Radiative Electroweak symmetry breaking in the MSSM and Low Energy Thresholds

K. Tamvakis[1]

Division of Theoretical Physics, University of Ioannina
Ioannina, GR - 451 10, GREECE

Abstract

We study Radiative Electroweak Symmetry Breaking in the Minimal Supersymmetric Standard Model (MSSM). We employ the 2-loop Renormalization Group equations for running masses and couplings taking into account sparticle threshold effects. The decoupling of each particle below its threshold is realized by a step function in all one-loop Renormalization Group equations (RGE). This program requires the calculation of all wavefunction, vertex and mass renormalizations for all particles involved. Adapting our numerical routines to take care of the succesive decoupling of each particle below its threshold, we compute the mass spectrum of sparticles and Higgses consistent with the existing experimental constraints. The effect of the threshold corrections is in general of the same order of magnitude as the two-loop contributions with the exception of the heavy Higgses.

[1]Talk presentend in Susy '95, Ecole Polytechnique, Paris, May 1995

effects in the Renormalization Group equations of the parameters of the MSSM in the framework of Radiative Electroweak breaking. Since we have employed the $\overline{DR}$ scheme in writting down the one-loop Renormalization Group equations, which is by definition mass-indepedent, we could "run" them from M_X down to M_Z without taking notice of the numerous sparticle thresholds existing in the neighborhood of the supersymmetry breaking scale near and above M_Z. This approach of working in the "full" theory consisting of particles with masses varying over 1-2 orders of magnitude has to overcome the technical problems of the determination of the pole masses. Our approach, also shared by other analyses, is to introduce a succession of effective theories defined as the theories resulting after we functionally integrate out all heavy degrees of freedom at each particle threshold. Above and below each physical threshold we write down the Renormalization Group equations in the $\overline{DR}$ scheme only with the degrees of freedom that are light in each case. This is realized by the use of a theta function at each physical threshold. The integration of the Renormalization Group equations in the "step approximation" keeps the logarithms $\ln(\frac{m}{\mu})$ and neglects constant terms. The physical masses are determined by the condition $m(m_{phys}) = m_{phys}$ which coincides with the pole condition if we keep leading logarithms and neglect constant terms. The great advantage of this approach is that the last step of determining the physical mass presents no extra technical problem and it is trivially incorporated in the integration of the Renormalization Group equations.

A dramatic simplification of the structure of the supersymmetry breaking interactions is provided either by Grand Unification assumptions or by Superstrings. The simplest possible choice at tree level is to take all sparticle and Higgs masses equal to a common mass parameter m_o, all gaugino masses equal to some parameter $m_{1/2}$ and all cubic couplings flavour blind and equal to A_o. This situation is common in the effective Supergravity theories resulting from Superstrings but there exist more complicated alternatives. For example Superstrings with massless string modes of different modular weights lead to different sparticle masses at tree level[2]. The equality of gaugino masses can also be circumvented in an effective supergravity theory with a suitable non-minimal gauge kinetic term[3]. Note however that such non-minimal alternatives like flavour dependent sparticle masses are constrained by limits on FCNC processes. In what follows we shall consider this simplest case of four parameters m_o, $m_{1/2}$, A_o and B_o. The scalar potential of the model is

$$\begin{aligned} V &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + \mu B(H_1 H_2 + c.c) \\ &+ \frac{1}{8}g'^2(|H_1|^2 - |H_2|^2)^2 + \frac{1}{8}g^2(|H_1|^4 + |H_2|^4 + 4|H_1^\dagger H_2|^2 - 2|H_1|^2|H_2|^2) + \ldots.. \end{aligned} \quad (1)$$

written in terms of

$$m_{1,2}^2 \equiv m_{H_{1,2}}^2 + \mu^2 \quad (2)$$

Only an unbroken minimum appears for $m_1^2 = m_2^2$. Replacing the appearing parameters with their running values $m_1^2(Q)$, $m_2^2(Q)$,... as defined by the Renormalization Group and adding the one-loop radiative corrections obtained in the $\overline{DR}$ scheme,

$$\Delta V_1 = \frac{1}{64\pi^2} Str\{\mathcal{M}^4(ln(\mathcal{M}^2/Q^2) - 3/2)\} \quad (3)$$

we end up with an Effective Potential that upon minimization supports a vacuum with spontaneously broken electroweak symmetry[1][4]. A reasonable approximation to (3) would be to allow only for the dominant top-stop loops. Note that although the Renormalization Group improved tree level potential depends on the scale Q this is not the case for the full 1-loop Effective Potential which is Q-independent up to, irrelevant for minimization, Q-dependent but field-independent terms.

We shall assume that at a very high energy scale M_X the soft supersymmetry breaking is represented by four parameters m_o, $m_{1/2}$, A_o and B of which we shall consider as input parameters only the first three and treat $B(M_Z)$ as determined by minimization conditions of the one loop effective potential. Actually we can treat $\beta(M_Z)$ as input parameter and both $B(M_Z), \mu(M_Z)$ are determined by solving the minimization conditions with the sign of μ left undetermined. The top-quark mass[5], or equivalently the top-quark Yukawa coupling, although localized in a small range of values should also be considered as an input parameter since the sparticle spectrum and the occurrance of symmetry breaking itself is sensitive to its value.Thus, the input parameters are m_o, $m_{1/2}$, A_o, $\beta(M_Z)$ and $m_t(M_Z)$ as well as the sign of μ.

In our notation, for a physical mass M,

$$\theta_M \equiv \theta(Q^2 - M^2) \tag{4}$$

Also t stands for $t = lnQ^2$ and $\beta_\lambda \equiv \frac{d\lambda}{dt}$ for each parameter λ. Note also that we assume diagonal couplings in family space.

As an example consider the one loop β-function of the trilinear coupling[6] A_τ ,

$$\begin{aligned} \frac{dA_\tau}{dt} &= \frac{1}{(4\pi)^2}\{-3{g_2}^2 M_2 \theta_{\tilde{W}\tilde{H}_1} - \frac{3}{5}{g_1}^2 M_1 (2+\theta_{\tilde{H}_1})\theta_{\tilde{B}} \\ &+ 3{Y_b}^2 A_b \theta_{\tilde{D}\tilde{Q}} + 4{Y_\tau}^2 A_\tau + A_\tau[Z_{\tau 1}{g_1}^2 + Z_{\tau 2}{g_2}^2 + Z_{\tau\tau}{Y_\tau}^2]\} \end{aligned} \tag{5}$$

Where $Z_{\tau 1}$, $Z_{\tau 2}$ and $Z_{\tau\tau}$ are displayed in Table I.

TABLE I

$$\begin{aligned} Z_{\tau 1} = \tfrac{3}{40}[11 + 10\theta_{\tilde{B}} - 8\theta_{\tilde{E}} - 4\theta_{\tilde{B}\tilde{E}} + 8\theta_{\tilde{B}\tilde{E}\tilde{H}_1} + \\ 2\theta_{H_1} - 8\theta_{H_1\tilde{E}} - 2\theta_{\tilde{L}} - \theta_{\tilde{B}\tilde{L}} - 8\theta_{\tilde{E}\tilde{L}} - 4\theta_{\tilde{B}\tilde{H}_1\tilde{L}} + 4\theta_{H_1\tilde{L}}] \end{aligned}$$

$$Z_{\tau 2} = \tfrac{1}{8}[-3 + 6\theta_{H_1} - 6\theta_{\tilde{L}} - 12\theta_{H_1\tilde{L}} + 6\theta_{\tilde{W}} - 3\theta_{\tilde{L}\tilde{W}} + 12\theta_{\tilde{W}\tilde{H}_1\tilde{L}}]$$

$$Z_{\tau\tau} = \tfrac{1}{4}[-16 + +6\theta_{\tilde{H}_1} - \theta_{\tilde{H}_1\tilde{E}} - 3\theta_{H_1} + 4\theta_{H_1\tilde{E}} + 4\theta_{\tilde{E}\tilde{L}} - 2\theta_{\tilde{H}_1\tilde{L}} + 8\theta_{H_1\tilde{L}}]$$

Table I: Threshold coefficients appearing in the renormalization group equation of the trilinear scalar coupling A_τ.

Note that the threshold corrections introduced in our approximation by the theta-functions at 1-loop are expected to be comparable to the standard 2-loop RG corrections. In our numerical analysis that we follow we shall employ the 2-loop RG equations which have not been presented here due to their complicated form but can be found elsewhere[7]. The problem at hand consists in finding the physical masses of the presently unobserved particles, i.e. squarks, sleptons, Higgses, Higgsinos and gauginos, as well as their physical couplings to other observed particles. This will be achieved by integrating the Renormalization Group equations from a superheavy scale M_X, taken to be in the neighbourhood of $10^{16} GeV$, down to a scale Q_o in the stepwise manner stated. If the equation at hand is the Renormalization Group equation by the condition $m(Q_o) = Q_o$. If the equation at hand is the Renormalization Group equation for a coupling the integration will be continued down to $Q_o = M_Z$. Acceptable solutions should satisfy the minimization conditions at M_Z, i.e. describe a low energy theory with broken electroweak symmetry at the right value of $M_Z \simeq 91.187 GeV$.

The boundary condition at high energy will be chosen as simple as possible, postponing for elsewhere the study of more complicated alternatives. Thus at the (unification) point M_X, taken to be 10^{16} GeV, we shall take

$$\begin{aligned} m_{\tilde{Q}}(M_X) &= m_{\tilde{D}^c}(M_X) = m_{\tilde{U}^c}(M_X) = m_{\tilde{L}}(M_X) = m_{\tilde{E}^c}(M_X) \\ &= m_{H_1}(M_X) = m_{H_2}(M_X) \equiv m_o \end{aligned} \tag{6}$$

and

$$M_1(M_X) = M_2(M_X) = M_3(M_X) \equiv m_{1/2} \tag{7}$$

In addition we take equal cubic couplings at M_X, i.e.

$$A_e(M_X) = A_d(M_X) = A_u(M_X) \equiv A_o \tag{8}$$

Our set of constraints includes the low energy experimental gauge coupling values which we have taken to be $M_Z = 91.187 Gev$, $\alpha(M_Z)^{-1}{}_{\overline{MS}} = 127.9 \pm 0.1$ and $(sin^2\theta_W)_{\overline{MS}} = 0.2316 - .8810^{-7}({M_t}^2 - 160^2) Gev^{-2}$. The knowing, average experimental value of α_3 is 0.117 ± 0.010. These $\overline{MS}$ values for the couplings are related to the relation, $\overline{DR}$[1]ones through the relations $g_{\overline{MS}} = g_{\overline{DR}}(1 - Cg^2/96\pi^2)$, where $C = 0, 2, 3$ respectively for the three factor gauge groups. For the b-quark and τ-lepton masses we have taken $m_b = 5.0$ Gev and $m_\tau = 1.8$ Gev.The recent evidence[5] for the top quark mass has motivated values in the neighborhood of 176 ± 8 Gev.The physical top quark mass M_t is related to the running top-quark mass through the approximate relation

$$m_t(M_t) = \frac{M_t}{(1 + \frac{5\alpha_3}{3\pi} + ...)} \tag{9}$$

As stated previously the B, μ are not inputs in the approach we are following but are determined through the equations minimizing the scalar potential. For their determination at the scale M_Z we take into account the one loop corrected potential considering the dominant top and stop contributions. This procedure modifies the tree level values $B(M_Z)$, $\mu(M_Z)$. It is well known that the value of μ affects the predictions for the physical masses especially those of the neutralinos and charginos. In approaches in which the effect of the thresholds is ignored in the RGE's the determination of B, μ is greatly facilitated by the near decoupling of these parameters from the rest of the RGE's. However with the effects of the thresholds taken into account such a decoupling no longer holds since the thresholds themselves depend on B, μ, or equivalently on μ, m_3^2. Thus, as initial inputs for $B(M_Z)$ and $\mu(M_Z)$ we take those arising from the minimization equations assuming that theshold effects are absent. At this stage our analysis is identical to those of other authors. Subsequently we run our numerical routines switcing on the threshold contributions to the RGE's keeping fixed the inputs for A_o,m_o, $m_{1/2}$, $\tan\beta$ and all couplings. This procedure corrects the initial inputs for $B(M_Z)$, $\mu(M_Z)$ in each run until convergence is reached. This is unecessary of course in cases where the thresholds are neglected. The next step regarding the mixing parameters μ, m_3^2 is to correct them taking into account the one loop effective potential in the way prescribed earlier.

[1]Note that at the 2-loop order the $\overline{DR}$ scheme needs to be modified so that no contribution to the scalar masses due to the"ϵ-scalars"[8] shows up.

We have displayed some of our results in tables II and III. We have taken $\mu(M_Z)$ positive and hence $B(M_Z)$ negative. Their mirror values $\mu(M_Z) < 0$, $B(M_Z) > 0$ lead to qualitatively similar results. In the table II, for a characteristic set of values $A_o = 400$ GeV, $m_o = 300$ GeV and $m_{1/2} = 200$ GeV we have varied $tan\beta$ between 2 and 25. Note the well known[9] approximate equality between the masses of one of the neutralinos and one of the charginos. The lightest Higgs turns out to be heavier than the Z - boson. Althought not displayed, for negative μ its mass drops below M_Z for small values of the angle $tan\beta \simeq 2$.

Finally, Table III compares for a characterestic choice of parameter values, three dinstict cases.Case [a] indicates one loop predictions, case [b] two loop predictions with thresholds only in couplings, and case [c] the complete two loop with thresholds everywhere.Comparison of the first two cases [a] and [b] point out the fact that thresholds in couplings affect only by small ammount ($1\% - 2\%$) the spectra, except from the neutralino and chargino states, where the differences are of order 10% due to different evolutions of the soft gaugino masses $M_{1,2}$.

Comparing cases [b] and [c], we observe quite large effects in states labeled as $\tilde{\chi}^o_{3,4}$, $\tilde{\chi}^c_1$ as well as in heavy Higgs states.This discrepancy is due mainly to the evolution of m_3^2, whose values affect substantially the masses of the pseudoscalar and charged Higgses,and in particular on it's dependense on the gaugino masses[10].

$m_t = 175$, $A_o = 400$, $m_o = 300$, $m_{1/2} = 200$, $\mu(M_Z) > 0$					
$tan\beta$	25	20	15	10	2
M_{GUT} $(10^{16} GeV)$	2.632	2.633	2.633	2.632	2.515
α_{GUT}	.04161	.04161	.04161	.04161	.04134
α_{em}^{-1}	127.9	127.9	127.9	127.9	127.9
$sin^2\theta_W$	.2311	.2311	.2311	.2311	.2311
α_3	.13128	.13129	.13129	.13126	.12909
M_t	177.0	177.0	177.0	177.0	176.8
$\tilde{g}$	495.6	495.8	495.9	495.9	492.4
$\tilde{\chi}_1^o$	77.1	77.0	76.8	76.4	74.7
$\tilde{\chi}_2^o$	139.1	138.9	138.5	137.6	136.0
$\tilde{\chi}_3^o$	350.5	352.3	354.7	359.1	487.8
$\tilde{\chi}_4^o$	-337.2	-338.6	-340.3	-343.2	-467.7
$\tilde{\chi}_1^c$	352.3	354.0	356.0	359.9	484.5
$\tilde{\chi}_2^c$	138.8	138.6	138.1	137.0	134.7
$\tilde{t}_1$,$\tilde{t}_2$	527.3,315.4	531.9,315.2	535.9,314.4	539.4,312.3	543.9,285.3
$\tilde{b}_1$,$\tilde{b}_2$	506.2,441.0	514.2,451.4	520.9,459.4	525.8,465.0	525.9,462.4
$\tilde{\tau}_1$,$\tilde{\tau}_2$	328.0,266.5	330.4,281.0	331.7,292.9	331.9,302.1	329.4,309.2
$\tilde{\nu}_\tau$	306.4	311.5	315.6	318.6	323.4
$\tilde{u}_{1,2}$,$\tilde{u}_{1,2}^c$	537.8,528.9	537.8,528.9	537.8,528.9	537.7,528.8	535.4,525.8
$\tilde{d}_{1,2}$,$\tilde{d}_{1,2}^c$	543.3,529.6	543.3,529.6	543.3,529.6	543.2,529.5	538.6,525.8
$\tilde{e}_{1,2}$,$\tilde{e}_{1,2}^c$	330.1,311.7	330.1,311.7	330.0,311.7	330.0,311.6	328.8,310.3
$\tilde{\nu}_{1,2}$	321.0	321.0	320.9	321.0	323.5
A	630.7	612.0	588.1	560.1	669.0
h_o,H_o	114.1,630.6	114.2,611.9	114.2,588.0	113.8,560.2	92.1,672.8
$H^\pm$	635.4	616.9	593.1	565.4	673.5

Table II: MSSM predictions for $m_t = 175\, GeV$, $A_o = 400\, GeV$, $m_o = 300$, $m_{1/2} = 200 GeV$ and for values of $\tan\beta$ ranging from 2 to 25. Only the $\mu > 0$ case is displayed.

$m_t = 175,\ tan\beta = 10,\ A_o = 250,\ m_o = 200,\ m_{1/2} = 150,\ \mu(M_Z) > 0$			
	Case [a] 1-loop	Case [b] 2-loop (thresholds in couplings)	Case [c] Complete 2-loop
M_{GUT} ($10^{16} GeV$)	2.1881	2.8876	2.8766
α_{GUT}	.04127	.04201	.04202
α_{em}^{-1}	127.9	127.9	127.9
$sin^2\theta_W$	.23105	.23110	.23110
α_3	.11767	.13284	.13289
M_t	181.0	177.0	177.0
$\tilde{g}$	398.4	381.4	382.6
$\tilde{\chi}_1^o$	59.2	55.0	54.4
$\tilde{\chi}_2^o$	109.0	98.3	96.7
$\tilde{\chi}_3^o$	302.8	304.1	279.6
$\tilde{\chi}_4^o$	-284.1	-287.7	-260.1
$\tilde{\chi}_1^c$	304.0	305.5	280.9
$\tilde{\chi}_2^c$	108.0	97.4	95.3
$\tilde{t}_1$,$\tilde{t}_2$	443.6,247.0	442.0,235.2	440.2,234.7
$\tilde{b}_1$,$\tilde{b}_2$	401.7,357.2	395.4,352.8	394.9,352.6
$\tilde{\tau}_1$,$\tilde{\tau}_2$	235.3,203.6	232.2,201.6	231.3,202.7
$\tilde{\nu}_\tau$	216.6	212.6	212.5
$\tilde{u}_{1,2}$,$\tilde{u}^c_{1,2}$	410.5,401.4	400.1,394.1	400.1,394.1
$\tilde{d}_{1,2}$,$\tilde{d}^c_{1,2}$	417.8,402.4	407.5,395.7	407.5,395.7
$\tilde{e}_{1,2}$,$\tilde{e}^c_{1,2}$	231.6,212.6	227.7,211.6	227.5,211.8
$\tilde{\nu}_{1,2}$	218.1	214.2	214.1
A	412.1	421.0	394.5
h_o,H_o	113.0,412.2	110.7,421.1	110.5,394.7
$H^\pm$	419.5	428.1	402.1

Table III: MSSM mass spectrum for the inputs shown in the first row ($\mu > 0$). We compare 1 - loop (case [a]), 2 - loop with thresholds in couplings (case [b]) and complete 2 - loop predictions (case [c]) with thresholds in both couplings and dimensionful parameters.

References

[1] G. Gamberini, G. Ridolfi and F. Zwirner, Nucl. Phys. B331(1990)331;

D. J. Castaño, E. J. Piard and P. Ramond, Phys. Rev. D49(1994)4882.

[2] D. Matalliotakis and H.-P. Nilles, Nucl.Phys.B435(1995)115;
A. Lleyda and C. Munoz, Phys.Lett.B317(1993)82;
N. Polonsky and A. Pomarol, Phys.Rev.D51(1995)6532;
Ph. Brax, U. Elwanger and C. A. Savoy, Phys.Lett.B347(1995)269;
J. Louis and Y. Nir, hep-ph/9411429.

[3] J. Ellis, K. Enqvist, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B155(1985)381.

[4] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257(1991)83 , B262(1991)477;
Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85(1991)1;
H. E. Haber and R. Hempfling, Phys. Rev. Lett. 66(1991)1815;
A. Brignole, Phys. Lett. B277(1992)313, Phys. Lett. B281(1992)284;
M. Drees and M. M. Nojiri, Phys. Rev. D45(1992)2482.

[5] F. Abe et.al, Phys.Rev.Lett. 74(1995)2676;
S. Abach et.al, Phys.Rev.Lett.74(1995)2632.

[6] A. B. Lahanas and K. Tamvakis, Phys.Lett.B348(1995)451.
"Radiative Electroweak symmetry breaking in the MSSM and low energy thresholds" A.Dedes, A.B. Lahanas and K.Tamvakis, hep-ph/9504239.

[7] S. P. Martin and M. T. Vaughn, Phys.Rev. D50(1994)2282;
Y.Yamada, Phys.Rev D50(1994)3537;
I. Jack and D. R. T. Jones, Phys.Lett.B333(1994)372.

[8] I. Jack, D. R. T Jones, S. P. Martin, M. T. Vaughn and Y. Yamada, Phys.Rev D50(1994)5481.

[9] R. Arnowitt and P. Nath, Phys. Rev. Lett. 69(1992)725, Phys. Lett. B 289(1992)368;
J. L. Lopez, D. V. Nanopoulos and H. Pois, Phys. Rev.D47(1993)2468.

[10] A. Dedes, A. B. Lahanas and K. Tamvakis, in preparation.