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NUCLEAR RESPONSES TO ELECTRO-WEAK PROBES AND IN-MEDIUM CHIRAL PERTURBATION THEORY[‡§]

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ABSTRACT

We discuss two topics concerning the application of chiral perturbation theory to nuclear physics: (1) the latest developments in the study of possible kaon condensation in dense baryonic systems; (2) nuclear responses to electro-weak probes

1. Introduction

Chiral perturbation theory (χPT) offers a valuable guiding principle in our attempt to relate nuclear dynamics to the fundamental QCD. The concept of chiral counting also gives a clear perspective in organizing our description of complicated nuclear dynamics. Indeed, a new line of nuclear physics based on χPT seems to be steadily gaining ground. In this talk, after giving a minimal sketch of χPT, we present two examples of the nuclear physics application of χPT. We first discuss the latest developments in the study of possible kaon condensation in dense matter. We then describe the use of χPT in calculating nuclear responses to electro-weak interaction probes.

The introduction of χPT follows a generic pattern to define an effective theory.[1,2,3] Consider the vacuum-to-vacuum amplitude in QCD in the presence of external fields

$$\mathrm{e}^{iZ[v,a,s,p]} = \int [dG][dq][d\bar{q}]\, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}(q,\bar{q},G;\, v,a,s,p)} \qquad (1)$$

where $\mathcal{L} = \mathcal{L}^0_{\mathrm{QCD}} + \bar{q}\gamma^\mu[v_\mu(x) - \gamma^5 a_\mu(x)]q - \bar{q}[s(x) - ip(x)]q$. The external fields, v_μ, a_μ, s and p, are assigned appropriate SU(3)×SU(3) transformation properties to make $\mathcal{L}$ chiral invariant. The effective lagrangian that describes low-energy phenomena of QCD ($E \lesssim \Lambda_\chi \sim 1$ GeV) involves the Goldstone bosons and is introduced through

$$\mathrm{e}^{iZ[v,a,s,p]} = \int [dU]\, \mathrm{e}^{\mathrm{i}\int d^4x\, \mathcal{L}_{\mathrm{eff}}(U;\, v,a,s,p)}, \qquad (2)$$

where $U \equiv \exp(i\sum_{a=1}^{8} \pi^a\lambda^a/f_\pi)$ with π^a the octet pseudo-scalar mesons. In χPT we expand $\mathcal{L}_{\mathrm{eff}}$ in powers of $\partial_\mu/\Lambda_\chi$ and the quark mass matrix $\mathcal{M}/\Lambda_\chi$ and, for a given order of expansion, retain all terms that are consistent with the symmetries. In extending this scheme to the baryon field N, we realize that ∂_0 acting on N yields $\sim m_{\mathrm{N}}$, which is not small compared with Λ_χ. The heavy-baryon chiral perturbation

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formalism (HBχPF) allows us to avoid this difficulty.[4] Here, instead of the ordinary Dirac field N we work with B defined by $B(x) \equiv \mathrm{e}^{\mathrm{i}v\cdot x}N(x)$ with $v \sim (1,0,0,0)$, shifting the energy reference point from 0 to $m_{\rm N}$. If we are only concerned with small energy-momenta Q around this new origin, the antibaryon can be "integrated away". $\mathcal{L}_{\rm eff}(B,U;\,v,a,s,p)$ describing this particle-only world may be defined similarly to Eq. (2). The corresponding equation of motion for B may be rewritten as coupled equations for the large and small components $B_\pm$ defined by $B_\pm \equiv P_\pm B$ with $P_\pm \equiv (1\pm \not{v})/2$. Eliminating B_- in favor of B_+ leads to an equation of motion for B_+. The HBχPF lagrangian $\mathcal{L}_{\rm HB}$ is defined as an effective lagrangian that reproduces the equation of motion for B_+ and U. Since $B_- \propto (Q/m_{\rm N})B_+$, $\mathcal{L}_{\rm HB}$ involves expansion in $\partial_\mu/m_{\rm N}$ as well as in $\partial_\mu/\Lambda_\chi$ and $\mathcal{M}/\Lambda$. We can organize this expansion as

$$\mathcal{L}_{\rm HB} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots \;\; ; \qquad \mathcal{L}^{(\nu)} = \mathcal{O}(Q^{\nu-1}) \tag{3}$$

The chiral order index ν is defined as $\nu = d + (n/2) - 2$, where n is the number of fermion lines involved in a vertex, and d is the number of derivatives (with $\mathcal{M} \propto m_\pi^2$ counted as two derivatives). The explicit expression relevant to the meson-baryon sector is[3]

$$\mathcal{L}_{\rm HB} \;=\; \bar{B}_+ \left[\mathcal{A}^{(1)} + \mathcal{A}^{(2)} \,+\, (\gamma_0 \mathcal{B}^{(1)}\gamma_0)\frac{1}{2m_{\rm N}}\mathcal{B}^{(1)}\right] B_+ + \mathcal{O}(Q^2), \tag{4}$$

The leading order term is given in terms of $u = \sqrt{U}$ and $S_\mu = i\gamma_5\sigma_{\mu\nu}v^\nu/2$ as

$$\mathcal{A}^{(1)} \;=\; i(v\cdot D) + g_{\rm A}(u\cdot S) \tag{5}$$

$$D_\mu \;=\; \partial_\mu + [u^\dagger, \partial_\mu u]/2 - i\,u^\dagger(v_\mu + a_\mu)u/2 - i\,u(v_\mu - a_\mu)u^\dagger/2 \tag{6}$$

The expressions for higher order terms can be found in Ref.[3]

Chiral counting can also be applied to Feynmann diagrams; the chiral order D of an irreducible Feynmann diagram is given by[2]

$$D \;=\; 2 - \frac{1}{2}N_E + 2L - 2(C-1) + \sum_i \nu_i, \tag{7}$$

where N_E is the number of external fermion lines, L the number of loops, C the number of disconnected parts, and the sum runs over vertices.

2. Kaon Condensation in Dense Baryonic Matter

Kaon condensation in dense baryonic matter has been discussed by many authors.[5,6] According to the latest calculation,[7] the critical density ρ_c for kaon condensation is $\rho_c \approx 4\rho_0$ (ρ_0 = normal nuclear matter density) and, with the Brown-Rho scaling[8] included, ρ_c can be as low as $2\rho_0$. Kaon condensation (as we are interested in here) is driven by the s-wave interactions, unlike pion condensation which depends on the p-wave interactions. The strong s-wave K-N attraction comes partly from the so-called σ-term, which is significantly stronger for the kaon than for the pion. Furthermore, the vector-meson exchange contributions can give rise to strongly attractive s-wave interactions for some K-N channels, whereas they are either repulsive or only weakly attractive for the πN channels. These features motivate us to examine the possibility of s-wave kaon condensation. As far as observational consequences are concerned, a kaon condensate (like a boson condensate in general) could enhance significantly neutrino emission from nascent neutron stars, cooling them much faster. Furthermore, the condensate can drastically soften the equation of state for collapsing stars. Brown and Bethe[9] argue that this softening leads to proliferation of mini blackholes, which resolves the long-standing puzzle that the observational value for the ratio $R \equiv$ [# of neutron stars]/[# of supernova events] is inexplicably low.

Two of the outstanding issues facing kaon condensation are the m_N^* effect and the off-mass-shell effects (both to be explained below). We wish to report here the progress we have made on these issues over the past year.

2.1. The $m_{\rm N}^*$ Effect

Several authors argued that in-medium nucleon mass reduction could strongly hinder kaon condensation.[10,11] As mentioned above, the K-N σ-term, $\sigma_{\rm KN}\bar{\psi}\psi\bar{K}K$, provides a significant part of the s-wave attraction. The σ-term attraction in baryonic matter is (in the mean-field approximation) proportional to the Lorentz scalar density $\rho_s \equiv <\bar{\psi}\psi>$. The earlier works, however, used the approximation $\rho_s \sim \rho$, where ρ is the baryon density, $\rho \equiv <\bar{\psi}\gamma_0\psi>$. This simplifies the calculation considerably, since ρ is a conserved quantity that can be specified as an external parameter, whereas ρ_s is known only after the whole dynamics is solved. For a nucleon of effective mass $m_{\rm N}^*$ and momentum $\mathbf{k}$, we have $\bar{u}_{\mathbf{k}}u_{\mathbf{k}} = [m_{\rm N}^*/(m_{\rm N}^{*2}+\mathbf{k}^2)^{1/2}]\, u_{\mathbf{k}}^\dagger u_{\mathbf{k}}$, which suggests that using ρ instead of ρ_s overestimates the σ-term contribution and that this overestimation becomes more serious for smaller values of $m_{\rm N}^*$. Detailed calculations[11] based on the Walecka model[12] indicate that, for $m_{\rm N}^* \lesssim 0.75\rho_0$, the effective kaon mass $m_{\rm K}^*$ does not any longer go down to zero but levels off as ρ increases, and $m_{\rm K}^*(\rho \to \infty) \gtrsim 0.45 m_{\rm K}$. For convenience we refer to this feature as the "$m_{\rm N}^*$ effect". If the $m_{\rm N}^*$ effect is indeed as strong as the Walecka model suggests, there would be no kaon condensation.

Does this argument invalidate Lee *et al.*'s conclusion[7] $\rho_c = (2{\sim}4)\rho_0$? This issue is connected to the choice of the nucleon field. The Walecka model uses the original Dirac field. For systematic chiral counting, however, it is more advantageous to work with the heavy baryon field B_+, and this is what Lee *et al.*[7] did. Now, for B_+, there is by construction no distinction between $\rho_s \equiv \bar{B}_+B_+$ and $\rho \equiv \bar{B}_+\gamma_0 B_+$. In this sense Lee *et al.*'s approach is free from the conventional approximation $\rho_s \approx \rho$. But this is of course not the whole story. In HBχPF the effects of the B_- responsible for $\rho_s \neq \rho$ are transformed into the higher order terms in $1/m_{\rm N}$ expansion. So we need to examine how this $1/m_{\rm N}$ expansion is handled in practice. The lowest-order term in HBχPF [*i.e.* $\mathcal{A}^{(1)}$ term in $\mathcal{L}_{\rm HB}$, Eq. (4)] applies to an infinitely heavy baryon, and hence the $m_{\rm N}^*$ effect is totally absent here. The next order contribution contains

$\nu = 1$ terms in ordinary chiral counting ($\mathcal{A}^{(2)}$ term) and terms that are first order in $1/m_{\rm N}$. We denote the latter by $\mathcal{L}_{1/m}$. $\mathcal{L}_{1/m}$ consists of the baryon kinetic energy term $\mathcal{L}_{1/m}^B \equiv \bar{B}_+(-\partial_\mu^2/2m_{\rm N})B_+$ and the meson-baryon interaction part $\mathcal{L}_{1/m}^{\rm int}$. Now, to understand the calculational scheme adopted by Lee *et al.*, let us rearrange $\mathcal{L}_{\rm HB}$ as

$$\begin{aligned}\mathcal{L}_{\rm HB} &= (\mathcal{L}_{\rm HB}(\text{non}-\text{strange sector}) + \mathcal{L}_{\rm HB}(\text{strange sector}))_{m_{\rm N}\to\infty} + \mathcal{L}_{1/m} + \cdots \\ &= \Big\{\mathcal{L}_{\rm HB}(\text{non}-\text{strange})_{m_{\rm N}\to\infty} + \mathcal{L}_{1/m}^B + \mathcal{L}_{1/m}^{\rm int}(\text{non}-\text{strange})\Big\} \\ &\qquad + \Big[\mathcal{L}_{\rm HB}(\text{strange})_{m_{\rm N}\to\infty} + \mathcal{L}_{1/m}^{\rm int}(\text{strange})\Big] + \cdots \qquad (8)\end{aligned}$$

We first discuss the non-strange sector corresponding to the terms in the curly brackets. In the existing calculations based on HBχPF the energy density for the non-strange sector is taken from nuclear matter calculations of the Brueckner-Hartree-Fock type. This effectively incorporates the $1/m_{\rm N}$ correction. In fact, since any realistic nuclear matter calculation takes account of the change $m_{\rm N}\to m_{\rm N}^*$, the use of the nuclear matter calculation results allows us to go beyond the $1/m_{\rm N}$ correction. This is in a sense a welcome feature but there is a problem too. In HBχPF the change $m_{\rm N}\to m_{\rm N}^*$ arises either from $(1/m_{\rm N})^n$ corrections ($n \geq 2$) or from vertices with $\nu \geq 2$, and we must deal with a great multitude of possible terms. By using the nuclear matter results containing the effective mass change one is selecting a very particular subset of the higher order effects, and at present there is no clear justification for doing so. On the other hand, the fact the change $m_{\rm N} \to m_{\rm N}^*$ features importantly in nuclear matter calculation does indicate that one cannot simply stop at the first correction term in $1/m_{\rm N}$ expansion.

We next discuss the strangeness sector, the terms in the square brackets in Eq. (8). Here we note that $\mathcal{A}^{(2)}$ terms contained in $\mathcal{L}_{\rm HB}$(strange) is of the same chiral order ($\nu = 1$) and that the coefficients appearing in $\mathcal{A}^{(2)}$ are in fact phenomenologically fixed in such a manner that observables for one-meson one-baryon systems be reproduced. Then the introduction of the $1/m_{\rm N}$ term just leads to a readjustment of these parameters. Therefore, the $m_{\rm N}^*$ effect in the Walecka model would correspond to terms of $\nu = 2$ or higher. Again, there are many such terms and, for consistency, one must retain all of them. The Walecka model represents a particular choice of a subset, and it remains to be seen whether the strong $m_{\rm N}^*$ effect suggested by the model survives a fully consistent treatment. On the other hand, no calculations so far done in HBχPF go beyond the $1/m_{\rm N}$ term in the strangeness sector. The only exception is a qualitative remark by Lee *et al.*[7] that a multifermion term such as $(\bar{B}_+\gamma_\mu B_+)(\bar{B}_+B_+)\bar{K}\partial_\mu K$ can lead to a in-medium ($m_{\rm N}^*$-dependent) modification of the K-N interaction. This $m_{\rm N}^*$ effect in fact enhances the K-N attraction quite in contrast to the $m_{\rm N}^*$ effect found in the Walecka model. Obviously, more systematic treatments of higher order terms are required before we can reach a solid conclusion on the $m_{\rm N}^*$ effect.

In this connection, one may worry that a plethora of multi-fermion vertices that can participate in dense matter will spoil the convergence of chiral expansion. In fact, this does not happen as easily as one naively expects. According to Eq. (7), a Feynmann diagrams with a given number of external lines N_E has a smaller value of C if it contains vertices with larger values of n, thus resulting in a higher chiral order index D. So, the actual contributions of vertices with large fermion numbers to a Feynmann diagram are more suppressed than the chiral counting of individual vertices would indicate. This implies that we probably need not deal with a tower of multi-fermion terms to understand the $m_{\rm N}^*$ effect in the framework of HBχPF. There have been interesting attempts at relating the Walecka model to HBχPF.[13]

2.2. Off-Shell-Effects

Since the main points of our discussion here can be described more conveniently for the pion than for the kaon, we shall discuss the pion case. According to the standard multiple scattering theory, the pion-nuclear optical potential, or pion self-energy, is given by

$$\Pi = \rho\, t_{\pi A} + \cdots, \tag{9}$$

where $t_{\pi A}$ is the t-matrix describing pion scattering off a nucleon in medium, and the dots represent processes involving more than a single scatterer. The pion propagator pertaining to $t_{\pi A}$ is a full A-body nuclear hamiltonian, not just the single nucleon hamiltonian. Note that, in order to use Π in the determination of the in-medium dispersion relation for a pion, we need information on $t_{\pi A}$ for off-shell as well as on-shell kinematics. In the low-density limit, we only need retain the $\rho\, t_{\pi A}$ term, and furthermore we can replace $t_{\pi A}$ with the on-shell t-matrix for free π-N scattering.

Now, the issue raised by Yabu *et al.*[14] is as follows. Consider a toy π-N lagrangian that contains only the σ term:[a]

$$\mathcal{L}_1 = \frac{1}{2}\left[-\phi(\Box + m_\pi^2)\phi + \frac{\sigma_{\pi N}}{f^2}\phi^2 \bar{N}N\right]. \tag{10}$$

For $\mathcal{L}_1$, the π-N scattering amplitude in tree approximation is simply a constant: $T_{\pi N}^{(1)} = \sigma_{\pi N}/f^2$. The corresponding pion effective mass m_π^* (in the mean-field approximation) is $[m_\pi^*(1)]^2 = m_\pi^2 - \rho\,(\sigma_{\pi N}/f^2)$. On the other hand, the PCAC plus current algebra gives the forward scattering amplitude $T_{\pi N}^{(2)} = [(k^2 + (k')^2 - m_\pi^2)/f^2 m_\pi^2]\sigma_{\pi N}$. The corresponding m_π^* is given by $[m_\pi^*(2)]^2 = m_\pi^2\,[1+\rho\,(\sigma_{\pi N}/m_\pi^2 f^2)]\cdot[1+2\rho\,(\sigma_{\pi N}/m_\pi^2 f^2)]^{-1}$. Although $m_\pi^*(1)$ and $m_\pi^*(2)$ are identical for low densities, they behave very differently for large values of ρ. In particular, $m_\pi^*(2) \to m_\pi/\sqrt{2}$ as $\rho \to \infty$. Yabu *et al.*, who pointed out this discrepancy, argued that the existing calculational frameworks did not allow one to resolve this problem.

[a] This is a highly simplified version of the Kaplan-Nelson lagrangian. Although recent calculations[7,16] take due account of energy-dependent terms of the same chiral order as the σ term, our points can be explained without those additional terms.

It behooves to remember here the following general points: (i) The formal definition of m_π^* is a value of the energy variable ω for which the exact in-medium Green's function $G_\rho(x;\phi) = <\rho|T\phi(x)\phi(0)|\rho>$ develops a pole (for zero momentum); (ii) For a given lagrangian, the physical observable m_π^* should not depend on the definition of interpolating fields ϕ; (iii) Although off-mass-shell π-N amplitudes vary for different choices of ϕ, this variation should not affect any observables including m_π^*; (iv) Although off-shell π-N amplitudes are unphysical in the sense of (iii) and also in that they cannot be observed in π-N scattering, they do constitute ingredients of larger Feynmann diagrams; (v) The statements (i)$\sim$(iii) hold true only if the whole calculation is done exactly. This last point is trivial but nonetheless worth emphasizing.

Now, within the framework of the leading order optical potential, the variance between $m_\pi^*(1)$ and $m_\pi^*(2)$ is a direct consequence of the fact that $T_{\pi N}^{(1)}$ and $T_{\pi N}^{(2)}$ have different off-shell behaviors. Referring to the above general statements, one could ask whether this is a manifestation of different dynamics, or just a spurious off-shell effect that fails to disappear because of the approximation used. Yabu *et al.* favored the first possibility, conjecturing that different treatments of multi-fermion terms are responsible for the different behaviors of $T_{\pi N}^{(1)}$ and $T_{\pi N}^{(2)}$. This interpretation, however, was criticized by Lee *et al.*[7] and by Thorsson and Wirzba (TW).[16] TW show explicitly that, starting from the same $\mathcal{L}_{HB}$, one can derive either of $T_{\pi N}^{(1)}$ and $T_{\pi N}^{(2)}$ by adding to $\mathcal{L}_{\rm HB}$ different pseudoscalar source terms. This ensures that, provided one can calculate $G_\rho(x;\phi) = <\rho|T\phi(x)\phi(0)|\rho>$ exactly, one would get the same m_π^* regardless of whether one uses $T_{\pi N}^{(1)}$ or $T_{\pi N}^{(2)}$. Beautiful !! (Please note, however, the underline attached to "exactly".) In practice, we must adopt some approximation, the crudest and most commonly used approximation being $\Pi \approx \rho\, t_{\pi N}$. In these approximate calculations, choice between $T_{\pi N}^{(1)}$ and $T_{\pi N}^{(2)}$ does matter, and TW's formal proof is not of immediate help in making this choice.

We must mention here, however, another important point made by TW. TW demonstrates that, within the mean-field approximation, the use of the effective action leads to the identical dispersion relation for an in-medium pion regardless of different choices of the pseudoscalar source. This is a remarkable result, but it seems important to examine to what extent this theorem is tied to the mean field approximation. In fact, if TW's result is valid beyond the mean filed approximation, that would give a tremendous impact to the "standard" multiple scattering formalism. We would be forced to conclude that the obvious off-shell dependence exhibited by the leading term in the Watson expansion is spurious (at least for a system the dynamics of which is strongly constrained by chiral symmetry). This point deserves a serious investigation quite apart from the specific problem of meson condensation.

3. Nuclear Responses to Electro-Weak Probes

The nuclear hamiltonian is normally taken to be $H_N = \sum_{i=1}^{A} T_i + \sum_{i,j}^{A} V_{ij}$, where T_i is the nucleon kinetic energy, and V_{ij} is the "realistic" N-N potential. Arriving at H_N starting from the fundamental QCD description involves: (i) translating the quark and gluon degrees of freedom into the effective degrees of freedom of hadrons; (ii) truncating the Hilbert space of hadrons down to that of non-relativistic nucleons interacting via potentials. The χPT allows us to carry out (i) and (ii) in a well-defined way, preserving the basic chiral properties of QCD. Construction of the realistic N-N potentials based on χPT was described by Weinberg[2] and by van Kolck *et al.*[17] These χPT potentials can reproduce the N-N observables almost as satisfactorily as the conventional boson-exchange potentials which contain many *ad hoc* parameters.

In the truncated nucleonic space, nuclear responses to external probes such as electromagnetic and weak currents involve not only single-nucleonic terms (= impulse approximation terms) but also multi-nucleonic contributions named the exchange currents. Here again, χPT provides a systematic framework for organizing exchange-current contributions according to their chiral counting orders.[23,24,28]

A problem in testing the exchange currents in complex nuclei is that exact solutions for the A-body Schrödinger equation $H_N\Psi = E\Psi$ are hard to obtain and therefore we are forced to work with truncated model wave functions Ψ_0. If the matrix element of a nuclear operator $\mathcal{O}$ is calculated using model wave functions, then $<\Psi^f|\mathcal{O}|\Psi^i> \neq <\Psi_0^f|\mathcal{O}|\Psi_0^i>$. This deviation represents the core-polarization effect. The core polarization effects need to be carefully sorted out before one can identify the exchange currents effects. Despite this non-trivial aspect, there is growing evidence that supports the χPT derivation of exchange currents. The best example is the nuclear axial-charge operator A_0. Warburton *et al.*'s systematic analyses[19,20] of the first-forbidden β transitions indicate that the ratio of the exchange-current contribution to the 1-body contribution is $\delta_{\rm mec} \equiv \langle A^0({\rm mec})\rangle/\langle A^0(\text{1-body})\rangle = 0.6 \sim 0.8$. (The semi-empirical method used in these analyses largely eliminates ambiguities due to the core-polariation effects.) The leading-order χPT term, *i.e.* the soft-pion exchange term,[18,23] can explain the bulk of $\delta_{\rm mec}$, and the next-order χPT term[24,25] gives an additional ~10% enhancement, bringing the theoretical value close to the empirical value.

It is informative to compare the above results with those obtained in the conventional meson-exchange approach.[21,22] Using the "hard-pion formalism" in conjunction with the lagrangian that engenders the phenemenological N-N interactions, Towner[22] finds that the pion-exchange contribution is reduced significantly by the phenomenological form factors, but the reduction is largely compensated by heavy-meson pair graphs. The net result is: $\delta_{\rm mec}^{\rm Towner} \sim \delta_{\rm mec}^{\rm CPhT}$. In fact, the former is slightly larger, but this small difference is qualitatively understood as follows. The largest heavy-meson pair contributions come from σ and ω mesons, and the σ-meson contribution can be effectively rewritten as the 1-body term with the nucleon mass replaced by an effective mass.[26] Thus the phenomenological σ-meson plays a role similar to the BR scaling.[8] Meanwhile, in χPT, the BR scaling is attributable to multi-fermion terms which have higher chiral orders than those appearing in the next-to-leading-order calculation of Park *et al.*[24,25] Then, we should qualitatively expect $\delta_{\rm mec}$ obtained by Park *et al.* to be somewhat smaller than $\delta_{\rm mec}^{\rm Towner}$. The above example demonstrates the usefulness of CPT in organizing complicated exchange-current contributions in a systematic manner.

For the two-nucleon systems we can obtain exact solutions for $H_N\Psi = E\Psi$, avoiding thereby the core-polarization problem. The A=2 systems therefore provide a clean case for checking the validity of the standard calculational framework based on the nucleonic Schrödinger equation supplemented with the exchange currents. A beautiful test is found in radiative capture of a thermal neutron by a proton: $n+p \to d+\gamma$. The observed capture rate for this process is $\sigma_{\rm exp} = 334.2 \pm 0.5$mb, which is $\sim$10% larger than the IA prediction $\sigma_{IA} = 302.5 \pm 4.0$mb. According to Riska and Brown,[27] the one-pion exchange current derived from the low-energy theorem can account for $\sim$70% of the missing capture rate. Recognizing that this contribution represents the leading order term in χPT, it is of great interest to examine what the next-order term will do. Park *et al.*'s recent calculation[28] that includes the next-to-leading order terms gives $\sigma = 334 \pm 2$mb, in perfect agreement with experiment. (Another impressive success of the exchange current calculations based on the low-energy theorem is known for the $e+d \to e+p+n$ reaction, see *e.g.* Ref.[29].)

Our last topic is neutrino reactions on the deuteron. The recent developments in the solar neutrino problem have further enhanced the importance of the MSW effect as a possible mechanism to explain the observed energy dependence of the solar neutrino deficit.[30] The SNO heavy-water Čerenkov counter[31] can provide crucial information on this issue because of its capability to register the charged- and neutral-current reactions simultaneously but separately. The SNO is also expected to be highly useful for studying supernova neutrinos. The neutrino-deuteron reactions relevant to the SNO are: $\nu+d \to \nu'+n+p$, $\bar\nu+d \to \bar\nu+n+p$, $\nu_e+d \to e^-+p+p$ and $\bar\nu_e+d \to e^+ + n+n$. Obviously, one needs reliable estimates of the cross sections for these reactions to extract useful astrophysical information from SNO data. The above discussion indicates that one can have enough confidence in the calculational framework that uses the nucleonic Schrödinger equation with realistic N-N interactions supplemented with the exchange currents. Although one may eventually be able to obtain all the ingredients from χPT, it is reasonable to use phenomenological input. There have been several calculations of this type[32,33], and the best available estimates (in our opinion) have been given by Kohyama *et al.*[33]

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