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A No-Go Theorem in String Cosmology

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Abstract

A no-go theorem pertaining to the graceful exit problem in Pre-Big-Bang inflation is reviewed. It is shown that dilaton self-interactions and string fluid sources fail to facilitate branch changing necessary to avoid singularities. A comment on the failure of the higher genus corrections to induce graceful exit is also included.

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Inflationary paradigm is one of the cornerstones of modern cosmology [1]. It states that there must have occurred a stage of rapid expansion of the Universe, in order to account for such large scale properties as homogeneity and isotropy, flatness, absence of topological relics etc. A simple argument illustrating the need for inflation is the so called horizon problem. Namely, we cannot reconcile the observed high degree of correlation in the background radiation (with the angular fluctuations in black-body temperature of the order $\delta T/T \propto 10^{-6}$) with the evolution of the Universe dictated by Einstein relativity and dominated by fluid sources. The Universe must have evolved out of roughly 10^5 causally disconnected domains at the time of decoupling of radiation from matter in order to achieve its present size. As the radiation did not interact from that time on, it is hard to see how the present correlation is attained. Inflationary scenario saves the day by postulating an era of rapid expansion prior to decoupling, producing a huge causally correlated region from which our Universe evolved.

We still need to construct a fully self-consistent dynamics of inflation, in agreement with observations and free of internal inconsistencies. A possibility may be found in string theory [2]. Up to date, it is the only candidate for a unified theory of interactions. If we accept it for its promise, it is important to see if the inflationary scenario, dictated by observations, can be incorporated naturally in string cosmology. This in fact could be viewed as a test of string theory. It turns out that this is not an easy task. Typically there appear new difficulties, such as the rolling scalar fields and induced variation of particle masses and couplings [3]. In response, an alternative approach towards string-driven inflation, dubbed the Pre-Big-Bang inflation, has been suggested [4, 5]. It strives to induce inflation deriving from genuinely stringy mechanisms, relying precisely on the rolling dilaton. The scenario is defined in the string world-sheet frame, where there exist two branches of solutions related by string scale factor duality. They are characterized by superexponential, pole-driven inflation for $t < 0$ and a milder, power-law expansion for $t > 0$. If the jump (branch change) at (or near) $t = 0$ could be made from the superinflationary phase to the power-law one, the dreaded singularity may be avoided. However, the scenario does not work in its simplest form [5, 6]. The desired branch changes cannot be catalyzed by dilaton self-interactions and/or string fluid sources. Here we will review this recent result, and also will comment on the failure of the higher genus corrections introduced by Damour and Polyakov [7] to induce such branch changes [6]. This review is based on the joint work of R. Madden, K.A. Olive and the author

[6], where a more detailed account can be found.

We start with the simplest case, defined by the action

$$S = \int d^4x \sqrt{g} e^{-2\phi} \left\{ \frac{R}{2} + 2\partial_\mu \phi \partial^\mu \phi - \Lambda(\phi) \right\} \quad (1)$$

Here ϕ is the dilaton and $\Lambda(\phi)$ is the dilaton potential. We have ignored the contribution of the axion since during expansion an axion dominated universe can be expected to quickly evolve to one dominated by the dilaton. Assuming a spatially flat Robertson-Walker geometry,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad (2)$$

we can write the two independent equations of motion in a relatively simple first order form:

$$\dot{\phi} = (3h \pm \sqrt{3h^2 + 2\Lambda(\phi)})/2 \quad (3)$$

$$\dot{h} = \pm h \sqrt{3h^2 + 2\Lambda(\phi)} - \Lambda'(\phi)/2 \quad (4)$$

with the \pm sign chosen for both equations simultaneously. These equations are easily solved when $\Lambda = 0$ resulting in four different solutions, two for each of the two branches corresponding to the choice of (+) and (-) sign. The (+) branch is defined in the domain $t < 0$ and the (-) branch in $t > 0$. The solutions are [5]

$$\begin{aligned} a = a_0 |t|^{\mp \frac{1}{\sqrt{3}}}, \quad h = \mp \frac{1}{\sqrt{3}t}, \quad \phi = \phi_0 + \frac{\mp \sqrt{3} - 1}{2} \ln |t| \quad & \text{for } t < 0 \text{ ((+) branch)} \\ a = a_0 t^{\pm \frac{1}{\sqrt{3}}}, \quad h = \pm \frac{1}{\sqrt{3}t}, \quad \phi = \phi_0 + \frac{\pm \sqrt{3} - 1}{2} \ln t \quad & \text{for } t > 0 \text{ ((-) branch)} \end{aligned} \quad (5)$$

We will focus here on the expanding solutions. The expanding solution for $t < 0$ begins in the weak coupling regime (ϕ large and negative) and evolves toward the strong coupling region (ϕ positive), compatible with the form of the action which represents a weak-coupling truncation of the full effective action of string theory. The expanding solution for $t > 0$ can be matched to a standard post-inflationary Robertson-Walker cosmology, by turning on dilaton self-interactions. We repeat here that for $t < 0$ we have a pole-driven expansion, reaching the singularity at $t = 0$, and for $t > 0$ we get a power-law expanding universe emerging from the singularity. If the singularity at the pole $t = 0$ were removed and the two branches joined smoothly, the resulting solution would represent a completely nonsingular cosmology.

In the region of the order of Planck scale the superinflating branch would metamorphose to a cooling, expanding universe which can be joined onto our own as $t \rightarrow \infty$ [5]. We note that the evolution should eventually lead to the decoupling of the dilaton by trapping it in a fixed point (potential well) in order to match the solution to a late time cosmology. The high curvature region around $t = 0$ would resemble the Big Bang and therefore, in addition to possibly solving the problems usually assigned to inflation, it would also give an elegant resolution to the question of initial singularity.

The mechanism of branch changing should be responsible for graceful exit. We now turn to it. It is obvious from the equations of motion that the two branches can never connect smoothly in the regions where the potential is positive (cf. eqs. (3) and (4)) since we must match derivatives at the point of contact. This requires that the potential is negative in a certain region. If we represent the dynamics by the phase space portrait in the phase plane (ϕ, h) , the regions where branch changes can occur are closed curves symmetric around the ϕ -axis, given by $3h^2 + 2\Lambda = 0$. They were conveniently named the “eggs” because of their concave shape in the regions containing a single negative minimum of Λ . Unfortunately, in our case although eggs lead to branch changes, there is no graceful exit since branch changes always occur in pairs. To see it, we define the egg function: $e = \sqrt{3h^2 + 2\Lambda(\phi)}$, which satisfies $\dot{e} = \pm(2h\dot{\phi} - \dot{h})$. By dividing both sides by $\dot{\phi}$ and integrating the result over ϕ along a trajectory, we obtain:

$$\pm(e(t_1) - e(t_0)) + h(t_1) - h(t_0) = 2 \int_{\phi(t_0)}^{\phi(t_1)} h d\phi \quad (6)$$

The integral here is to be understood as a line integral along the path of the system between $\phi(t_0)$ and $\phi(t_1)$. In fact this is the equation of phase trajectory in integral form. The positivity of the integral in relevant cases is the input needed to obtain the no-go theorem.

This comes about as follows. From looking at the flows in the phase space, we see that any trajectory hitting the top of the egg must have come from the left, and any trajectory hitting below the egg must have come from the right. A case when a trajectory flows around the egg without hitting it for “half” a cycle is also consistent with the above remark. Then we can show that 1) no $(-)$ branch trajectory, originating from anywhere on the upper side of the egg can escape over the right end of the egg but must rehit it, and 2) no $(+)$ trajectory coming from the right and flowing below the egg hits the egg below, or on, the ϕ -axis. From this, we conclude that any $(+)$ branch entering an egg region from the left must go over the

top of the egg, possibly experiencing several branch changes, and must exit the region of the egg while still on the (+) branch. No (+) branch entering an egg region from the right can hit below, and it must remain (+) while flowing under the egg. Thus any (+) trajectory entering the egg region cannot leave on a (−) branch, and there is no “graceful exit”. The egg can only convert (−) to (+). Clearly, multiple eggs cannot change this conclusion.

To prove the above statements, one notices that for the first case the integral formula applied for the (−) bounce between the hit point (t_0) and the terminus of the egg (t_1) rules this option out. In this region the flow is to the right and $h \geq 0$. Therefore, the integral is equal to the area between the segment and the ϕ -axis and hence strictly positive $\int_{\phi(t_0)}^{\phi(t_1)} h d\phi = \mathcal{A} > 0$. Substituting the corresponding parameters in the integral formula we arrive at the sought contradiction:

$$0 < 2\mathcal{A} = -(\sqrt{3} - 1)h(t_1) - h(t_0) \leq 0 \quad (7)$$

Therefore, the (−) bounce emerging from the upper side of the egg must rehit it, as we claimed. A similar contradiction shows the (+) trajectory cannot hit below the egg. Furthermore, there may be several complications regarding the pathological trajectories hitting the singular egg points, corresponding to inflections of the potential or local maxima at $\Lambda = 0$. Although these are not covered by the above arguments, they do not lead to graceful exit either [6].

As we have mentioned before, this no-go theorem can be generalized to the case when stringy fluid sources are present. In this case, the phase space of the model should be extended to three dimensions, the third coordinate being the energy density of the fluid ρ . The associated equations of motion are given by the following generalization of (3-4) [5, 6]:

$$\begin{aligned} \dot{\phi} &= (3h \pm \sqrt{3h^2 + 2\Lambda(\phi) + \rho \exp(2\phi)})/2 \\ \dot{h} &= \pm h \sqrt{3h^2 + 2\Lambda(\phi) + \rho \exp(2\phi)} - \Lambda'(\phi)/2 + \frac{\gamma}{2} \rho \exp(2\phi) \\ \dot{\rho} &= -3(1 + \gamma)h\rho \end{aligned} \quad (8)$$

Here $\gamma = p/\rho \in (-1/3, 1/3)$ is a constant representing the fluid equation of state. The physical restriction $\rho \geq 0$ is consistent with the equations of motion, as the ρ flow terminates at the $\rho = 0$ plane, which is like a potential barrier. The trajectories completely confined in this plane are governed by our previous theorem, so there is no graceful exit for them.

Now we look at the fully three-dimensional trajectories. The egg function is given by $e = \sqrt{3h^2 + 2\Lambda(\phi) + \rho \exp(2\phi)}$ and the modified integral formula when sources are present is

$$\pm(e(t_1) - e(t_0)) + h(t_1) - h(t_0) = 2 \int_{\phi(t_0)}^{\phi(t_1)} h d\phi + \frac{1 + \gamma}{2} \int_{t_0}^{t_1} \rho e^{2\phi} dt \quad (9)$$

This equation differs from (6) only in the presence of the last term, which is a nonnegative quantity for all trajectories. The other relevant characteristics of the phase space are also easily generalized. Without delving into the details, we note that qualitatively the picture remains the same: the egg is now a two-dimensional compact surface cut by the plane $\rho = 0$. Trajectories flow around the egg along helical paths, which if projected onto the $\rho = 0$ plane turn clockwise. We then need to consider trajectories crossing the cylindrical surface enclosing the egg, obtained by translating the curve representing the boundary of the egg in the $h = 0$ plane vertically upwards. We can see that the first integral on the LHS, representing the area enclosed by the projection of the trajectory onto the $\rho = 0$ plane, remains positive for all such trajectories. The second integral is always nonnegative, as we mentioned above. Thus all the arguments for the sourceless case extend to this case, again preventing favorable branch changes from occurring. As no new pathologies appear, we conclude that the no-go theorem must hold for this case too.

We note in passing that dilaton-dependent corrections to couplings, coming from the higher genus terms [7, 8], also fail to induce the desired graceful exit [6]. In that case, the failure is caused solely by the flows around the eggs which preclude even the possibility for the needed hits on the egg. Thus the only option to consistently incorporate Pre-Big-Bang cosmology which remains open is to resort to higher derivative terms in the α' expansion [9]. However this approach must consider systematically all the terms in the α' expansion, and thus should be implemented via the conformal field theory approach. At this moment, it appears that this goal is still beyond our means.

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