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Low–Energy Expansion of the Pion–Nucleon Lagrangian*

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Abstract

The renormalized pion–nucleon Lagrangian is calculated to $O(p^3)$ in heavy baryon chiral perturbation theory. By suitably chosen transformations of the nucleon field, the Lagrangian is brought to a standard form.

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1. The effective field theory of the pion–nucleon interaction in the one–nucleon sector can be formulated in terms of a chiral Lagrangian [1]

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \dots \tag{1}$$

The chiral counting rules for constructing the Lagrangians $\mathcal{L}_{\pi N}^{(n)}$ of $O(p^n)$ are different for pions and nucleons because the nucleon mass does not vanish in the chiral limit. Unlike in the meson sector, there is no direct correspondence between the chiral expansion and the loop expansion with the Lagrangian (1).

Heavy baryon chiral perturbation theory (HBCHPT) [2] eliminates this drawback at the price of introducing a special reference frame characterized by a time–like unit four–vector v. In constructing the chiral Lagrangian of HBCHPT, care must be exercised to satisfy Lorentz invariance. The key observation in this context is that in the presence of external objects like the four–vector v Lorentz covariance alone is not sufficient to guarantee Lorentz invariance.

The purpose of this letter is to construct the complete renormalized chiral Lagrangian for the pion–nucleon system to $O(p^3)$ in HBCHPT. To ensure Lorentz invariance, we start from the fully relativistic Lagrangian (1) to perform the frame–dependent decomposition of HBCHPT. With such a procedure, reparametrization invariance [3] is automatically fulfilled.

Both the non–relativistic decomposition and the loop expansion generate so–called equation–of–motion terms. By successive transformations of the (frame–dependent) nucleon field, those terms will be eliminated from the chiral Lagrangian. After these transformations, the final Lagrangian yields directly all one–particle–irreducible vertices in the usual manner.

2. Our starting point is QCD with two massless quarks u, d (all others are massive) coupled to external fields [4]:

$$\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu \left(v_\mu + \frac{1}{3} v_\mu^{(s)} + \gamma_5 a_\mu \right) q - \bar{q}(s - i\gamma_5 p)q, \qquad q = \left(\begin{array}{c} u \\ d \end{array} \right). \tag{2}$$

The isotriplet vector and axial–vector fields v_μ, a_μ are traceless hermitian and s, p are hermitian matrix fields. The isosinglet vector field $v_\mu^{(s)}$ is needed to generate the electromagnetic current. Explicit chiral symmetry breaking is implemented by setting $s = \mathcal{M} = {\rm diag}\,(m_u, m_d)$.

The Lagrangian (2) exhibits a local chiral symmetry $SU(2)_L \times SU(2)_R \times U(1)_V$ that breaks spontaneously to $SU(2)_V \times U(1)_V$. It is realized non–linearly on the Goldstone pion fields ϕ and on the nucleon field Ψ [5]:

$$\begin{aligned} & u(\phi) \stackrel{g}{\to} g_R u(\phi) h(g,\phi)^{-1} = h(g,\phi) u(\phi) g_L^{-1} \\ & \Psi = \begin{pmatrix} p \\ n \end{pmatrix} \stackrel{g}{\to} h(g,\phi)\Psi \\ & g = (g_L, g_R) \in SU(2)_L \times SU(2)_R\ . \end{aligned} \tag{3}$$

The matrix field $u(\phi)$ is an element of the chiral coset space and the compensator field $h(g,\phi) \in SU(2)_V$ characterizes the non–linear realization. The baryon number symmetry $U(1)_V$ acts only on the nucleon field.

The following ingredients are needed to construct the chiral Lagrangian (1):

$$\begin{aligned} u_\mu &= i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\} \\ \Gamma_\mu &= \frac{1}{2}\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\} \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u \ , \qquad \chi = 2B(s+ip) \\ f_\pm^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu} u \\ v_{\mu\nu}^{(s)} &= \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)}. \end{aligned} \tag{4}$$

Here, B is a parameter of the meson Lagrangian of $O(p^2)$ [4] and $r_\mu = v_\mu + a_\mu$, $\ell_\mu = v_\mu - a_\mu$ are the external gauge fields with associated non–Abelian field strengths

$$\begin{aligned} F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] \\ F_L^{\mu\nu} &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu, \ell^\nu] \ . \end{aligned} \tag{5}$$

With a covariant derivative

$$\nabla_\mu = \partial_\mu + \Gamma_\mu - iv_\mu^{(s)}, \tag{6}$$

the lowest–order chiral Lagrangian in (1) takes the form [1]

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i \not\nabla - m + \frac{g_A}{2} \not u \gamma_5\right)\Psi, \tag{7}$$

where m, g_A are the nucleon mass and the axial–vector coupling constant in the chiral limit.

HBCHPT rewrites the Lagrangian (1) in terms of velocity–dependent fields N_v, H_v defined as [6]

$$\begin{aligned} N_v(x) &= \exp[imv\cdot x]P_v^+\Psi(x) \\ H_v(x) &= \exp[imv\cdot x]P_v^-\Psi(x) \\ P_v^\pm &= \frac{1}{2}(1 \pm \not v) \ , \qquad v^2 = 1 \ . \end{aligned} \tag{8}$$

The pion–nucleon Lagrangian takes the form

$$\mathcal{L}_{\pi N} = \bar{N}_v A N_v + \bar{H}_v B N_v + \bar{N}_v \gamma^0 B^\dagger \gamma^0 H_v - \bar{H}_v C H_v \tag{9}$$

where A, B, C are mesonic field operators with a straightforward chiral expansion, e.g.,

$$\begin{aligned} A &= A_{(1)} + A_{(2)} + A_{(3)} + \dots \\ A_{(1)} &= iv\cdot\nabla + g_A S\cdot u \end{aligned} \tag{10}$$

with the spin matrix $S^\mu = i\gamma_5\sigma^{\mu\nu}v_\nu/2$. In the generating functional of Green functions [1], the fields H_v are integrated out to produce a non–local action in terms of the fields N_v [7, 8, 9]

$$S_{\pi N} = \int d^4x\ \widehat{\mathcal{L}}_{\pi N} = \int d^4x\ \bar{N}_v(A + \gamma^0 B^\dagger \gamma^0 C^{-1} B)N_v \ . \tag{11}$$

Expanding C^{-1} in inverse powers of the nucleon mass, one arrives finally at the chiral Lagrangian of HBCHPT. Up to $O(p^3)$, its general form is given by

$$\begin{aligned} & A + \gamma^0 B^\dagger \gamma^0 C^{-1} B = A_{(1)} \\ & \quad + A_{(2)} + \frac{1}{2m}\gamma^0 B^\dagger_{(1)}\gamma^0 B_{(1)} \\ & \quad + A_{(3)} + \frac{1}{2m}(\gamma^0 B^\dagger_{(2)}\gamma^0 B_{(1)} + \gamma^0 B^\dagger_{(1)}\gamma^0 B_{(2)}) - \frac{1}{4m^2}\gamma^0 B^\dagger_{(1)}\gamma^0 (iv\cdot\nabla + g_A S\cdot u) B_{(1)} \\ & \quad + O(p^4). \end{aligned} \tag{12}$$

The chiral Lagrangian of $O(p)$ is determined by the operator $A_{(1)}$ in (10):

$$\widehat{\mathcal{L}}^{(1)}_{\pi N} = \bar{N}_v (iv\cdot\nabla + g_A S\cdot u) N_v \ . \tag{13}$$

In order to obtain the Lagrangians of $O(p^2)$ and $O(p^3)$, we are going to construct the most general operators $A_{(2)}$, $A_{(3)}$ and $B_{(2)}$ compatible with chiral symmetry, parity, charge conjugation and Lorentz invariance.

3. The relativistic Lagrangian of $O(p^2)$ was considered in Refs. [1, 10]. Our procedure is slightly different since we include all field monomials of the form $\bar{\Psi} P_2 \Psi$ that contribute to $A_{(2)}$ and/or $B_{(2)}$. The complete list of independent operators P_2 is as follows:

$$\begin{array}{rcl} \langle u_\mu u^\mu\rangle,\ \langle u_\mu u_\nu\rangle \nabla^\mu\nabla^\nu + \text{h.c.},\ \langle\chi_+\rangle,\ \chi_+ - \frac{1}{2}\langle\chi_+\rangle & \rightarrow & A_{(2)} \\ i\sigma^{\mu\nu} u_\mu u_\nu,\ \sigma^{\mu\nu} f_{+\mu\nu},\ \sigma^{\mu\nu} v^{(s)}_{\mu\nu},\ \langle u_\mu u_\nu\rangle i\gamma^\mu\nabla^\nu + \text{h.c.} & \rightarrow & A_{(2)}, B_{(2)} \\ \chi_-\gamma_5,\ \langle\chi_-\rangle\gamma_5,\ i\gamma_5[\nabla_\mu, u_\nu]\{\nabla^\mu,\nabla^\nu\} + \text{h.c.} & \rightarrow & B_{(2)} \end{array} \tag{14}$$

where $\langle \ldots \rangle$ stands for the trace. We refer to the paper of Krause [10] for a thorough exposition of how to arrive at a minimal set of such operators. Note in particular that all equation–of–motion terms proportional to $(i\not{\nabla} - m)\Psi$ can be eliminated by a transformation of the nucleon field Ψ.

The two pieces of $O(p^2)$ in (12) are then found [8] to be

$$\begin{aligned} A_{(2)} \ =\ & \frac{c_1}{m}\langle\chi_+\rangle + \frac{c_2}{m}(v\cdot u)^2 + \frac{c_3}{m} u\cdot u + \frac{c_5}{m}\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right) \\ & + \frac{1}{m}\varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma [ic_4 u_\mu u_\nu + c_6 f_{+\mu\nu} + c_7 v^{(s)}_{\mu\nu}] \end{aligned} \tag{15}$$

and

$$\begin{aligned} \frac{1}{2m}\gamma^0 B^\dagger_{(1)}\gamma^0 B_{(1)} = & \frac{1}{2m}\{(v\cdot\nabla)^2 - \nabla\cdot\nabla - ig_A\{S\cdot\nabla, v\cdot u\} \\ & - \frac{g_A^2}{4}(v\cdot u)^2 + \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma [iu_\mu u_\nu + f_{+\mu\nu} + 2v^{(s)}_{\mu\nu}]\} \ . \end{aligned} \tag{16}$$

The present status of the scale–independent, dimensionless low–energy constants (LECs) $c_1, \ldots, c_7$ is summarized in Refs. [11, 12].

In order to incorporate all 1PI vertices in the Lagrangian, the operator $(v\cdot\nabla)^2/2m$ in (16) can be eliminated by a field transformation [1]. Up to $O(p^3)$, we will encounter such equation–of–motion terms of the following variety:

$$\mathcal{L}_{\rm EOM} = \bar N_v\{X(iv\cdot\nabla)^3 + iv\cdot\overleftarrow{\nabla}\, Y iv\cdot\nabla + Ziv\cdot\nabla - iv\cdot\overleftarrow{\nabla}\, Z^\dagger\}N_v \ . \tag{17}$$

The mesonic operators $Y=Y^\dagger$ and Z are at most of $O(p)$ and $O(p^2)$, respectively and $X=X^*$ is a constant. Applying a nucleon field transformation

$$\begin{aligned} N_v &= \Big\{1-\frac{X}{2}(iv\cdot\nabla)^2+\frac{1}{2}(Y+g_AXS\cdot u)iv\cdot\nabla+\frac{g_A}{2}X[iv\cdot\nabla,S\cdot u] \\ &\quad -\frac{g_A^2}{2}X(S\cdot u)^2-\frac{g_A}{2}YS\cdot u-Z^\dagger\Big\}N_v' \end{aligned} \tag{18}$$

to $\widehat{\mathcal{L}}^{(1)}_{\pi N}$ in (13) and $\mathcal{L}_{\rm EOM}$ in (17) eliminates the equation–of–motion terms and generates the following Lagrangian of the same chiral order as $\mathcal{L}_{\rm EOM}$:

$$\begin{aligned} \mathcal{L}_{\rm induced} &= \overline{N_v'}\{-g_A^3X(S\cdot u)^3+\frac{g_A^2}{2}X[S\cdot u,[iv\cdot\nabla,S\cdot u]] \\ &\quad -g_A^2S\cdot uYS\cdot u-g_A(ZS\cdot u+S\cdot uZ^\dagger)\Big\}N_v' \ . \end{aligned} \tag{19}$$

Of course, the transformation (18) must be applied to the complete Lagrangian to the order considered and it will therefore generate additional terms. For the case at hand (with $X=Z=0$, $Y=1/2m$), the complete Lagrangian of $O(p^2)$ in HBCHPT assumes its final form

$$\begin{aligned} \widehat{\mathcal{L}}^{(2)}_{\pi N} &= \bar N_v\Big(-\frac{1}{2m}(\nabla\cdot\nabla+ig_A\{S\cdot\nabla,v\cdot u\}) \\ &\quad +\frac{a_1}{m}\langle u\cdot u\rangle+\frac{a_2}{m}\langle(v\cdot u)^2\rangle+\frac{a_3}{m}\langle\chi_+\rangle+\frac{a_4}{m}\Big(\chi_+-\frac{1}{2}\langle\chi_+\rangle\Big) \\ &\quad +\frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma[ia_5u_\mu u_\nu+a_6f_{+\mu\nu}+a_7v^{(s)}_{\mu\nu}]\Big)N_v \ . \end{aligned} \tag{20}$$

To facilitate comparison, we display the relations between the LECs a_i and the previously defined c_i:

$$\begin{aligned} &a_1=\frac{c_3}{2}+\frac{g_A^2}{16}, \qquad a_2=\frac{c_2}{2}-\frac{g_A^2}{8}, \qquad a_3=c_1, \qquad a_4=c_5, \\ &a_5=c_4+\frac{1-g_A^2}{4}, \qquad a_6=c_6+\frac{1}{4}, \qquad a_7=c_7+\frac{1}{2} \ . \end{aligned} \tag{21}$$

The LECs a_6, a_7 are related to the nucleon magnetic moments in the chiral limit:

$$\begin{aligned} a_6 &= \frac{\mu_v}{4}=\frac{1}{4}(\mu_p-\mu_n) \\ a_7 &= \frac{\mu_s}{2}=\frac{1}{2}(\mu_p+\mu_n) \ . \end{aligned} \tag{22}$$

[1]The heavy–nucleon propagator in the presence of external fields is given by the inverse of $v\cdot\nabla$.

The two terms in (20) with fixed coefficients are a manifestation of the difference between Lorentz invariance and covariance and they are of course consistent with reparametrization invariance [3]. In particular, they generate the Thomson limit in nucleon Compton scattering [8] and the $O(p^2)$ amplitude for pion photoproduction at threshold [13].

The field transformation (18) produces also terms of $O(p^3)$ and even higher orders. The terms of $O(p^3)$ will be included in the final Lagrangian (23).

4. As shown in Eq. (12), there are three different pieces in the chiral Lagrangian $\widehat{\mathcal{L}}_{\pi N}^{(3)}$ of $O(p^3)$. The term $A_{(3)}$ is obtained from the non–relativistic decomposition of the relativistic Lagrangian $\mathcal{L}_{\pi N}^{(3)}$. We refrain from writing down the Lagrangian $\mathcal{L}_{\pi N}^{(3)}$ as it coincides with the $SU(2)$ version of Krause's Lagrangian of $O(p^3)$ [10]. Some of the couplings in $A_{(3)}$ must absorb the divergences of the one–loop functional with vertices from $\widehat{\mathcal{L}}_{\pi N}^{(1)}$ and from the lowest–order mesonic Lagrangian. Those divergences were calculated in Ref. [14]. Since they contain seven independent equation–of–motion terms of the type (17), we perform another field transformation to get rid of those terms. At $O(p^3)$, this transformation modifies the coefficients of the remaining monomials. In addition, this transformation also influences the divergence structure at higher orders. In particular, applying it to $\widehat{\mathcal{L}}_{\pi N}^{(2)}$ in (20) generates divergent terms of $O(p^4)$ that have to be included in the full (one–loop) functional of $O(p^4)$ to be calculated eventually.

Here, we concentrate on the Lagrangian of $O(p^3)$. Altogether, we find 22 independent terms for $A_{(3)}$. However, two more terms with arbitrary coupling constants emerge from the terms in $\widehat{\mathcal{L}}_{\pi N}^{(3)}$ involving $B_{(2)}$ [cf. Eq. (14)]. The complete list of independent field monomials of $O(p^3)$ is displayed in Table 1 (the additional terms due to $B_{(2)}$ correspond to $i = 19, 20$).

Calculation of the remaining terms in (12) of $O(p^3)$ with coefficients defined at $O(p)$ or $O(p^2)$ is straightforward. Once again, one encounters equation–of–motion terms involving $v \cdot \nabla N_v$. Another basis transformation (not affecting the Lagrangian to $O(p^2)$, of course) brings the Lagrangian of $O(p^3)$ into its final form

$$\begin{aligned}
\widehat{\mathcal{L}}_{\pi N}^{(3)} \;=\; & \bar{N}_v \left(\frac{g_A}{8m^2}[\nabla_\mu, [\nabla^\mu, S\cdot u]] + \frac{1}{2m^2}\left[\left\{ i\left(a_5 - \frac{1-3g_A^2}{8}\right) u_\mu u_\nu \right.\right.\right. \\
& + \left.\left(a_6 - \frac{1}{8}\right) f_{+\mu\nu} + \left(a_7 - \frac{1}{4}\right) v_{\mu\nu}^{(s)} \right\} \varepsilon^{\mu\nu\rho\sigma} S_\sigma i \nabla_\rho + \frac{g_A}{2} S\cdot\nabla u \cdot \nabla \\
& - \left. \frac{g_A^2}{8}\{v\cdot u, u_\mu\}\varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma \nabla_\nu - \frac{i g_A}{16}\varepsilon^{\mu\nu\rho\sigma} f_{-\mu\nu} v_\rho \nabla_\sigma + \text{h.c.}\right] \\
& + \left. \frac{1}{(4\pi F)^2}\sum_{i=1}^{24} b_i O_i \right) N_v \; . \qquad (23)
\end{aligned}$$

Some of the b_i are divergent to absorb the divergences of the one–loop functional of $O(p^3)$ [14]. Decomposing them in the standard way (using dimensional regularization) as

$$\begin{aligned}
b_i &= b_i^r(\mu) + (4\pi)^2 \beta_i \Lambda(\mu) \qquad (24) \\
\Lambda(\mu) &= \frac{\mu^{d-4}}{(4\pi)^2}\left\{ \frac{1}{d-4} - \frac{1}{2}[\ln 4\pi + 1 + \Gamma'(1)] \right\}
\end{aligned}$$

Table 1: Field monomials of $O(p^3)$ in the Lagrangian (23) and their β–functions.

i	O_i	β_i
1	$i[u_\mu,[v\cdot\nabla,u^\mu]]$	$-g_A^4/6$
2	$i[u_\mu,[\nabla^\mu,v\cdot u]]$	$-(1+5g_A^2)/12$
3	$i[v\cdot u,[v\cdot\nabla,v\cdot u]]$	$(3+g_A^4)/6$
4	$i\langle u_\mu v\cdot u\rangle\nabla^\mu$+ h.c.	0
5	$iv_\lambda\varepsilon^{\lambda\mu\nu\rho}\langle u_\mu u_\nu u_\rho\rangle$	0
6	$[\chi_-,v\cdot u]$	$(1+5g_A^2)/24$
7	$[\nabla^\mu,f_{+\mu\nu}]v^\nu$	$-(1+5g_A^2/6$
8	$\partial^\mu v_{\mu\nu}^{(s)}v^\nu$	0
9	$\varepsilon^{\mu\nu\rho\sigma}\langle f_{+\mu\nu}u_\rho\rangle v_\sigma$	0
10	$\varepsilon^{\mu\nu\rho\sigma}v_{\mu\nu}^{(s)}u_\rho v_\sigma$	0
11	$S\cdot u\langle u\cdot u\rangle$	$g_A(1+5g_A^2+4g_A^4)/2$
12	$u_\mu S_\nu\langle u^\mu u^\nu\rangle$	$g_A(3-9g_A^2+4g_A^4)/6$
13	$S\cdot u\langle(v\cdot u)^2\rangle$	$-g_A(2+g_A^2+2g_A^4)$
14	$v\cdot uS_\mu\langle u^\mu v\cdot u\rangle$	$g_A^3+2g_A^5/3$
15	$\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma\langle[v\cdot\nabla,u_\mu]u_\nu\rangle$	$g_A^4/3$
16	$\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma\langle u_\mu[\nabla_\nu,v\cdot u]\rangle$	0
17	$S\cdot u\langle\chi_+\rangle$	$g_A/2+g_A^3$
18	$S^\mu\langle u_\mu\chi_+\rangle$	0
19	$iS^\mu[\nabla_\mu,\chi_-]$	0
20	$iS^\mu\langle\partial_\mu\chi_-\rangle$	0
21	$iS^\mu v^\nu[f_{+\mu\nu},v\cdot u]$	$g_A+g_A^3$
22	$iS^\mu[f_{+\mu\nu},u^\nu]$	$-g_A^3$
23	$S^\mu[\nabla^\nu,f_{-\mu\nu}]$	0
24	$\varepsilon^{\mu\nu\rho\sigma}S_\mu\langle u_\nu f_{-\rho\sigma}\rangle$	0

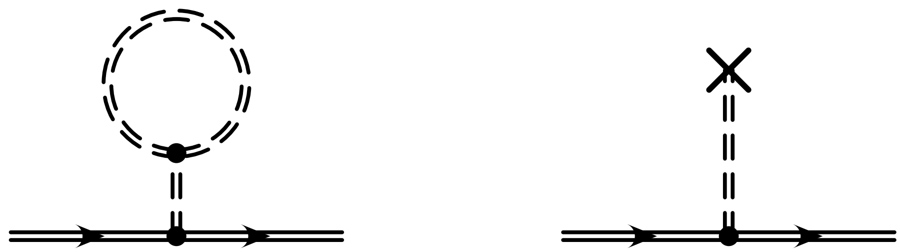

Figure 1: Reducible functional diagrams of $O(p^3)$. The full (dashed) lines denote the nucleon (meson) propagators. The propagators and the vertices have the full tree–level structure attached as functionals of the external fields. The cross denotes a vertex of $O(p^4)$ from the mesonic Lagrangian $\mathcal{L}_4$.

gives rise to 24 measurable renormalized LECs $b_i^r(\mu)$. The β_i are dimensionless functions of g_A listed in Table 1 and they govern the scale dependence of the $b_i^r(\mu)$:

$$b_i^r(\mu) = b_i^r(\mu_0) + \beta_i \ln \frac{\mu_0}{\mu} \ . \tag{25}$$

It is important to realize that the LECs $b_i^r(\mu)$ and their β–functions depend on the choice of the mesonic Lagrangian of $O(p^4)$. In addition to the action for the Lagrangian (23), the generating functional of $O(p^3)$ consists of four pieces corresponding to two irreducible and two reducible diagrams [1, 14]. It can be shown by explicit calculation that the sum of the two reducible diagrams (reproduced in Fig. 1) is finite and scale independent if the mesonic Lagrangian $\mathcal{L}_4^{\rm GSS}$ of Ref. [1] is used for the functional tree diagram in Fig. 1. The divergences of the one–loop functional are therefore all contained in the irreducible diagrams [14] and they give rise to the β_i of Table 1.

However, the mesonic Lagrangian of $O(p^4)$ is not unique. One can always use the equations of motion to transform it into different, but physically equivalent forms. In fact, the original Lagrangian $\mathcal{L}_4^{\rm GL}$ [4] differs from $\mathcal{L}_4^{\rm GSS}$ by precisely such an equation–of–motion term:

$$\mathcal{L}_4^{\rm GL} - \mathcal{L}_4^{\rm GSS} = \frac{l_4}{16}\left(4i\langle u_\mu \chi_-^\mu\rangle - 4\langle \chi_+ u_\mu u^\mu\rangle - 2\langle \chi_-^2\rangle + \langle \chi_-\rangle^2\right) \tag{26}$$

$$\chi_-^\mu = u^\dagger D^\mu \chi u^\dagger - u D^\mu \chi^\dagger u \ , \qquad D^\mu \chi = \partial^\mu \chi - i r^\mu \chi + i \chi \ell^\mu \ .$$

Applying partial integration and the equations of motion

$$[\nabla^\mu, u_\mu] = \frac{i}{2}\chi_- - \frac{i}{4}\langle \chi_-\rangle \ , \tag{27}$$

the difference of the corresponding actions can be shown to vanish. The problem, which also appears in the calculation of the mesonic functional of $O(p^6)$ [11], is that use of the equations of motion (27) in the construction of $\mathcal{L}_4$ does not commute with calculating vertices from $\mathcal{L}_4$. To obtain the relevant vertex in the tree diagram of Fig. 1, one expands the action to first order in the fluctuation variables ξ around the classical solution. In the notation of Ref. [14], one finds

$$S_4^{\rm GL} - S_4^{\rm GSS} = \frac{i l_4}{4}(\chi_- - \frac{1}{2}\langle \chi_-\rangle)_j (d_\mu d^\mu + \sigma)_{ji}\xi_i + O(\xi^2) \ , \tag{28}$$

where all quantities except ξ are to be taken at the classical solution. The meson propagator in the tree diagram of Fig. 1 is given by the inverse of the differential operator $d_\mu d^\mu + \sigma$ (times a normalization factor $2/F^2$). Thus, as expected on general grounds [11], the difference between the two functionals is a local action. Recalling the functional nucleon–nucleon–meson vertex [14] appearing in the diagram in question,

$$V_i = \frac{i}{4\sqrt{2}}[v\cdot u, \tau_i] - \frac{g_A}{\sqrt{2}}\tau_j S^\mu d_{\mu,ji} \ , \tag{29}$$

the difference between the tree–diagram functionals for the two choices of $\mathcal{L}_4$ is

$$Z^{\rm GL}_{\rm tree} - Z^{\rm GSS}_{\rm tree} = \frac{l_4}{8F^2}\int d^4x \bar{N}_v \left([\chi_-, v\cdot u] - 4ig_A S_\mu[\nabla^\mu,\chi_-] + 2ig_A S_\mu\langle\partial^\mu\chi_-\rangle\right) N_v \ . \tag{30}$$

Since l_4 is divergent [4], the sum of the two functional diagrams in Fig. 1 diverges for $\mathcal{L}_4^{\rm GL}$ because it is finite for $\mathcal{L}_4^{\rm GSS}$. Therefore, the entries in Table 1 must be modified if the mesonic Lagrangian $\mathcal{L}_4^{\rm GL}$ is used:

$$\beta_6^{\rm GL} = -\frac{5(1-g_A^2)}{24} \ , \qquad \beta_{19}^{\rm GL} = g_A \ , \qquad \beta_{20}^{\rm GL} = -\frac{g_A}{2} \ , \tag{31}$$

all other coefficients being identical to the ones in Table 1.

The main lesson from this analysis is that the LECs b_i depend in general on the choice of pion fields. For the two particular choices $\mathcal{L}_4^{\rm GSS}$ and $\mathcal{L}_4^{\rm GL}$, one finds in addition to (31)

$$\begin{aligned} b_6^{{\rm GL},r}(\mu) &= b_6^{{\rm GSS},r}(\mu) - \frac{(4\pi)^2}{8} l_4^r(\mu) \\ b_{19}^{{\rm GL},r}(\mu) &= b_{19}^{\rm GSS} + \frac{(4\pi)^2}{2} g_A l_4^r(\mu) \\ b_{20}^{{\rm GL},r}(\mu) &= b_{20}^{\rm GSS} - \frac{(4\pi)^2}{4} g_A l_4^r(\mu) \end{aligned} \tag{32}$$

for the renormalized LECs. All other coupling constants remain unchanged.

5. Although different parts of the Lagrangian (23) have been used before, the complete Lagrangian of $O(p^3)$ is given here for the first time. Our knowledge of the b_i is still rather limited. For an up–to–date account, we refer to Ref. [12].

Of special interest are the terms in the Lagrangian (23) with coefficients defined at $O(p)$ or $O(p^2)$. As an example, consider the terms with coefficients a_6, a_7 that contribute to the spin–dependent amplitude in nucleon Compton scattering. Projecting out the electromagnetic field produces an amplitude proportional to

$$k_3 = \frac{1}{2}(1+\tau_3)\left[\left(a_6 - \frac{1}{8}\right)\tau_3 + \frac{1}{2}\left(a_7 - \frac{1}{4}\right)\right] = \frac{1}{4}\begin{pmatrix} 1+2\kappa_p & 0 \\ 0 & 0 \end{pmatrix} \ , \tag{33}$$

with κ_p the anomalous magnetic moment of the proton (in the chiral limit). There is an additional tree–level contribution [8] with two vertices of $O(p^2)$, each proportional to [cf. Eq. (20)]

$$k_2 = a_6\tau_3 + \frac{a_7}{2} = \frac{1}{2}\begin{pmatrix} 1+\kappa_p & 0 \\ 0 & \kappa_n \end{pmatrix} . \tag{34}$$

The leading contribution to the spin–dependent Compton amplitude in the forward direction for small photon energies is of $O(p^3)$ and it is proportional to

$$k_2^2 - k_3 = \frac{1}{4}\begin{pmatrix} \kappa_p^2 & 0 \\ 0 & \kappa_n^2 \end{pmatrix} \tag{35}$$

in accordance with a classic low–energy theorem [15].

The spin–dependent nucleon Compton scattering amplitude is an example for the class of amplitudes in the one–nucleon sector that are insensitive to the LECs b_i. All such amplitudes are uniquely determined by LECs of at most $O(p^2)$ with possible loop contributions being necessarily finite. Another prominent example is the electric dipole amplitude E_{0+} for π^0 photoproduction at threshold that receives also a finite one–loop contribution [13, 16].

As a final observation, we note that all terms with fixed coefficients in (23) except one contain the spin matrix S. The odd term without an explicit spin matrix has at least one external gauge field through the tensor field $f_-^{\mu\nu}$.

6. By suitably chosen nucleon field transformations, we have brought the pion–nucleon Lagrangian of $O(p^3)$ in HBCHPT to a standard form

$$\widehat{\mathcal{L}}_{\pi N}^{(1)} + \widehat{\mathcal{L}}_{\pi N}^{(2)} + \widehat{\mathcal{L}}_{\pi N}^{(3)} , \tag{36}$$

with the three Lagrangians given by Eqs. (13), (20) and (23). In addition to g_A and m, this Lagrangian contains 7 scale–independent LECs a_i of $O(p^2)$ and 24 renormalized, in general scale–dependent LECs $b_i^r(\mu)$ of $O(p^3)$. By construction, it is fully Lorentz invariant despite the dependence on an arbitrary four–vector v. All equation–of–motion terms have been transformed away. Consequently, all 1PI vertices can be read off directly from (36). More work is needed to determine and to interpret the coupling constants of the $O(p^3)$ Lagrangian.

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References

[1] J. Gasser, M.E. Sainio and A. Švarc, Nucl. Phys. B307 (1988) 779.

[2] E. Jenkins and A.V. Manohar, Phys. Lett. B255 (1991) 558.

[3] M. Luke and A.V. Manohar, Phys. Lett. B286 (1992) 348.

[4] J. Gasser and H. Leutwyler, Ann. Phys. 158 (1984) 142.

[5] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239;
C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.

[6] H. Georgi, Phys. Lett. B240 (1990) 447.

[7] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B368 (1992) 204.

[8] V. Bernard, N. Kaiser, J. Kambor and U.-G. Meißner, Nucl. Phys. B388 (1992) 315.

[9] G. Ecker, Czech. J. Phys. 44 (1994) 405.

[10] A. Krause, Helvetica Phys. Acta 63 (1990) 3.

[11] G. Ecker, Progr. Part. Nucl. Phys., vol. 35, pp. 1–80, Ed. A. Fäßler, Elsevier Science Ltd. (Oxford, 1995).

[12] V. Bernard, N. Kaiser and U.-G. Meißner, Chiral dynamics in nucleons and nuclei, hep-ph/9501384, to appear in Int. J. Mod. Phys. E.

[13] V. Bernard, J. Gasser, N. Kaiser and U.-G. Meißner, Phys. Lett. B268 (1991) 291.

[14] G. Ecker, Phys. Lett. B336 (1994) 508.

[15] F. Low, Phys. Rev. 96 (1954) 1428;
M. Gell-Mann and M.L. Goldberger, Phys. Rev. 96 (1954) 1433.

[16] G. Ecker and U.-G. Meißner, What is a low–energy theorem ?, hep-ph/9409442, to appear in Comments Nucl. Part. Phys.