Bose-Einstein correlations in thermal field theory [1]

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Abstract. Two-particle correlation functions are calculated for bosons emitted from a localized thermal source (the "glow" of a "hot spot"). In contrast to existing work, non-equilibrium effects up to first order in gradients of the particle distribution function are taken into account. The spectral width of the bosons is shown to be an important quantity: If it is too small, they do not equilibrate locally and therefore strongly increase the measured correlation radius.

In memoriam of Eugene Wigner and Hiroomi Umezawa.

1. Introduction

In the field of relativistic heavy-ion collisions the analysis of Bose-Einstein correlations [1] has attracted much attention recently. The general hope is to extract information about the size of a source radiating mesons by studying their two-particle correlation function [2, 3]. These correlations are typical quantum effects, hence quantum field theory is a proper framework to describe the problem theoretically. Such efforts have been undertaken with great success for many years [4, 5, 6, 7], with a big emphasis on the quantum properties of mesons that were emitted from a distribution (in space and time) of classical currents. The merger of these theoretical considerations with space-time distributions of quasi-particles generated by simulation codes for relativistic heavy ion collisions has led to predictions of correlation radii that are in rough agreement with experimental data.

The problem of a free quantum field radiating from a classical current is exactly solvable [8, pp.438], also at finite temperature [9]. However, in the original field where Bose-Einstein correlations have been used to measure source radii, i.e., in the Hanbury-Brown–Twiss analysis of star"light", as well as in the analysis of

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relativistic heavy-ion collisions, the radiation source is a localized *thermal* distribution. Such a state is a non-equilibrium state, hence somewhat difficult to handle theoretically.

As was pointed out above, the usual way to circumvent this problem is the approximation of a non-equilibrium state as the superposition of an infinite number of classical currents, with a certain current distribution in space and time (mostly assumed to be gaussian) [5]. From the viewpoint of quantum field theory this approximation is somewhat unsatisfactory, if not questionable: True *non-equilibrium* effects on the propagating particles are neglected, only *local* equilibrium effects are maintained.

Since also in relativistic heavy-ion collisions the radiating system is off equilibrium, one question arises immediately: Are there non-equilibrium effects on the quantum Bose-Einstein correlation ? A secondary question is, whether the correlation effects generated by a distribution of classical currents and those generated by a thermal source are equivalent or not.

To address these questions we consider a stationary temperature distribution with limited spatial extension, and calculate the two-particle correlation function of bosons emitted thermally from such a distribution. In other words, we are investigating correlations in the "glow" from a hot spot – in close analogy to the physical situation present in astrophysics, but on the quantum level.

To this end we formulate the two-particle correlation problem in a field theoretical method suited to handle non-equilibrium states. However, since the solution of the full problem is beyond our capabilities, we restrict ourselves to the same order of accuracy that is reached in standard transport theory: Our correlation function incorporates non-equilibrium effects beyond the local equilibrium function, but only up to first order in the gradients of the "temperature" distribution of this system. Furthermore, the discussion is limited to a static non-equilibrium system.

In another sense we treat the correlation problem more consistently than in standard transport theory: We take into account a nonzero spectral width for the particles we consider. This is necessary, because at nonzero temperature *every* excitation acquires a finite lifetime due to collisions with the medium [10]. We describe this finite lifetime by attributing a certain spectral width $\gamma > 0$ *inside the hot spot* also to asymptotically stable particles. For strongly interacting particles, like e.g. pions, we may assume that such a spectral width is due to the coupling to Δ_{33}-resonances.

Quantum field theory for non-equilibrium states comes in two flavors: The Schwinger-Keldysh method [11] and thermo field dynamics (TFD, see [12, 13] for references to the original work). For the purpose of the present paper, we prefer the latter method: The problem of an inhomogeneous temperature distribution has been solved explicitly in TFD up to first order in the temperature gradients [12]. This solution includes a nontrivial spectral function of the quantum field under consideration. It employs a perturbative expansion in terms of *generalized free* *fields* with continuous mass spectrum [10].

The paper is organized as follows. The next section contains a brief introduction to the formalism of thermo field dynamics for spatially inhomogeneous systems. In section 3 we derive expressions for the two-boson correlation function in non-equilibrium states as well as its local equilibrium approximation. Section 4 contains a study of the effect that is exerted on the correlation function by a nonzero spectral width, section 5 contains an example with semi-realistic pion spectral function in hot nuclear matter. Finally we draw some conclusions and discuss the experimental relevance.

2. Outline of the method

In "ordinary" quantum mechanics, a statistical state of a quantum system is described by a statistical operator (or density matrix) W, and the measurement of an observable will yield the average

$$\left\langle \mathcal{E}(t,\boldsymbol{x}) \right\rangle = \frac{\mathrm{Tr}\left[\mathcal{E}(t,\boldsymbol{x})\; W\right]}{\mathrm{Tr}\left[W\right]}\;, \tag{1}$$

where the trace is taken over the Hilbert space of the quantum system and $\mathcal{E}$ is the hermitean operator associated with the observable. In thermo field dynamics (TFD), the calculation of this trace is simplified to the calculation of a matrix element

$$\left\langle \mathcal{E}(t,\boldsymbol{x}) \right\rangle = \frac{(\!(1\|\; \mathcal{E}(t,\boldsymbol{x})\; \|W)\!)}{(\!(1\|W)\!)}\;, \tag{2}$$

with "left" and "right" statistical state defined in terms of the two different commuting representations (see refs. [13, 12] for details).

In this state, we consider a complex, scalar boson field describing spinless charged excitations in a statistical system not too far from equilibrium. In the spirit of the first remark, one could think of this field as describing positive and negative pions in nuclear matter.

According to the reasoning above, this "thermal" boson field is described by two field operators ϕ_x, $\widetilde{\phi}_x$ and their adjoints $\phi^\star_x$, $\widetilde{\phi}^\star_x$, with canonical commutation relations

$$\begin{aligned} \left[\phi(t,\boldsymbol{x}),\partial_t\phi^\star(t,\boldsymbol{x}')\right] &= \mathrm{i}\delta^3(\boldsymbol{x}-\boldsymbol{x}') \\ \left[\widetilde{\phi}(t,\boldsymbol{x}),\partial_t\widetilde{\phi}^\star(t,\boldsymbol{x}')\right] &= -\,\mathrm{i}\delta^3(\boldsymbol{x}-\boldsymbol{x}') \end{aligned} \tag{3}$$

but commuting with each other. These two fields may be combined in a statistical doublet, see ref. [12, 14] for details.

The free as well as the interacting scalar field can be expanded into momentum eigenmodes

$$\begin{aligned}\phi_x &= \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3}} \left(a^\dagger_{k-}(t)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{kx}} + a_{k+}(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{kx}} \right) \\ \widetilde{\phi}_x &= \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3}} \left(\widetilde{a}^\dagger_{k-}(t)\, \mathrm{e}^{\mathrm{i}\boldsymbol{kx}} + \widetilde{a}_{k+}(t)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{kx}} \right) \ . \end{aligned} \tag{4}$$

$\boldsymbol{k}$ is the three-momentum of the modes, therefore in this notation $a^\dagger_{k-}(t)$ creates a negatively charged excitation with momentum $\boldsymbol{k}$, while $a_{k+}(t)$ annihilates a positive charge. Henceforth the two different charges are distinguished by an additional index $l = \pm$ whenever possible.

For the free case the commutation relations of the a-operators at different times are simple, while they are unknown for the interacting fields. However, we want to go only one step beyond the free field case, i.e., we approximate the fully interacting quantum fields by *generalized free fields* [10]. In this formulation, the operators a,$\widetilde{a}$ do not excite stable on-shell pions. Rather, they are obtained as an integral over more general operators ξ, $\widetilde{\xi}$ with a continuous energy parameter E [15, 14]:

$$\begin{aligned}\left(\begin{array}{c} a_{kl}(t) \\ \widetilde{a}^\dagger_{kl}(t) \end{array} \right) &= \int_0^\infty dE \int d^3\boldsymbol{q}\, \mathcal{A}_l^{1/2}(E,\boldsymbol{k})\, \left(\mathcal{B}_l^{-1}(E,\boldsymbol{q},\boldsymbol{k})\right)^\star \left(\begin{array}{c} \xi_{Eql} \\ \widetilde{\xi}^\#_{Eql} \end{array} \right) \mathrm{e}^{-\mathrm{iEt}} \\ \left(\begin{array}{c} a^\dagger_{kl}(t) \\ -\widetilde{a}_{kl}(t) \end{array} \right)^T &= \int_0^\infty dE \int d^3\boldsymbol{q}\, \mathcal{A}_l^{1/2}(E,\boldsymbol{k}) \left(\begin{array}{c} \xi^\#_{Eql} \\ -\widetilde{\xi}_{Eql} \end{array} \right)^T \mathcal{B}_l(E,\boldsymbol{q},\boldsymbol{k})\, \mathrm{e}^{\mathrm{iEt}} \ , \end{aligned} \tag{5}$$

where $\mathcal{B}$ is a 2×2 matrix, the weight functions $\mathcal{A}_l(E,\boldsymbol{k})$ are positive and have support only for positive energies, their normalization is

$$\int_0^\infty dE\, E\, \mathcal{A}_l(E,\boldsymbol{k}) = \frac{1}{2} \qquad \int_0^\infty dE\, \mathcal{A}_l(E,\boldsymbol{k}) = Z_{kl} \ . \tag{6}$$

The principles of this expansion have been discussed in ref. [10], its generalization to non-equilibrium states was introduced in ref. [12]. For equilibrium states the combination

$$\mathcal{A}(E,\boldsymbol{k}) = \mathcal{A}_+(E,\boldsymbol{k})\Theta(E) - \mathcal{A}_-(-E,-\boldsymbol{k})\Theta(-E) \tag{7}$$

is the spectral function of the field ϕ_x and the limit of free particles with mass m is recovered when

$$\mathcal{A}(E,\boldsymbol{k}) \longrightarrow \mathrm{sign}(E)\, \delta(E^2 - \boldsymbol{k}^2 - m^2) = \mathrm{sign}(E)\, \delta(E^2 - \omega_k^2) \ . \tag{8}$$

For non-equilibrium systems, the existence of a spectral decomposition cannot be guaranteed [13]. We may expect however, that close to equilibrium the field properties do not change very much. Thus, with this formalism we study a quantum system under the influence of small gradients in the temperature, with *local* spectral function $\mathcal{A}(E,\boldsymbol{k})$. Corrections to such a picture only occur in second order of temperature gradients [14, 12].

A thorough discussion of the 2×2 Bogoliubov matrices was carried out in ref. [12]. For the purpose of the present paper, we simply state their explicit form as

$$\mathcal{B}_l(E,\boldsymbol{q},\boldsymbol{k}) = \left(\begin{array}{ll} \left(\delta^3(\boldsymbol{q}-\boldsymbol{k}) + N_l(E,\boldsymbol{q},\boldsymbol{k})\right) & -N_l(E,\boldsymbol{q},\boldsymbol{k}) \\ -\delta^3(\boldsymbol{q}-\boldsymbol{k}) & \delta^3(\boldsymbol{q}-\boldsymbol{k}) \end{array}\right)\,, \tag{9}$$

where $N(E,\boldsymbol{q},\boldsymbol{k})$ is the Fourier transform of a space-local Bose-Einstein distribution function

$$\begin{aligned} N_l(E,\boldsymbol{q},\boldsymbol{k}) &= \frac{1}{(2\pi)^3}\int\!d^3\boldsymbol{z}\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{q}-\boldsymbol{k})\boldsymbol{z}}\,n_l(E,\boldsymbol{z}) \\ n_l(E,\boldsymbol{z}) &= \frac{1}{\mathrm{e}^{\beta(\boldsymbol{z})(E-\mu_l(\boldsymbol{z}))}-1}\,. \end{aligned} \tag{10}$$

Here we have assumed a distribution function that only depends on the energy parameter E and on the space coordinate z. The expansion allows for a generalization of this, to more general distribution functions depending also on the momentum $(\boldsymbol{q}+\boldsymbol{k})/2$.

We have argued, that our ansatz for the fields gives rise to a local spectral function. A moving particle however feels an influence also of the *gradients* of this local equilibrium distribution. Consequently also the propagator for the fields we consider is correct beyond a local equilibrium situation, to be precise it is correct to first order in the gradients of $n_l(E,\boldsymbol{z})$.

To complete the brief description of the TFD formalism, we specify the commutation relation of the various operators in our expressions. The ξ-operators have commutation relations

$$\left[\xi_{Ekl},\xi^{\#}_{E'k'l'}\right] = \delta_{ll'}\,\delta(E-E')\,\delta^3(\boldsymbol{k}-\boldsymbol{k}')\,. \tag{11}$$

Similar relations hold for the $\widetilde{\xi}$ operators, all other commutators vanish, see [10]. It follows from these definitions, that

$$\begin{aligned} \left[a_{kl}(t),a^{\dagger}_{k'l'}(t)\right] &= Z_{kl}\,\delta_{ll'}\,\delta^3(\boldsymbol{k}-\boldsymbol{k}') \\ \left[\widetilde{a}_{kl}(t),\widetilde{a}^{\dagger}_{k'l'}(t)\right] &= Z_{kl}\,\delta_{ll'}\,\delta^3(\boldsymbol{k}-\boldsymbol{k}') \end{aligned} \tag{12}$$

are the equal-time commutation relations for the a, $\widetilde{a}$ operators.

The ξ, $\widetilde{\xi}$ operators act on the "left" and "right" statistical state according to

$$\xi_{Ekl}\|W\rangle\!\rangle = 0,\;\; \widetilde{\xi}_{Ekl}\|W\rangle\!\rangle = 0,\;\; \langle\!\langle 1\|\xi^{\#}_{Ekl} = 0,\;\; \langle\!\langle 1\|\widetilde{\xi}^{\#}_{Ekl} = 0 \quad \forall\, E,\boldsymbol{k},l=\pm 1\,. \tag{13}$$

With these rules, all bilinear expectation values can be calculated exactly. Higher correlation functions have a perturbative expansion in the spectral function.

3. Two-particle correlation function

Of the higher correlation functions, we are interested in the two-particle correlation function, which is the probability to find in the system a pair of pions with momenta p and q. For the non-equilibrium system we are considering, this correlation function is

$$c_{ll'}(\boldsymbol{p},\boldsymbol{q}) = \frac{\left\langle a^\dagger_{pl}(t) a^\dagger_{ql'}(t) a_{ql'}(t) a_{pl}(t) \right\rangle}{\left\langle a^\dagger_{pl}(t) a_{pl}(t) \right\rangle \left\langle a^\dagger_{ql'}(t) a_{ql'}(t) \right\rangle} = 1 + \delta_{ll'} \frac{\mathcal{F}(\boldsymbol{p},\boldsymbol{q})\mathcal{F}(\boldsymbol{q},\boldsymbol{p})}{\mathcal{F}(\boldsymbol{p},\boldsymbol{p})\mathcal{F}(\boldsymbol{q},\boldsymbol{q})} \; . \qquad (14)$$

For simplicity, we abbreviate the mean momentum of this pair by $\boldsymbol{Q} = (\boldsymbol{q}+\boldsymbol{p})/2$. The function $\mathcal{F}(\boldsymbol{q},\boldsymbol{p})$ is calculated using the standard rules of thermo field dynamics given above. One obtains

$$\mathcal{F}(\boldsymbol{p},\boldsymbol{q}) = \int\limits_0^\infty dE \int d^3\boldsymbol{z}\; \left(\mathcal{A}_l(E,\boldsymbol{p})\mathcal{A}_l(E,\boldsymbol{q})\right)^{\frac{1}{2}}\; \mathrm{e}^{\mathrm{i}(\boldsymbol{p}-\boldsymbol{q})\boldsymbol{z}}\, n_l(E,\boldsymbol{z}) \; , \qquad (15)$$

where one may also insert a $\boldsymbol{z}$-dependent spectral function without violating the accuracy to first order in the gradients.

How these gradients enter the above expressions may be seen when performing an expansion of $\mathcal{A}$ around the mean momentum $\boldsymbol{Q}$.

$$\begin{aligned} \mathcal{F}(\boldsymbol{p},\boldsymbol{q}) &= \mathcal{F}^0(\boldsymbol{p},\boldsymbol{q}) \\ &+ \int\limits_0^\infty dE \int d^3\boldsymbol{z}\, \mathrm{e}^{\mathrm{i}(\boldsymbol{p}-\boldsymbol{q})\boldsymbol{z}} \left(\mathrm{i}\nabla_{\boldsymbol{Q}}\mathcal{A}_l(E,\boldsymbol{Q},\boldsymbol{z})\, \nabla_{\boldsymbol{z}} n_l(E,\boldsymbol{z})\right) \\ &+ \mathcal{O}(\nabla^2_z n) \; . \end{aligned} \qquad (16)$$

Here, the lowest order term

$$\mathcal{F}^0(\boldsymbol{p},\boldsymbol{q}) = \int\limits_0^\infty dE \int d^3\boldsymbol{z}\, \mathrm{e}^{\mathrm{i}(\boldsymbol{q}-\boldsymbol{p})\boldsymbol{z}}\, \mathcal{A}_l(E,\boldsymbol{Q},\boldsymbol{z})\, n_l(E,\boldsymbol{z}) \qquad (17)$$

is the *local equilibrium* contribution to the $\mathcal{F}$ we have obtained above.

For a possible generalization, i.e., to explicitly momentum dependent n_l, it is worthwhile to note that the gradient term in (16) is just one half of the Poisson bracket of $\mathcal{A}$ and n [12]. Furthermore, we find that $\mathcal{F}(\boldsymbol{p},\boldsymbol{p}) = \mathcal{F}^0(\boldsymbol{p},\boldsymbol{p})$, i.e., the denominator of the correlation function is not affected by the gradient expansion.

We therefore obtain as the *local equilibrium two-particle correlation function* the expression

$$c^{\mathrm{loc}}_{ll'}(\boldsymbol{p},\boldsymbol{q}) = 1 + \delta_{ll'} \frac{\mathcal{F}^0(\boldsymbol{p},\boldsymbol{q})\mathcal{F}^0(\boldsymbol{q},\boldsymbol{p})}{\mathcal{F}^0(\boldsymbol{p},\boldsymbol{p})\mathcal{F}^0(\boldsymbol{q},\boldsymbol{q})} \; . \qquad (18)$$

However, the full *non-equilibrium correlation function* $c_{ll'}(\boldsymbol{p},\boldsymbol{q})$ is the one measured experimentally.

The exact correspondence between the local equilibrium result and other calculations of the correlator [4, 5, 6, 7] may be found when inserting the free spectral function from (8):

$$c_{ll'}^{\text{free}}(\boldsymbol{p},\boldsymbol{q}) = 1 + \delta_{ll'} \frac{\left| \int d^3\boldsymbol{z}\, \mathrm{e}^{\mathrm{i}(\boldsymbol{q}-\boldsymbol{p})\boldsymbol{z}}\, n_l(\omega_{\boldsymbol{Q}},\boldsymbol{z}) \right|^2}{\left(\int d^3\boldsymbol{z}\, n_l(\omega_{\boldsymbol{p}},\boldsymbol{z})\right)\left(\int d^3\boldsymbol{z}'\, n_l(\omega_{\boldsymbol{q}},\boldsymbol{z}')\right)} \,. \tag{19}$$

Obviously, one may not insert the free spectral function into equation (15) for $\mathcal{F}$. This is *not* a flaw of the derivation, but suggests – as expected – that the limit of zero spectral width at finite temperature is ill-defined [16].

4. Simple spectral function

In this section we study the difference between the local equilibrium correlation function (18) and the non-equilibrium result (15) in more detail. To this end we calculate the correlation functions with a simple parameterization of a boson (pion) spectral function,

$$\mathcal{A}_l(E,\boldsymbol{p}) = \frac{2E\gamma}{\pi}\frac{1}{(E^2-\Omega_p^2)^2+4E^2\gamma^2} \tag{20}$$

where $\Omega_p = \sqrt{m_\pi^2+\boldsymbol{p}^2+\gamma^2}$ and $m_\pi = 140$ MeV. To gain information about the *maximal* influence exerted by the occurrence of a nonzero spectral width, we use an energy and momentum independent γ equal for both charges.

The temperature distribution is taken as a radially symmetric gaussian,

$$T(\boldsymbol{z}) = T(r) = T_0 \exp\left(-\frac{r^2}{2R_0^2}\right) \,, \tag{21}$$

with chemical potential $\mu = 0$ and $R_0 = 5$ fm.

The local equilibrium pion distribution for a given momentum $\boldsymbol{k}$ is obtained by folding $n(E,\boldsymbol{z})$ with the spectral function. Hence, the mean radius of the *particle* distribution function acquires a γ-dependence. We define the rms radius *orthogonal* to the direction of $\boldsymbol{k}$ as

$$R_{\text{rms}} = \sqrt{\frac{I_2}{I_0}} \qquad I_j = \int_0^\infty dr\, r^j \int_0^\infty dE\, \mathcal{A}(E,\boldsymbol{k}) \left(\mathrm{e}^{E/T(r)}-1\right)^{-1} \,. \tag{22}$$

Note, that R_{rms} is *not* the 3-dimensional rms radius of the distribution function (which would be I_4/I_2). Rather, R_{rms} is half the product of angular diameter and distance between detector and source. A constant temperature over a sphere of radius R_0 would yield an $R_{\rm rms} = R_0/\sqrt{3}$, while its 3-D rms radius is $R_0\sqrt{3/5}$.

In fig. 1 we have plotted the correlation functions for two different constant values of the parameter γ. Clearly, for γ=50 MeV the correlation function $c_{ll'}(\boldsymbol{p},\boldsymbol{q})$ agrees quite well with the local equilibrium result, and very closely resembles a gaussian.

However, for smaller γ=5 MeV the non-equilibrium correlation function is much narrower in momentum space than the local equilibrium result, it also deviates from a gaussian form. Nevertheless we may approximate it by such a simple functional form in order to extract quantitative information, i.e.,

$$c_{ll'}(\boldsymbol{p},\boldsymbol{q}) \approx 1 + \exp\left(-R^2(\boldsymbol{p}-\boldsymbol{q})^2\right) \tag{23}$$

and similarly for $c^{\rm loc}_{ll'}(\boldsymbol{p},\boldsymbol{q})$ with parameter $R_{\rm loc}$.

In figure 2 we show the two fit parameters R and $R_{\rm loc}$ as function of γ, together with the γ-dependent rms radius of the particle distribution. We find that the *measured* correlation radius R is always larger than $R_{\rm loc}$, with a minimum reached at $\gamma \approx 26.7$ MeV.

The plot may be divided in two regions, with a boundary at $2\gamma R_{\rm rms}[\gamma] = 1 \Leftrightarrow \gamma \approx 30.2$ MeV. For these two regions we find

$$\begin{array}{lllllll} 2\gamma R_{\rm rms} \ll 1 & \Rightarrow & R \approx R_{\rm loc} + 1/\gamma & \gg & R_{\rm rms} & > & R_{\rm loc} \\ 2\gamma R_{\rm rms} \gg 1 & \Rightarrow & R \approx R_{\rm loc} & \gtrsim & R_{\rm rms} & > & R_{\rm loc}\,. \end{array} \tag{24}$$

For larger γ, the small differences between R, $R_{\rm loc}$ and $R_{\rm rms}$ may be attributed to our use of a gaussian temperature distribution: $n(T(r))$ is not strictly gaussian, only in the (unphysical) limit $\gamma \rightarrow \infty$ one reaches $R = R_{\rm loc} = R_{\rm rms} = R_0/\sqrt{2}$

The interpretation of this result is straightforward: A finite lifetime or nonzero spectral width $\gamma > 0$ of the bosons *inside the source* is essential, if one wants to infer the *thermal source radius* R_0 from correlation measurements. To be more precise, only for $2\gamma R_{\rm rms} \geq 1$ the correlation function measures the mean diameter of the particle distribution function $n(E,\boldsymbol{z})$.

This result is in agreement with our view of the *relaxation process* of a non-equilibrium distribution function: The relaxation rate is, to lowest order, given by the spectral width of the particle [12]. Consequently, *zero* γ corresponds to a system without dissipation. In such a system, only quantum coherence effects exist, and thus the correlation function approaches the "quantum limit"

$$\lim_{\gamma\rightarrow 0} c_{ll'}(\boldsymbol{p},\boldsymbol{q}) = 1 + \delta_{ll'}\delta_{\boldsymbol{p}\boldsymbol{q}}\;. \tag{25}$$

For this case, the correlation radius obtained by a gaussian fit becomes infinite. We may also view, for a given energy, $1/\gamma$ as a measure for the spatial size of the pion "wave packet", which must be smaller than the object to be resolved. In other words, the mean free path of the bosons must not exceed the object size to produce correlations.

$f^{\pi}_{N\Delta}$	g'	m_π	M_N	M_Δ	Γ	ρ_0
2	0.5	0.14 GeV	0.938 GeV	1.232 GeV	0.12 GeV	0.155 fm $^{-3}$

Table 1: Coupling constants and masses used in the calculations of this work. The value of g' was chosen to allow for a direct comparison with simulation codes for heavy-ion collisions, a more realistic value to describe pion scattering data would be $g' \approx 1/3$.

5. Semi-realistic spectral function

To get a more realistic result for the non-equilibrium two-pion correlation function measured in the thermal radiation from a hot spot in nuclear matter, we use the the spectral function derived in [12, 17]. It includes the coupling of pions to Δ_{33}-resonances in nuclear matter, which are taken to have a constant spectral width Γ by themselves.

It was argued in ref. [12], that using such an approximate spectral width for the Δ_{33} resonances constitutes the only way to achieve an analytical solution of the Δ-hole polarization problem in nuclear matter. An even more realistic energy-momentum dependent spectral width for the Δ_{33} resonance can be treated only in a fully self-consistent numerical treatment involving dispersion integral techniques.

To first order in such a constant Γ, the pionic spectral function is

$$\mathcal{A}(E,\boldsymbol{k},\boldsymbol{z}) = \frac{\boldsymbol{k}^2 C(\boldsymbol{z})}{\pi}\,\frac{\Gamma\,E\,\omega_\Delta}{(E^2-\omega_+'^2)^2\,(E^2-\omega_-'^2)^2+\Gamma^2\,E^2\,(E^2-E_\pi^2)^2}\;. \tag{26}$$

Note, that this function is coordinate dependent. The energies in the denominator are

$$\omega_\pm'^2 = \frac{1}{2}\left(E_{N\Delta}^2+(\Gamma/2)^2+E_\pi^2 \pm \sqrt{\left(E_{N\Delta}^2+(\Gamma/2)^2-E_\pi^2\right)^2+4\boldsymbol{k}^2C(\boldsymbol{z})\omega_\Delta}\right)\;, \tag{27}$$

with functions

$$\begin{aligned}
\omega_\Delta &= E_\Delta(\boldsymbol{k})-M_N=\sqrt{\boldsymbol{k}^2+M_\Delta^2}-M_N\\
C(\boldsymbol{z}) &= \frac{8}{9}\left(\frac{f^\pi_{N\Delta}}{m_\pi}\right)^2\left(\rho^0_N(\boldsymbol{z})-\frac{1}{4}\rho^0_\Delta(\boldsymbol{z})\right)\\
E_{N\Delta}(\boldsymbol{k}) &= \sqrt{\omega_\Delta(\boldsymbol{k})\left(\omega_\Delta(\boldsymbol{k})+g'\,C(\boldsymbol{z})\right)}\;.
\end{aligned} \tag{28}$$

The baryon number in each small volume and hence the baryon density is a constant parameter of the calculations, for free particles with bare "on-shell" energies

$$E_N(\boldsymbol{p})=\sqrt{\boldsymbol{p}^2+M_N^2}\quad\text{and}\quad E_\Delta(\boldsymbol{p})=\sqrt{\boldsymbol{p}^2+M_\Delta^2} \tag{29}$$

this baryon density is obtained as

$$\begin{aligned}\rho_b^0 &= \rho_N^0(\boldsymbol{z}) + \rho_\Delta^0(\boldsymbol{z}) \\ &= 4\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, n_N(E_N(\boldsymbol{p}),\boldsymbol{z}) + 16\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, n_\Delta(E_\Delta(\boldsymbol{p}),\boldsymbol{z})\ .\end{aligned} \tag{30}$$

The distribution functions are taken as local Fermi-Dirac functions

$$n_{N,\Delta}(E,\boldsymbol{z}) = \frac{1}{\mathrm{e}^{(E-\mu_{N,\Delta})/T(\boldsymbol{z})}+1}\ , \tag{31}$$

with temperature $T(\boldsymbol{z})$. As temperature distribution we use the same as in the previous section, eq. (21), but with different central temperatures T_0.

Since the temperature depends on the spatial coordinate, fixed baryon density implies that the "baryochemical" composition of the hot spot changes with coordinate $\boldsymbol{z}$. In the center of the hot spot baryons are, to a large extent, present in the form of Δ-resonances. Outside the hot region baryons are "only" nucleons – and as shown before, the pion properties there do not influence our calculation.

In fig. 3, we have plotted the correlation function for a pair momentum of $\boldsymbol{Q} = (\boldsymbol{p}+\boldsymbol{q})/2 = 100$ MeV at the two values T_0= 100 MeV and T_0 = 160 MeV. In this momentum region, the spectral width of the pion is small due to its pseudovector coupling with baryons. This we assume to be a general feature of pions in nuclear matter, although one may argue about the exact value of γ. We find, that at higher temperature the non-equilibrium correlation function (which we had assumed to be the one measured experimentally) is much narrower in momentum space than the local equilibrium function.

The local equilibrium correlation function however is close to the the rms-radius of the thermal source. The correlation radii obtained by a gaussian fit to the non-equilibrium as well as the local equilibrium distribution are given in table 2. Following these results we conclude, that the effect we propose is absent at higher momentum of the pions, where the p-wave coupling to nuclear matter is big enough to give it a sufficiently large spectral width for local equilibration. In the low momentum region, the *measured* correlation radius overestimates the source size by as much as 30 - 40 %.

T	$(\boldsymbol{p}+\boldsymbol{q})/2$	R	R_{loc}	R_{rms}	R/R_{rms}
100 MeV	100 MeV	3.99 fm	2.89 fm	3.14 fm	1.27
160 MeV	100 MeV	4.86 fm	3.23 fm	3.51 fm	1.38
100 MeV	350 MeV	3.23 fm	3.16 fm	3.34 fm	0.97
160 MeV	350 MeV	3.57 fm	3.49 fm	3.73 fm	0.96

Table 2: Correlation radii obtained by gaussian fit to the correlation function with semi-realistic spectral function.

6. Conclusions

Before we draw a final conclusion from our work, we must emphasize that it is still too early to use our result for the correlation function in a direct comparison to experimental data. For any realistic situation, we certainly have to take into account also the partially *coherent* production of pions: A substantial fraction of pions arriving in a detector stems from the free-space decay of Δ_{33} resonances, thus forcing $c_{ll'}(\boldsymbol{p},\boldsymbol{p}) < 2$.

Furthermore it seems worthwhile to note that our derivation does not contradict existing work on the correlation function. In an early semiclassical treatment, an infinite lifetime (equivalent to $\gamma \equiv 0$) of an excitation was found to produce a correlation function $c \equiv 1$ [4], whereas we find an infinitely narrow peak in this "quantum limit", see eq. (25.

Another paper based on the Schwinger-Keldysh method also finds that the total production rate of particles is proportional to an energy integral over the off-diagonal self-energy components, i.e., to the integral over $\gamma \cdot n$ in our formalism [7]. However, due to their explicit quasi-particle approximation the authors of ref. [7] lose the consistent treatment of the non-equilibrium effects to first order in the temperature gradients.

Let us briefly remark on the physical situation present in *stars* emitting photons: For these we may assume a thermal width of the photon which is very small, e.g., $\gamma \approx \alpha T \approx 0.5/137$ eV. However, the source radius $R_{\rm rms}$ of a star is, in general, so big that the condition $2\gamma R_{\rm rms} \gg 1$ is always satisfied. Hence our results do *not* affect correlation radii measured for astronomical objects.

Having in-lined our calculation with existing work, we may now carefully conclude the following physical effect relevant for relativistic heavy-ion collisions: When pion pairs with a relatively low momentum are created in hot nuclear matter, their p-wave interaction with the surrounding medium is small. Hence, in their movement outwards from the hot zone they do not have a sufficiently short mean free path to be locally equilibrated. Thus, their correlation length is dominated by their mean free path ($\approx 1/\gamma$) rather than by the thermal source radius.

As we have shown, this leads to a correlation function which is *narrower* in momentum space than expected in a local equilibrium situation: Compare the solid and the dashed curves in fig. 3. Consequently, measuring the correlation function of low momentum pions in a non-equilibrium state *overestimates* the source radius. For a semi-realistic pion spectral function and a pair momentum of 100 MeV, we find this effect to be as large as 30 – 40%, depending strongly on the actual source size.

Turning to experimental results of NA44, we learn that correlation measurements of pions and K-mesons emanating from relativistic heavy-ion collisions yield comparable fitted correlation radii of 3–4 fm [2]. As we have shown, it might be premature to conclude from these measurements that the *source size* for both mesons is similar: One would have to compare the mean free path of both particle species before such a conclusion.

More recently the experiment NA49 has measured pion correlation functions in central Pb+Pb collisions at 33 TeV. Resulting was a correlation radius of 6-7 fm, as compared to 4.5 fm in S+Au collisions [3]. It is a particularity of our semi-realistic pion spectral function that it is narrower for higher temperature and low momenta. Physically, the first effect stems from the higher Δ_{33} abundance in the hotter system: Δ-particle/nucleon-hole pairs are less easily polarized. The second effect is due to the p-wave coupling of pions to nucleons, as was pointed out above.

Consequently, the effective γ of the pions drops with increasing temperature – and one measures a higher correlation radius. If we presume this effect, which was deduced for *equilibrated* matter, also to hold in the highly non-equilibrium situation present in the experiments, we can state that the higher correlation radius of the NA49 data might indicate a higher temperature of the reaction zone rather than its bigger size. However, this interpretation depends crucially on the momentum range where the pions are measured. The uncertainties in this statement indicate the importance of calculations for mesonic spectral functions in hot nuclear matter.

The effect of narrowing the correlation function might be even more pronounced due to the short time-scales of relativistic heavy-ion collisions, which may suppress the spectral width ($\simeq$ collision rate) of pions even more. Indications for such a behavior are obtained in simulation codes, where at high enough collision energies one may reproduce experimental data with *free* particle collision rates. Thus, instead of speaking about "temperature", which always is connected with the notion of a partial equilibrium, we may also reformulate our conclusion even more sharply: Measured correlation functions become narrower due to non-equilibrium effects in statistically radiating sources. A bigger correlation radius therefore is an indication, that a system of colliding heavy-ion collisions is farther from local equilibrium in the collision zone.

Finally we point out that in common calculations of the correlation function one has to introduce ad-hoc random phases between several classical sources and then obtains only the local equilibrium correlator $c^{\mathrm{loc}}_{ll'}(\boldsymbol{p},\boldsymbol{q})$. Instead, we relied on a proper field theoretical treatment which incorporates *non-equilibrium effects* correctly up to first order in gradients of the temperature. We found, that the non-equilibrium character of the system must be taken serious when calculating the correlation function – which answers the secondary question we asked in the introduction.

Coda

A large part of the work of E.Wigner was dedicated to the study of symmetries in physical systems. The fact, that particles in a thermal state must have a nontrivial spectral function rather than a sharp "mass-shell constraint", is connected to the breaking of such a symmetry: A thermal state has a preferred rest frame and therefore violates the Lorentz invariance of the usual field theoretical ground state

[18].

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Figure 1: Correlation function for a pion "hot spot" with temperature T_0=100 MeV.
Simple spectral function from eq. (20), $(\boldsymbol{p}+\boldsymbol{q})/2$= 100 MeV.
Thin lines: γ=50 MeV, Thick lines: γ=5 MeV.
Solid lines: non-equilibrium correlation function (14).
Dashed lines: local equilibrium correlation function (18).

Figure 2: Correlation radius of a pion "hot spot" with temperature T_0=100 MeV.
Simple spectral function from eq. (20), $(\boldsymbol{p}+\boldsymbol{q})/2$= 100 MeV.
Thick solid line: gaussian fit radius R of the correlation function (14).
Thick dashed line: gaussian fit radius R_{loc} of the correlation function (18).
Thin solid line: γ-dependent rms radius R_{rms}.

Figure 3: Correlation function for pions from a "hot spot" with different central temperature.
Semi-realistic spectral function from eq. (26), $(\boldsymbol{p}+\boldsymbol{q})/2 = 100$ MeV.

Top panel: Central temperature $T_0 = 100$ MeV,
Bottom panel: Central temperature $T_0 = 160$ MeV,

Solid thick line: non-equilibrium correlation function (14).
Dashed line: local equilibrium correlation function (18).
Solid thin line: gaussian with correlation radius equal to $R_{\rm rms}$ (see table).