)$.

string fragmentation can mimick the effects of parton showering to some extent, it is not able to produce additional jets. It is therefore not surprising that a recent analysis [33] found larger jet rates in $e\gamma$ scattering than the MC predicted.

Some of these points have also been raised in ref.[38]. We would like to emphasize that none of these problems is intrinsic to the formalism of ref.[36]. Moreover, the input model used for the unfolding always turned out to be consistent within errors with the extracted F_2^γ, if t_0 is chosen in the vicinity of 0.5 GeV^2.

The point of this lengthy discussion is not to criticize our experimental colleagues. Rather, we hope that it might serve as a starting point for further work by people familiar with the design of MC event generators, which we are not. However, this discussion shows that present data on F_2^γ have to be taken with a grain of salt. In particular, the shortcomings listed above are usually *not* addressed in the estimates of the systematic error due to the unfolding procedure; this error only includes things like the choice of binning [29]. This might help to explain the apparent discrepancy between different data sets [15]. Fortunately, new ideas for improved unfolding algorithms [39] are now under investigation; this should facilitate the measurement of F_2^γ at small x, especially at high energy (LEP2).

In spite of this, measurements of F_2^γ probably still provide the most reliable constraints on the input distributions $\vec{q}_0^{\,\gamma}(x)$; they are certainly the only data that have been taken into account when constructing existing parametrizations of $\vec{q}^{\,\gamma}(x,Q^2)$. In the remainder of this subsection we will briefly describe these parametrizations.

The simplest and oldest parametrizations [40, 41] are based on the "asymptotic" LO prediction [3, 42]. While the theoretical basis for this prediction is weak, as described in the previous subsection, their simplicity makes these parametrizations useful for first estimates of reaction rates, as long as one stays away from very small x. Refs.[40, 41] only give parametrizations for $N_f = 4$ active flavours. More recently, Gordon and Storrow [43] provided different, more accurate fits for $N_f = 3,\ 4$ and 5.

All other parametrizations involve some amount of data fitting. However, due to the rather large experimental errors of data on F_2^γ, additional *assumptions* always had to be made. In particular, quark and antiquark distributions (of the same flavour) are always assumed to be identical, which guarantees that the photon carries no flavour. This assumption is eminently reasonable. The $\gamma q\bar{q}$ vertex treats quarks and antiquarks symmetrically, and we do not know of any effect that could destroy this symmetry.

The DG parametrization [44] was the first to start from input distributions. At the time of this fit, only a single measurement of F_2^γ at fixed $Q^2 \simeq 5.3$ GeV^2 existed. In order to determine three, in principle independent, input distributions (for nonsinglet and singlet quarks as well as gluons), two assumptions were made: All input quark densities were assumed to be proportional to the squared quark charges, i.e. $u^\gamma = 4d^\gamma = 4s^\gamma$ at $Q_0^2 = 1$ GeV^2; and the gluon input was generated purely radiatively, i.e.

$$G^\gamma_{0,\mathrm{DG}} = \frac{2}{\beta_0} \Sigma^\gamma_{0,\mathrm{DG}} \otimes P_{Gq}, \tag{12}$$

where $\beta_0 = 9$ is the coefficient of the 1–loop QCD β function. This parametrization only exists in LO. Moreover, it treats flavour thresholds by introducing three independent sets of distributions for $N_f = 3,\ 4,\ 5$, so that the transition across a threshold is not automatically smooth. Nevertheless it continues to describe data reasonably well, although a combined fit would probably give a higher χ^2 than for more recent parametrizations.

The LAC parametrizations [45] are based on a much larger data set. The main point of these fits was to demonstrate that data on F_2^γ constrain G^γ very poorly. In particular, they allow a very hard gluon, with xG^γ having a maximum at $x \simeq 0.9$ (LAC3), as well as very soft gluon distributiuons, with xG^γ rising very steeply at low x (LAC1, LAC2). The LAC parametrizations only exist for $N_f = 4$ massless flavours and in LO. *No* assumptions about the relative sizes of the four input quark densities were made in the fit. LAC3 has been clearly excluded by data on jet production in ep scattering (Sec. 3a) as well as in real $\gamma\gamma$ scattering (Sec. 4a); the experimental status of LAC1,2 is less clear.

The recent WHIT parametrizations [46] follow a similar philosophy as LAC, at least regarding the gluon input; however, their choices for G_0^γ are much less extreme. In the WHIT1,2,3 parametrizations, gluons carry about half as much of the photon's momentum as quarks do (at the input scale $Q_0^2 = 4$ GeV^2), while in WHIT4,5,6 gluons and quarks carry about the same momentum fraction. These two groups of fits also have slightly different valence quark densities.[†] The input gluon density is assumed to have the simple shape $xG_0^\gamma \propto (1-x)^{c_g}$, with $c_g = 3$ for WHIT1,4; $c_g = 9$ for WHIT2,5; and $c_g = 15$ for WHIT3,6. Recall that the normalization is adjusted such that $\int xG_0^\gamma(x)dx$ is constant within each group of parametrizations; a larger c_g therefore also means a larger G^γ at small x. The input distributions for the sea quarks are computed from the cross section for $\gamma^* g \to q\bar{q}$, regularized by light quark masses $m = 0.5$ GeV. The WHIT parametrizations only exist in LO, but great care has been taken to treat the ($x-$dependent) charm threshold correctly. This is much more important here than for nucleonic parton densities, since the photon very rapidly develops an "intrinsic charm" component from $\gamma \to c\bar{c}$ splitting.

The GRV parametrization [47] is the first NLO fit of $\vec{q}^\gamma$; a LO version is also available. This parametrization is based on the same "dynamical" philosophy as the earlier fits of protonic [48] and pionic [49] parton densities by the same authors. The idea is to start from a very simple input at a very low Q_0^2 (0.25 GeV^2 in LO, 0.3 GeV^2 in NLO); this scale is assumed to be the same for p, π and γ targets. The observed, more complex structure is then generated dynamically by the evolution equations. In case of the proton, only valence quarks were originally assumed to be present at scale Q_0^2. While the gluon density does evolve fast enough to carry approximately half the proton's momentum at Q^2 of a few GeV^2, it was found to be too soft in shape. The ansatz therefore had to be modified [50] by introducing "valence–like" gluon and even sea quark densities already at the input scale, thereby giving up much of the original simplicity of the idea. Their pionic input distributions also include a "valence–like" gluon density, which is in fact strictly proportional to the valence quark density, but no sea quarks at scale Q_0^2.

In the photonic case,

$$\vec{q}^{\,\gamma}_{0,\rm GRV}(x) = \kappa \frac{4\pi\alpha_{\rm em}}{f_\rho^2} \vec{q}^{\,\pi}_{0,\rm GRV}(x). \tag{13}$$

[†]The definition of "valence" and "sea" quarks used here differs from the more common "nonsinglet" and "singlet" distributions. The Q^2 evolution of the valence density is independent of G^γ, i.e. obeys eq.(8a), while a non–zero sea quark density is produced only through $g \to q\bar{q}$ splitting. In the absence of mass effects, the valence distributions are proportional to the squared quark charges, while the sea distributions are independent of the quark charge.

This input is motivated from VDM ideas, where f_ρ^2 determines the $\gamma \to \rho$ transition probability ($f_\rho^2/4\pi \simeq 2.2$), and κ has been introduced to describe contributions from heavier vector mesons ($\omega, \phi, \ldots$). In fact, κ is the *only* free parameter in this ansatz; it was determined to be $\kappa = 2$ (1.6) in LO (NLO). However, it should be clear that eq.(13) is an *assumption* that has to be tested experimentally; in particular, it is not obvious that the parton densities in a pion resemble those in a vector meson, or that QCD is applicable at such small input scales.[‡] On the positive side, the GRV parametrization ensures a smooth onset of the charm density, using an $x-$independent threshold. Moreover, care has been taken to split the NLO parametrizations for $\vec{q}^\gamma$ into LO and NLO pieces; only the former should be multiplied with NLO pieces of hard cross sections.[§]

The GS parametrizations [43] were developed shortly after GRV, but follow a quite different strategy. Problems with low input scales [53] are avoided by choosing $Q_0^2 = 5.3$ GeV^2. This is certainly in the perturbative region, but necessitates a rather complicated ansatz for the input distributions:

$$\vec{q}^\gamma_{0,\mathrm{GS}}(x) = \kappa \frac{4\pi\alpha_{\mathrm{em}}}{f_\rho^2} \vec{q}^\pi_0(x, Q_0^2) + \vec{q}^\gamma_{\mathrm{QPM}}(x, Q_0^2). \tag{14}$$

The free parameters in the fit are the momentum fractions carried by gluons and sea–quarks in the pion, the parameter κ, and the light quark masses. In the GS2 parametrization, G_0^γ is assumed to come entirely from the first term in eq.(14), while in GS1 the second term also contributes via radiation, see eq.(12). The fit gives $m_u = m_d = 0.29$ GeV and $\kappa = 1.96$. These values are not unreasonable. However, the ansatz (14) might be somewhat suspect: It holds in the perturbative region of Q^2, but its form is not invariant under the evolution equations. On the other hand, for practical purposes it includes sufficiently many free parameters to allow a decent description of data on F_2^γ. This parametrization is now being updated [54]. The new version uses a slightly reduced input scale $Q_0^2 = 3$ GeV^2, and for the first time includes data on jet production in two–photon collisions (see Sec. 4a) in the fit; unfortunately this still does not allow to pin down G^γ with any precision.

The AGF parametrization [55] is (in its "standard" form) quite similar to GRV. In particular, they also assume that at a low input scale $Q_0^2 = 0.25$ GeV^2 the photonic parton densities are described by the VDM. There are some differences, however. First, AGF point out that in NLO the input densities are scheme–dependent, if physical quantities like F_2^γ are to be scheme–independent. GRV use their ansatz (13) in their DIS$_\gamma$ scheme, since it is perturbatively more stable than the more commonly used $\overline{\mathrm{MS}}$ scheme. AGF point out that this treatment includes certain process–dependent terms in $\vec{q}^\gamma$; they therefore prefer to use the $\overline{\mathrm{MS}}$ scheme, and define their input distributions to be the difference of a (regular) "VDM" term and a process–independent term containing a $\log(1-x)$ divergence, so that $F_2^\gamma(x, Q_0^2)$ is well–behaved in the limit $x \to 1$. Secondly, they include $\rho - \omega - \phi$ interference

[‡]It has been shown previously [51] that an ansatz like eq.(13) cannot be brought into agreement with the data if one insists on $Q_0^2 \geq 1$ GeV^2. One way out [28] would be to multiply the resulting F_2^γ with something like $Q^2/(Q^2+\mu^2)$, but this goes beyond the leading–twist partonic contributions.

[§]This distinction is more important in the photonic case, since possible $\log(1-x)$ divergencies are *not* always regularized by parton densities falling like a power of $1-x$. However, even in the photonic case this problem is greatly ameliorated if one uses the "DIS$_\gamma$" scheme introduced [52] by the same authors.

when specifying the "VDM" input, so that

$$\begin{aligned} u^\gamma_{0,\mathrm{AGF}}(x) &= K\alpha_{\mathrm{em}}\left[\frac{4}{9}u^\pi_{\mathrm{valence}}(x,Q_0^2)+\frac{2}{3}u^\pi_{\mathrm{sea}}(x,Q_0^2)\right]; \\ d^\gamma_{0,\mathrm{AGF}}(x) = s^\gamma_{0,\mathrm{AGF}}(x) &= K\alpha_{\mathrm{em}}\left[\frac{1}{9}u^\pi_{\mathrm{valence}}(x,Q_0^2)+\frac{2}{3}u^\pi_{\mathrm{sea}}(x,Q_0^2)\right]; \\ G^\gamma_{0,\mathrm{AGF}}(x) &= K\alpha_{\mathrm{em}}\frac{2}{3}G^\pi(x,Q_0^2), \end{aligned} \tag{15}$$

where the pion distribution functions are taken from ref.[56]. (Recall that in NLO the input quark densities have to be modified by a subtraction term.) Perhaps the for practical purposes most important difference from the GRV parametrization is that the coefficient K is left free, i.e. separate fits are provided for the "anomalous" (or "pointlike") and "nonperturbative" contributions to $\vec{q}^\gamma$, allowing the user to specify the absolute normalization (although not the shape) of the latter.

Finally, two of the SaS parametrizations [38] are based on a similar philosophy as the GRV and AGF parametrizations, by assuming that at a low $Q_0 \simeq 0.6$ GeV the perturbative component vanishes (SaS1). However, while the normalization of the nonperturbative contribution is taken from the VDM (including ρ, ω and ϕ contributions with fixed normalization), the shapes of the quark and gluon distributions are fitted from data. Although the SaS parametrizations are available in LO only, the authors attempt to estimate the scheme dependence (formally an NLO effect) providing a parametrization (SaS1M) where the non leading–log part of the QPM prediction for F_2^γ has been added to eq.(7), while SaS1D is based on eq.(7) alone. There are also two parametrizations with $Q_0 = 2$ GeV; however, in this case the normalization of the fitted "soft" contribution had to be left free, and its shape is much harder than one what expects from hadronic parton densities. Again two sets of parametrizations are available, using different schemes (SaS2D, SaS2M).

The SaS1 sets preferred by the authors are quite similar to AGF; the real significance of ref.[38] is that it carefully describes the properties of the hadronic state X for both the hadronic and "anomalous" contributions, as needed for a full event characterization. We will come back to this aspect of their work in Sec. 5.

In Fig. 3 we compare various LO parametrizations of F_2^γ at $Q^2 = 15$ GeV^2 with recent data taken by the OPAL [29] and TOPAZ [33] collaborations; present data are not able to distinguish between LO and NLO fits. In order to allow for a meaningful comparison, we have added a charm contribution to the OPAL data, as estimated from the QPM; this contribution had been subtracted in their analysis. We have used the DG and GRV parametrizations with $N_f = 3$ flavours, since their parametrizations of c^γ are meant to be used only if $\log Q^2/m_c^2 \gg 1$; the charm contribution has again been estimated from the QPM.¶ As discussed earlier, WHIT provides a parametrization of c^γ that includes the correct kinematical threshold, while LAC treat the charm as massless at all Q^2.

We see that most parametrizations give quite similar results for F_2^γ over most of the relevant $x-$range; the exception is LAC1, which exceeds the other parametrizations both at large and at very small x. It should be noted that the data points represent averages over the respective x bins; the lowest bin starts at $x = 0.006$ (0.02) for the OPAL (TOPAZ) data.

¶We have ignored the small contribution [27] from $\gamma^* g \to c\bar{c}$ in this figure.

The first OPAL point is therefore in conflict [35] with the LAC1 prediction. Unfortunately there is also some discrepancy between the TOPAZ and OPAL data at low x. As discussed above, one is sensitive to the unfolding procedure here; for this reason, WHIT chose not to use these (and similar) points in their fit. (The other fits predate the data shown in Fig. 3.) This ambiguity in present low$-x$ data is to be regretted, since in principle these data have the potential to discriminate between different ansätze for G_0^γ. This can most clearly be seen by comparing the curves for WHIT4 (long dashed) and WHIT6 (long–short dashed), which have the *same* valence quark input, and even the same $\int xG_0^\gamma dx$: WHIT4 has a harder gluon input distribution, and therefore predicts a larger F_2^γ at $x \simeq 0.1$; WHIT6 has many more soft gluons, and therefore a very rapid increase of F_2^γ for $x \leq 0.05$, not unlike LAC1. Finally, we should mention that the GS, AGF and SaS parametrizations also reproduce these data quite well.

Discriminating between these parametrizations would be much easier if one could measure the gluon density directly. This is demonstrated in Fig. 4a, where we show results for xG^γ at the same value of Q^2; we have chosen the same LO parametrizations as in Fig. 3, and included the LAC3 parametrization with its extremely hard gluon density. Note that, for example, WHIT4 and WHIT6 now differ by a factor of 5 for x around 0.3. The gluon distribution of WHIT6 is rather similar in shape to the one of LAC1, but significantly smaller in magnitude. Indeed, in all three LAC parametrizations, gluons carry significantly more momentum than quarks for $Q^2 \leq 20$ GeV^2; this is counter–intuitive [43], since in known hadrons, and hence presumably in a VMD–like low$-Q^2$ photon, gluons and quarks carry about equal momentum fractions, while at very high Q^2 the inhomogeneous evolution equations (8) predict that quarks in the photon carry about three times more momentum than gluons. Notice finally that GRV predicts a relatively flat gluon distribution. This results partly from the low value of the input scale $Q_0^2 = 0.25$ GeV^2, compared to 1 GeV^2 for DG and 4 GeV^2 for WHIT and LAC1; a larger Q^2/Q_0^2 allows for more radiation of relatively hard gluons off large$-x$ quarks. Recall also that their (pionic) input distribution includes a valence–like (hard) gluon density.

Finally, Fig. 4b shows that some parametrizations also differ substantially in the flavour structure. Both DG and WHIT assume $d^\gamma = s^\gamma$ at all Q^2. Moreoever, DG assumes $q_{i,0}^\gamma \propto e_{q_i}^2$ for the entire input quark distribution, while WHIT assumes this only for the valence input. Nevertheless the smaller value of Q_0^2 assumed by DG leads to a strangeness content quite similar to that predicted by WHIT: At very low x, sea quarks dominate, which have $u^\gamma = d^\gamma = s^\gamma$, so that $s^\gamma/(u^\gamma + d^\gamma) \simeq 1/2$; at high x, $q_i^\gamma \propto e_{q_i}^2$, so that $s^\gamma/(u^\gamma + d^\gamma) \simeq 1/5$.[‖] Since sea quarks are produced by gluon splitting, the transition between the sea–dominated and valence–dominated regions depends on G^γ.

In contrast, GRV assumes $u_0^\gamma = d_0^\gamma$ and $s_0^\gamma = 0$ at the input scale; for given F_2^γ, the former assumption increases $u^\gamma + d^\gamma$ and the latter reduces s^γ, compared to the ansatz $q_i^\gamma \propto e_{q_i}^2$. This explains the smallness of the strangeness content of the photon predicted by GRV, which persists to surprisingly large values of x. It is worth mentioning that AGF, which is otherwise quite similar to GRV [at least for the standard normalization of the nonperturbative contribution, $K = 1$ in eqs.(15)], also assumes the input valence quark

[‖]The ratio of 4:1 between large$-x$ (valence) u and d quark densities is implicit to the structure of the WHIT fits, while in DG, u^γ and d^γ are parametrized independently; this explains the $\sim 2\%$ deviation between the two parametrizations shown in Fig. 4b at large x.

distributions to be proportional to the squared quark charges; its predictions for $s^\gamma/(u^\gamma+d^\gamma)$ are therefore quite similar to those of the WHIT group. GS falls between WHIT and GRV, since here a small but nonzero s_0^γ is assumed, from the pionic sea quarks as well as the QPM part in eq.(14). On the other hand, LAC treats all four quark input distributions as completely independent quantities, without imposing any constraints between them. This results in the erratic behaviour of the strangeness content depicted in Fig. 4b; the ratio even exceeds unity for $x \simeq 0.1$. Clearly LAC should therefore not be used when the flavour structure of the photon is important, e.g. in W and Z boson production at HERA.

3) Resolved Photon Processes in γp Scattering

In this section we discuss γp scattering reactions that are sensitive to the hadronic structure of the photon. Most of our numerical results will be for the *ep* collider HERA, with appropriate (anti–)tagging conditions for the outgoing electron, in order to make sure that the virtuality of the exchanged photon is small; we will also make a few comments on fixed target γp scattering. We follow the terminology of ref.[57] in distinguishing between *direct* and *resolved photon* contributions to a given process; the existence of these physically distinct contributions had already been emphasized in ref.[42]. The former are defined as reactions where the incident photon participates directly in the relevant hard scattering process; some examples are shown in Fig. 5a. In contrast, in resolved photon contributions the incoming photon takes part in the hard scattering via one of its constituents, a quark or gluon, as shown in Fig. 5b. Notice that in direct processes the entire photon energy goes into the hard partonic final state, while in resolved photon processes only a fraction x_γ of the photon energy is available for the hard scattering. This also leads to different topologies for the two classes of contributions. In LO, the direct contribution to jet production will give rise to two high$-p_T$ jets and the remnant jet from the proton, see Fig. 6a; there will be little or no hadronic activity in the direction of the incident photon. In contrast, resolved photon contributions are characterized by a second spectator jet, which is formed when a coloured parton is "taken out" of the photon; this photonic remnant or spectator jet will usually go approximately in the direction of the incident photon, which coincides with the electron direction at HERA. This additional jet in principle allows distinction between direct and resolved photon contributions on an event–by–event basis.

Occasionally one sees the erroneous statement in the literature that resolved photon contributions are NLO corrections to the corresponding direct process. However, by counting powers of coupling constants in Figs. 5a and 5b one can easily convince oneself that this is not the case. The direct contribution to di–jet production is obviously of order $\alpha_{\rm em}\alpha_s$. The resolved photon contribution is of order $\vec{q}^\gamma \cdot \alpha_s^2$, where $\vec{q}^\gamma$ stands for any photonic parton density. We have seen early in Sec. 2 that these densities are of first order in $\alpha_{\rm em}$. Moreover, we also saw that they grow logarithmically with the relevant scale Q^2 of the given process; in leading–log summed perturbation theory this $\log Q^2$ has to be counted as a factor $1/\alpha_s$, see eq.(5).* Altogether the photonic parton densities therefore have to be counted as $\mathcal{O}\left(\alpha_{\rm em}/\alpha_s\right)$, so that $\vec{q}^\gamma \cdot \alpha_s^2$ is again of order $\alpha_{\rm em}\alpha_s$, just like the direct contribution. It

*Recall that this expression remains approximately correct even beyond the "asymptotic" approximation, if we allow a weak Q^2 dependence of the functions a and b.

should be emphasized that LO QCD is *always* leading log summed, i.e. $\alpha_s(Q^2) \cdot \log Q^2$ is always counted as $\mathcal{O}(1)$, not $\mathcal{O}(\alpha_s)$. To give one example, the resummation of such leading logarithms leads to the scaling violations of nucleonic structure functions; it should be clear that one is *not* performing an NLO analysis by simply using Q^2-dependent parton densities when estimating $p\bar{p}$ cross–sections.

Part of the confusion is caused by the fact that in NLO, direct and resolved photon contributions mix. This can, e.g., be seen from the Feynman diagram of Fig. 7, which shows an NLO contribution to direct jet production. If no restrictions are imposed on the transverse momentum of the outgoing antiquark, it could go in the photon direction and thus form a "remnant jet". In fact, the total contribution from the diagram of Fig. 7 to the inclusive jet (pair) cross–section will be dominated by configurations where the transverse momentum of the antiquark is small, due to the $1/t$ pole associated with the exchanged quark. The crucial point is that the contribution from this pole has already been included in the resolved LO qq' scattering process, where the exchanged quark is treated as being on–shell. In order to avoid double–counting, the contribution of the $1/t$ pole therefore has to be subtracted from the NLO contribution of Fig. 7; in other words, the corresponding collinear divergence is absorbed in the quark distribution function in the photon. We emphasize that this treatment is completely analogous to the calculation of NLO corrections to jet production in $p\bar{p}$ scattering. For example, the incident photon in Fig. 7 could be replaced by a gluon coming from the $\bar{p}$. The above argument then tells us that qq' and gq scattering mix in NLO, i.e. a part of the NLO contribution from gq scattering has to be absorbed in qq' scattering; nevertheless nobody would consider one to be an NLO correction to the other.†

We do not attempt to split resolved photon contributions into those coming from the "anomalous", "pointlike" or "perturbative" part of $\vec{q}^\gamma$ and those due to the "hadronic" or "nonperturbative" part. We have seen in the previous section that it is not easy to separate these parts consistently; indeed, it should be clear that in reality there is a smooth transition from the one to the other. Nevertheless we will see later (in Sec. 5) that the existence of the "pointlike" contribution may have some impact on overall event characteristics.

After these preliminaries, we are ready to discuss various hard γp reactions. We start with jet production in Sec. 3a, where both NLO calculations and high–energy data from the ep collider HERA are available. Open heavy quark production (Sec. 3b) has also been treated in NLO, and first HERA data have started to appear. Direct photon production (Sec. 3c) also also been treated in NLO, but no HERA data have yet been published. We then discuss the production of J/ψ mesons in Sec. 3d, and lepton–pair (Drell–Yan) production in Sec. 3e.

3a) Jet Production in γp Collisions

The production of high$-p_T$ jets offers the largest cross–section of all hard γp scattering reactions. It was therefore the first such process for which resolved photon contributions were calculated [58], and also among the first reactions to be studied experimentally at

†The diagram of Fig. 7 also has a divergence when the exchanged gluon goes on–shell; this is absorbed in the gluon density in the proton, i.e. in direct γg scattering. Finally, in principle both the exchanged quark and the gluon can be (nearly) on–shell. However, in this case none of the final state partons has large transverse momentum; this configuration therefore only contributes to soft processes, which cannot be treated perturbatively.

HERA [59, 60]. Moreover, many properties of direct and resolved photon contributions to jet production carry over to the production of heavy quarks and direct photons, to be discussed in subsequent subsections. We therefore treat jet production in somewhat more detail than other photoproduction processes.

Feynman diagrams contributing to jet production in γp scattering are sketched in Fig. 5. In leading order (LO), direct contributions (Fig. 5a) come from either the "QCD Compton" process (left diagram), or from photon–gluon fusion (right diagram); notice that, unlike DIS, this latter process is sensitive to the gluon content of the proton already in LO. The resolved photon contributions (Fig. 5b) involve the same matrix elements for $2 \to 2$ QCD scattering processes that appear in calculations of jet production at purely hadronic ($p\bar{p}$ or pp) scattering. Note that, in contrast to the direct processes, many of these QCD scattering processes can proceed via the exchange of a gluon in the $t-$ or $u-$channel; processes like $gg \to q\bar{q}$ that only proceed via the exchange of a quark in the $t-$ or $u-$channel, and/or the exchange of a gluon in the $s-$channel, contribute only little to the inclusive jet cross–section.

In LO, the cross–section for the electroproduction of two (partonic) jets with transverse momentum p_T and (pseudo)rapidities* $\eta_1,\ \eta_2$ can be written as:

$$\frac{d^3\sigma(ep \to e j_1 j_2 X)}{dp_T d\eta_1 d\eta_2} = 2 p_T x_e x_p \sum_{i,j,k,l} f_{i|e}(x_e) f_{j|p}(x_p) \frac{d\hat{\sigma}_{ij\to kl}(\hat{s},\hat{t},\hat{u})}{d\hat{t}}. \tag{16}$$

Here, i stands for a photon, quark or gluon, and $j,\ k,\ l$ stand for a quark or gluon. If $i = \gamma$ (direct contribution), the function $f_{i|e}$ is just the Weizsäcker–Williams photon flux (11); otherwise it is given by

$$f_{i|e}(x_e) = \int_{x_e}^{1} \frac{dz}{z} f_{\gamma|e}(z) f_{i|\gamma}\left(\frac{x_e}{z}\right). \tag{17}$$

The resolved photon contribution to the cross–section (16) therefore only depends on the product x_e of the fraction z of the electron energy carried by the incident photon, and the fraction x_γ ($= x_e/z$) of the photon energy carried by the parton in the photon. Of course, $f_{i|\gamma}$ and $f_{j|p}$ are nothing but the parton densities in the photon and proton, respectively.

The (pseudo)rapidities of the jets are related to these Bjorken variables by:

$$x_p = \frac{1}{2} x_T \sqrt{\frac{E_e}{E_p}} \left(e^{\eta_1} + e^{\eta_2}\right); \tag{18a}$$

$$x_e = \frac{1}{2} x_T \sqrt{\frac{E_p}{E_e}} \left(e^{-\eta_1} + e^{-\eta_2}\right), \tag{18b}$$

with $x_T = 2p_T/\sqrt{s}$, where $\sqrt{s}$ is the ep centre–of–mass energy; note that in our convention positive rapidities correspond to the direction of the proton. Finally, the subprocess cross–sections $\hat{\sigma}$ for the direct [61, 58] and resolved photon [62] contributions depend on the Mandelstam variables describing the hard partonic scattering, with $\hat{s} = x_e x_p s$ and

$$\hat{t} = -\frac{\hat{s}}{2}\left(1 \pm \sqrt{1 - \frac{4p_T^2}{\hat{s}}}\right); \tag{19}$$

*There is no difference between the partonic rapidity and pseudorapidity in LO.

both solutions in eq.(19) have to be included when evaluating eq.(16), which can be accomplished by simply symmetrizing the subprocess cross–sections under $\hat{t} \leftrightarrow \hat{u}$.

Direct and resolved photon contributions to the photoproduction of jets were first compared by Owens [58] in 1979 for fixed target energies, where the resolved photon contributions were found to be subdominant, except at very small p_T. Although a few theoretical analyses [63, 64] of the photoproduction of jets appeared in the first half of the 1980's, the importance of resolved photon contributions was fully appreciated only in 1987, when it was realized [57, 65, 66] that at HERA energies, they could exceed the direct contribution by as much as a factor of ten at $p_T \simeq 5$ GeV, and remained dominant out to $p_T \simeq 35$ to 40 GeV.

An update [67] of this result is shown in Fig. 8, where the ratio of resolved photon and direct contributions to the single jet inclusive cross–section[†] is plotted for the nominal HERA energy $\sqrt{s} = 314$ GeV. Unlike in ref.[66], we have imposed some acceptance cuts, taken from a recent ZEUS analysis [68]: The jet has to fall in the pseudorapidity range $-1 \leq \eta_{\rm jet} \leq 2$, and the "antitag" requirement that the outgoing electron is not seen in the main detector implies that the photon virtuality $Q^2 \leq 4$ GeV^2. Most results in Fig. 8 have been obtained using the MRSD-' parametrization [69] for the parton densities in the proton; comparison between the solid and the dotted curve, which is for the MRSD0' parametrization, shows that even the pre–HERA uncertainty of nucleon densities only leads to an uncertainty of a few percent in the ratio of Fig. 8, except at very small p_T. HERA data on deep–inelastic scattering have since then improved our knowledge of the nucleon structure considerably; the impact of these data on parametrizations of $G^p(x)$ is still under investigation [70], but it is already clear that very soon the uncertainty from the parton densities in the nucleon will be negligible compared to the differences between predictions based on various parametrization of $\vec{q}^{\gamma}$. In particular, HERA data clearly favour MRSD-' over MRSD0'; we therefore take the former as our standard choice.

We see that implementation of the acceptance cuts reduces the region where resolved photon contributions are dominant to $p_T \leq 25$ to 30 GeV; this is mostly due to the upper limit on $\eta_{\rm jet}$, which reduces the resolved photon contribution much more than the direct one (see below). Nevertheless, the former still exceeds the latter by a factor between 5 and 11 at $p_T = 5$ GeV; at present our lack of knowledge of $\vec{q}^{\gamma}$ does not allow us to predict this ratio more precisely. This dominance of resolved photon contributions at small p_T can partly be explained by the fact that they get contributions from gluon exchange in the $t-$channel, see Fig. 5b; this enhances the squared matrix elements for resolved photon processes by a factor $\hat{s}/|\hat{t}| > 2$, compared to those for direct processes. Colour factors generally also favour the former over the latter. Finally, we saw in Figs. 4 that for small x_γ, the parton densities in the photon can actually exceed $1/\alpha_{\rm em}$ substantially; for small x_e, which contribute only at small p_T, the integral in eq.(17) can therefore enhance resolved photon contributions even further.[‡] On the other hand, this convolution integral decreases more rapidly with increasing x_e than the photon flux factor $f_{\gamma|e}$ does; this explains the more rapid decrease of resolved photon contributions with increasing p_T, and thus the shape of the curves in Fig. 8.

Obviously parametrizations with sizable G^{γ} (LAC1, WHIT4) predict a considerably larger resolved photon contribution at small p_T than those with smaller G^{γ} [WHIT1, WHIT3;

[†]Recall that by definition, events with two accepted jets count twice here.

[‡]Recall that the factor of $\alpha_{\rm em}$ contained in $\vec{q}^{\gamma}$ is cancelled in the ratio by the explicit factor of $\alpha_{\rm em}$ appearing in the $\hat{\sigma}$ for direct processes.

DG, GRV, AGF and GS2 (not shown) also belong to this class]. Note that the prediction from the WHIT1 parametrization exceeds that using WHIT3 even at the smallest p_T shown; this indicates that the p_T distribution integrated over jet rapidities is not sensitive to very small x_γ, where the WHIT3 gluon density exceeds that of WHIT1, see Fig. 4b. Finally, we should warn the reader that additional experimental cuts can change the ratio of direct and resolved photon contributions substantially. This is demonstrated by the dot–dashed curve, which has been obtained with the same parametrizations of parton densities in the proton and photon as the solid curve, but where we have demanded that the outgoing electron be detectable in the ZEUS luminosity monitor; this implies $Q^2 < 0.01$ GeV^2 and, more importantly, $0.2 \leq z \leq 0.75$. The lower cut on the scaled photon energy z greatly reduces direct contributions at low p_T, while the upper cut reduces resolved photon contributions at high p_T more than direct ones; as a result, the p_T-dependence of the ratio of the two contributions becomes considerably steeper.

In Fig. 9 we further split the resolved photon (9a) and direct (9b) contributions depending on whether the two high$-p_T$ partons in the final states are of two quarks, two gluons, or a quark and a gluon. Since the contributions from $q\bar{q} \rightarrow gg$ and $gg \rightarrow q\bar{q}$ are very small, the curves in Fig. 9a can also be read as coming from qq, gg and qg initial states, while in Fig. 9b the qg and qq final states come from γq and γg initial states, respectively. We have used the WHIT1 parametrization for $\vec{q}^\gamma$, and applied the same acceptance cuts as in Fig. 8.

WHIT1 assumes a relatively small G^γ; as a result, the gg final state is dominant only for $p_T \leq 5$ GeV. However, even the WHIT4 parametrization, which assumes a two times larger input distribution G_0^γ, predicts the cross–section for the gg final state to be well below that for the qg final state for $p_T > 10$ GeV. Note that in the majority of qg events the quark comes from the photon and the gluon from the proton; this is partly because the photon is assumed to be relatively poor in gluons, and partly because the cut $\eta_{\rm jet} \leq 2$ removes many events with a gluon in the photon in the initial state, as discussed below. We saw in sec. 2 that the quark densities in the photon are much better known than G^γ, except at small x_γ. Together with the result of Fig. 9a, this explains the rapid convergence of the curves in Fig. 8, although some difference between the LAC1 and WHIT predictions persists even at large p_T (see also Fig. 3[§]). Finally, the comparison of Figs. 9a and 8 shows that two quark final states dominate resolved photon contributions only for values of p_T where the total jet cross section is already dominated by direct contributions.

As expected from Fig. 8, the direct contributions shown in Fig. 9b have a considerably flatter transverse momentum spectrum than the resolved photon contributions. Unfortunately, photon–gluon fusion (the two quark final state) dominates the direct contribution only at relatively small p_T; this is because the gluon density in the proton is softer (i.e., decreases more rapidly with increasing x_p) than the valence quark distributions. Since at small p_T the inclusive jet cross section is dominated by resolved photon contributions, a direct study of photon–gluon fusion, which might allow to further constrain the gluon density in the proton even at rather small x_p [71], will be difficult unless the resolved photon contribution can be suppressed by additional cuts.

As shown in Figs. 10, the (pseudo)rapidity distribution of the jet can be used to help

[§]Notice, however, that all quark flavours contribute equally to the jet cross–section, while contributions to F_2^γ are weighted by the squared charge. Two parametrizations can therefore have very similar F_2^γ and yet lead to different predictions for quark–initiated jet production at HERA.

disentangle direct and resolved photon contributions. In Fig. 10a we have used the present HERA energy $\sqrt{s} = 296$ GeV, and applied the same cuts on the parton level that the ZEUS collaboration applied [68] on their reconstructed jets. Unfortunately the p_T cut chosen is still too low to allow a direct comparison between partonic and jet cross sections, since a substantial part of the jets still comes from the "underlying event" (beam fragments, initial state radiation, and possibly multiple interactions producing minijets [72, 73]; see sec. 5); these effects are expected to become more important as one approaches the proton beam direction, which corresponds to $\eta = +\infty$ in our convention, and can thus distort the jet rapidity distribution compared to the parton–level distribution shown in Figs. 10. We nevertheless expect that the *differences* between the curves shown in these figures will not be washed out by the underlying event.

We see that in this single–differential cross section the direct contribution is always subdominant; however, one can enhance its relative importance by requiring $\eta_{\rm jet} < 0$. Of more interest for us is the opposite region of positive, sizable $\eta_{\rm jet}$, which is sensitive to small x_e, see eq.(18b), and thus allows to probe the parton densities in the photon at small x_γ, see eq.(17). Indeed, the figure shows that in this region one is most sensitive to differences between the various parametrizations of $\vec{q}^\gamma$. We also observe again that these differences are much larger than even the pre–HERA uncertainty from nucleonic structure functions.

We have already noted above that the jet rapidities only depend on the product $x_e = z \cdot x_\gamma$; events with large $\eta_{\rm jet}$ can come from soft partons in hard photons (small x_γ, large z), but also from hard partons in soft photons (large x_γ, small z). Fortunately, ZEUS has demonstrated [68] the ability to (approximately) determine z purely from the longitudinal momenta of particles in the main detector, without having to detect the outgoing electron in the luminosity monitor, which would reduce the accepted event rate by about a factor of four. This should allow one to increase the lower cut on z from 0.2 to 0.5, which enhances the relative importance of contributions with small x_γ, so that the differences between predictions based on different ansätze for $\vec{q}^\gamma$ become larger, as illustrated in Fig. 10b. (H1 prefers to only use photoproduction events where the outgoing electron is tagged in the small–angle detector; z can then be determined from its energy.) Although this stronger cut on z reduces the resolved photon contribution by slightly more than a factor of two for $\eta_{\rm jet} \geq 1$, it should still enhance the discriminative power of this measurement, given that the ZEUS data sample [68] with the looser cuts contains almost 20,000 events with reconstructed jets.

The integration over the rapidity of the second jet in single–jet inclusive cross sections leads to a substantial spread in x_e and x_p, see eqs.(18)¶; in order to further increase the sensitivity of the jet rate to the region of small x_γ in general and the gluon density in the photon in particular, one therefore has to study more differential cross–sections. In Figs. 11a,b we show predictions for the triple–differential di–jet cross section, as given by eq.(16), for $\sqrt{s} = 314$ GeV, $p_T = 10$ GeV and $\eta_1 = \eta_2 \equiv \eta$. As in Fig. 9a we display resolved photon contributions with different final states separately, as predicted from the WHIT1 parametrization. In Fig. 11a we have applied the antitag cut $Q^2 \leq 4$ GeV^2 on the virtuality of the photon, but we have not restricted the allowed range of the scaled photon energy z. We see that the direct contribution now dominates at $\eta < 0$, and remains sizable even for $\eta = 2$; in the region of negative η it is mostly due to photon–gluon fusion. Note that our

¶The integration over p_T is less important here, since most events will have $p_T \simeq p_{T,{\rm cut}}$ anyway.

choice $\eta_1 = \eta_2$ implies $\hat{t} = \hat{u} = -\hat{s}/2$, which minimizes the dynamical enhancement factor $\hat{s}/|\hat{t}|$ of most resolved photon contributions. Together with the slightly larger value of p_T this explains why direct contributions appear more prominent in Fig. 10a than in Fig. 9a.

Contributions from the gluon in the photon peak at $\eta \simeq +2$. The reason is that a gluon usually only carries a rather small fraction of the photon energy, see Fig. 4a; for given $\hat{s} \geq 4p_T^2$, a large contribution to the total energy in the hard sub–process then has to come from the proton, leading to a rather strong boost of the high$-p_T$ partons in the proton direction. The quarks in the photon can be substantially more energetic, and therefore already contribute at $\eta \simeq 0$.

Although according to the WHIT1 parametrization, the contribution from gluons in the photon is clearly enhanced at large η, at $\eta = 2$ slightly more than half of the total cross–section still comes from quarks in the photon or direct processes. As before, the sensitivity to the region of small x_γ can be further enhanced by imposing cuts on the scaled photon energy z. This is illustrated in Fig. 11b, where we have required $0.3 \leq z \leq 0.8$. The lower limit has been chosen such that the direct contribution vanishes identically for $\eta > 0$, see eq.(18b). (Recall that $x_e = z$ for direct contributions, while $x_e = zx_\gamma$ otherwise.) Moreover, the relative importance of contributions from gluons in the photon is clearly enhanced compared to Fig. 11a; in particular, the gg final state is now dominant for $\eta \simeq 2$, where only $\sim 20\%$ of the total cross section is predicted to come from quarks in the photon. Recall that G^γ might well be larger than assumed in the WHIT1 parametrization, so that the contributions from gluons in the photon might be even larger. It should therefore be possible to derive stringent constraints on G^γ even using jets with relatively large p_T, by focussing on events with large rapidity and large z. Of course, the sensitivity to resolved photon contributions involving soft partons in the photon is even larger [74] if one can measure a cross–section for fixed z; however, this will need a rather large event sample.

If z and both jet rapidities have been measured, one can in principle reconstruct both x_p and x_γ, using eqs.(18). In practice, HERA experiments use an estimator for x_γ to separate the events into "direct" and "resolved" samples; this estimator reduces to x_γ in LO QCD, but only uses the measured rapidities and transverse energies of the jets, and can thus also be computed in NLO (where it will differ from the partonic x_γ). The ZEUS collaboration [75] has used 284 reconstructed di–jet events from the first run of HERA to show that the x_γ distribution has a peak near $x_\gamma = 1$, as expected from direct contributions. The H1 collaboration [76] has gone one step further and subtracted the contribution from quarks in the photon, estimated using the GRV parametrization. They find evidence for a non–vanishing contribution from gluons in the photon only for $x_\gamma \leq 0.2$, thereby ruling out the LAC3 parametrization (see Fig. 4a). H1 reconstructs G^γ in the range $0.02 \leq x_\gamma \leq 0.2$ at an effective scale $\mu^2 \simeq 60$ GeV^2; the result is in good agreement with GRV, but disfavours LAC1. However, a fairly sophisticated Monte Carlo analysis was necessary to extract G^γ even using the LO formalism. For example, we saw in the discussion of Fig. 10a that parts of the "underlying event" contribute to the reconstructed jets, which obscures the relation between hard partons and jets. The H1 analysis [76] depends on such details quite sensitively. It might therefore be somewhat premature to exclude the LAC1 parametrization on the basis of this evidence alone.

The ZEUS collaboration has also published [77] an updated jet analysis, based on 12,000 reconstructed di–jet events with $E_T(\text{jet}) \geq 6$ GeV. They study $d\sigma/d\overline{\eta}$ for 4,000 events with

$|\eta_1 - \eta_2| \leq 0.5$, where $\overline{\eta}$ is the average pseudorapidity of the two jets; this quantity is closely related to the double differential cross section $d^2\sigma/(d\eta_1 d\eta_2)$ at $\eta_1 = \eta_2$. They find reasonable agreement between their data and LO QCD predictions in the "direct" sample (events with large measured x_γ), but most parametrizations of q^γ give too low predictions for the "resolved" sample. This again indicates that at this rather low value of E_T the "underlying event" plays quite an important role; as we will discuss in more detail in Sec. 5, this aspect is not well described by present standard QCD Monte Carlo event generators.

Direct and resolved photon contributions are also expected to have different distributions in $\cos\theta^*$, where θ^* is the cms scattering angle [78]. Due to diagrams where a gluon is exchanged in the $t-$ or $u-$channel, resolved photon contributions are more strongly peaked at small θ^* than direct contributions. The increasing importance of the latter over the former at higher p_T means that the $\cos\theta^*$ distribution of di–jet events with large p_T, will be flatter than at low p_T.

As remarked earlier, the most obvious distinction between direct and resolved photon events is that only the latter contain a photonic spectator (or remnant) jet, see Fig. 6. In Fig. 12 we show the average energy of this jet in the lab frame, as predicted from the WHIT1 parametrization; on the parton level, this energy is simply given by $E_e \cdot z \cdot (1 - x_\gamma)$. For negative η (of the high$-p_T$ jets), the spectator jet is rather soft, since both z and x_γ have to be large, so that $1 - x_\gamma$ is quite small. For $\eta \simeq 0$, events with a gluon from the photon usually have quite large z, but moderate x_γ, yielding a high spectator jet energy; it declines at large η since the average z becomes smaller. Quark–initiated events typically have considerably larger x_γ and hence a softer photonic spectator jet. Obviously the average spectator jet energy will increase (decrease) if a lower (upper) cut on z is applied.

In principle the results of Fig. 12 offer another possibility to enhance the contributions from gluons in the photon, by requiring the presence of an energetic remnant jet in the electron direction. However, in practice the energy of this jet cannot be measured very accurately, since some part of it will usually be lost in the beam pipe. It should nevertheless be emphasized that the first analysis of jet data [59] taken during the HERA pilot run found substantial energy deposition in the backward calorimeter even if all high$-p_T$ jets have positive rapidities; this can be understood only if the photonic remnant jet is included in the MC simulation. This jet has recently been studied in more detail by the ZEUS collaboration [79]; we will discuss their results in Sec. 5.

Recently new data from the Fermilab fixed target photoproduction experiment E683 have been published [80]. It uses a tagged photon beam with mean lab energy of 260 GeV, giving a mean $\sqrt{s}$ of slightly over 20 GeV. A Monte Carlo analysis suggest that their di–jet sample, required to have two reconstructed jets with average $p_T > 4$ GeV, gets approximately equal contributions from direct and resolved photon contributions, in agreement with theoretical expectations. However, unlike at HERA, no direct experimental evidence for the existence of resolved photon contributions could be established (other than the overall event rate). In particular, the hadronic energy flow in the very forward direction was found to be quite similar for direct and resolved photon events, and even for di–jet events produced from a pion beam; this somewhat counter–intuitive result can be explained in terms of jet fluctuations.

So far all our predictions have been computed in LO in QCD. As well known, the overall normalization of such predictions is uncertain, since in the leading log approximation used here, one cannot with certainty determine the values of the factorization scales in the parton distribution functions and the renormalization scale appearing in the running QCD coupling constant. These scales have no physical significance; predictions would be independent of them if all orders of perturbation theory could be summed. One therefore expects reduced scale dependence already in next–to–leading order (NLO). Moreover, many quantities can be predicted meaningfully only if one allows at least three high$-p_T$ partons in the final state; these include the dependence of jet cross–sections on the jet definition, and the distribution in the transverse opening angle between the two (hardest) jets in events with (at least) two jets.

The first step towards a full NLO calculation of jet photoproduction was taken in 1980 with a calculation [81] of direct $2 \to 3$ cross sections (e.g. $\gamma q \to qgg$ and $\gamma g \to q\bar{q}g$). A first complete NLO calculation of the direct contribution, including virtual (1–loop) corrections, was performed [82] in 1986. These results were applied to jet production at fixed target energies in [83], and at HERA energies in [84]. An NLO prediction of resolved photon contributions to jet production become possible only after corrections to the hard partonic QCD cross–sections had been computed [85]. These results were applied to jet production from resolved photons at HERA in ref.[86]. Finally, in refs.[87, 88, 89, 90], complete NLO calculations for single–jet inclusive jet cross–sections were presented, including both direct and resolved photon contributions.

Typical results are presented in Fig. 13, adapted from Bödeker et al. [89]. We show the scale dependence of the predicted jet cross–section at HERA for $E_T = 25$ GeV and $\eta_{\rm jet} = 1.5$. Note that E_T and p_T are in general no longer identical in NLO, since now a jet might be made up of two partons. In Fig. 13 two partons have been merged into a single jet if $\Delta R \equiv \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \leq 0.7$.

The solid and short dashed curves have been obtained by setting all three scales (the renormalization scale, and the factorization scales for the photonic and nucleonic parton densities) equal to each other. As expected, the LO prediction exhibits a much stronger scale dependence than the NLO result. It is worth noting, however, that for the "natural" choice $\mu = p_T$ (or E_T in NLO), LO and NLO predictions almost coincide. Generally the difference between LO and NLO predictions was found to be quite small at HERA energies if $\mu = p_T$ has been chosen and jets are defined with ΔR between about 0.7 and 1.* This means that the results of Figs. 8–12 should not be affected too much by NLO corrections.

As already discussed in the beginning of this section, in NLO the distinction between direct and NLO contributions is blurred. Diagrams like the one shown in Fig. 7 contribute to direct jet production in NLO, but they also contain a logarithmically divergent piece which has already been included in the LO resolved photon contribution; this piece therefore has to be subtracted from the NLO direct contribution. This subtraction term grows logarithmically with the photonic factorization scale M_γ, which is also the scale appearing in the photonic parton distribution functions. In NLO the direct contribution (dotted curve) therefore decreases with increasing M_γ, while the resolved photon contribution (dot–dashed curve) increases. The sum of the two (long dashed) is nearly independent of M_γ; the dependence does not cancel completely since in NLO the subtraction term is exactly proportional to $\log M_\gamma$, while the photonic parton densities only increase approximately like $\log M_\gamma$, as

*There appears to be some discrepancy between the results of ref.[88] and the earlier calculations [86, 87]; this is now being sorted out (M. Greco, private communication).

discussed in Sec. 2.

We saw above that di–jet cross sections that are differential in both jet rapidities are more powerful discriminators of photonic parton densities than single–jet inclusive cross sections. Unfortunately, in conventional NLO calculations going from single–jet to di–jet cross sections introduces considerable complications;[†] as a result, only the direct production of jet pairs has been treated in NLO to date [91]. The urgent need for a full calculation is emphasized by the recent ZEUS data [77], which have the potential to discriminate between different parametrizations of photonic parton densities once the cross section can be predicted with some confidence.

Recently the calculation of jet rates at HERA has been further refined by including two additional effects. First, eq.(18a) shows that (direct) jet production at negative rapidities probes the proton structure at quite low values of x_p; for $p_T \leq 10$ GeV, partons with x_p as low as 10^{-3} contribute. Small$-x$ effects may then become important, and one may have to use [92] the so–called k_T factorization [93]. Secondly, if one defines the photoproduction sample with a rather moderate (no–tag) cut on the outgoing electron, one includes contributions where the photon virtuality Q^2 may not be entirely negligible; recall, for example, that the ZEUS cuts [68] include events with Q^2 up to 4 GeV^2. On the other hand, when using the simple Weizsäcker–Williams approximation (17), one assumes Q^2 to be small compared to all other scales in the problem. This is still a good approximation for the direct contribution in this case, but the parton densities in the photon become suppressed [30, 94] once $Q^2 > \Lambda^2_{\rm QCD}$. The simple factorization (17) then breaks down, but one can still define a "parton density in the electron", which will depend on the experimental cut on Q^2 [95]. For the ZEUS cuts, this suppression only amounts to a few percent.

Finally, as mentioned in the discussion of Fig. 10, so far jet production at HERA could only be investigated experimentally at rather moderate transverse momenta, where the "underlying event" can still contribute significantly to the reconstructed jets. The influence of the underlying event might be less problematic when one studies the production of high$-p_T$ particles [96, 82] rather than jets. On the other hand, one now has to specify not only parton distribution functions, but also fragmentation functions, before a prediction can be made. Also, the cross–section falls off very rapidly with p_T, forcing one to work at scales of only a few GeV where the convergence of the perturbative expansion might be rather slow. Moreover, one either has to experimentally identify different particle species (chiefly pions and kaons, if only charged particles are counted), or make assumptions about the relative abundances of these species. Recent measurements of the cross–section for the production of charged particles with $p_T \geq 1.5$ GeV, by both the H1 and ZEUS collaborations [97], find good agreement with a theoretical NLO prediction [98] as far as the p_T spectrum is concerned; the pseudorapidity distribution measured by H1 is less well described, but the experimental errors do not allow to draw a definite conclusion at this point.

3b) The Photoproduction of Heavy Quarks

The production of heavy quarks offers two theoretical advantages over the production of light partons (jets) discussed in the previous subsection. First, their large mass $m_Q \gg \Lambda_{\rm QCD}$

[†]This step should be much easier using the Monte Carlo method of ref.[83].

ensures that QCD perturbation theory is applicable in all of phase space, although non-perturbative corrections $\propto (\Lambda_{\rm QCD}/m_Q)^{n\geq 1}$ might not be negligible for charm quarks. In particular, the total cross–section without any cuts could now be predicted with some reliability if the values of certain parameters (m_Q, $\Lambda_{\rm QCD}$ and the parton distribution functions) were known precisely. Secondly, at least in leading order the number of contributing partonic processes is much smaller, making heavy quark production easier to analyze. Specifically, in LO only photon–gluon fusion contributes to $Q\bar{Q}$ production from direct photons, while the relevant resolved photon processes are gg fusion and light $q\bar{q}$ annihilation.*

In LO, the $Q\bar{Q}$ production cross–section in ep scattering is still given by eqs.(16)–(18). However, $\eta_{1,2}$ now have to be interpreted as true rapidities, which differ from the pseudorapidity for massive particles; moreover, x_T in eqs.(18) is now given by $2\sqrt{p_T^2 + m_Q^2}/\sqrt{s}$, and eq.(19) has to be replaced by

$$\hat{t} = \frac{\hat{s}}{2}\left[\frac{2m_Q^2}{\hat{s}} - 1 \pm \sqrt{1 - \frac{4\left(m_Q^2 + p_T^2\right)}{\hat{s}}}\right]. \tag{20}$$

The partonic cross–sections $\hat{\sigma}$ for $\gamma g \rightarrow Q\bar{Q}$ and $gg, q\bar{q} \rightarrow Q\bar{Q}$ can be found in refs.[100] and [101, 102], respectively.

Resolved photon contributions to heavy quark production were first treated in ref.[44], for the case of the top quark, whose mass was then believed to be in the vicinity of 40 GeV. We now know [103] that the top quark is too heavy to be produced at HERA, but $c\bar{c}$ and $b\bar{b}$ pairs will be produced copiously. It was pointed out in refs.[57, 104] that (for most parametrizations of q^γ) resolved photon contributions to the total cross–sections are subdominant, but not negligible; e.g., they amount to $\sim 20\%$ for the DG parametrization. Resolved photon contributions to the production of heavy quark pairs are therefore considerably less important than for high$-p_T$ jet production. The reason is that now the resolved photon processes also only involve gluon exchange in the $s-$channel or (heavy) quark exchange in the $t-$ or $u-$channel; there is no enhancement factor $\hat{s}/|\hat{t}|$, unlike for jet production. Moreover, the two sub–processes involving the parton content of the photon have rather small colour factors, again in contrast to the matrix elements appearing in resolved photon contributions to jet production. These analyses also showed that gg fusion is predicted to dominate over $q\bar{q}$ annihilation, by a factor of 10 (3.5) in case of the DG parametrization and $c\bar{c}$ ($b\bar{b}$) production. The resolved photon contribution therefore offers a good opportunity to constrain G^γ, while (in LO) the direct contribution is proportional to G^p.

We saw in the previous subsection that at HERA, contributions from gluons in the photon are most important at sizable, positive rapidities, but are suppressed at negative rapidity. The rapidity distribution therefore offers a good handle for separating the two contributions to $Q\bar{Q}$ production. In Fig. 14 we show the p_T spectrum of c and b quarks at central rapidity,

*It can be argued that at very high transverse momentum, $p_T \gg m_Q$, $\alpha_s \log p_T/m_Q$ should be counted as $\mathcal{O}(1)$, rather than $\mathcal{O}(\alpha_s)$. In this case the "excitation" processes $Qg \rightarrow Qg$ and $Qq \rightarrow Qq$ also contribute in LO, since the $Q-$quark density in the photon grows logarithmically with the hard scale of the process. In these reactions the heavy quark jet is balanced by a light quark or gluon jet, while in $Q\bar{Q}$ creation events two heavy quark jets occur with equal and opposite p_T. These "flavour excitation" contributions are now under study [99].

$y_1 = y_2 = 0$. We see that the resolved photon contribution (difference between solid and dashed curves) only amounts to 10–15% at small p_T, and becomes entirely negligible for $p_T > 5$ GeV. In fact, as shown in ref.[66], the resolved photon contribution to heavy quark production at HERA can always be suppressed to an insignificant level by requiring (at least) *one* of the two heavy quarks to emerge at $y < 0$.[†] Note that we have chosen the WHIT4 parametrization in Fig. 14, which is characterized by a rather large and hard G^γ; it predicts that resolved photon contributions amount to more than 20% (30%) of the total $c\bar{c}$ ($b\bar{b}$) cross–section at HERA. This contribution is more important for $b\bar{b}$ production, since the direct contribution is suppressed by the small charge of the b quark; for parametrizations with rather hard G^γ (WHIT1,4, DG, GS2, GRV) this suppression is stronger than the relative reduction of the resolved photon contributon with increasing m_Q, which is caused by the additional convolution (17) with the gluon density in the photon.

Since resolved photon contributions to $Q\bar{Q}$ production are insignificant at high p_T (unless their importance is enhanced by specific cuts, as discussed below), the ratio of $c\bar{c}$ to $b\bar{b}$ cross–sections in the region $p_T \geq 10$ GeV simply reflects the ratio of their squared charges. This relative suppression of the $b\bar{b}$ cross–section means that $b-$tagging at HERA will be significantly more difficult than it is at $p\bar{p}$ colliders, where $b\bar{b}$ and $c\bar{c}$ cross–sections become equal at high p_T. At $p\bar{p}$ colliders the harder fragmentation function of $b-$flavoured hadrons [105] means that inclusive high$-p_T$ muon production is dominated by $b\bar{b}$ events; this will not be true at HERA, however, except at very high p_T where the cross–section is quite small. Charm quarks will also be a serious background to $b-$tagging by micro–vertex detectors at HERA.

As mentioned earlier, the resolved photon contribution is concentrated at positive rapidities. Just as in the case of jet production it can be isolated by a cut on the scaled incident photon energy z; e.g., requiring $z > 0.3$ at $p_T = 10$ GeV removes all direct contributions with $y_1 = y_2 > 0$. Fig. 15 shows that the remaining resolved photon contribution is indeed very sensitive to the gluon content of the photon. Even according to the WHIT1 parametrization the $q\bar{q}$ annihilation contribution (short dashed) is considerably below the one from gg fusion (long dashed). The WHIT4 parametrization therefore predicts a considerably larger cross–section, but the shape of the rapidity distribution is similar to that predicted from WHIT1. In contrast, the LAC1 prediction differs in both normalization and shape. However, we should warn the reader that without the cut on z, even at $y_1 = y_2 = 2$ the direct contribution would be at least ten times larger than the resolved one; the experimental implementation of this cut therefore has to be very efficient. It might even be necessary to require the presence of a photonic remnant jet to extract the resolved photon contribution; recall that this jet is expected to be quite energetic in events that originate from the gluon content of the photon. Recall also that there will be a large contribution from "charm excitation" [99] if only one of the two high$-p_T$ jets is tagged as a heavy quark. Finally, the cross–section shown in Fig. 15 is quite small, even though we have not yet required any specific charm signal (e.g., a hard muon or reconstructed D^* meson). Clearly HERA will have to accumulate significantly more data than the present 6 pb^{-1} (as of the end of 1994) to measure such triple–differential cross–sections even at lower p_T.

The predictions shown in Figs. 14 and 15 were computed in LO. NLO calculations of

[†]This result does not hold for the LAC3 parametrization; fortunately, this parametrization is excluded by other data, as discussed in secs. 3a and 4a.

the photoproduction of heavy quarks exist [104, 106, 107]; unlike for jet production, even the fully differential cross–section is available in NLO [108]. It has been demonstrated [109] that the direct contribution can be extracted reliably also in NLO, i.e. the sensitivity to the gluon content of the proton is not degraded. In view of their smaller size, extraction of the resolved photon contribution might prove more difficult. In particular, in direct events with an additional hard parton in the final state ($\gamma g \rightarrow Q\bar{Q}g,\ \gamma q \rightarrow Q\bar{Q}q$) the heavy quarks can occur at large positive rapidity even after a cut on z has been applied; one may have to veto the occurence of additional high$-p_T$ jets, and/or require the two highest p_T jets to be back–to–back in the transverse plane, in order to efficiently suppress such backgrounds to the cross–section shown in Fig. 15.

The considerable body of data on photoproduction of charm at fixed target energies ($\sqrt{s}\ \leq 20$ GeV) is well described by NLO QCD calculations [110]. However, at these low energies the resolved photon contribution is quite small; it is significant only on the backward direction (opposite to the incident photon) [108], where the experimental acceptance is poor.

Very recently, first data on charm production at HERA have become available. The ZEUS collaboration [111] searched for fully reconstructed $D^{\pm *}$ mesons. They observe a signal of 48 ± 11 events within the acceptance region $p_T(D^*) > 1.5$ GeV, $|\eta(D^*)| < 1.5$; this corresponds to $\sigma(ep \rightarrow D^{\pm *}X) = (32 \pm 7\ ^{+4}_{-7})$ nb at $\sqrt{s}\ = 296.7$ GeV with $Q^2 \leq 4$ GeV^2 and $0.15 \leq z \leq 0.86$. They then attempt to estimate the *total* $c\bar{c}$ cross–section from this measurement; this, however, sensitively depends on the extrapolation of the cross–section into kinematic regions where it has not been measured, which introduces a strong dependence of the "measured" $c\bar{c}$ cross–section on the assumed parton distribution functions in the proton and photon as well as on m_c. This not only greatly increases the quoted (systematic) error; even the central value depends on these assumptions. It does not make much sense to compare this "measured" cross–section with different theoretical predictions, since "measurement" and "prediction" depend on the same quantities! Notice also that the prediction for the total $c\bar{c}$ cross section suffers from large uncertainties [112]. Since $\alpha_s(m_c)$ is still quite large, the perturbative expansion only converges slowly, which manifests itself in a rather strong dependence of even the NLO prediction on the factorization and renormalization scales. Moreover, the prediction is very sensitive to m_c, decreasing by more than a factor of three when m_c is increased from 1.2 to 1.8 GeV. Finally, small$-x$ effects might be sizable [112, 113]. All these sources of theoretical uncertainties are reduced once we require the charm quarks, or their fragmentation and decay products, to have significant transverse momentum. One should therefore directly compare QCD predictions for the cross–section in the experimentally accessible region with the data.

The same remarks also apply to the as yet preliminary analysis of charm production by the H1 collaboration [114], which is based on events with a hard muon. They find 484 events where at least one muon satisfies $p_T(\mu) > 1.5$ GeV and $30^\circ \leq \theta(\mu) \leq 130^\circ$; some 280 of these events are expected to contain fake muons, or muons from π and K decays. This gives an accepted cross–section $\sigma(ep \rightarrow \mu^\pm X) = (2.03 \pm 0.43 \pm 0.7)$ nb; about 95% of this signal is expected to come from $c\bar{c}$ events, the rest coming from $b\bar{b}$ production.

We attempted to reproduce the cross–sections measured by the ZEUS and H1 collaborations with a parton–level MC generator based on LO QCD expressions. We take the renormalization and factorizations scales to be $\sqrt{m_c^2 + p_T^2}$ and $m_c = 1.6$ GeV; as mentioned earlier, the p_T cuts greatly reduce the sensitivity to m_c. However, these cuts also introduce an additional difficulty in the theoretical treatment. They are sufficiently high so that fragmentation effects will play a role. On the other hand, p_T cannot safely be assumed to be much larger than m_c here, so factorizing the result into a hard production cross section and a fragmentation function may not yet be a good approximation. We therefore ran our MC programs with two different options, using the standard Peterson et al. fragmentation functions [105] or no fragmentation at all. In the former case we also include contributions from the charm in the photon ($qc \to qc$ and $gc \to gc$);[‡] after all, the use of both fragmentation and structure functions rests on the factorization theorem, so it seems reasonable to treat them symmetrically.

In our comparison with the ZEUS result we always include a factor of 0.26, which is [111] the probability for a charm quark to fragment into a charged D^* meson. We find [99] that if we leave out both fragmentation and the contribution from the charm in the photon, we can reproduce the experimental cross section only if we assume a gluon distribution that increases rapidly at small x; MRSD-' works well, while the prediction of MRSD0' is too low by nearly a factor of two for all reasonable choices of momentum scale. Due to the rapidity cut, the contribution from resolved photon processes (chiefly gg fusion) only amounts to 20% or less. On the other hand, if we include fragmentation effects in the standard way, the average p_T of the charm quarks in accepted events increases by nearly a factor of two, thereby reducing the sensitivity to very small x; moreover, the total cross section is now actually dominated by contributions involving the charm content in the photon, chiefly cg scattering. As a result, we can reproduce the experimental cross section using either MRSD0' or MRSD-' partons in the proton, and LAC1 or WHIT partons for the photon; the DG parametrization now predicts a far too large cross section, since it assumes $c^\gamma = u^\gamma$, which is manifestly a bad approximation at these rather low momentum scales.

The H1 sample is less sensitive to the gluon in the proton at small x, due to both the $p_T(\mu)$ cut (which leads to a considerably higher mean transverse momentum for accepted charm quarks than in the ZEUS data sample), and the requirement that $\theta(\mu) \leq 130°$. The data therefore do not even allow to discriminate between MRSD0' and MRSD-' if we ignore both fragmentation and the charm content of the photon. In this case the resolved photon contribution is quite small, so that any (reasonable) combination of photon and proton structure functions is in agreement with the data, yielding a LO cross section of about 1.4 to 2.4 nb. If we include both charm fragmentation and the charm content of the photon, the sensitivity to soft gluons in the proton is reduced even further, and predictions using MRSD0' differ from those using MRSD-' by less than 0.1 nb. The LAC1 and WHIT parametrizations yield a predicted cross–section of about 1.5 nb, very close to the MRSD0' prediction without fragmentation and without the contribution from charm in the photon. The DG parametrization gives a prediction of about 2.9 nb; as already discussed, this parametrization over–estimates the charm content of the photon, but even this high number is not inconsistent with the experimental result.

3c) Direct Photon Production in γp Scattering

Interest in the "deep–inelastic Compton process" $\gamma p \to \gamma X$ dates back to the early days of the quark–parton model [115]; among other things, the cross–section for the simplest

[‡]There is also a very small contribution of this kind from the charm in the proton, which we neglect.

contributing sub–process, $\gamma q \to \gamma q$, depends on the fourth power of the quark charge, and thus allows an independent determination of these charges. However, very soon after the introduction of the concept of the parton content of the photon it was realized [42] that other subprocesses contribute to the production of direct photons* at high p_T: Not only resolved photon processes like $q\bar{q} \to g\gamma$ and $gq \to q\gamma$ have to be included in a consistent LO QCD treatment, but also fragmentation processes, where a high$-p_T$ parton fragments into a hard photon and an additional (nearly collinear) jet; note that the $q \to \gamma$ and $g \to \gamma$ fragmentation functions [41] are $\mathcal{O}(\alpha_{\rm em}/\alpha_s)$, just like the parton densities in the photon. A full LO treatment therefore has to include direct processes like $\gamma q \to gq \to gq\gamma$ and resolved photon processes like $qq' \to qq' \to qq'\gamma$; these processes involve the same partonic matrix elements that appear in direct and resolved photon contributions to the production of high$-p_T$ jets. Hence direct photon production is in general actually more complicated to analyze theoretically than jet production, contrary to statements often found in the literature. On the other hand, photons are easier to study experimentally than jets; in particular, their energy can be measured considerably more precisely.

In LO the triple differential cross–section for the production of a direct photon in processes that do not involve parton $\to$ photon fragmentation is still given by eq.(16); the cross–section for fragmentation processes contain a convolution with a fragmentation function:

$$\frac{d^3\sigma_{\rm frag}(ep \to e\gamma jX)}{dp_T d\eta_\gamma d\eta_j} = \sum_{i,j,k,l} \int \frac{dz'}{z'} f_{i|e}(x_e) f_{j|p}(x_p) D_{k\to\gamma}(z') \frac{d\hat{\sigma}_{ij\to kl}}{d\hat{t}}. \tag{21}$$

Eqs.(18),(19) also still apply here, but for the fragmentation contribution one has to replace p_T by $p'_T = p_T/z'$. Moreover, unlike the case of di–jet production, the final state particles are now distinguishable. Note that eqs.(18) allow a two–fold ambiguity for $\eta_{1,2}$ in terms of the Bjorken$-x$ variables; taking $\eta_1 \equiv \eta_\gamma$, one has

$$\eta_\gamma = \ln\left[\sqrt{\frac{E_p}{E_e}}\frac{x_p}{x_T}\left(1 \pm \sqrt{1 - \frac{x_T^2}{x_e x_p}}\right)\right]. \tag{22}$$

In case of jet production one can arbitrarily fix the sign in eq.(22), since this only corresponds to the definition of which of the two partons gives "jet 1". However, in case of direct photon production the sign in eq.(22) is correlated with the choice of sign in $\hat{t}$, eq.(19). In particular, if $\hat{t}$ is defined as the momentum transfer from the incident photon, or parton in the incident photon, to the final photon, or parton fragmenting into the final photon, taking a + sign in eq.(19) means $|\hat{t}| > \hat{s}/2$, which implies that one has to take the + sign in eq.(22); recall that we define the proton direction has having positive rapidity.

The first quantitative estimates of cross–sections for the production of direct photons in γp scattering, including all sub–processes listed above, were presented in refs.[116], using very simple ansätze for parton distribution and fragmentation functions. The first partial NLO analysis for fixed–target energies was published in ref.[41], where only NLO corrections to $\gamma q \to \gamma q$, as well as the direct NLO process $\gamma g \to q\bar{q}\gamma$, were included; all other contributions were treated in LO. This was justified by the observation that at these (low) energies a rather modest p_T cut on the outgoing photon suffices to greatly suppress contributions

*As opposed to photons stemming from the decay of hadrons, mostly π^0 and η mesons.

involving photonic parton densities and/or fragmentation functions. Notice that only processes involving both the parton content of the photon and parton to photon fragmentation can proceed via gluon exchange in the $t-$ or $u-$channel; one needs to convolute the cross section with *two* additional functions, compared to simple $\gamma q \to \gamma q$ scattering, before the $\hat{s}/|\hat{t}|$ enhancement characteristic of these gluon exchange processes becomes available. It is therefore not surprising that resolved photon contributions are not quite as important in the case of high$-p_T$ direct photon production as they are for jet production.

The analysis of ref.[41] was repeated independently, as well as extended to HERA energies, in ref.[117]. However, here the resolved photon contributions involving fragmentation (which have comparatively soft p_T spectra) were omitted, while the contribution from $\gamma g \to \gamma g$ (via a box diagram [118]) was included; this last contribution was shown to be significant in certain regions of phase space (see below), even though it is formally of next–to–next–to–leading order (NNLO).

In contrast, the emphasis of refs.[119] was on processes involving the parton content of the photon. In particular, it was shown that a clear signal from the gluon content of the photon (largely from $gq \to \gamma q$) can be extracted from a sample of events with *fixed energy* of the incident photon. Due to the large Lorentz boost from the parton–parton cms to the lab frame these photons can be quite energetic if they emerge at large rapidity, even at rather small p_T.[†]

A further step towards a full NLO analysis was taken in ref.[120], where corrections to the resolved photon contributions were included; however, all fragmentation contributions were still treated in LO. Notice that inclusion of NLO corrections to non–fragmentation contributions are necessary to reduce the dependence on the scale appearing in the fragmentation functions. In NLO, fragmentation and non–fragmentation contributions mix, just like direct and resolved photon processes do. For example, $gq \to gq\gamma$ is an NLO correction to $gq \to q\gamma$, but also part of the LO contribution involving $q \to \gamma$ fragmentation *if* the final state q and γ are (nearly) collinear. In order to avoid double counting, this collinear (divergent) contribution therefore has to be subtracted from the NLO correction. This subtraction term is proportional to the logarithm of the scale appearing in the fragmentation function. Since the subtraction term (obviously) appears with a negative sign in the final result it largely cancels the dependence on this "fragmentation scale"; the cancellation is not perfect, since the fragmentation function resums all orders of this leading logarithm, while the subtraction term does not. On the other hand, since the fragmentation contributions have only been treated in LO in ref.[120], a rather strong dependence on the renormalization scale appearing in α_s remains.

In Fig. 16 we show some LO predictions for the rapidity dependence of the direct photon cross–section, adapted from numerical results of ref.[120]. We see that the "Born" cross–section (from $\gamma q \to \gamma q$) peaks at negative rapidity (in the direction of the incoming photon); even though the $1/\hat{u}$ pole of the hard matrix element favours configurations where the final photon is emitted in the proton direction, negative rapidities are favoured since they probe the proton at small x, where the (sea) quark densities increase quickly. In the resolved photon process $g^\gamma q^p \to \gamma q$ the final photon is also preferentially emitted in the proton direction; moreover, at large positive rapidity one becomes sensitive to the gluon density in

[†]The same is true for very forward jets, of course. However, it should be significantly easier to detect a photon at small angle than to reconstruct a jet just a few degrees away from the proton beam.

the photon at small x, where it is (presumably) large. As a result, contributions $\propto G^\gamma$ are expected to dominate at large, positive η_γ, as first noted in refs.[119]. Notice that resolved photon contributions involving parton to photon fragmentation have not been included in Fig. 16; however, we already know from the discussion of Sec. 3a that at large positive rapidity they are also dominated by contributions $\propto G^\gamma$. Finally, the box contribution, $\gamma g \to \gamma g$, is important only at very negative η_γ, where one probes the gluon density in the proton at small x.

The results of Fig. 16 have been computed using LO expressions. While NLO corrections are significant (e.g., they change the rapidity distribution of the direct contributons [41, 117]), they do not change the conclusion that direct photon production offers a good handle for constraining G^γ. More recently this conclusion has been challenged on different grounds [122, 123]: The integral over the photon spectrum (11) tends to smear out the distributions, which in Fig. 16 were shown for a fixed energy (10 GeV) of the incident photon. In particular, direct contributions from rather soft initial photons also populate the region of positive rapidity. These papers also complete the NLO calculations by including corrections to the fragmentation processes, but these corrections have little bearing on the question whether direct photon production is useful for constraining the gluon content of the photon.

Ref.[123] introduces two additional refinements. It makes use of an NLO parametrization [124] of the parton to photon fragmentation functions, which updates the "asymptotic" expressions of ref.[41]. More importantly, this analysis for the first time introduces an isolation requirement. At fixed target energies, backgrounds from π^0 and η decays can be suppressed quite reliably, since they contain two photons with (usually) substantial opening angle [125]. At higher energy, and higher p_T^γ, this opening angle becomes much smaller, making it more difficult to detect both photons individually. At $p\bar{p}$ colliders a direct photon signal could therefore only be detected [126] if the photons were isolated, i.e. after events were discarded if more than some maximal (small) amount of energy was found in a cone around the photon. Such a cut reduces the fragmentation contribution considerably; in particular, the contribution from $g \to \gamma$ fragmentation becomes very small, since here most of the energy usually goes into the accompanying jet rather than the photon. Nevertheless, at hadron colliders contributions from $q \to \gamma$ fragmentation can still be significant [127]. Strictly speaking, the necessity to impose an isolation cut at HERA has not yet been demonstrated. Indeed, the only background study we are aware of [128] reaches the conclusion that even without isolation cut the background in the most interesting region of positive rapidity can be suppressed to the 25% level, which might be tolerable. However, this study ignores resolved photon contributions to the background (they are included for the signal); we saw in Sec. 3a that these contributions will increase the total jet cross–section (and hence also the cross–section for the production of high$-p_T$ particles) by a large factor at positive rapidity. In the absence of a more complete background study we are therefore inclined to believe that an isolation cut will indeed be necessary at HERA.

In Fig. 17, which has been adapted from numerical results of ref.[123], we show the rapidity dependence of $\sigma(ep \to e\gamma X)$ after requiring that the hadronic energy in a cone $\delta \equiv \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}/\cosh\eta_\gamma = 0.4$ around the outgoing photon be less than 10% of the energy of that photon. No cut on the energy of the incident photon has been imposed. We see that, except at very large η_γ, the cross–section is dominated by the direct contributions and resolved photon contributions involving the quark content of the photon only. In particular, in sharp contrast to the "Born" cross–section of Fig. 16, after integration over the incident photon spectrum the direct contribution has a very flat rapidity distribution. The authors concluded that, while one might be able to further constrain the quark densities in the photon from this measurement (see the difference between the dotted curve, obtained from the GS parametrization, and the solid one, which is for the GRV parametrization), there is very little sensitivity to the gluon content of the photon.

However, we saw in Sec. 3a that even a rather modest lower cut on the energy of the incident photon greatly suppresses direct photon contributions at positive rapidity; see e.g. Fig. 10. Such a cut would also reduce contributions $\propto q_i^\gamma$ substantially, but would have much less effect on contributions $\propto G^\gamma$. The sensitivity to the gluon content of the photon would presumably be enhanced even more if the rapidity of the jet balancing the direct photon can be measured as well [122][‡]. We therefore believe that the conclusions of refs.[122, 123] are probably too pessimistic. On the other hand, HERA will have to accumulate a large body of data before a substantial number of direct photon events with known (fixed) energy of the incident photon will become available; only then can predictions like those shown in Fig. 16 be compared with experiment.

So far no HERA data on direct photon production have been published. Data from a fixed target photoproduction experiment exist [125]; they agree with theoretical predictions [129], but the energy is too low to extract a signal for the resolved photon contribution.

Before closing this subsection we briefly mention some work on closely related topics. In ref.[130] the cross–section for $c + \gamma$ production has been computed (ignoring fragmentation contributions, however), the main motivation being that this might allow to constrain the charm content of the photon. In ref.[131] it has been suggested that one might learn something about the parton content of polarized photons from the study of direct photon production. Indeed, longitudinally polarized electron beams should become available at HERA in a few years; part of this polarization will be passed on to photons emitted from these electrons, if they carry a substantial fraction of the electron's energy. However, one would also have to measure the polarization of the *outgoing* photon, which seems quite difficult.

Finally, ref.[132] is a first study of the production of *two* direct photons. The cross–section is substantially smaller (by a factor $\propto e_q^2 \alpha_{\rm em}/\alpha_s$) than the one for single direct photon production. Apart from fragmentation contributions, only the resolved photon process $q\bar{q} \to \gamma\gamma$ contributes in LO; processes like $\gamma q \to q\gamma\gamma$ are formally of NLO, since $q_i^\gamma \propto \alpha_{\rm em}/\alpha_s$, and are indeed found to be subdominant numerically. Due to the large charge factor, this process might be useful for probing the up–quark density in the photon at small x. If p_T^γ values as low as 3 GeV are experimentally accessible, one might even be able to extract the contribution from the box diagram, $gg \to \gamma\gamma$; since it is proportional to the gluon density in the photon, it should be concentrated at larger rapidities than the contribution from $q\bar{q}$ annihilation. This close analogue of the famous light–by–light scattering process [133] has yet to be studied experimentally.

3d) J/ψ Production in γp Scattering

The inelastic production of J/ψ mesons in (virtual or real) γp collisions has long been regarded as one of the cleanest methods to constrain the shape of the gluon distribution in

[‡]Unfortunately, no NLO calculation for γ+jet production exists as yet.

the proton [134]. "Inelastic" here means that the quantity

$$Z \equiv \frac{p_p \cdot p_{J/\psi}}{p_p \cdot p_\gamma} \tag{23}$$

is significantly below unity. In the proton rest frame, eq.(23) reduces to $Z = E_{J/\psi}/E_\gamma$; $Z = 1$ therefore means that the incident photon transmits its entire energy to the J/ψ meson. The cross–section for inelastic J/ψ production is nowadays usually computed using the "colour singlet" model of ref.[135], see Fig. 18. In this model one computes the cross–section for $\gamma g \to c\bar{c}g$, and projects out the contribution where the $c\bar{c}$ system is in a colour–singlet, $s-$wave, $J = 1$ state. One further assumes that the relative momentum of the c and $\bar{c}$ in the J/ψ are negligible; the matrix element for J/ψ production is then proportional to the wave function at the origin $\Psi(0)$, which can be determined from the leptonic decay width of J/ψ.

This model seems to describe the Z distribution of fixed target data better [136] than the alternative "dual model" [137, 101]. The agreement in the high$-Z$ region becomes even better if one introduces a small but nonvanishing relative momentum between the c and $\bar{c}$ quarks [138]. Until recently the overall normalization of the cross–section was not well reproduced by theoretical calculations, i.e. a sizable "K–factor" had to be introduced. Fortunately NLO corrections to the direct diagram of Fig. 18 have recently been calculated [139]. Together with the NLO corrections to the leptonic decay width [140] they give an overall inelastic cross–section in agreement with fixed target photoproduction experiments; however, the photoproduction cross–section extracted from fixed target leptoproduction experiments seems to be somewhat higher [136]. The calculation of ref.[139] predicts that the K–factor should be smaller at high (HERA) energies.

As usual, resolved photon contributions are small at the energies of fixed target experiments, but they could be quite important at HERA [141, 142]. In addition to the process $gg \to J/\psi g$, one also has to consider $gg \to \chi_c$ and $gg \to \chi_c g$, with subsequent decay of the heavier χ_c states into a J/ψ and a (soft) photon [143]; the latter process has to be considered if one imposes a cut on the transverse momentum of the J/ψ. For $p_T(J/\psi) > 3$ to 5 GeV, the contribution from $b \to J/\psi$ decays becomes important, and eventually even dominates the direct contribution [144]. There might also be sizable contributions from $cg \to J/\psi c$ [145]. However, the typical momentum scale in most J/ψ events is too small to reliably use charm distribution functions in the photon or proton; we therefore neglect this contribution here.

As usual, resolved photon contributions are characterized by the presence of the photonic remnant jet, and by a rapidity distribution that peaks at large, positive values. In addition, direct and resolved photon contributions have very different Z distributions. The direct contribution is peaked at $Z \simeq 1$, which corresponds to a small energy of the outgoing gluon. In contrast, for fixed x_γ the resolved photon contribution peaks at $Z \simeq x_\gamma$; after convolution with the photon spectrum (11) this leads to a Z distribution that quickly rises with decreasing Z.

This is shown in Fig. 19, which we adapted from numerical results of ref.[146]. Since only a mild cut on the p_T of the J/ψ meson has been applied, the contribution from b decays is relatively small and has been neglected. LO expressions have been used everywhere; neither NLO nor non–relativistic corrections to the resolved photon contribution are as yet known. The B1 parametrization of ref.[147] has been used for the gluon density in the proton, and

DG [44] for the photon. Finally, although the cross–section has not been multiplied with any branching ratio, it is clear that only the leptonic decays $J/\psi \to e^+e^-, \mu^+\mu^-$ are detectable at HERA; the combined branching ratio for these modes is about 12%. The results of Fig. 19 have been obtained under the condition that both leptons can be reconstructed by the ZEUS tracking system. Unfortunately this removes events at large rapidity, which greatly reduces the resolved photon contribution. Including the leptonic branching ratio, it only amounts to about 7 pb after integration over Z, compared to a direct contribution of about 110 pb.* Requiring $Z > 0.2$ leaves a very pure direct sample, which should allow to determine the shape of the gluon density in the proton for $2 \cdot 10^{-4} \leq x \leq 0.1$ [146].

Unfortunately the extraction of G^γ seems to be much less straightforward. The simulation of ref.[146] indicated that measurement errors would smear out the direct contribution into the region of small Z, totally obscuring the resolved photon contribution. It remains to be seen whether an unfolding procedure, and/or the application of additional cuts (e.g., tagging of the photon remnant jet) will improve the situation sufficiently to allow to extract new information on G^γ from J/ψ production at HERA.†

Both H1 and ZEUS have published first results [148] on J/ψ production. However, in both analyses events were rejected if any particle in addition to the two leptons resulting from J/ψ decay was observed. This cut removes most if not all inelastic contributions, but is sensitive to elastic or diffractive J/ψ production. This allows to test Pomeron–based models [146, 138, 149], but teaches us nothing about the parton content of the photon. Very recently, ZEUS has announced [150] preliminary results on inelastic J/ψ production. However, they required $Z > 0.2$, which again removes most of the resolved photon contribution.

Finally, we briefly mention associate $J/\psi+\gamma$ production. As first pointed out by Fletcher et al. [142], in LO only the resolved process $gg \to J/\psi+\gamma$ contributes, the direct contribution being forbidden by colour conservation. In principle this process therefore allows a clean determination of (the shape of) G^γ [151]. Unfortunately the cross–section at HERA is quite small, very roughly of order 0.1 to 1 pb after acceptance cuts and multiplication with the leptonic branching ratio.

3e) Production of Lepton Pairs in γp Collisions

In this subsection we discuss the production of lepton pairs, either due to the exchange of a virtual photon or from the decay of an on–shell W or Z boson. The corresponding cross–sections are quite small even at HERA energies, so we will be brief here.

In the theoretical treatment of these reactions one has to distinguish the cross–section integrated over the transverse momentum of the lepton pair (not to be confused with the p_T of the individual leptons) from the cross–section for the production of a high$-p_T$ lepton pair. In the former case, the only LO contribution comes from the resolved photon process $q\bar{q} \to l^+l^-$, which produces a lepton pair with vanishing transverse momentum. The corresponding ep cross–section is $\mathcal{O}(\alpha_{\rm em}^3/\alpha_s)$ since, as emphasized repeatedly, the parton distribution functions in the photon are $\mathcal{O}(\alpha_{\rm em}/\alpha_s)$. In contrast, both the direct process $\gamma q \to l^+l^-q$ and the

*Recall that these are LO predictions, and hence somewhat uncertain; the cross–sections also depend on the gluon densities in the photon and proton, of course.

†Previous analyses [142, 74], which had led to very optimistic conclusions, had used much milder acceptance cuts on the leptons, or none at all.

resolved photon process $gq \to l^+l^-q$ contribute in LO to the production of a lepton pair with sizable p_T. The corresponding ep cross–sections are both $\mathcal{O}(\alpha_{\rm em}^3)$.

This distinction has been well understood in existing treatments of the "Drell–Yan" production of l^+l^- pairs via the exchange of a virtual photon; see ref.[141] for a LO estimate, and [107, 152, 153] for NLO calculations. Compared to direct photon production this process has the theoretical advantage that one does not have to worry about fragmentation contributions. Moreover, backgrounds are low, and the final state can be reconstructed cleanly even at rather low $p_T(l)$, allowing one to probe quite small x values [153]. As usual, the region of positive rapidity corresponds to small x_γ and moderate x_p, while negative rapidities correspond to large x_γ and very small x_p. The production of lepton pairs at positive rapidity and sizable p_T is sensitive to the gluon content of the photon [152]. Unfortunately the expected event rates are quite low: $\frac{d\sigma(ep \to l^+l^-X)}{dM_{l^+l^-}} \simeq 1$ pb/GeV at $M_{l^+l^-} = 4$ GeV after integration over the transverse momentum of the pair and before any acceptance cuts have been imposed [153].

In contrast, the fact that the *total* photoproduction cross–section for W and Z bosons is, in LO, given by the resolved photon contribution *only* has not been appreciated in the existing literature.* The resolved photon contribution has first been estimated in ref.[155], but here this contribution was considered to be an addition to the direct process even at vanishing p_T of the heavy gauge boson. This is not correct, since the direct contribution has a $u-$channel singularity which has to be absorbed into the resolved photon contribution; simply adding both contributions implies double–counting. While current calculations [156, 154] of the total W and Z photoproduction cross–sections are in our view not entirely satisfactory, since they mix LO and NLO contributions in an ill–controlled manner, they should predict the production of high$-p_T$ gauge bosons quite accurately.† This high$-p_T$ region is sensitive to the form of the $W^+W^-\gamma$ vertex [156]. The total cross–sections for W^+ and W^- production at HERA amount to approximately 0.5 pb each. This cross–section is to be divided by another factor of five if only the clean $e\nu_e$ and $\mu\nu_\mu$ final states are observable; indeed, a first study [157] has concluded that the detection of hadronically decaying W and Z bosons at HERA is quite challenging, although perhaps not impossible. Finally, we mention that recently the H1 collaboration has announced [158] observation of one event that can be interpreted as the production of a leptonically decaying W boson at high p_T; within the SM this interpretation is quite unlikely, due to the smallness of the predicted cross–section, but all other interpretations seem even less likely. Clearly much more data have to be analyzed before any definite conclusion can be drawn.

4) Real $\gamma\gamma$ Scattering at e^+e^- Colliders

In this section we discuss hard processes with two (quasi–)real photons in the initial state and a hadronic final state. At present, and in the near future, such reactions can only be

*The total W and Z cross–sections in ep collisions also get contributions where the heavy gauge boson is radiated off the electron line; in case of Z production at HERA this contributes roughly 50% of the total cross–section [154].

†In principle the resolved photon contribution from $gq \to Wq'$ should be included in a full LO treatment, but it is strongly suppressed at HERA energies due to phase space constraints.

studied at e^+e^- storage rings, where the effective photon flux is given by eq.(11). At future linear e^+e^- colliders the photon spectrum might receive large additional contributions from "beamstrahlung" [159], which is emitted when a particle is accelerated in the field produced by the opposite bunch; beamstrahlung photons are exactly on–shell. The produced spectrum sensitively depends on the size and shape of the electron and positron bunches; see ref.[160] for a handy parametrization of the spectrum in terms of a few machine parameters. Finally, in recent years the possibility has been discussed to convert a linear e^+e^- collider into a "$\gamma\gamma$ collider" by scattering laser photons off the incident $e^\pm$ beams [161]. The achievable luminosity is predicted to be comparable to the (geometrical) e^+e^- luminosity prior to conversion of the beams; in contrast, at existing e^+e^- storage rings one has $\mathcal{L}_{\gamma\gamma} \sim \left(\frac{\alpha_{\rm em}}{\pi} \ln \frac{s}{m_e^2} \right)^2 \sim 10^{-3} \mathcal{L}_{ee}$. The photon spectrum of such a photon collider again depends on the geometrical set–up, and also on the polarization of the incident electron and laser photon beams. Most of the results presented here will be for present and near–future colliders; these are obviously of more immediate interest, and already allow to illustrate most physics principles. For comparison we will also give a few results for the more "futuristic" colliders.

Since we now have two photons in the initial state, we have to distinguish three physically distinct event classes [42], see Fig. 20. Following the notation of ref.[162] we call reactions where the entire energy of both photons goes into the hard subprocess "direct", see Fig. 20a. If only one of the photons couples in a pointlike manner while the other participates via its quark and gluon content, as in Fig. 20b, the process is called "once resolved" ("1–res" for short). Finally, processes where both photons are resolved into their partonic constituents are called "twice resolved" ("2–res"); an example is shown in Fig. 20c. Recall that each resolved photon produces a remnant or "spectator" jet, which goes approximately in the direction of the incident $e^\pm$ beams; the three event classes are therefore characterized by having zero, one or two of these photonic remnant jets. Since the parton distribution functions in the photon are $\mathcal{O}(\alpha_{\rm em}/\alpha_s)$, all three contributions are of the same order in coupling constants, and have to be treated on the same footing [42].

We saw in the previous section that direct and resolved photon contributions to photoproduction processes mix in NLO QCD. Similarly, the three event classes contributing to real $\gamma\gamma$ scattering mix once higher–order QCD corrections are included. Parts of the NLO direct (1–res) contributions have already been included in the LO 1–res (2–res) terms; these parts therefore have to be subtracted from the NLO contributions. Note that mixing between direct and twice resolved contributions only occurs in NNLO in QCD.

Following our preceding discussion of γp scattering, we discuss different final states in separate sub–sections. Jet production is treated in Sec. 4a, open heavy flavour production in Sec. 4b, J/ψ production in Sec. 4c, and direct photon production in Sec. 4d.

4a) Jet Production in $\gamma\gamma$ Collisions

As in case of γp scattering, the production of jets offers the largest cross–section of all hard $\gamma\gamma$ collisions that lead to hadronic final states. In LO, the cross–section can be written as [see eq.(16)]:

$$\frac{d^3\sigma(e^+e^- \to e^+e^- j_1 j_2)}{dp_T d\eta_1 d\eta_2} = 2 p_T x_1 x_2 \sum_{i,j,k,l} f_{i|e}(x_1) f_{j|e}(x_2) \frac{d\hat{\sigma}_{ij\to kl}(\hat{s},\hat{t},\hat{u})}{d\hat{t}}. \qquad (24)$$

If i or j is a quark or gluon, $f_{i|e}$ can again be obtained by convoluting the photon flux function $f_{\gamma|e}$ with the quark or gluon density in the photon, see eq.(17). However, antitag requirements at current e^+e^- collider experiments often allow larger photon virtualities than one has in photoproduction events at HERA; at the same time, the scale $|\hat{t}|$ or p_T^2 of the hard process is usually smaller. This means that the suppression of resolved photon contributions due to the reduced parton content of virtual photons is usually larger at e^+e^- colliders than at HERA. In our numerical estimates to be presented below we have included this effect using the formalism of ref.[95].

The scaling variables $x_{1,2}$ in eq.(24) are related to the jet (pseudo)rapidities $\eta_{1,2}$ by:

$$x_1 = \frac{x_T}{2}\left(e^{\eta_1} + e^{\eta_2}\right); \tag{25a}$$

$$x_2 = \frac{x_T}{2}\left(e^{-\eta_1} + e^{-\eta_2}\right), \tag{25b}$$

with $x_T = 2p_T/\sqrt{s}$ as before. The Mandelstam variable $\hat{s}$ of the hard scattering sub–process is again given by $\hat{s} = x_1 x_2 s$, and $\hat{t}$ is as in eq.(19). Most of the sub–processes contributing to jet production in $\gamma\gamma$ collisions also contribute to jet production at HERA. In particular, if both i and j are partons (twice–resolved contribution), $\hat{\sigma}$ is the same as in resolved γp collisions, and the case where either i or j is a parton while the other is a photon (single–resolved contribution) corresponds to the direct contribution at HERA. Finally, if $i = j = \gamma$ (direct contribution), then $k = q,\ l = \bar{q}$ and one has

$$\frac{d\hat{\sigma}(\gamma\gamma \to q\bar{q})}{d\hat{t}} = 3e_q^4 \frac{2\pi\alpha_{\rm em}^2}{\hat{s}^2}\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right), \tag{26}$$

where e_q is the electric charge of quark q in units of the proton charge, and the factor of 3 is due to colour.

The first quantitative estimate of high$-p_T$ jet production in (quasi-)real $\gamma\gamma$ scattering has been presented by Brodsky et al. [163]. However, gluon–initiated processes were omitted, and a rather crude parametrization for the quark densities in the photon was used.* Gluon–initiated contributions have been included in refs.[165], but again very simple parametrizations for the parton densities in the photon were used. In spite of their shortcomings, these early studies clearly demonstrated that resolved photon contributions are quite important, and often even dominant, if $x_T \leq 0.2$. Notice that jet production from two–photon collisions at present e^+e^- colliders cannot be studied experimentally if $x_T \geq 0.4$; the cross–section becomes too small, and annihilation backgrounds too large. Hence resolved photon contributions were predicted to be sizable for at least half the experimentally accessible range of x_T.

Nevertheless data [15] on multi–hadron production from $\gamma\gamma$ collisions taken at the PEP and PETRA storage rings were usually only compared to the direct (QPM) contribution.† Not surprisingly, a significant excess of data over MC prediction was observed. Given that the

*Ref.[163] also finds quite large higher twist contributions to the production of high$-p_T$ mesons and jets. It was shown later [164] that the "constituent interchange model" used in ref.[163] over–estimates the normalization of these terms by as much as a factor of a thousand.

†The PLUTO collaboration [166] attempted to include 1–res qg or 2–res qq final states in their analysis. However, they made many simplifying assumptions, some of which are incorrect. In particular, they assumed

importance of resolved photon contributions had been emphasized quite early on [163, 165], it is surprising that it took ten years before the origin of this excess of data over QPM prediction was clarified experimentally.

In ref.[162] it was pointed out that the characteristics of these excess events at least qualitatively agreed with those expected from resolved photon contributions: Their p_T spectrum is softer than that of the direct contribution, just as in case of photoproduction events. In addition, resolved photon events are less two–jet like, i.e. have smaller thrust T, than direct events; in fact, the PLUTO collaboration had shown [166] that the excess could be removed by requiring $T \geq 0.9$. Ref.[162] also contains the prediction that, for fixed p_T, resolved photon events should rapidly become more important as the beam energy is increased. The reason is that raising $\sqrt{s}$ decreases x_T, and hence $x_{1,2}$ in eqs.(25); due to the additional convolution with photonic parton distribution functions, the quark and gluon density in the electron is always considerably softer, i.e. grows faster with decreasing x, than the photon flux in the electron. Resolved photon contributions were therefore expected to play an even more important role at TRISTAN and LEP than at PEP and PETRA. This was confirmed soon afterwards by the AMY collaboration [6], who showed that their data on multi–hadron production in antitagged $\gamma\gamma$ collisions could be reproduced by their QCD–based Monte Carlo program, in both shape and normalization, if and only if it included the full set of resolved photon contributions, including those with a gluon in the initial state. This was the first unambiguous observation of resolved photon interactions other than in deep–inelastic $e\gamma$ scattering, and the first direct experimental evidence for a nonzero gluon density in the photon.

Early experimental analyses of multi–hadron production in $\gamma\gamma$ collisions [15], including the AMY analysis [6], were not amenable to direct comparison with theoretical calculations, since the experiments defined a "jet" as a thrust hemisphere. The production of actual, reconstructed jets (using a cone algorithm) has been studied only quite recently, first by the TOPAZ collaboration [167] and then by AMY [168]. This is an important development, since it allows a much more direct comparison between theory and experiment.

In the meantime theoretical estimates are also becoming more sophisticated, by including NLO corrections. This process was actually already startd in 1979 with two calculations [169] of $\gamma\gamma \to gg$ via a quark box diagram. NLO corrections to the direct process $\gamma\gamma \to q\bar{q}$ (for massless quarks) were calculated soon afterwards [170]. However, the result contains collinear divergencies, which have to be absorbed in the 1–res contribution; in NLO it therefore makes little sense to consider the direct contribution in isolation. This was recognized in ref.[171], which deals with the production of high$-p_T$ hadrons. Here the direct contribution was treated in NLO (including the contribution from $\gamma\gamma \to gg$, which is formally NNLO), but at the time resolved photon contributions could only be included in leading order.‡ This shortcoming could be remedied only after the NLO corrections to the hard partonic sub–process had been calculated [85]. A full NLO calculation of the single–jet inclusive cross–

that the cross–sections for these resolved photon contributions drops like the square of the $\gamma\gamma$ invariant mass W; in contrast, for fixed p_T QCD predicts these cross–sections to increase with W. Moreover, PLUTO did not attempt to include 1–res and 2–res contributions simultaneously.

‡Notice that one only needs the LO 1–res contribution from γq scattering in order to absorb the collinear divergencies of the NLO direct term. The procedure of ref.[171] should therefore be quite accurate at high x_T, where the direct contribution dominates.

section has become available only recently [172]; it finds only modest NLO corrections to the total cross–section ($\leq 25\%$, for a jet cone size $\Delta R = 1$ used by both TOPAZ and AMY, and renormalization and factorization scale $\mu = p_T$), but the relative weights of direct, single– and double–resolved contributions can change by larger amounts.[§] Finally, in ref.[173] the 2–res contribution has been studied in NLO, with emphasis on constraining the gluon content of the photon.

Since NLO corrections appear to be quite modest, and since many more LO parametrizations of parton densities in the photon exist, we will only present results from LO analyses here. As mentioned earlier, we do include the suppression of resolved photon contributions due to the finite virtuality of the photons, using the simplest ansätze in ref.[95]. (Quark and gluon densities have to be treated separately.) We remind the reader that, unlike ref.[172], these ansätze do not assume a power–like suppression of the "hadronic" contribution to the parton densities of virtual photons; they might therefore slightly under–estimate the (in any case only modest) size of the correction.

In Fig. 21 we compare our LO calculation of the single–jet inclusive cross–sections as measured by the TOPAZ (a) and AMY (b) collaborations at TRISTAN ($\sqrt{s} = 58$ GeV). TOPAZ requires the jet to have pseudorapidity $|\eta_{\rm jet}| \leq 0.7$, and rejects an event if it contains an electron or positron at an angle $\theta > 3.2^\circ$ relative to the beam pipe and with energy $E > 0.25 E_{\rm beam} = 7.2$ GeV. AMY accepts jets out to $|\eta_{\rm jet}| = 1.0$; moreover, it can antitag an event only if it contains an electron or positron with $E > 0.25 E_{\rm beam}$ at an angle $\theta > 14.1^\circ$.

We see from Fig. 21 that the DG, LAC1 and WHIT1 parametrizations reproduce the TOPAZ data at $p_T \leq 4.5$ GeV quite well, and also fit the AMY data over the entire p_T range. Notice, however, that we had to use $N_f = 4$ active flavours already at $p_T(\text{jet}) = 2.5$ GeV in order to achieve this agreement. This is not really justifiable for the DG and LAC parametrizations, which treat the charm quark as massless; on the other hand, WHIT explicitly includes some charm mass effects, which leads to a greatly reduced contribution from charm in the photon, in agreement with expectations. (It makes up for the shortfall by slightly larger light quark and gluon densities.) The GRV parametrization seems to fall below the data at low p_T; this is mostly because we have used $\Lambda_{\rm QCD}(N_f = 3) = 0.4$ GeV for DG and WHIT, as compared to 0.2 GeV for GRV and LAC, as is implicit in these parametrizations. NLO corrections have been found to be most important at low p_T [172]; they might very well bring the GRV prediction into agreement with the data. In contrast, the WHIT4 prediction lies above the data at low p_T, since it assumes a quite large and hard gluon density in the photon. This discrepancy might be reduced if the "hadronic" piece of the parton density of virtual photons is suppressed by a power of the virtuality, but it seems unlikely that this will restore full agreement with the data. All the other WHIT parametrizations seem acceptable, however.

Fig. 21 shows that the LAC3 parametrization clearly over–estimates the cross–section, while the direct contribution by itself falls well below the data. Notice that the resolved photon contributions remain non–negligible out to the highest p_T where data exist, although their importance clearly diminishes with increasing p_T, as discussed above. Finally, we

[§]However, there seems to be a discrepancy between the LO result of ref.[172] and our calculation. In particular, Aurenche et al. find that 2–res processes still contribute about 25% to the jet cross–section measured by TOPAZ at $p_T = 7.5$ GeV, while in our calculation it amounts to less than 10% at this rather large value of x_T (unless we use the LAC3 parametrization).

observe that our calculation falls somewhat below the TOPAZ data for $p_T > 4.5$ GeV. This is disturbing, since in principle our neglect of the charm mass should be better justified if $p_T^2 \gg m_c^2$. Notice, however, that our calculation agrees very nicely with the AMY data. This hints at an experimental discrepancy between these two data sets. Indeed, from the increased acceptance in rapidity as well as the much looser antitag requirements used by AMY, one expects that the cross–section measured by AMY should exceed that measured by TOPAZ by a factor of about 1.7 to 1.8; here we have made use of the fact [167, 168] that $d\sigma/d\eta_{\rm jet}$ is quite flat over the observed range, and have used eq.(11) to estimate the effect of the tagging criteria on the photon flux. On the other hand, the cross–section measured by AMY only exceeds that reported by TOPAZ by a factor of 1.36 ± 0.26 (1.29 ± 0.20) at $p_T = 4.55$ (6.5) GeV, where we have assumed all errors to be independent. Although this procedure probably over–estimates the true error, since part of the systematics should be common to both experiments, this comparison indicates that the discrepancy in any one p_T bin is not very significant statistically. Due to bin–to–bin correlations produced by the unfolding procedure, it is difficult to combine different bins to arrive at an overall significance of the discrepancy. At the lowest p_T bin, the ratio of the two experimental cross–sections is 1.55, which agrees very well with expectations.[¶] Moreover, both collaborations also publish di–jet cross–sections, although with somewhat larger relative errors; our LO calculation reproduces both these measurements over the entire p_T range.

Note that the published cross–sections are actually partonic cross–sections, which have been obtained by unfolding the observed p_T(jet) and η(jet) distributions using a LO MC program. We will argue in Sec. 5 that such a procedure might be more reliable at TRISTAN than at HERA, since events with multiple partonic interactions should not pose much of a problem here. Nevertheless this procedure does produce some dependence on the details of the Monte Carlo program, e.g. via the predicted jet reconstruction efficiency. It might also be dangerous to directly compare these extracted cross–sections with NLO calculations, since the higher order corrections seem to change [172] the relative weights of the three classes of contributions compared to what has been assumed in the LO MC.[*] Both experiments also give the measured rate of jet events as a function of p_T; however, at these rather low transverse momenta the jet reconstruction efficiency is still rather small and p_T-dependent. A fair amount of MC work is therefore (unfortunately) necessary before a comparison with theoretical predictions can be made.

The published data are based on an integrated luminosity of about 90 pb^{-1} for TOPAZ, and 27 pb^{-1} for AMY. The total available data samples are more than three times larger than this. Notice that the error bars in Fig. 21 are already quite small; if the slight discrepancy between AMY and TOPAZ can be resolved, the full data set should therefore allow a quite precise comparison between theory and experiment.

[¶]The effect of the looser antitag requirement of AMY is smaller at low p_T, due to the dynamical upper bound $P^2 \leq p_T^2$ that has to be imposed on the virtuality of the exchanged photons in order to meaningfully speak of parton or photon densities in the electron.

[*]It is not obvious how the "theoretical" definition of the three classes of contributions used in ref.[172] relates to a more experimental definition relevant for event reconstruction, which could e.g. be based on the presence of absence of (remnant) jets close to the beam pipes. For example, the relative weights of the three classes of contributions in the NLO calculation strongly depend on the factorization scale, which is an unphysical parameter. Unfortunately it is quite difficult to write an MC generator based on a full NLO calculation; no such generator for jet events produced in hadronic or photonic collisions exists as yet.

The larger data sets should also allow to investigate more differential distributions, using events with two reconstructed jets. As an example we show in Fig. 22 the triple–differential cross–section for events with two jets with equal (pseudo)rapidity $\eta_1 = \eta_2 \equiv \eta$. This distribution is quite flat for the direct contribution, at least over the range covered by TRISTAN detectors. For parametrizations with a relatively hard gluon density (WHIT1, DG) the total single–resolved contribution also depends only weakly on η; for the given choice of p_T, the decrease of the contributions $\propto q_i^\gamma$ (qg final state) is more or less balanced by the increase of those $\propto G^\gamma$ ($q\bar{q}$ final state). Increasing η means increasing x_1 but decreasing x_2 in eqs.(25). Since the gluon content in the electron is always quite strongly peaked at small x while the photon flux function $f_{\gamma|e}$ is relatively hard, the increase of $f_{G|e}$ more than compensates the decrease of $f_{\gamma|e}$ as η is increased. (Note that $f_{q|e}$ increases much more slowly with decreasing x than $f_{G|e}$ does.) If the gluon density in the photon is large but strongly peaked at small x (WHIT6, LAC1), the rise of the cross–section for $\gamma g \rightarrow q\bar{q}$ even leads to a total 1–res contribution that peaks at $\eta \simeq 2$, rather than at $\eta = 0$.

Turning to the twice–resolved contributions shown in Fig. 22b, we observe that the cross–sections for processes with two gluons in the final state, which most of the time also have two gluons in the initial state, is quite strongly peaked at $\eta = 0$, even for the comparatively hard gluon density of WHIT1. The same parametrization predicts the cross–section from $qg \rightarrow qg$ scattering to remain quite flat for $|\eta| \leq 1$, since the decrease of $f_{q|e}(x_1)$ is compensated by the increase of $f_{G|e}(x_2)$.† Nevertheless, parametrizations with relatively hard gluon density still predict a significant decrease of the total 2–res cross–section as η is increased from 0 to 1. In contrast, parametrizations with a large and soft gluon content (WHIT6, LAC1) predict the 2–res contribution to rise with increasing η, reaching a maximum at $\eta \simeq 1.3$. This increase is entirely due to the qg final state, where the increase of $f_{G|e}$ now over–compensates the decrease of $f_{q|e}$.

As discussed in the next subsection, the TOPAZ collaboration has proven capable of detecting photon remnant jets with an efficiency of about 70%; this ability was already implicit in their observation [167] of considerable energy flow at small angles relative to the beam pipes. They are now beginning to exploit this capability also in the jet analysis [174]. This shows that the 1–res, 2–res and direct contributions can indeed be studied separately. On the other hand, Fig. 22 shows that even the sum over all contributions should allow to discriminate between some existing parametrizations of the parton content of the photon. In particular, at $p_T = 3$ GeV, parametrizations with a large but soft gluon density predict a flat or even slowly rising jet cross–section as η is increased away from zero, in contrast to parametrizations with a small or hard gluon density. Gluon–initiated processes are expected to contribute more than 50% of the total 2–res cross–section, and still some 25 to 35% of the total di–jet cross–section at small rapidity. This fraction, and hence the difference between predictions using different parametrizations, will be smaller (larger) for $p_T > (<) 3$ GeV.

Experiments at the LEP storage ring should also be able to contribute significantly to our understanding of (almost) real $\gamma\gamma$ collisions. At LEP1 the vicinity of the Z peak means that multi–jet annihilation events are a much more severe background than at TRISTAN; however, a parton–level investigation [175] concluded that this should not be much of a problem as long as the invariant mass M_{jj} of the two high$-p_T$ jet system is below 15

†Of course, there is also a contribution $\propto f_{q|e}(x_2) f_{G|e}(x_1)$, but it decreases very quickly with increasing, positive η.

to 20 GeV. Indeed, some experimental results have already been published. The ALEPH collaboration repeated [176] the AMY analysis [6] of multi–hadron production. Although some of the details differ, as will be discussed in Sec. 5, ALEPH also finds that the description of the data is greatly improved once resolved photon interactions are included in the Monte Carlo model.

DELPHI [177] reached similar conclusions regarding "minimum bias" multi–hadron production. In addition, they analyzed a subsample of events containing two jets with $|\eta| \leq 1.0$ and $p_T > 1.75$ GeV, where the jets were defined using a cluster algorithm. At these small transverse momenta the jet reconstruction efficiency is rather low $[\sigma(2-\text{jet}) \sim 0.1\sigma(\text{parton})]$, which introduces a strong dependence on the details of the MC program. DELPHI finds that DG falls below the data for $p_T \leq 3$ GeV, while the LAC1 and GS parametrizations work well. Note that their event sample is mostly sensitive to direct events, as well as resolved photon events with large x, i.e. soft remnant jets, since they require $W_{\text{vis}} < 13$ GeV and $E_{\text{vis}} < 20$ GeV; the calculation of these quantities includes information from the small–angle taggers, which cover angles down to 2.5° and should therefore see parts of the remnant jets. Fig. 3 shows that the DG parametrization does indeed fall well below LAC1 for large x and small Q^2.

More recently, DELPHI has published [178] a study of hadronic $\gamma\gamma$ events where either the e^+ or the e^- is tagged at a very small angle, corresponding to a photon virtuality $P^2 \sim 0.1$ GeV^2. Since even stronger cuts were imposed on the hadronic system than in the first DELPHI analysis, the data sample was quite small (491 events); nevertheless it was sufficient to once again prove the necessity to include resolved photon contributions if the data are to be described by the MC program. This small angle tagging technique holds much promise, since the tag helps to reconstruct the kinematics of $\gamma\gamma$ events by determining the energy of one of the two photons. If (as at HERA) the energy of the second photon can be determined using the Jacquet–Blondel method, a measurement of the jet rapidities in di–jet events would allow to directly reconstruct the Bjorken$-x$ variables of the partons in the photons. If, as TOPAZ results indicate, the DELPHI small angle detectors (not to be confused with the very small angle taggers used for the electron tagging) can be used for detecting the presence or absence of remnant jets, DELPHI (and similar detectors) should be able to perform quite detailed analyses of hard two–photon reactions. In order to fully exploit this potential, the strong upper limits on the visible energy and invariant mass used in refs.[177, 178] should be relaxed; this should certainly be possible at LEP2 energies, where the annihilation background is much smaller.

The LEP energy is expected to soon be increased to $\sqrt{s} \simeq 180$ GeV; LEP experiments will then have the unique opportunity to study two–photon cross–sections over a wide range of energies. As illustrated in Fig. 23, the energy dependence can be quite strong. In this figure we show the triple–differential cross–section at $p_T = 5$ GeV as a function of $\eta_1 = \eta_2 \equiv \eta$, imposing an antitag condition similar to that used by ALEPH [176]. For $\sqrt{s} = 90$ GeV, x_T is similar to the value used in Fig. 22, which leads to a similar shape of the pseudorapidity distribution; the kinks at $\eta \simeq 1.5$ occur because the ALEPH antitag becomes ineffective if the outgoing electron has less than half the beam energy, i.e for scaled photon energy $z > 0.5$. Raising $\sqrt{s}$ from 90 to 180 GeV increases the direct contribution at $\eta = 0$ only by a factor of 1.45, while the 1–res and 2–res contributions increase by factors of 2.0 and 3.2, respectively. This again demonstrates the strong dependence of the cross–section for resolved photon processes on the available phase space, or, equivalently, on the Bjorken$-x$ variables of eqs.(25). Note that the single–jet inclusive cross–section at central rapdity grows even faster with energy, since the kinematical integration limits for the rapidity of the second jet also increase with $\sqrt{s}$. Finally, we remind the reader that the shape and normalization of the solid curves in Fig. 22 could be quite different if the photon has a large but soft gluon component; this would also result in an even more rapid increase of the cross–section for resolved photon events with $\sqrt{s}$.

LEP will almost certainly be the highest energy e^+e^- storage ring ever. In order to reach even higher energies, one will need to build a linear collider. In such a device each bunch can most likely only be used once; the repitition rate (number of bunch collisions per second) will therefore almost certainly be smaller than at LEP. At the same time the total luminosity must increase like the square of the beam energy in order to maintain a roughly constant rate of e^+e^- annihilation events. The luminosity per bunch crossing will therefore have to be much larger than at existing storage rings, forcing one to use very dense bunches. The correspondingly large charge density gives rise to strong electromagnetic fields. When particles inside one bunch enter the field produced by the opposite bunch, they will be accelerated, and will therefore radiate real photons. This is known as "beamstrahlung" [159].

The flux of these beamstrahlung photons depends quite sensitively on the design characteristics of the collider; this is not surprising, since this radiation is due to the field of the entire bunch, not due to individual e^+e^- collisions (unlike the only quasi–real "bremsstrahlung" photons we have dealt with up to now). Beamstrahlung can be reduced by using flat beams, and by splitting large bunches into "trains" of smaller ones. On the other hand, for a given class of designs, beamstrahlung increases quite rapidly with increasing beam energy.

This is ilustrated in Figs. 24, which show the single–jet inclusive cross–section as a function of p_T, where we have accepted jets with $|\eta| \leq 2.0$. We have used the WHIT1 parametrization, and imposed the antitag requirement $\theta < 10^\circ$ when computing the bremsstrahlung contribution to the total photon flux. The beamstrahlung spectrum has been calculated using the analytical expressions of ref.[160], for the JLC design as specified at the 1993 international linear collider conference [179]. Designs for linear colliders are still evolving; the results of Fig. 24 should therefore be considered as indicative only.

At $\sqrt{s} = 0.5$ TeV (Fig. 24a) the beamstrahlung spectrum is considerably softer than the equivalent bremsstrahlung spectrum (11). This enhances the relative importance of the direct contribution, since the cross–section for resolved photon contributions increases with the two–photon invariant mass W while that for the direct contribution decreases. For $p_T > 100$ GeV, the total cross–section is dominated by directly interacting bremsstrahlung photons. Notice that this design leads to a luminosity of about 50 fb^{-1} per year; the two–photon cross–section should therefore remain measureable for jets with p_T well above 100 GeV. Moreover, one expects of the order 10^8 events per year with a jet with $p_T > 5$ GeV. This sounds like a large number, but corresponds to a trigger rate of 10 Hz or less, which should be easily manageable.

Increasing $\sqrt{s}$ to 1.0 TeV (Fig. 24b) greatly increases the flux of beamstrahlung photons, and also makes it harder. This enhances the relative importance of resolved photon contributions, as can be seen from the x_T value where direct and resolved photon cross–sections are equal. Notice also that now beamstrahlung increases the cross–section by about a factor of 5.5 even at $p_T = 200$ GeV. Finally, comparing Figs. 24a and b, we see that the jet cross–section increases by a factor of about 7 (20) for $p_T = 5$ (100) GeV; without beamstrahlung, the corresponding factors would "only" have been 3 and 5, respectively. We remind the reader that these results depend on the specific machine design; see refs.[180, 181] for further discussions of this point.

As mentioned in the beginning of this section, one might be able to convert a linear e^+e^- collider into a $\gamma\gamma$ collider by back–scattering laser photons off the e^+ and e^- beams [161]. The spectrum and luminosity of such a collider depend quite strongly on details such as the polarization of the laser photons and incident electrons beams. The production of jets at such a collider has first been discussed in ref.[182]. In Fig. 25 we present a result for the simple case of unpolarized beams and small distance between the conversion and interaction points. This leads to a photon spectrum which peaks at $z = 0.828$, where it also cuts off; the two–photon luminosity is then quite flat over a wide range of W. Most modifications that have been discussed in the literature [161] give even harder photon spectra. By comparing Fig. 25 with Fig. 24a we see that, as expected, the much harder photon spectrum of the $\gamma\gamma$ collider has greatly increased the relative importance of resolved photon contributions. Moreover, the hard photon spectrum allows to efficiently access soft partons in the photon via 1–res contributions at large rapidity. As a result, the cross–section increases with η even for parametrizations with rather modest gluon content, like WHIT1; this is to be contrasted with the situation at present and future e^+e^- colliders, see Figs. 22 and 23. This also explains the large difference between the predictions from the LAC1 and WHIT1 parametrizations close to the kinematical maximum of η, inspite of the rather large value of x_T.

Finally, we should warn the reader that from LEP2 energies onwards, multiple interactions could substantially increase the true jet cross-section, compared to the simple parton–level estimates of Figs. 23 to 25; we already saw in Sec. 3a that this phenomenon seems to play an important role in jet production from resolved photons at HERA. This will be discussed in more detail in Sec. 5.

4b) Heavy Quark Production in $\gamma\gamma$ Collisions

Apart from the production of jets discussed in the previous subsection, the production of heavy quarks is the only hard QCD process that has been studied experimentally in two–photon collisions. As discussed in Sec. 3b, the main advantage of heavy $Q\bar{Q}$ pair production is that perturbative QCD should be applicable over the entire phase space. However, as we already saw in the discussion of the photoproduction of heavy quarks, predictions for total $c\bar{c}$ production rates are at present still quite uncertain, due to unknown higher order corrections (which lead to a strong scale dependence) and the uncertainty in the value of m_c to be used here.

Progress in the theoretical treatment of heavy quark production in two–photon processes has been relatively slow. The first complete LO calculation, including resolved photon processes, was only performed in 1989 [162]. The contribution from twice resolved processes was found to be very small at TRISTAN energies, but the 1–res contribution (from $\gamma g \to c\bar{c}$) is quite sizable. Further LO predictions, for newer parton densities in the photon, were published in [46]. A full NLO analysis of the direct and 1–res contributions has been performed in ref.[183].

In Fig. 26 we show updated predictions for the total $c\bar{c}$ cross–section in the PETRA to LEP2 energy range. We have included NLO corrections to the direct contribution, using a simple parametrization.* In ref.[183] the direct contribution to the total cross–section is written as

$$\sigma_{\rm dir}(\gamma\gamma \to Q\bar{Q}(g)) = \frac{\alpha_{\rm em}^2 e_Q^4}{m_Q^2}\left(c_{\gamma\gamma}^{(0)} + 4\pi\alpha_s c_{\gamma\gamma}^{(1)}\right). \tag{27}$$

Here $c_{\gamma\gamma}^{(0)}$ describes the well–known [184] tree–level (QPM) prediction, while $c_{\gamma\gamma}^{(1)}$ can be parametrized as:

$$\begin{aligned} c_{\gamma\gamma}^{(1)} &= \frac{\pi}{2} - \sqrt{r}\left(\frac{5}{\pi} - \frac{\pi}{4}\right), \qquad r < 2.637 \\ &= 0.35 r^{-0.3}, \qquad\qquad\qquad r \geq 2.637 \end{aligned} \tag{28}$$

where $r = W_{\gamma\gamma}^2/(4m_Q^2) - 1$. This parametrization is exact at threshold $r \to 0$, and describes the full NLO result to better than 10% accuracy for all $r \leq 100$. On the other hand, the predictions for the 1–res contribution shown in Fig. 26 have been computed in LO only. One reason is that here NLO corrections are considerably smaller than for the direct contribution [183], largely because the threshold ("Sommerfeld") corrections are negative for a colour–octet $Q\bar{Q}$ state. Moreover, the uncertainty of the prediction is much larger than for the direct cross–section, making it less important to include 10% corrections.

In Fig. 26 this uncertainty is given by the width of the bands defined by two curves with the same pattern. Following ref.[162] we have in all cases required $W_{\gamma\gamma} > 2m_D = 3.74$ GeV, but the "dynamical" charm quark mass appearing in the expressions for the hard cross–sections is less well defined. In case of the direct cross–section we have varied m_c between 1.3 GeV (upper dotted curve) and 1.6 GeV (lower dotted curve). Another uncertainty arises from the choice of scale in α_s and (for the 1–res contribution) in G^γ. In Fig. 26 we have estimated this uncertainty by varying this scale between $M_{c\bar{c}}/4$ (upper curve) to $M_{c\bar{c}}$ (lower curve). In case of the direct contribution the combined uncertainty only amounts to slightly over 20%, almost independently of $\sqrt{s}$. Note, however, that other authors [112, 183] prefer to use an even wider range of values for m_c.

Unfortunately the uncertainty for the prediction of the 1–res contribution is considerably larger than this. One reason is that now α_s already appears in the tree–level cross–section, leading to a stronger dependence on the renormalization scale; there is also a factorization scale dependence in this case. As usual, the inclusion of NLO corrections should reduce these scale uncertainties. However, the biggest uncertainty comes from the choice of m_c, and of the minimal allowed $M_{c\bar{c}}$. In case of the direct contribution one always has $W_{\gamma\gamma} = M_{c\bar{c}}$, at least in leading order. On the other hand, 1–res events have $W_{\gamma\gamma} > M_{c\bar{c}}$; it is then not clear whether one has to require $M_{c\bar{c}} > 2m_D$ in order to describe open charm production, or whether it is sufficient to have $W_{\gamma\gamma} > 2m_D$. If $M_{c\bar{c}} < 2m_D < W_{\gamma\gamma}$, some energy has to be transferred from the remnant jet to the "hard" $c\bar{c}$ pair; it has to be remembered that in any case colour needs to be exchanged between these two systems in the hadronization step. It seems unlikely to us that this soft energy exchange can exceed 1 GeV, however. We have therefore used $m_c = 1.4$ GeV, $M_{c\bar{c}} > 2m_c$ and scale $\mu = M_{c\bar{c}}/4$ in order to estimate the

*Note that here direct and 1–res contributions remain well–defined even in NLO, since there is no LO 1–res contribution from the quarks in the photon.

upper limit of the 1–res contribution, and $m_c = 1.6$ GeV, $M_{c\bar{c}} > 2m_D$ and $\mu = M_{c\bar{c}}$ for the lower limit.

Fig. 26 shows that this uncertainty makes it impossible to distinguish between the DG and WHIT1 parametrizations based on the total $c\bar{c}$ production rate alone. Note that the uncertainty is larger for LAC1, which has a very soft gluon density and therefore reacts very sensitively to changes of the lower bound of $M_{c\bar{c}}$. Even for the more conservative parametrizations, the uncertainty amounts to roughly a factor of two. Nevertheless, some sensitivity to the parton densities does remain; we will come back to this point shortly. Finally, we note that even at LEP2 the 2–res contribution amounts to at most 5% of the 1–res one [183]; we have therefore not shown it in this figure.

In the last few years several experimental studies of charm production in $\gamma\gamma$ collisions have been published. The first analysis, by the JADE collaboration [185], looked for fully reconstructed charged D^* mesons in single–tag events. They reported a considerable excess over QPM predictions, but this has not been confirmed by the TASSO [186] or TPC/2γ [187] collaborations, who took data at similar energies. TASSO also measured the cross section for exclusive $D^0\overline{D^0}$ production. The total $c\bar{c}$ production cross–sections derived by these two experiments agree with expectations [183].

The first study of charm production in two photon collisions at TRISTAN, by the TOPAZ collaboration [188], also searched for fully reconstructed D^* mesons. Unfortunately the reconstruction efficiency is quite low, leading to rather poor statistics (a few dozen events per experiment). TOPAZ therefore also published a study [189] based on $D^{\pm *} \to \pi_s^\pm D^0$ decays, where only the soft pion needs to be detected; here the signal after background subtraction consists of 372 ± 54 events. VENUS [190] and TOPAZ [191] have also published analyses of hadronic two–photon events containing an electron or positron (electron–inclusive analysis); after subtracting backgrounds (mostly from Dalitz decays and photon conversions), these experiments extract a charm signal with $\mathcal{O}(100)$ events each. Finally, very recently ALEPH presented [192] results of an analysis based on 33 fully reconstructed $D^{\pm *}$ mesons.

Unfortunately these experimental results are somewhat contradictory. All TRISTAN experiments find some excess of events with high p_T; e.g., TOPAZ [189] reports a 2.9 σ excess of events with $p_T(D^*) \geq 3.6$ GeV. This is actually not all that surprising, given that we had to include contributions from the charm in the photon in order to reproduce the production of central jets with $p_T \geq 2.5$ GeV; such "charm excitation" contributions are not included in present MC programs.[†] On the other hand, ALEPH [192] finds a cross–section for the production of charged D^* mesons with $p_T \geq 2.0$ GeV that agrees with the *lower* range of predictions; no excess is visible here.

Most of these experimental analyses are only sensitive to charmed hadrons with significant transverse momentum. The notable exception is the TOPAZ inclusive electron analysis [191]; electrons with momentum as low as 400 MeV are accepted, so that even charm quarks at rest can contribute to the signal. Moreover, TOPAZ used their forward calorimeter, which covers angles down to 3.2° with respect to the beam pipe, to look for the presence of photon remnant jets in the event; their MC predicts a jet tagging efficiency of $73 \pm 2\%$. This allows them to study direct and resolved photon contributions separately. The direct contribution

[†]The NLO correction to the direct process should describe $\gamma c \to gc$ scattering accurately at these energies; however, it is not clear whether the parametrized form of the NLO corrections used by TOPAZ [188, 189] treats such contributions properly. Moreover, 2–res excitation contributions are not included at all.

agrees roughly with the upper range of NLO predictions. The resolved photon contribution agrees with NLO predictions using the LAC1 parametrization, while the DG prediction seems to be at least two standard deviations below the data. This demonstrates that this kind of analysis can lead to significant constraints on the gluon density in the photon.

In order to compare experimental results on charm production with theoretical calculations, one has to model the quark to hadron transition. This is more difficult here than in the more familiar high–energy e^+e^- annihilation events, since for the low values of $M_{c\bar{c}}$ or $p_T(c)$ relevant for present two–photon data the effect of hard gluon radiation can *not* be absorbed into fragmentation functions; this is only possible if gluons are predominantly emitted collinear to the quarks, which requires $p_T \gg m_c$. In this context it is interesting to note that the CLEO collaboration [193], working at values of $M_{c\bar{c}}$ only slightly above the upper range probed by present two–photon experiments, found that popular fragmentation functions could describe the observed D^* meson spectrum *only* if the emission of hard gluons was allowed explicitly. In particular, it was not possible to describe the spectrum by simply convoluting the tree–level $c\bar{c}$ cross–section with the Peterson et al. fragmentation function [105]. Other fragmentation functions gave a better fit, but only if the fragmentation parameters were chosen differently from those used for data at higher energy; this is not surprising, since fragmentation functions are scale dependent, just like parton distribution functions. It is therefore encouraging to note that an NLO event generator for heavy quark production in two photon collisions, written by J. Zunft, is now publicly available [194].[‡] Once hard gluon radiation has been included explicitly, the fragmentation function only has to include nonperturbative effects, which should indeed factorize to good approximation. Studies like that by the TOPAZ collaboration [191], which are sensitive to charmed hadrons at rest, can play an important role in testing this formalism, since by definition fragmentation functions can only change the spectrum, but not the total cross–section. Once the fragmentation process has been fully understood, direct $c\bar{c}$ pair production might be the best way to measure the value of m_c to be used in perturbative QCD calculations, since the scale uncertainty is rather small here. A good knowledge of m_c would help to sharpen predictions for 1–res $c\bar{c}$ production, as well as photo– and hadro–production of charm.

Fig. 26 shows that the total $c\bar{c}$ pair production cross–section is expected to grow quite rapidly with energy. However, at higher energies it might be necessary to impose quite stringent cuts in order to extract a charm signal; this will reduce the detectable cross–section significantly. For example, in ref.[183] it was found that requiring one of the charm quarks to have rapidity $|y| \leq 1.7$ and $p_T \geq 5$ GeV reduces the cross–section by almost a factor of 50. This still leaves us with at least 5,000 $c\bar{c}$ events in 500 pb^{-1} of data; however, at this point we have not yet required anything that would actually identify these events as being due to charm, e.g. a hard lepton or a reconstructed D^* meson.

At a 500 GeV e^+e^- linear collider even the DG parametrization predicts [181, 195] the total $c\bar{c}$ pair cross–section to reach a value between 5 and 50 nb, depending on the amount of beamstrahlung generated. However, requiring the event to contain at least one muon with rapidity $|y| \leq 2$ and $p_T(\mu) \geq 5$ GeV reduces this cross–section by at least a factor of 2,000.

As in case of jet production, cross–sections for the production of heavy quarks can be boosted considerably by converting an e^+e^- collider into a $\gamma\gamma$ collider by means of laser

[‡]This generator only allows the emission of a single hard gluon; the generator used by CLEO to model e^+e^- annihilation events included $2 \to 4$ matrix elements.

backscattering. However, at such a collider QCD processes will likely be regarded primarily as backgrounds to "new physics" signals. In particular, one advantage of a $\gamma\gamma$ collider is that it can produce a Higgs boson as an $s-$channel resonance. If $m_H < 150$ GeV, this Higgs boson will predominantly decay into into a $b\bar{b}$ pair [196]. The direct (tree–level) background from $\gamma\gamma \to b\bar{b}$ can be reduced significantly by chosing the two incident photons to be (predominantly) in a $J_z = 0$ state [197]. There is still a direct background from $\gamma\gamma \to b\bar{b}g$, but this appears to be manageable [198]. However, at least if one uses a collider with a broad photon spectrum, which allows to search a wide range of Higgs masses simultaneously, the dominant background will actually come from resolved photon events. This has been pointed out in ref.[199], where the 1–res $\gamma g \to b\bar{b}$ contribution is studied. As emphasized in ref.[200], the exact size of this background will also depend on the extent to which a gluon in a polarized photon is itself polarized.

The to date most complete study of resolved photon backgrounds to the Higgs signal at a broad–band $\gamma\gamma$ collider has recently been performed by Baillargeon et al. [201]. They ignore polarization effects on the parton densities in the photon (which amounts to the assumption $\Delta G^\gamma = 0$), but include all possible processes, including b and c excitation contributions; these are very large in the relevant kinematical region. They include production of c quarks, as well as light partons, in order to estimate the effects of imperfect $b-$tagging.

An example is shown in Fig. 27, which shows the signal for a 120 GeV Standard Model Higgs boson [196] (assuming a $b\bar{b}$ invariant mass resolution of 5 GeV), as well as various backgrounds. The electron beams have 175 GeV energy, and 90% polarization; 100% polarized laser beams have been assumed. This suppresses the direct backgrounds by about an order of magnitude. Moreover, a p_T cut of 30 GeV has been applied. We see that the largest contribution to the single$-b$ inclusive cross–section comes from 2–res b excitation processes (labelled as "bX_{2-res}"). Depending on details of the $b-$tagging efficiency, it is therefroe often advantageous to require both high$-p_T$ jets to be tagged as b quarks, or at least as heavy flavours.

Another conclusion of ref.[201] is that Higgs searches at a $\gamma\gamma$ collider become easier with increasing Higgs mass and *de*creasing beam energy. The reason is that, as we have seen several times already, cross–sections for resolved photon processes increase rapidly with $\sqrt{s}$, but decrease equally quickly when the invariant mass of the hard system is increased for fixed $\sqrt{s}$. In particular, for the case shown in Fig. 27, even with a $b-$tagging efficiency comparable to present LEP detectors, a 7 σ Higgs signal could be extracted from 10 fb^{-1} of data, which roughly corresponds to one year's running; for the same parameters, one could at best hope to extract a 3.5 σ signal at a 500 GeV collider. Since one here probes the photon at very high momentum scales these conclusions do not depend very sensitively on the parametrization chosen [201]. Nevertheless, Fig. 27 clearly shows that a good understanding of heavy quark production in resolved photon interactions is mandatory if we ever want to look for Higgs bosons with mass below 150 GeV at a $\gamma\gamma$ collider.

4c) J/ψ Production in $\gamma\gamma$ Collisions

J/ψ production in principle offers a clean method for constraining G^γ in two photon collisions [162]. At least in the framework of the colour singlet model [135] and to leading order in α_s, only resolved photon processes can contribute to $\gamma\gamma \to J/\psi$+hadrons. Moreover, the

2–res contribution is expected to be very small, just as in case of open heavy flavour production discussed in the previous subsection. In LO the cross–section is therefore essentially proportional to G^γ; the hard sub–process, $\gamma g \to J/\psi + g$, is the same as in direct inelastic photoproduction of J/ψ mesons, see Sec. 3d. As mentioned in that subsection, NLO corrections to J/ψ production in γg fusion have recently been computed [139]. We use the results of this calculation to update our previous LO predictions [162] for J/ψ production at TRISTAN energies, and also extend the calculation into the LEP energy range.

In order to simplify this calculation, we again use a parametrized form of the NLO corrections. According to ref.[139] the total cross–section for $\gamma g \to J/\psi + X$, integrated over the region $Z \leq 0.9$ where Z is the inelasticity defined in eq.(23), can be written as:*

$$\hat{\sigma}(\gamma g \to J/\psi + X) = \frac{\alpha_{\rm em}\alpha_s^2}{m_c^2} \frac{8 e_c^2 |\psi(0)|^2}{m_{J/\psi}^3} \left\{ c^{(0)}(r) + 4\pi\alpha_s \left[c^{(1)}(r) + \bar{c}^{(1)}(r) \log \frac{\mu^2}{m_c^2} \right] \right\}, \qquad (29)$$

where $r = \hat{s}/m_{J/\psi}^2 - 1$. The wave function at the origin $|\psi(0)|^2$ can be derived from the measured leptonic partial width [140, 139]. The function $c^{(0)}(r)$ describes the tree–level cross–section [135]; we do not impose any cut on Z for this part, since it remains finite in the limit $Z \to 1$, unlike the NLO correction, which diverges in this limit. These corrections are given by the two functions $c^{(1)}(r)$, $\bar{c}^{(1)}(r)$, which can be parametrized as:

$$c^{(1)}(r) = \sqrt{r - 0.1} \left[\frac{(r-0.8)(r-6.2)}{0.85 + 1.1r - 0.1r^2 + 2.7r^{2.5}} - \frac{1}{1.22 - 0.90r + 1.14r^{1.5}} \right]; \qquad (30a)$$

$$\bar{c}^{(1)}(r) = (r - 0.1)^{0.7} \frac{2.6 - r}{1.4 - 1.2r + 3r^{1.7}}. \qquad (30b)$$

Note that the NLO corrections (30) remain finite as $r \to \infty$, while the tree–level cross–section drops like $1/r^2$ in this limit.

In Fig. 28 we show results of a partial NLO calculation of J/ψ production at current e^+e^- colliders based on eqs.(29) and (30). We did not include the small NLO contribution from γq scattering [139]; moreover, the NLO direct contribution from $\gamma\gamma \to J/\psi + gg$ is not yet available. Note also that ψ' production with subsequent $\psi' \to J/\psi$ decay should increase the total J/ψ yield by another 15% or so. The results of Fig. 28 are for a no–tag situation, and include the suppression of G^γ due to the virtuality of the photon [95]. We checked that our parametrization of the NLO cross–section shows significantly reduced scale dependence, as expected, at least as long as the scale $\mu^2 \geq 1.5 m_c^2$; the cross–section drops quickly for even smaller μ. The same behaviour has been observed in [139] for the case of photoproduction of J/ψ. We therefore estimate the scale dependence by varying μ^2 between m_c^2 (lower curves) and $2m_c^2$ (upper curves); the cross–section slowly decreases again for larger values of μ. In addition, we have varied m_c in eq.(29) from 1.6 GeV (lower curves) to 1.3 GeV (upper curves). Since we are now explicity producing a colour singlet state already in

*In ref.[139] the factor $8/m_{J/\psi}^3$ has been written as $1/m_c^3$. However, in the framework of the colour singlet model it seems more natural to us to have the partonic cross–section scale like $1/m_c^2$, rather than $1/m_c^5$. In the latter case the uncertainty related to the value of m_c discussed below would obviously be significantly larger.

the hard scattering process we have always required the γg cms energy to exceed $m_{J/\psi}$, even if $2m_c < m_{J/\psi}$.[†]

Fig. 28 shows that the uncertainty estimated in this way is still quite sizable; most of this uncertainty, however, is due to the variation of the factor $1/m_c^2$ in eq.(29). Notice also that the LO estimate (dotted curve, for scale $\mu^2 = 2m_c^2$ and $m_c = m_{J/\psi}/2$) is quite close to the lower range of NLO predictions. This indicates that the perturbative result is relatively stable. Recall also that ref.[139] finds good agreement of their prediction with low–energy photoproduction data, although leptoproduction data favour larger cross–sections. However, there might be sizable corrections to the non–relativistic approximation intrinsic to the colour singlet model [138]; this has not been included in our estimate of the theoretical uncertainty.

We see that the total expected cross–section for J/ψ production at TRISTAN lies in the range from about 0.4 to 2.5 pb, depending on the parametrization of G^γ. This corresponds to several hundred events per experiment in the total TRISTAN data sample. However, one will almost certainly have to demand that the J/ψ meson decays into an e^+e^- or $\mu^+\mu^-$ pair; this reduces the rate by about a factor of 7.5 even before any acceptance cuts have been applied. The cross–section increases quite rapidly with energy, possibly exceeding 10 pb at LEP2; however, the acceptance at these higher energies might also be poorer. A detailed MC study is necessary to decide whether the study of J/ψ production in two photon events at current colliders is feasible.

As usual, the cross–section at future linear e^+e^- colliders is expected to be even larger, partly due to contributions from beamstrahlung photons. Since the invariant mass of the produced final state is quite small, the cross–section is now largest for designs giving a large number of rather soft photons. For example, at the 500 GeV TESLA collider (we again use the 1993 design [179]) one expects a total J/ψ production cross–section between 0.3 and 2.0 μb, or some 10^7 events per year. At the same collider the $\Upsilon(1s)$ cross–section is expected to lie between 0.25 and 0.85 pb, which still gives around 10,000 events per year. It is at present not clear what fraction of these events would be detectable in the considerably "dirtier" environment of such linear colliders.

4d) Direct Photon Production in $\gamma\gamma$ Scattering

The production of a direct photon in the collision of two (quasi–)real photons, $\gamma\gamma \to \gamma\, X$, is another reaction that in LO only receives contributions from resolved photon processes: The direct $2 \to 3$ process $\gamma\gamma \to q\bar{q}\gamma$ is $\mathcal{O}(\alpha_{\rm em}^3)$, and thus of next–to–leading order compared to the 1–res contribution from $\gamma q \to \gamma q$, as well as the 2–res contributions from $gq \to \gamma q$ and $q\bar{q} \to g\gamma$, which are formally $\mathcal{O}(\alpha_{\rm em}^3/\alpha_s)$; the relevant hard scattering cross–sections can be found in ref.[202]. There are additional LO contributions involving parton $\to$ photon fragmentation; as discussed in Sec. 3c, the corresponding fragmentation functions are $\mathcal{O}(\alpha_{\rm em}/\alpha_s)$, which compensates for the relative factor of $\alpha_s/\alpha_{\rm em}$ that appears in the hard sub–process cross–sections for these contributions. However, these fragmentation contributions will in general lead to softer photons, which are accompanied by a jet.

[†]In ref.[139] r has also been written as a function of m_c^2, rather than a function of $m_{J/\psi}^2$. However, this results in negative cross–sections for $m_c > m_{J/\psi}/2$. It seems to us that the m_c dependence of the c functions should be treated on a par with corrections to the non–relativistic approximation used in the colour singlet model.

The only existing calculation [202] for this process omitted the fragmentation contributions; this roughly corresponds to imposing stringent isolation requirements for the direct photon. Ref.[202] uses LO expressions; an NLO calculation is now under way [54].

In Fig. 29 we show updated LO estimates*. We have required $p_T^\gamma \geq 1.5$ GeV, since QCD perturbation theory would be very unreliable for even softer photons. We have also applied the acceptance cut $|\eta_\gamma| \leq 1$ on the (pseudo)rapidity of the emitted photon. In case of the WHIT1 parametrization, 2–res and 1–res contributions are shown separately, together with the sum; for all other parametrizations only the total prediction is given. We see that WHIT1 predicts the 2–res contribution to be very small in the PEP to TRISTAN energy range, and to remain sub–dominant even at the highest LEP2 energy. On the other hand, according to the LAC1 parametrization the 2–res contribution should be dominant for $\sqrt{s} > 130$ GeV, due to the rapid rise of the cross–section for $gq \to \gamma q$, which profits from the steep gluon density assumed in this parametrization. In contrast, the GRV prediction for the 2–res contribution at $\sqrt{s} = 200$ GeV is about a factor of 1.6 below the WHIT1 result, largely due to the smaller value of $\Lambda_{\rm QCD}$ that has been assumed for GRV; note that we are probing QCD at rather small momentum scales, where α_s depends quite sensitively on $\Lambda_{\rm QCD}$.

The similarity of the various predictions shown in Fig. 29 is therefore somewhat misleading; clearly one could gain more information if the 2–res and 1–res contributions can be separated experimentally, e.g. by remnant jet tagging. Due to the large charge factor, the 1–res contribution mostly probes the $u-$quark density in the photon. Its measurement might therefore yield valuable information about the flavour structure of the photon at rather low momentum scales, where nonperturbative contributions to the photonic parton densities are still important. Whenever the 2–res contribution is sizable, it is dominated by gq scattering; it could therefore give us an additional handle on the gluon content of the photon.

Unfortunately the cross–section is not large even if we allow p_T^γ to be as small as 1.5 GeV; the NLO calculation now being performed [54] will show how well QCD perturbation theory converges at such low momentum scales.† Furthermore, the Bjorken$-x$ of the partons (or photon) "in" the electron can only be reconstructed if the rapidity of the jet balancing the photon is also known. However, requiring $|\eta_{\rm jet}| \leq 1$ reduces the cross–section by another factor of 1.7 (2.0) at $\sqrt{s} = 30$ (200) GeV; moreover, the reconstruction efficiency for such a soft jet might be quite low. On the positive side, Fig. 29 shows that each TRISTAN experiment should have seen about 100 $\gamma\gamma$ events with an isolated hard central photon in the final state; at LEP2, several hundred such events should be accumulated by each experiment. They are well worth looking for.

5) Beyond Perturbation Theory

In this section we will address several topics that go beyond QCD perturbation theory. We will be rather brief here, due to lack of both space and expertise on our part. Nevertheless,

*As discussed in Sec. 3c, care has to be taken when computing the rapidities in this case. This has not been described properly in ref.[202], although the numerical results of this paper are correct.

†When calculating the LAC1 prediction shown in Fig. 29 we had to "freeze" the parton densities, i.e. use $\min(4 \text{ GeV}^2, (p_T^\gamma)^2)$ as momentum scale, since this parametrization produces negative parton densities if used at scales below the input scale $Q_0^2 = 4$ GeV^2.

we feel that we should at least comment on these issues. For one thing, when comparing perturbative QCD calculations with data, one nearly always has to correct for nonperturbative effects, usually with the aid of Monte Carlo event generators; we have already commented on the MC dependence of certain results in the preceding sections. Secondly, as remarked in the Introduction, the study of reactions involving real photons might provide us with new insight into the interplay of soft and hard QCD; given that perturbative QCD is by now generally accepted as the theory of hard hadronic interactions, much future research in strong interactions will presumably be focussed on this transition region.

nonperturbative effects clearly play an important role in so–called minimum bias events. This name derives from the (idealized) definition that in a minimum bias event sample, each inelastic scattering event should be included, without any experimental (trigger) bias. This is very hard to achieve in practice, in particular for final states with low particle multiplicity and small energy deposition in the detector. Measuring total inelastic cross–sections is therefore not easy.

One usually distinguishes three classes of events in γp scattering. In quasi–elastic events, $\gamma p \rightarrow Vp$, the proton remains intact while the photon is transformed into a single vector meson V; usually only ρ, ω, ϕ, and occasionally J/ψ, are included here. In diffractive events either the photon or the proton (or both) gets broken up into several hadrons, but no colour is exchanged between the two systems. In such events there is usually a gap in rapidity space between the two diffractive systems (in double diffractive events), or between the hadronic system and the single p or V (in single diffraction). In contrast, in non–diffractive events colour is thought to be exchanged between the photon and the proton. Both get broken up (in case of direct interactions, the photon is absorbed), and soft particles usually fill rapidity space more or less uniformly.

All these components have to be modelled accurately for a complete understanding of minimum bias events, and in particular if one wants to extract the total γp cross–section from the observed event rate. Both HERA experiments have performed [203, 204] such measurements. H1 [204] has so far only published an analysis of 1992 data, while ZEUS has also published [203] results based on the much larger 1993 sample. Both experiments use different MC models to describe the various event classes, and then try to fit the relative weights to the data. For example, H1 determines that about 26% of all events are quasi–elastic or diffractive; their result for the total γp cross–section at $\sqrt{s} = 195$ GeV is* $\sigma_{\gamma p}^{\rm tot}({\rm H1}) = (171 \pm 7 \pm 22)$ μb, where statistical and systematic errors are listed separately. The 1993 ZEUS data at $\sqrt{s} \simeq 180$ GeV favour a slightly larger quasi–elastic + diffractive event fraction ($\simeq 35\%$), but a lower total cross–section: $\sigma_{\gamma p}^{\rm tot}({\rm ZEUS}) = (143 \pm 4 \pm 17)$ μb. Very recently, H1 announced [205] preliminary results from a special 1994 run with a minimum bias trigger. Their analysis indicates an even larger quasi–elastic + diffractive event fraction (around 40%), and $\sigma_{\gamma p}^{\rm tot}({\rm H1,\ prelim.}) = (172 \pm 3 \pm 10)$ μb.

Note that the systematical uncertainties of these measurements are significantly larger than the statistical errors. This indicates that our understanding of minimum bias photoproduction events still needs to be improved. Note also that the best description of non–diffractive events found by ZEUS [203], based on a superposition of "soft" and "semi–hard" events (see below), still has $\chi^2/d.o.f. > 2$. The detection efficiencies, and hence the extracted

*We have increased the published number by 8%, since H1 used an inaccurate expression for $f_{\gamma|e}$ when converting ep into γp cross–sections value of $\sigma_{\gamma p}^{\rm tot}$, determined from the same MC program may therefore also not yet be entirely reliable.

So far little effort has been made to understand diffractive events in the framework of QCD. A possible connection [206] might be derived from the observation of jets in single–diffractive $p\bar{p}$ events by the UA8 collaboration [207], as well as the recent observation of di–jet events with rapidity gap between the jets by the D0 collaboration [208]. Very recently the ZEUS collaboration announced [209] preliminary results indicating the presence of such di–jet events with gap between the jets at HERA; ZEUS interprets them as resolved photon events with large x_γ.

Much more effort has been devoted to the understanding of non–diffractive inelastic reactions. This is necessary even for the study of hard processes, since the "underlying event" in events with high$-p_T$ jets is closely related to minimum bias events without any (obvious) hard interactions. nonperturbative effects again play an important role in the description of such events; however, it by now seems quite likely that semi–hard QCD interactions, with partonic transverse momenta of order 1 to 2 GeV, are also important here. Such interactions are said to lead to "minijets": Even though a perturbative (hard) scattering took place, the resulting "jet" might be too soft to pass experimental jet identification cuts.

The idea that such minijets might play an important role in minimum bias hadronic physics dates back to 1973, when Cline et al. [7] proposed that the rapid increase of the inclusive jet cross–section with energy might be the root cause of the observed increase of total hadronic cross–sections. This jet cross–section is given by

$$\sigma_{ab}^{\rm jet}(s) = \int_{p_{T,\rm min}}^{\sqrt{s}/2} dp_T \int_{4p_T^2/s}^{1} dx_1 \int_{4p_T^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|a}(x_1) f_{j|b}(x_2) \frac{d\hat{\sigma}_{ij\to kl}(\hat{s})}{dp_T}, \tag{31}$$

where subscripts a and b denote particles ($\gamma,\ p,\ldots$) and $i,\ j,\ k,\ l$ are partons. $\hat{s} = x_1 x_2 s$ as usual, and $\hat{\sigma}$ are hard partonic scattering cross–sections. Note that $d\hat{\sigma}/dp_T \propto p_T^{-3}$; the cross–section defined in eq.(31) therefore depends very sensitively on $p_{T,\rm min}$, which is supposed to parametrize the transition from perturbative to nonperturbative QCD.

If $\sqrt{s} \gg p_{T,\rm min}$, the integral in eq.(31) receives its dominant contribution from $x_{1,2} \ll 1$. The relevant parton densities can then be approximated by a simple power law, $f \propto x^{-J}$. In case of pp or $p\bar{p}$ scattering, $a = b$ and the cross–section asymptotically scales like [210]

$$\sigma^{\rm jet} \propto \frac{1}{p_{T,\rm min}^2} \left(\frac{s}{4p_{T,\rm min}^2} \right)^{J-1} \log \frac{s}{4p_{T,\rm min}^2}, \tag{32}$$

if $J > 1$. For $J \simeq 1.3$, as measured by HERA, the jet cross–section will therefore grow much faster than the total $p\bar{p}$ cross–section, which only grows $\propto \log^2 s$ (Froissart bound [211]), or, phenomenologically [212] for $\sqrt{s} \leq 2$ TeV, $\propto s^{0.08}$. Eventually the jet cross–section (31) will therefore exceed the total $p\bar{p}$ cross–section.

This apparent paradox is solved by the observation that, by definition, inclusive cross–sections include a multiplicity factor. Since a hard partonic scattering always produces a pair of (mini–)jets, we can write

$$\sigma_{ab}^{\rm jet} = \langle n_{\rm jet\ pair} \rangle \sigma_{ab}^{\rm inel}, \tag{33}$$

where $\langle n_{\rm jet\ pair}\rangle$ is the average number of (mini–)jet pairs per inelastic collision. $\sigma_{ab}^{\rm jet} > \sigma_{ab}^{\rm inel}$ then implies $\langle n_{\rm jet\ pair}\rangle > 1$, which means that, on average, each inelastic event contains more than one hard partonic scatter. The simplest possible assumption about these multiple partonic interactions is that they occur completely independently of each other, in which case $n_{\rm jet\ pair}$ obeys a Poisson distribution. At a slightly higher level of sophistication, one assumes these interactions to be independent only at fixed impact parameter b; indeed, it seems natural to assume that events with small b usually have larger $n_{\rm jet\ pair}$. This leads to the eikonal formalism [213], where one writes the cross–section as an integral over the two–dimensional impact parameter b:

$$\sigma_{ab}^{\rm inel}(s) = P_{ab}^{\rm had} \int d^2b \left[1 - \exp\left(- \frac{A_{ab}(b)\chi_{ab}(s)}{P_{ab}^{\rm had}} \right) \right]. \tag{34}$$

Here $P_{ab}^{\rm had}$ is the probability that both initial particles are in a hadronic state when they interact (see below), and A_{ab} describes the transverse overlap of the partons. In realistic analyses [214] it is necessary to introduce both nonperturbative (soft) and hard contributions to the eikonal χ_{ab}:

$$\chi_{ab}(s) = \sigma_{ab}^{\rm soft}(s) + \sigma_{ab}^{\rm jet}(s), \tag{35}$$

where $\sigma_{ab}^{\rm jet}$ is given by eq.(31). The connection to the Poisson distribution then becomes evident from the identity

$$\begin{aligned} \sigma_{ab}^{\rm inel} &= P_{ab}^{\rm had} \int d^2b \left[1 - \exp\left(- \frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right) \right] \\ &+ P_{ab}^{\rm had} \int d^2b \exp\left(- \frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right) \left[1 - \exp\left(- \frac{A_{ab}(b)\sigma_{ab}^{\rm soft}}{P_{ab}^{\rm had}} \right) \right] \\ &= P_{ab}^{\rm had} \sum_{n=1}^{\infty} \int d^2b \exp\left(- \frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right) \cdot \left(\frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right)^n \frac{1}{n!} \quad + \cdots, \end{aligned} \tag{36}$$

where the dots stand for the second line in eq.(36). Each term in the sum corresponds to the cross–section for events with exactly n hard partonic scatters; the second line gives the cross–section for events without any hard scatter [probability = $\exp\left(- \frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right)$, for fixed impact parameter b], but with a soft interaction. Note that the inclusive jet cross–section (31) can be recovered from eq.(36) by introducing a multiplicity factor n in the sum:

$$\begin{aligned} \sigma_{ab}^{\rm jet} &= P_{ab}^{\rm had} \sum_{n=1}^{\infty} n \cdot \int d^2b \exp\left(- \frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right) \cdot \left(\frac{A_{ab}(b)\sigma_{ab}^{\rm jet}}{P_{ab}^{\rm had}} \right)^n \frac{1}{n!} \\ &= \int d^2b A_{ab}(b) \sigma_{ab}^{\rm jet}, \end{aligned} \tag{37}$$

since the function A_{ab} describing the transverse overlap of the partons in the two projectiles is by definition normalized such that $\int d^2b A_{ab}(b) = 1$.

We have already emphasized that, for fixed impact parameter, the ansatz (34) assumes hard scatters to occur independently of each other. This cannot be strictly true, since energy conservation implies that the maximal Bjorken$-x$ for the partons in the second hard interaction is less than 1, but it might still be a good approximation at high energies. A second assumption is that the dependence of the parton densities on Bjorken$-x$ and on the impact parameter b factorizes; only in this case can the dependence on b be parametrized by a single function $A_{ab}(b)$. At present we are not (yet?) able to derive these properties from first principles. The ansatz (34) therefore goes beyond perturbation theory, even though it postulates that perturbative interactions play an important role in minimum bias events; it clearly needs to be checked against experimental data to assess its validity.

It has been shown [214] that eq.(34) with $P^{\rm had}_{p\bar{p}} = 1$ can be brought into agreement with data on total $p\bar{p}$ cross–sections, with $p_{T,{\rm min}}$ around 1.5 GeV. In these calculations $A_{p\bar{p}}(b)$ is usually computed from the Fourier transform of the charge form factor of the proton; this amounts to the assumption that colour charges track the distribution of electric charge in the nucleon. Moreover, the soft cross–section is usually parametrized as

$$\sigma^{\rm soft}_{p\bar{p}}(s) = \sigma_0 + \frac{\sigma_1}{\sqrt{s}}, \tag{38}$$

where $\sigma_{0,1}$ are constants. Including $p_{T,{\rm min}}$, one therefore has three new free paramters (beyond those describing high$-p_T$ jet production) to fit total and inelastic cross–sections (the elastic cross–section can be calculated in this model using the optical theorem); the success of such a fit is not entirely trivial. In particular, the rise of the total cross–section appears very naturally here, although the rate of increase is determined by $p_{T,{\rm min}}$, and is thus fitted rather than predicted. These fits do have some weaknesses, however. They basically ignore the existence of diffractive events; more exactly, if the diffractive cross–section is supposed to be part of $\sigma^{\rm soft}$, eq.(38), then eq.(36) leads to the prediction that the fraction of diffractive events (which are part of the second term, without hard scatters) decreases quickly with energy. A remedy within the framework of the minijet model is possible [215], but only at the cost of introducing additional free parameters. Secondly, existing analyses typically assume that parton densities only increase slowly with decreasing x, at least at scale $\mu^2 \simeq p^2_{T,{\rm min}}$; HERA DIS data favour a much steeper behaviour [14]. It is not clear whether the faster increase of $\sigma^{\rm jet}$ predicted by such parton densities can be compensated by re–fitting the free parameters of the model. We should also keep in mind that total cross–section data can just as well be fitted in a more conventional Pomeron picture [212].

Fortunately there is more direct evidence that an ansatz like eq.(34) can describe some features of hadronic interactions. Using eq.(36), and the standard machinery to describe parton $\rightarrow$ hadron transitions, the idea of having multiple partonic interactions within a single $p\bar{p}$ reaction can be built into an event generator [216, 217, 218]. In ref.[216] it was shown that many features of $p\bar{p}$ scattering as seen at the CERN SpS can be understood in such a picture. These authors did not try to fit total cross–section data; instead, $p_{T,{\rm min}}$ was determined from the measured multiplicity distribution. It is encouraging that this again leads to a value around 1.5 GeV. Multiple interactions then naturally lead to the observed broad (non–Poissonian) multiplicity distribution. The perhaps greatest success of this ansatz is that it reproduces the "jet pedestal" effect. It had been observed that events with hard jets ($E_T > 15$ GeV or so) also contain more hadronic activity far away from the jet cores (in the "pedestal") than minimum bias events do. Eq.(34) predicts such a behaviour, since events with hard jets are likely to have small impact parameter b, which significantly enhances the probability of having additional semi–hard interactions (minijets) in the same events.

Further qualitative and quantitative successes of this ansatz are described in refs.[216, 217]. However, it should be noted that these analyses again assume a rather flat small$-x$ behaviour of the parton distribution functions.

The most direct confirmation of eq.(34) would obviously be the observation of events with (at least) two independent jet pairs due to multiple interactions. These jets should be pairwise back–to–back in the transverse plane; the transverse opening angle should have a flat distribution for these events. Both properties are quite distinct from QCD $2 \rightarrow 4$ events (usually called "double bremsstrahlung"). The cross–section for the production of at least two independent jet pairs can be computed by inserting the multiplicity factor $\left(\begin{array}{c} n \\ 2 \end{array} \right)$ into the sum in eq.(36), and summing all $n \geq 2$:

$$\begin{aligned} \sigma_{ab}(\geq 2 \text{ jet pairs}) &= \frac{1}{2 P_{ab}^{\rm had}} \left[\sigma_{ab}^{\rm jet}(s) \right]^2 \int d^2b A_{ab}^2(b) \\ &\equiv \frac{1}{2} \frac{\left[\sigma_{ab}^{\rm jet}(s) \right]^2}{\sigma_{\rm eff}}. \end{aligned} \tag{39}$$

Energy conservation will again necessitate a slight modification of this simple expression.

The first experimental search for multiple interactions was performed by the AFS collaboration [219] at the CERN ISR (pp collisions at $\sqrt{s} = 63$ GeV). They required $\sum E_T(\text{jet}) > 28$ GeV, which implies $\sum_i x_i \geq 0.45$. It is doubtful whether multiple interactions at these rather large Bjorken$-x$ can be treated as independent; indeed, implementing energy conservation here reduces the expected event rate by about a factor of 4. Moreover, no complete calculation of the QCD $2 \rightarrow 4$ background processes was available at the time, so that this background had to be modelled using a leading logarithmic approximation in a region of phase space where no large logarithms occur. The large signal reported by the AFS collaboration, corresponding to $\sigma_{\rm eff} \simeq 5$ mb, therefore should be taken with a grain of salt.

More recently the UA2 collaboration [220] found some indication for the existence of multiple interactions, but due to its rather small statistical significance (~ 3 standard deviations) they prefer to only quote the lower bound $\sigma_{\rm eff} > 8.3$ mb. Finally, the CDF collaboration [221] found a 2.7 σ signal, corresponding to $\sigma_{\rm eff} \simeq 12$ mb, in the range expected by model calculations.

In the first theoretical studies [222] of minijet production in γp collisions, eq.(34) with $P_{\gamma p}^{\rm had} = 1$ was assumed to hold also for the case of photoproduction. In this case eikonalization effects are very small even at HERA energies, and a rather rapid increase of the total cross–section was predicted. However, as first pointed out by Collins and Ladinsky [8], $P_{\gamma p}^{\rm had}$ should be $\mathcal{O}(\alpha_{\rm em})$. This enhances the exponent in eq.(34) by a factor of order $1/\alpha_{\rm em}$, so that eikonalization does become relevant at HERA energies if $p_{T,\rm min} \leq 2.5$ GeV. The necessity to have $P_{\gamma p}^{\rm had} \sim \mathcal{O}(\alpha_{\rm em})$ can most easily be seen from eq.(36), once we recognize that $\sigma_{\gamma p}^{\rm jet}$ is $\mathcal{O}(\alpha_{\rm em})$; if $P_{\gamma p}^{\rm had} = 1$, the production of additional minijet pairs would therefore be suppressed by powers of $\alpha_{\rm em}$, even though only strong interactions are involved once the photon has been transformed into a hadronic state. Notice also that $P_{ab}^{\rm had}$ cancels out in eq.(37).

In ref.[8], $P_{\gamma p}^{\rm had}$ was taken to be the $\gamma \rightarrow \rho$ transition probability $\simeq 1/300$. In ref.[223] it was suggested to instead estimate it as the momentum fraction carried by partons in the photon at scale $\mu^2 \simeq p_{T,\rm min}^2$; this gives slightly larger values $P_{\gamma p}^{\rm had} \simeq 1/200$. One can (roughly) reproduce the HERA measurements with either number.

One might argue that $P_{\gamma p}^{\rm had}$ should really be of order $\alpha_{\rm em}/\alpha_s$; after all, in eq.(36) one wants the production of a second jet pair to be suppressed by a factor α_s^2, not just a single power of α_s.[†] However, it should be recognized that $P_{\gamma p}^{\rm had}$ cannot be discussed independently of $A_{\gamma p}(b)$. In refs.[223, 224, 225] $A_{\gamma p}(b)$ was computed from the Fourier transform of the pion form factor; it is not at all clear whether this describes the transverse distribution of partons in a vector meson properly, and it is certainly not applicable to the perturbative ("anomalous") component of the photon structure functions. Mathematically, eq.(34) only depends on the combination $A_{ab}/P_{ab}^{\rm had}$; one can see this by using the substitution $b' = b \cdot \sqrt{P_{ab}^{\rm had}}$:

$$\sigma_{ab}^{\rm inel}(s) = \int d^2b' \left[1 - \exp\left(-A'_{ab}(b')\chi_{ab}(s)\right)\right], \tag{40}$$

with $A'_{ab}(b') = A_{ab}\left(b'/\sqrt{P_{ab}^{\rm had}}\right)/P_{ab}^{\rm had}$. Note that $\int d^2A(b) = 1$ implies $\int d^2b' A'(b') = 1$ as well; nevertheless b' cannot be interpreted as the physical impact parameter. Eq.(40) shows that one can always compensate an increase in $P_{ab}^{\rm had}$ by a steeper decrease of $A_{ab}(b)$ at large b, which implies an increase of $A(b)$ at small b. One can therefore simply fix $P_{\gamma p}^{\rm had}$ to some value and fit $A_{\gamma p}(b)$ to data, or vice versa; this approach was followed in ref.[225].

However, in case of photons the ansatz (34) might in any case be too simple. As already discussed in the Introduction, the hadronic structure of the photon is generally assumed to have both perturbative and nonperturbative components. The corresponding contributions to the parton densities have quite different x and Q^2 dependence. It therefore makes sense to split the cross–section (34) into (at least) two terms, characterized by different values of $P^{\rm had}$, different parton densities [and hence different $\sigma^{\rm jet}$, see eq.(31)], and presumably also different overlap functions $A(b)$. Yet another term has to be introduced to describe direct interactions; these cannot be eikonalized, since here the photon is "used up" after the first interaction.

Two different ansätze of this kind have been proposed so far. Both Honjo et al. [226] and Schuler and Sjöstrand [227, 38] split the hadronic photon into a discrete sum over vector mesons, and a perturbative ("anomalous") contribution; symbolically

$$|\gamma\rangle = |\gamma\rangle_{\rm bare} + \sum_{\rho,\omega,\phi} \frac{e}{f_V}|V\rangle + \sqrt{P_{q\bar q}}|q\bar q\rangle, \tag{41}$$

where $|\gamma\rangle_{\rm bare}$ is the "direct" photon[‡], e/f_V are the $\gamma \to V$ transition amplitudes, $|V\rangle$ is a vector meson state, and $|q\bar q\rangle$ is a state that develops from a hard $\gamma \to q\bar q$ splitting. The sum in eq.(41) is assumed to be incoherent, so that each term also contributes separately to the parton densities and to the total γp cross–section. In refs.[226, 227] the vector meson contributions to the parton content of the photon were described in terms of pion structure functions, while in ref.[38] the shapes of these contributions were fitted from F_2^γ data (SaS parametrization; see Sec. 2b). The perturbative contribution to eq.(41) is really a continuum

[†]Recall that even the resolved photon contribution to $\sigma_{\gamma p}^{\rm jet}$ is $\mathcal{O}(\alpha_{\rm em}\alpha_s)$, since the parton densities in the photon are $\mathcal{O}(\alpha_{\rm em}/\alpha_s)$.

[‡]Strictly speaking the coefficient of the first term in eq.(41) should differ from unity by an amount of order $\alpha_{\rm em}$; this effect can safely be ignored.

of states; its contribution to the parton densities can be written as [38]

$$f_{i|\gamma}^{\rm pert}(x,\mu^2) = \frac{\alpha_{\rm em}}{\pi} \sum_q \int_{k_0^2}^{\mu^2} \frac{dk^2}{k^2} f_{i|q\bar{q}}(x,\mu^2,k^2). \tag{42}$$

Note that the perturbative $\gamma \to q\bar{q}$ transition has been factored out here; it gives rise to the factor dk^2/k^2. The "state distributions" $f_{i|q\bar{q}}$ therefore obey *homogeneous* evolution equations in the factorizatioon scale μ^2, with the (leading order) boundary conditions [38]

$$f_{q|q\bar{q}}(x,k^2,k^2) = f_{\bar{q}|q\bar{q}}(x,k^2,k^2) = \frac{3}{2}\left[x^2+(1-x)^2\right]; \tag{43a}$$

$$f_{q'|q\bar{q}}(x,k^2,k^2) = f_{G|q\bar{q}}(x,k^2,k^2) = 0. \tag{43b}$$

At this point the treatment of refs.[226] and [227, 38] diverges. Schuler and Sjöstrand do not attempt to predict the total γp cross–section. Rather, the emphasis of their work is on the properties of photoproduction events, both minimum bias events and events containing hard jets. They therefore assume a parametrization of the total cross–section as in ref.[212]. The overall normalization of the nonperturbative contributions to the photon structure functions, given by the f_V, is fixed by low energy data. This also determines the contribution of this component to the total cross–section, since the Vp cross–sections are assumed to be identical to the πp cross–section. Moreover, they assume that the perturbative component (42) only contributes to hard scattering events, which are very rare at $\sqrt{s(\gamma p)} \simeq 10$ GeV. However, their choice of the f_V gives a contribution to $\sigma_{\gamma p}^{\rm tot}$ which falls about 20% below the data at these low energies. The difference then has to come from direct interactions. This forces them to include direct γp scattering with partonic transverse momentum as low as 0.5 GeV.[§] The same value is also used for the cut–off parameter k_0 in eq.(42); this is in accordance with ref.[226], as well as (approximately) with the GRV [47] and AGF [55] parametrization.

Ref.[227] devotes much attention to the description of quasi–elastic and diffractive events, which are entirely due to contributions from the $|V\rangle$ states. These states also contribute to soft and semi–hard (minijet) non–diffractive γp interactions, where the cut–off $p_{T,\rm min} \simeq 1.3$ GeV at $\sqrt{s} = 200$ GeV is fixed from multiplicity distributions of $p\bar{p}$ events at the same energy. Schuler and Sjöstrand allow for multiple interactions in this sector, but assume them to be completely independent of each other; in contrast, the ansatz (34) assumes independent scattering *only* at fixed impact parameter b. We have seen above that the correlation between the presence of hard jets and small b offered a natural explanation of the jet pedestal effect. Assuming completely independent interactions means that every partonic interaction occurs with probability

$$P_{\rm jet}(p_T) = \frac{1}{\sigma_{Vp}^{\rm nd}} \frac{d\sigma_{Vp}^{\rm jet}}{dp_T}, \tag{44}$$

where the superscript "nd" stands for non–diffractive. Finally, in this model no multiple interactions are allowed to originate from the perturbative contribution (42) to the photon

[§] In ref.[227] Schuler and Sjöstrand modify the proton structure functions for scales $\mu^2 < 5$ GeV^2 and/or small x, in order to enforce a smooth transition between DIS and photoproduction. However, in ref.[38] they use standard leading twist QCD to describe F_2^γ for momentum scales down to about 1 GeV^2.

structure functions. In order to maintain the assumed $s^{0.08}$ behaviour of the total γp cross–section it then becomes necessary to increase $p_{T,\min}$ (in this sector only!) linearly with $\sqrt{s}$, so that $p_{T,\min} \simeq 2.2$ GeV for HERA ($\sqrt{s(\gamma p)} \simeq 200$ GeV).*

The great advantage of the model of refs.[227, 38] is that is comes pre–packaged in the successful LUND/PYTHIA Monte Carlo event generator [228]. This allows to test many aspects of the model against data from HERA, as well as from experiments at lower energies. On the positive side, not only does the total γp cross–section seem to follow the "universal" $s^{0.08}$ behaviour, but the relative sizes of the contributions from quasi–elastic, diffractive, and non–diffractive events also match more or less the model predictions. The recent observation by the ZEUS collaboration [79] that the transverse momentum k_T of the photon remnant jet in events with at least two high$-p_T$ jets is much harder than the Gaussian with width ~ 0.4 GeV characteristic for hadronic remnant jets can also be regarded as a success of this model; indeed, it has been predicted more than ten years ago [229] that the perturbative contribution should have a k_T distribution $\propto dk_T^2/k_T^2$ for large k_T, which is directly related to the ansatz (42). Moreover, the observation [97] that in the region below about 1 GeV, the shape of the p_T-distribution of charged particles at HERA closely matches that of $p\bar{p}$ events at the same energy indicates the presence of a component in the hadronic photon that behaves similarly to other hadrons. At higher p_T, the spectrum becomes harder, in agreement with perturbative calculations [98] that include direct contributions as well as contributions from the perturbative part of the photon structure.

There also appear to be some problems with this model in its present form, however. We already mentioned that the properties of the ZEUS minimum bias sample [203] cannot be described properly by PYTHIA. Moreover, when trying to extract partonic cross–sections (and, ultimately, G^γ) from the measured di–jet cross–section, the H1 collaboration observed [70] that switching on multiple partonic interactions in PYTHIA is not sufficient to describe the energy flow in resolved photon events.† It is at present not clear whether this problem can be solved by replacing the simple ansatz (44) by something like eq.(34) (for the contribution from the $|V\rangle$ states), or whether a more substantial modification of the model will be necessary.

Finally, we find the introduction of three different scales that are meant to separate perturbative and nonperturbative interactions not very appealing. One might argue that $p_{T,\min}$ ought to be larger for the pertubative $|q\bar{q}\rangle$ component than for the $|V\rangle$ components, since the state that develops from a hard $\gamma \to q\bar{q}$ splitting may have a smaller transverse size (as we will see shortly, this is assumed in ref.[226]), so that a larger momentum transfer becomes necessary to resolve individual colour charges. (A similar argument is used in

*In an alternative version of this model, $p_{T,\min}$ is held fixed in the perturbative sector, while the f_V are assumed to decrease with energy. Clearly this can at best be a temporary solution, since eventually the minijet contribution from the perturbative sector alone will exceed the assumed total γp cross–section unless it is unitarized in some way. Schuler and Sjöstrand therefore favour the variant of their model with constant f_V. They also discuss a few other versions, not of all of which are meant to be potentially realistic.

†Since including multiple interactions improved agreement with the data, H1 implicitly assumed that the remaining difference is due to even more partonic interactions; since these produce a jet pedestal that is uniform in ϕ, they can then simply subtract this pedestal from the measured E_T of the jet to arrive at the partonic p_T (up to showering and fragmentation effects, which are presumably described adequately by the model). However, this treatment would not give the correct answer if the additional energy flow was (partly) due to, e.g., enhanced initial state radiation, which is not uniform in ϕ.

ref.[225] for the photon as a whole.) However, this does not explain why k_0, which is also the minimal p_T of direct interactions, is so much smaller than even the value of $p_{T,\min}$ used for the semi–hard interactions of the $|V\rangle$ components. Alternatively, one might argue that quark exchange (which dominates direct interactions and, obviously, $\gamma \to q\bar{q}$ splitting) is perturbative down to significantly smaller momentum scales than gluon exchange (which dominates minijet production in all resolved photon interactions). This could explain why k_0 is smaller than $p_{T,\min}$, but the difference between the $p_{T,\min}$ values used for the $|V\rangle$ and $|q\bar{q}\rangle$ components cannot be explained by such an argument. The different energy dependence of the two $p_{T,\min}$ values is also difficult to understand intuitively.

In this respect the model of ref.[226] seems somewhat more appealing. Here a similar value for the cut–off k_0 in eq.(42) is used, but all minijet production, from direct photons as well as the different classes of resolved photons, is assumed to be regularized by a single, energy independent parameter $p_{T,\min}$. A fit to low energy data, which show a small but significant increase of the total γp cross section between 10 and about 18 GeV (the highest energy that had been reached in fixed target experiments), gives $p_{T,\min} \simeq 1.5$ GeV. Since this is significantly larger than k_0, Honjo et al. also allow the $|q\bar{q}\rangle$ states to have nonperturbative interactions, which are assumed to scale $\propto 1/k^2$; the idea here is that the "hardness" k^2 of the $\gamma \to q\bar{q}$ splitting determines the transverse size of the state that develops from this splitting. Minijet production from this state is also eikonalized, where the typical transverse size [which determines $A(b)$] again scales like $1/k^2$; that is, eikonalization à la eq.(34) occurs for each value of k independently. As written in ref.[226] the model predicts a total γp cross–section at $\sqrt{s} = 200$ GeV between 190 and 250 μb, the lower end of which is compatible at least with the H1 measurement [204]; presumably the agreement could be improved if one allowed the x and/or b dependence of the partons in the $|V\rangle$ states to differ from those in the pion. However, given the assumptions involved, we do not think that total cross–section data can be used to constrain parton distribution functions. Note that this model does predict a jet pedestal effect; it might even be stonger than in $p\bar{p}$ collisions, since for the perturbative contribution to the photon structure functions a very hard interaction not only implies small b, but also (at least on average) a larger value of k^2 and hence a narrower $A(b)$, which increases the chances for additional semi–hard interactions. Unfortunately this model has not yet been built into an event generator.

We finally mention two slightly different approaches to γp scattering. In ref.[230] it was observed that, at least in a single–component model of the photon‡, where $P_{\gamma p}^{\rm had}$ and $A_{\gamma p}(b)$ are determined from "canonical" VDM ideas, it is difficult to simultaneously describe the rise of $\sigma_{\gamma p}^{\rm tot}$ between $\sqrt{s} = 10$ and 18 GeV, and the rather small value of the total cross–section measured at HERA (or at least the smaller ZEUS result [203]). These authors therefore allowed the *soft* contribution to the cross–section to grow $\propto s^{0.058}$, similar to the behaviour found earlier in ref.[232]. Adding a minijet contribution estimated using the DG parametrization with $p_{T,\min} = 3.0$ GeV (where eikonalization effects are still quite small for HERA energies) then gives $\sigma_{\gamma p}^{\rm tot}(\sqrt{s} = 200 \text{ GeV}) \simeq 160$ μb. A similar approach is taken in

‡A single component description was used here since in ref.[231] the perturbative component of the photon structure functions had been found to be too small to have much impact on minijet production. However, comparison with the explicit calculation of ref.[38] shows that the simple estimate of ref.[231], which used the double scaling limit of QCD, greatly under–estimates the perturbative contribution to G^γ at small x and scale $\mu^2 \simeq p_{T,\min}^2$.

ref.[233]. This work attempts a comprehensive description of γp scattering, including quasi–elastic and diffractive events, in the framework of the Dual Parton Model [234]; it is now being implemented in the PHOJET event generator. Since the rise of the soft cross–section is now also fitted from data, in these models one can no longer claim to explain the rise of total cross–sections in terms of minijets. Moreover, $p_{T,\min} \simeq 3.0$ GeV seems rather large to us; there is evidence from J/ψ decays, DIS, and e^+e^- annihilation that perturbative QCD is applicable at lower momentum scales. This is also supported by analyses of $\gamma\gamma$ data, to which we turn next.

Once the description of the photon has been fixed, e.g. by fitting the free parameters of models like those of refs.[226, 227, 233] from γp data, the properties of $\gamma\gamma$ interactions can in principle be predicted unambiguously [235]. However, two–photon experiments have needed comprehensive event generators well before the recent sophisticated models of γp scattering were developed. We already discussed in Sec. 2b that the determination of Bjorken$-x$ in deep–inelastic $e\gamma$ scattering necessitates a model of the hadronic final state; in Sec. 4 we mentioned that the event generators that were used to model quasi–real $\gamma\gamma$ scattering prior to the pioneering AMY study [6] did not include resolved photon processes at all.

The situation has certainly improved a great deal since then; however, there are still some problems. At present most experiments use the same basic ansatz to describe real $\gamma\gamma$ scattering, which contains three separate contributions. Soft interactions, characterized by an exponential p_T spectrum, are modelled using VDM ideas.§ The second component is estimated from the QPM. At high p_T it coincides with the direct contribution introduced in Sec. 4, but the $p_T \rightarrow 0$ divergence is here regularized by constituent quark masses (typically $m_u = m_d \simeq 300$ MeV, $m_s \simeq 500$ MeV), rather than by a momentum cut–off. This QPM contribution therefore extends to (partonic) $p_T = 0$, and is hence not entirely perturbative.¶ In ref.[236] it has been shown that data on total $\gamma\gamma$ cross–sections and F_2^γ can be fitted from these two components only. However, the description of multi–hadron and jet production in $\gamma\gamma$ collisions is only possible if one also introduces single and double resolved contributions, which form the third component of current $\gamma\gamma$ event generators.

So far all currently used generators agree. There are some differences in the details, however. For example, the TRISTAN experiments [6, 167, 168], as well as DELPHI [177, 178] use parameter values for the description of the soft ("VDM") component that have been determined by experiments at lower energies. This might be dangerous, since these earlier analyses did not allow for resolved photon interactions; there is a strong correlation between the assumed size of the soft component and the parameter $p_{T,\min}$, which essentially fixes the size of the resolved photon contribution to the minimum bias event sample. The ALEPH collaboration [176] therefore preferred to fit both $p_{T,\min}$ and the parameters describing the soft contribution from their own data. This might explain the rather large value of $p_{T,\min}$, 2.5 GeV, obtained by ALEPH for the DG parametrization, compared to 1.45 GeV for DELPHI and 1.6 to 2.0 GeV for AMY and TOPAZ. The expression for the total $\gamma\gamma$ cross–section

§This "VDM component" is not to be confused with the nonperturbative contribution to photon structure functions, which is often also estimated from the VDM. As emphasized in refs.[227, 235], these "VDM partons" also participate in hard resolved photon interactions.

¶Recall that in the model of refs.[227, 235] direct interactions with partonic p_T as low as 500 MeV are allowed; in practice this should give a similar contribution as the QPM prediction using constituent quark masses.

derived by ALEPH also differs significantly from conventional VDM expectations. It should be noted that the jet cross–sections derived by TRISTAN experiments [167, 168] are based only on high$-p_T$ events; these analyses are therefore not sensitive to the assumed value of $p_{T,\mathrm{min}}$. Furthermore, the experimental definition (trigger conditions) of the "minimum bias" sample differs quite significantly between the various experiments, with ALEPH having the loosest cuts and largest visible cross–section, while AMY uses quite stringent requirements. Of course, a complete event generator should still give the same values (within errors) for its free parameters even when quite different data samples are used for the fit. The discrepancies between the values of $p_{T,\mathrm{min}}$ determined by different experiments therefore indicates that (some) present generators are not complete.

One problem of these generators is that they do not include initial and final state radiation (parton showers). ALEPH states [176] that generators like PYTHIA [228] and HERWIG [237] with parton showering switched on did not describe the data well. However, QCD tells us that such radiation must exist at some level. It might, for example, change the p_T-dependence of the partonic cross–sections extracted by TOPAZ [167] and AMY [168], although the string fragmentation scheme used in these analyses may mimick some of these effects; a preliminary TOPAZ study [174] finds that their MC program gives a reasonable, but not perfect description of the distribution of the transverse opening angle ϕ between the jets in di–jet events. Note that initial state radiation should be treated differently for photons than for real hadrons [227, 238], due to the presence of the hard $\gamma q \bar{q}$ vertex. Whenever one reaches this vertex in the standard backward evolution [239] of initial state radiation, the shower has to be terminated, even if the parton's virtuality is still quite high at this point. This feature is included in the latest versions of PYTHIA and HERWIG, at least in an average ($x-$independent) sense.

Another shortcoming of present $\gamma\gamma$ event generators is that they do not allow for the hard transverse momentum distribution of the remnant jets that has been predicted by theory [229] and seen experimentally by ZEUS [79]. This seems to have little impact on the p_T-spectrum of the hard jets [79], but might change the predicted efficiency for detecting remnant jets, e.g. in the TOPAZ detector [240, 167, 191].

Finally, these event generators at present do not allow for multiple partonic interactions. Even if only the contribution from the nonperturbative part of photon structure functions is eikonalized, multiple interaction effects will play a significant role from LEP energies onward [73]. In a single–component model of the photon, eq.(34) with $P_{\gamma\gamma}^{\mathrm{had}} \sim \alpha_{\mathrm{em}}^2$, four jet events due to multiple parton scattering might already be detectable at TRISTAN*, and ought to become a prominent feature of two–photon events at LEP2 energies [241]. The study of $\gamma\gamma$ collisions at the highest available energies should therefore tell us how to unitarize the contribution to minijet production from the perturbative ("anomalous") component of the photon structure.

Apart from its intrinsic interest, a good understanding of minimum bias $\gamma\gamma$ events is necessary for a realistic evaluation of possible hadronic backgrounds at future linear e^+e^- colliders. In ref.[242] it was pointed out that in designs with strong beamstrahlung, several hadronic $\gamma\gamma$ collisions might occur in each bunch crossing, thereby effectively giving rise to an underlying event. This conclusion has been criticized [243] on the grounds that the

*Even in this case they are not expected to significantly affect the properties of events with one or two high$-p_T$ jets; they do therefore not invalidate the analyses of refs.[167, 168] minijet contribution had not been eikonalized in ref.[242]. However, in refs.[180, 244] is has been shown that eikonalization does not significantly change predictions for the next generation of linear colliders, planned to operate at $\sqrt{s} \simeq 500$ GeV. The "hadron crisis" has not been solved by improved calculations, but by improved machine designs, which reduce beamstrahlung by employing a larger number of flat bunches. It should be emphasized that hadronic backgrounds are not the only, and perhaps not even the most severe, problem of designs with high beamstrahlung; they also suffer from large backgrounds from soft e^+e^- pairs [245], as well as from a poorly defined beam energy, which complicates the study of thresholds [246].

So far only one Monte Carlo study of the properties of these soft hadronic backgrounds has been published, by Chen et al. [244]. They assume that the total $\gamma\gamma$ cross–section tracks the measured $p\bar{p}$ cross–section at high energies. It should be pointed out that at present even the value of the total $\gamma\gamma$ cross–section at low energies is only poorly determined experimentally. Chen et al. then modelled minimum bias events using ISAJET [247], which contains a parametrization of the underlying event as determined in pp and $p\bar{p}$ collisions. Using the same model for $\gamma\gamma$ collisions may not be a bad first approximation, given the similarities of some features of HERA and $p\bar{p}$ events discussed earlier. However, Chen et al. did not attempt to compare their model with actual data. They also used a rather large value for $p_{T,\min}$, 3.6 GeV, this being (approximately) the smallest partonic p_T leading to observable jets at the UA1 experiment [248] at the SpS. However, TRISTAN experiments have demonstrated [167, 168, 174] that much softer jets can be reconstructed in $\gamma\gamma$ collisions. The analysis by Chen et al., which uses the simplified unitarization scheme (44), therefore probably under–estimates the effect of minijets on event characteristics.† Chen et al. conclude that the average $\gamma\gamma$ event only deposits about 3.5 GeV of hadronic energy in the central detector ($|\cos\theta| \leq 0.9$) at a typical design for a 500 GeV e^+e^- collider. It would be interesting to repeat their analysis, using an event generator that has been tuned to describe real $\gamma\gamma$ data.

Finally, we briefly mention that eikonalization effects [243] do certainly play an important role if the next e^+e^- collider is converted into a $\gamma\gamma$ collider, which greatly increases the average $\gamma\gamma$ cms energy. The analysis of ref.[244] indicates that even here one can stay well below one hadronic event per bunch crossing, at least at a 500 GeV collider; the problem rapidly gets worse at higher energies, mostly due to the need to increase the luminosity $\propto s$ in order to achieve a constant rate of hard events. Since at those colliders $\gamma\gamma$ scattering occurs at energies well beyond the reach of current colliders, and in view of the incomplete description even of these accessible events, any prediction of event properties at future $\gamma\gamma$ colliders should be taken with a grain of salt.

6) Outlook

In this article we have reviewed the present knowledge of resolved photon interactions. Great progress has been made in the last few years, both experimentally and theoretically. We now

†As noted above, ISAJET simply fits the properties of the underlying event in $p\bar{p}$ collisions, including effects that can be explained in terms of minijets. However, it is known that γp events are more "jetty" than $p\bar{p}$ events, and hard partonic reactions are expected [235] to play an even more prominent role in $\gamma\gamma$ scattering.

know for a fact that partons "in" (quasi–)real photons play an important role in both γp and $\gamma\gamma$ interactions. Theory tells us that the hard $\gamma q\bar{q}$ vertex should lead to (a component of the) photonic parton densities that have a much harder $x-$dependence than the more familiar nucleonic parton distribution functions, and which should grow approximately linearly with the logarithm of the momentum scale at which the photon is being probed. Both properties have been confirmed experimentally, at least for the quark densities. Studies of photonic remnant jets also indicate that resolved photons do not always act like ordinary hadrons.

At the same time many properties of minimum bias photoproduction and two photon events do resemble those of purely hadronic collisions at similar energies. Moreover, the total γp cross–section seems to show approximately the same energy dependence as total hadronic cross–sections. We therefore have good evidence for the existence of both perturbative and nonperturbative contributions to the hadronic structure of the photon.

Qualitatively speaking this summarizes the present level of understanding of resolved photons. In the preceding sections we have reviewed the various pieces of information that contributed to this understanding; we have also described attempts to cast this understanding in a quantitative form. It seems rather futile to try and summarize this summary. We therefore decided to conclude this article with a list of open problems, which might become the foci of future work.

The most pressing problems in deep–inelastic $e\gamma$ scattering (Sec. 2) have to do with the description of the final state:

- The measurement of Bjorken$-x$ has traditionally relied on a reconstruction of the invariant mass W of the hadronic final state from the measured quantity $W_{\rm vis}$. An improvement of this procedure has recently been suggested [39]. It might also be possible to determine the energy of the target photon using the Jacquet–Blondel method, as is done by HERA experiments. Detailed Monte Carlo work, using realistic detector resolutions and acceptances, is needed to decide how well these ideas work in practice.

- There are some discrepancies between existing data on F_2^γ. Improved reconstruction methods might be of some help here. It is not clear whether older PEP and PETRA data can be re–analyzed in this way, but new measurements and/or new analyses of F_2^γ at small x_γ might help to resolve the discrepancy between recent results by the TOPAZ and OPAL collaborations. Such studies have the potential to discriminate between existing parametrizations of photonic parton densities.

A list of open problems in hard γp scattering (Sec. 3) includes:

- No complete NLO treatment of di–jet production exists. Recall that one needs to measure the rapidities of both high$-p_T$ partons/jets in a hard event in order to reconstruct the Bjorken$-x$ variables.

- The measured jet cross–sections should be extended both in rapidity and in p_T. The former increases the sensitivity to the interesting region of small x_γ, while the latter should allow to test theory cleanly, since a detailed understanding of the underlying event (see below) is less crucial at high p_T, and differences between parametrizations of photonic parton densities are small at large x_γ and large momentum scale.

- It might be interesting to try to correlate properties of the photonic remnant jet with those of the high$-p_T$ jets. In the usual "nonperturbative + anomalous" description of the hadronic photon, the nonperturbative component should always have a remnant jet with very small k_T; this component is also characterized by soft parton densities. In this picture one therefore expects nontrivial correlations between x_γ and k_T.

- Studies of heavy flavour production hold great potential. We do not think it very interesting to try to derive total cross–sections from measurements covering only a limited region of phase space, which contains only a small fraction of all produced heavy quarks. It might be more fruitful to attempt to extract the resolved photon contribution, which is sensitive to the as yet poorly constrained gluon density in the photon. At high p_T, "excitation" contributions from the charm in the photon have to be taken into account. An important open problem is the fragmentation of rather soft (low$-p_T$) charm quarks, which contribute most to the total charm cross–section. Theoretical predictions are more reliable for b production, but it might be difficult to find a clean signal.

- The production of direct photons is by now quite well understood, although an NLO calculation of photon + jet production would certainly be welcome. Realistic background studies are also needed, but can presumably only be performed by members of HERA experiments.

- In spite of the recent NLO calculation [139] of direct J/ψ production in the colour singlet model, much needs to be done here: The resolved photon contribution is only known to leading order in the colour singlet model. Nothing is known about the contribution from the colour octet component of the wave function of the J/ψ [249], which is accessible to resolved photons already in LO. In addition, there are contributions at high p_T coming from charm and gluon fragmentation [250].

- It is important to test our understanding of the hadronic photon in as many different channels as possible. The production of Drell–Yan lepton pairs, two photon final states, and associate $J/\psi + \gamma$ final states are all plagued by rather small cross–sections, but this should at least partly be compensated by the cleanliness of the final states.

We should emphasize that many of the measurements proposed here need more luminosity than HERA has delivered so far. Fortunately new data are being collected at an ever accelerating pace.

Some open problems in <u>hard $\gamma\gamma$ scattering</u> (Sec. 4) are:

- An NLO calculation of di–jet production is still lacking.

- There is some discrepancy in the high$-p_T$ end between the partonic cross–sections published by AMY and TOPAZ. Also, it is not clear how these cross–sections, which have been extracted with the aid of leading order event generators, can be compared to NLO calculations.

- More detailed tests of theory should be possible if the kinematic reconstruction of events with high$-p_T$ jets can be improved. One possibility might be to determine the

Bjorken$-x$ variables using small angle electron taggers and/or the Jacquet–Blondel method. A study of the energy distribution of the remnant jet(s), and possible correlations with the hard jet(s), might also prove rewarding. All these measurements are probably quite difficult, however.

- Measurements of the cross–sections for the production of high$-p_T$ particles should be relatively straightforward; they test similar aspects of the theory as studies of jet final states do.

- The excess of high$-p_T$ charm events seen by TRISTAN experiments might well be explainable in terms of charm excitation from the photon; in the kinematical regime that is being probed by these experiments, this should be well described by the NLO event generator that has become available recently. Recall, however, that ALEPH does not see any excess.

- J/ψ and direct photon production has so far only been treated in LO. No data on these final states exists as yet.

Finally, some of the most challenging problems go beyond perturbation theory (Sec. 5):

- We have to understand the energy flow in resolved photoproduction events. This is crucial for the comparison of measured jet rates with theoretical (partonic) calculations, especially for presently accessible values of p_T.

- The modelling of minimum bias photoproduction events needs to be improved. Among other things, this is necessary for an accurate determination of total γp cross–sections.

- The total hadronic $\gamma\gamma$ cross–section is at present only determined very poorly. An accurate measurement can probably only be made if at least one of the outgoing electrons is tagged at a small angle. This is important for the assessment of hadronic backgrounds at future linear e^+e^- colliders, which may play a role in the decision which of several competing designs should be pursued.

- The role of multiple partonic interactions needs to be clarified. They will probably (help to) explain the energy flow in hard γp events mentioned above. Theoretically least understood is the treatment of contributions from the perturbative component of photon structure functions. Studies of multi–jet production in high energy $\gamma\gamma$ collisions might yield important clues here. This also has ramifications for a theory (as opposed to parametrization) of total hadronic cross–sections.

- $\gamma\gamma$ event generators in present use are known to be incomplete: They do not include parton showers, and assume too soft a k_T distribution of the remnant jets. At higher energies multiple interactions might also affect the properties of the underlying event substantially. Finally, some standardization might help to decide whether putative differences in published results signal real discrepancies in the data, or are merely a reflection of different MC generators.

- It is important to test the (potentially) complete event generators for γp and $\gamma\gamma$ scattering that are now being developed in as many different ways as possible. Not only should things like the energy flow, multiplicity distributions, particle p_T spectra, and strangeness production be investigated, there might also be interesting correlations between some of these quantities.

This list is almost certainly not complete. Moreover, new questions are likely to arise as soon as current problems are solved. Much needs to be done before we can say with confidence that we understand the structure of light.

Acknowledgements

We greatly benefitted from numerous discussions with many people working in this field. In particular, we wish to thank P. Aurenche, G. Bélanger, F. Borzumati, F. Boudjema, J. Butterworth, R. Enomoto, J. Forshaw, L. Gordon, M. Greco, H. Hayashii, M. Iwasaki, D. Miller, T. Nozaki, G. Schuler, M. Seymour, T. Sjöstrand and J. Storrow for their replies to our questions. The work of M.D. was supported in part by the U.S. Department of Energy under grant No. DE-FG02-95ER40896, by the Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, as well as by a grant from the Deutsche Forschungsgemeinschaft under the Heisenberg program. R.M.G. wishes to acknowledge research grant no. 03(0745)/94/EMR-II, from the Council of Scientific and Industrial Research.

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Figure Captions

Fig. 1 Generic Feynman diagram for deep–inelastic $e\gamma$ scattering: A probing photon with large virtuality $Q^2 \equiv -q^2$ scatters off the (quasi–)real target photon (with virtuality $P^2 \simeq 0$) to produce a hadronic final state X.

Fig. 2 Feynman diagrams that contribute to $\gamma^*\gamma$ scattering, i.e. the blob in Fig. 1. (a) shows a tree–level (QPM) contribution, while (b) and (c) are examples of perturbative QCD corrections in the flavour non–singlet and flavour singlet sector, respectively, and (d) shows nonperturbative contributions as estimated in the VDM. All these contributions are formally summed in (e), where the quark densities in the photon q_i^γ are introduced.

Fig. 3 Predictions of various existing parametrizations of photonic parton densities for the structure function F_2^γ at $Q^2 = 15$ GeV^2 are compared with data from the OPAL [29] (squares) and TOPAZ [33] (diamonds) collaborations.

Fig. 4 Predictions of various existing parametrizations of photonic parton densities for the gluon content in the photon (a), and for the ratio of strange to non–strange light quark densities (b). Notice that we have used a logarithmic scale for the $y-$axis in (a), while in (b) x has been shown on a logarithmic scale. Both figures are for $Q^2 = 15$ GeV^2.

Fig. 5 Feynman diagrams contributing to the photoproduction of high$-p_T$ jets. Direct contributions from the QCD Compton scattering and photon–gluon fusion are shown in (a), while (b) gives examples of resolved photon contributions involving the scattering of two quarks, a quark and a gluon, or two gluons.

Fig. 6 The topology of photoproduction events. In (a) the incident photon couples directly, leading to a final state with two high$-p_T$ jets (in leading order), and a remnant jet from the proton. In resolved photon contributions (b) the photon gives rise to a second remnant jet.

Fig. 7 Example of a diagram that can be interpreted as either a leading order direct or resolved photon contribution, or an NLO direct contribution, depending on the virtualities of the exchanged quark and gluon; see the text for a detailed discussion.

Fig. 8 The ratio of resolved photon and direct contributions to the inclusive jet cross–section at HERA, for jet pseudorapidity between –1.0 and +2.0. We have used the MRSD-' parametrization for the proton, and various parametrizations for the photon as indicated. These are leading order predictions, for momentum scale $\mu^2 = p_T^2$. While most curves have been computed using the simple antitagging requirement $Q^2 < 4$ GeV^2, the dot–dashed curve has been calculated demanding $Q^2 < 0.01$ GeV^2 and scaled initial photon energy z between 0.20 and 0.75.

Fig. 9 Resolved photon (a) and direct (b) contributions to the inclusive jet cross–section at HERA. Contributions from qq, gq and gg final states are shown separately, where q stands for a quark or antiquark of any flavour. We have used the same antitagging and jet rapidity cuts as in Fig. 8.

Fig. 10 The rapidity distribution of photoproduced jets with transverse momentum exceeding 8 GeV at HERA. The dotted curves show the contribution from directly interacting photons only, while the other curves show the total prediction using various parametrizations of photonic parton densities. The requirements for the scaled initial photon energy z differ in (a) and (b); requiring larger z clearly increases the differences between the predictions.

Fig. 11 The triple differential di–jet cross–section at HERA, for the case of equal jet pseudorapidities. The direct and three subclasses of resolved photon contributions are shown separately. In (a) no cut on z is applied, while in (b) only events with $0.3 \leq z \leq 0.8$ are accepted; in particular, the requirement $z \geq 0.3$ removes the direct contribution for $\eta_1 = \eta_2 > 0.2$.

Fig. 12 The average energy of the photonic remnant jet for resolved photon events at HERA with $p_T = 10$ GeV, as a function of the common pseudorapidity of the high$-p_T$ jets. No cuts on the incident photon energy have been applied. The contributions from qq, qg and gg final states are again shown separately.

Fig. 13 The scale dependence of the inclusive jet cross section at HERA, for jets with transverse energy $E_T = 25$ GeV and pseudorapidity $= +1.5$, as predicted using the GRV parametrization for the photon and MRSD0 for the proton. The dotted, dot–dashed and long dashed curves have been obtained by varying the photonic factorization scale M_γ only, while the short dashed and solid curves are for the case where both factorization scales and the renormalization scale are identical. Adapted from Bödeker et al. [89].

Fig. 14 The transverse momentum spectrum of $c\bar{c}$ and $b\bar{b}$ pairs, for the case where both heavy quarks are produced centrally (rapidity $y = 0$). Note that at high p_T the single charm (or bottom) inclusive cross–section will receive substantial "excitation" contributions involving the heavy flavour content of the photon, which are not included here. Even though we have used a parametrization with a hard G^γ, the resolved photon contribution is clearly very small.

Fig. 15 The triple differential cross–section for the photoproduction of $c\bar{c}$ pairs at HERA, for $p_T(c) = 10$ GeV, as a function of the rapidities of the two heavy quarks, which are taken to be equal. These are LO results; MRSD-' has been used for the nucleonic parton densities. For the WHIT1 parametrization contributions from gg fusion and $q\bar{q}$ annihilation are shown separately, while only the sum is shown for the WHIT4 and

LAC1 parametrizations. Note that for the range of rapidities shown here, the cut on the scaled incident photon energy z removes the direct contribution completely (in leading order).

Fig. 16 The rapidity dependence of the direct photon cross–section for incident photon energy $E_\gamma = 10$ GeV and transverse momentum of the final photon $p_T^\gamma = 5$ GeV, as predicted from a leading order calculation [120] using the ABFOW parametrization [121] for the proton and AGF for the photon. Note that the rapidity η_γ is measured in the γp centre–of–mass frame here. The contribution from the gluon in the photon clearly dominates for $\eta_\gamma \geq 1$.

Fig. 17 The rapidity dependence of the direct photon cross–section at HERA for $p_T^\gamma = 5$ GeV, as predicted from an NLO calculation [123] using the GRV parametrization [50] for the proton. Note that η_γ is measured in the HERA lab frame. No cut on the energy of the incident photon has been applied; the contribution from the gluon in the photon is sub–dominant everywhere. Some sensitivity to the quark densities in the photon still remains, as demonstrated by the difference between the solid (GRV) and dotted (GS) curves.

Fig. 18 A Feynman diagram contributing to the photoproduction of J/ψ mesons in the colour singlet model. For the resolved photon contribution, the incident photon has to be replaced by another gluon.

Fig. 19 The J/ψ production cross–section at HERA is shown as a function of the elasticity parameter Z of eq.(23), as predicted from a leading order calculation [146]. Acceptance cuts have been applied on the transverse momentum of the J/ψ meson and on the angle of both leptons that originate from J/ψ decay, but the leptonic branching ratio has not been included.

Fig. 20 Typical Feynman diagrams (left) and topologies (right) of the three classes of contributions to the production of high$-p_T$ jets in (quasi–)real $\gamma\gamma$ collisions: direct (a), single resolved (b), and double resolved (c). Note that each resolved photon gives rise to a remnant jet.

Fig. 21 Leading order predictions for the inclusive jet cross–section at TRISTAN. The lower dotted curve in (a) shows the direct contribution only, while the other curves show the total prediction using various parametrizations of photonic parton densities, as indicated. The cuts on the pseudorapidity of the jet and on the maximal scattering angle of the outgoing $e^\pm$ differ, (a) corresponding to the cuts used by TOPAZ and (b) to those used by AMY; note that the antitagging cut is only effective if $z \leq 0.75$. Data by these two experiments [167, 168] are also shown.

Fig. 22 The triple differential di–jet cross–section at TRISTAN, as predicted in leading order. (a) shows the direct and 1–res contributions, while (b) shows 2–res contributions. For the WHIT1 parametrization the contributions from different final states are shown separately, while only the sum is given for the DG and WHIT6 parametrizations. The results are for TOPAZ antitagging conditions.

Fig. 23 The triple differential di–jet cross–section at LEP1 and LEP2, as predicted in LO using the WHIT1 parametrization. The steps at pseudorapidities $\eta_1 = \eta_2 \simeq 1.5$ (for $\sqrt{s} = 90$ GeV) or 2.2 (for $\sqrt{s} = 180$ GeV) occur since the ALEPH antitagging condition has been used, which limits the scattering angle of the outgoing $e^{\pm}$ only if it carries more than 50% of the beam energy.

Fig. 24 The transverse momentum distribution of jets produced in $\gamma\gamma$ events at two stages of the planned JLC collider, as predicted in LO using the WHIT1 parametrization. The dotted curves show the total result in the absence of beamstrahlung, while the dashed curves show different contributions as indicated, and the solid curves their sum, for realistic beamstrahlung spectra. The comparison between the solid and dotted curves show that the beamstrahlung spectrum is expected to be significantly harder at $\sqrt{s} = 1$ TeV (b) than at 0.5 TeV (a).

Fig. 25 The triple differential di–jet cross–section at a $\gamma\gamma$ collider, based on a 500 GeV e^+e^- collider with unpolarized beams. Due to the hardness of the photon spectrum, there are substantial differences between predictions using WHIT1 and LAC1 even at the high value of p_T chosen here.

Fig. 26 Predictions for the total cross–section for $c\bar{c}$ pair production in $\gamma\gamma$ collisions at e^+e^- colliders. The direct contribution (dotted) includes the parametrized form (28) of NLO corrections, whereas the resolved photon contribution (dashed and solid curves) has been computed in LO only. Two curves of a given pattern show the uncertainty of the theoretical prediction, as described in the text.

Fig. 27 The signal for a Standard Model Higgs boson of mass $m_H = 120$ GeV decaying into a $b\bar{b}$ pair, as seen at a $\gamma\gamma$ collider based on a 350 GeV e^+e^- collider, as well as various backgrounds. 90% electron beam polarization has been assumed in order to suppress the direct background. Both $c\bar{c}$ and $b\bar{b}$ pair production processes and c and b excitation processes are included in the background calculation. The width of the Higgs signal, and of the Z peak, is determined by the resolution, not the intrinsic widths of these particles. From Baillargeon et al. [201].

Fig. 28 The total cross–section for the production of J/ψ mesons in $\gamma\gamma$ collisions, as predicted by the colour singlet model. The dotted curve shows a LO prediction for scale $\mu^2 = m^2_{J/\psi}$, while for the dashed and solid curves NLO corrections to the 1–res γg fusion contribution have been included using the parametrization of eqs.(29), (30), but the

NLO direct and γq scattering contributions have been omitted. Two curves of a given pattern show the theoretical uncertainty of our estimate, as described in the text.

Fig. 29 Leading order predictions for the production of a hard, central photon in no–tag two–photon collisions (no antitag condition has been imposed). Fragmentation contributions are not included. The long and short dashed curves show 2–res and 1–res contributions as predicted from the WHIT1 parametrization; they add up to the solid curve. For all other parametrizations, only the total sum has been shown.