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# Hydrodynamical Simulations of Corotating Interaction Regions and Discrete Absorption Components in Rotating O-Star Winds

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## ABSTRACT

We present two-dimensional hydrodynamical simulations of corotating stream structure in the wind from a rotating O star, together with resulting synthetic line profiles showing discrete absorption components (DACs). An azimuthal variation is induced by a local increase or decrease in the radiative driving force, as would arise from a bright or dark "star spot" in the equatorial plane. Since much of the emergent wind structure seems independent of the exact method of perturbation, we expect similar morphology in winds perturbed by localized magnetic fields or nonradial pulsations, as well as by either rotationally-modulated structure or transient mass ejections.

We find that bright spots with enhanced driving generate high-density, low-speed streams, while dark spots generate low-density, high-speed streams. Corotating interaction regions (CIRs) form where fast material collides with slow material – e.g. at the leading (trailing) edge of a stream from a dark (bright) spot, often steepening into shocks. The unperturbed supersonic wind obliquely impacts the high-density CIR and sends back a nonlinear signal which takes the form of a sharp propagating discontinuity ("kink" or "plateau") in the radial velocity gradient. These features travel inward in the co-moving frame at the radiative-acoustic characteristic speed, and thus slowly outward in the star's frame. We find that these slow kinks, rather than the CIRs themselves, are more likely to result in high-opacity DACs in the absorption troughs of unsaturated P Cygni line profiles. Because the hydrodynamic structure settles to a steady state in a frame corotating with the star, the more tightly-spiraled kinks sweep by an observer on a longer time scale than material moving with the wind itself. This is in general accord with observations showing slow apparent accelerations for DACs.

*Subject headings:* circumstellar matter – hydrodynamics – radiative transfer – stars: early-type – stars: mass loss – stars: rotation

## 1. Introduction

The radiatively-driven winds of early-type (O, B, Wolf-Rayet) stars are observed to vary on time scales ranging from hours to years. In addition, as in the highly complex solar wind, the mass outflows from hot stars are presumably not spherically symmetric. There are many physical mechanisms that can lead to wind structure and variability, and it is useful to distinguish between (1) *small-scale* stochastic fluctuations, intrinsic to the wind itself, and (2) *large-scale* quasi-regular variability, induced by changes in the underlying star. In the former category is the the shocked structure arising from the strong instability of the line-driving mechanism (Rybicki 1987; Owocki 1992), which may explain black troughs in saturated UV P Cygni lines in OB stars (Lucy 1982; Puls, Owocki, & Fullerton 1993, hereafter POF), shock-heated X-ray emission

(Cooper & Owocki 1994), and moving "bumps" in Wolf-Rayet optical emission lines (Robert 1994). On the other hand, the larger-scale structure could be attributed to the dynamical effects of rotation, magnetic fields, or nonradial pulsations, which may produce the recurring discrete absorption components (DACs) and blue-edge variability observed in ultraviolet P Cygni lines. There has been much progress in using radiation hydrodynamics to model the small-scale intrinsic wind instability, but considerably less attention has been given to the problem of how line-driven winds respond to larger star-induced variations. This paper reports on initial results from radiation hydrodynamics models of winds affected by such laterally coherent and rotationally modulated perturbations.

Large-scale wind structure in hot stars is inferred most directly from time variability in the blueshifted absorption troughs of UV P Cygni profiles. The most conspicuous variations are the DACs, which appear as narrow and localized optical depth enhancements in unsaturated lines, in some stars even dominating the "mean wind" absorption. DACs are present in a majority of O-star (Howarth & Prinja 1989) and Be-star (Grady, Bjorkman, & Snow 1987) winds, and are typically seen to accelerate to the blue wing of the profile over a few days, becoming narrower as they approach an asymptotic velocity. Prinja (1988) and Henrichs, Kaper, & Zwarthoed (1988) found an apparent correlation between both the recurrence and acceleration time scales of DACs (typically of the same order as each other) and the projected rotational velocity of the star, $V_{\rm eq} \sin i$. Corresponding and often temporally-correlated variability is seen in the blue edge fluctuations of saturated UV P Cygni lines, in the low-velocity variability of subordinate-level P Cygni lines, and in optical lines such as H$\alpha$ and He II $\lambda$4686, suggesting a single dynamical phenomenon reaching down to very near the photosphere (Henrichs, Kaper, & Nichols 1994).

Attempts to model DACs have been progressively constrained by better observations. By studying lines of different ionization species, Lamers, Gathier, & Snow (1982) ruled out the early supposition that DACs might be caused by ionization gradients in an otherwise spherically symmetric and time steady wind. The episodic ejection of spherical "shells" of increased mass loss was an often-invoked model for a time, but the lack of both *emission* variability in UV P Cygni lines (Prinja & Howarth 1988) and significant infrared variability (Howarth 1992) seems to rule out a spherically-symmetric disturbance. On the other hand, to produce the observed strong absorption dips, the structure must be large enough to cover a substantial fraction of the stellar disk. This seems to rule out the small-scale wind instability as the source of most DAC clumpiness, since global averaging would weaken the observable signature (Owocki 1994). Also, Rybicki, Owocki, & Castor (1990) showed that small-scale, lateral velocity perturbations should be strongly damped, and so should not disrupt the horizontal scale size set by base variations. Altogether, these constraints suggest that DACs originate from moderate size wind structures, e.g., spatially-localized clouds, streams, or "blobs."

Of particular interest is the apparent acceleration rate of DACs. When compared to the acceleration of the mean wind inferred from line-driven flow theory and detailed profile fitting, some (typically weaker) DACs seem to be passively carried along the same velocity law (see, e.g., Kaper 1993). But most strong DACs accelerate much more *slowly* (Prinja 1994), suggesting they may not represent a single mass-conserving feature, but rather might arise from a slowly evolving *pattern* or perturbation through which wind material flows. The enhanced optical depth could result from either a higher density or a lower wind velocity gradient (a "plateau"), or by a combination of the two (Fullerton & Owocki 1992; Owocki, Fullerton, & Puls 1994), as is found in the dynamical models below (§ 3).

Mullan (1984a, b; 1986) proposed that DACs and related phenomena could arise from "corotating interaction regions" (CIRs) analogous to those commonly observed *in situ* in the solar wind. In the solar corona, regions of open magnetic field cause the flow from coronal holes to be accelerated faster than the mean ecliptic-plane wind, resulting in colliding fast and slow streams strung into spiral CIR patterns by rotation (Hundhausen 1972; Zirker 1977). These nonlinear interacting streams eventually steepen into oblique corotating shocks, through which the wind flows nearly radially. Because hot stars do not have the strong surface convection and coronae known to exist in the sun, the "seed" mechanism for large-scale azimuthal perturbations may be quite different.

In this paper we do not adhere to any particular model for these photospheric variations, but several plausible scenarios have been proposed. Underhill & Fahey (1984) and Henrichs et al. (1994) suggested that small patches of enhanced magnetic field could exist undetected on early-type stars and produce corotating wind structure. Also, nonradial pulsations have been observed in many OB stars, and have been shown to be able to induce localized increased mass loss and outward angular momentum transfer (Castor 1986; Willson 1986; Ando 1991). Circumstellar disks exhibit many natural large-scale instabilities, e.g. Okazaki's (1991) global one-armed normal modes, which may be correlated with DAC variability in Be stars (Telting & Kaper 1994). Dowling & Spiegel (1990) discuss the possible existence of Jupiter-like zonal bands and vortices in the atmospheres of hot stars, and give order-of-magnitude estimates of the flux enhancement over a "Great Red Spot" type of shear pattern.

Several qualitative attempts have been made to apply the CIR picture to observations of time variability in early-type stellar winds, but all have been *kinematic* in nature. Prinja & Howarth (1988) fit slowly-accelerating spiral streamlines to DACs in time series spectra from the O7.5 giant 68 Cyg, and showed that the narrowing of the absorption feature as it accelerates can be explained roughly by the decrease in the line-of-sight velocity gradient of the CIR. Harmanec (1991) extended this analysis and discussed possible observational signatures of CIRs in other classes of early-type stars. Rotationally-modulated gas streams or "spokes" have been proposed in models of Be star circumstellar material (see, e.g., Štefl et al. 1995) and in the time variability of Herbig Ae star spectra (Catala et al. 1991). Of course, the physics of circumstellar streams in Be and Herbig Ae stars will most likely be very different from that of corresponding structures around O stars, and we will focus mainly on the latter.

The principal goal of this paper is to model the *dynamical* effect of radiative driving on the formation of CIRs in a hot star wind, and to compute synthetic observational diagnostics to see if, e.g., slow DAC-like signatures can be theoretically produced. Our computational approach is to apply a reasonable parameterization for a localized "star spot" perturbation in the radiation force near the stellar surface, and allow the wind to respond consistently. Although the actual photospheric structure perturbing the base of the wind is likely to be different, we suspect the characteristic response of a radiatively-driven medium will be insensitive to the details of this physical mechanism, and mainly depend on base changes in the fluid velocity and density.

The remainder of this paper is organized as follows. We first (§ 2) describe our radiation hydrodynamics code and the details of the induced azimuthally-dependent force enhancement. Next (§ 3) we present results for a series of O star models with varying rotation rates and wind parameters, and discuss the emergent corotating structure. We then (§ 4) compute synthetic UV P Cygni line profile time series for the various models in our parameter study. Finally, a discussion and conclusion section (§ 5) summarizes our results and outlines directions for future work.

## 2. Numerical Radiation Hydrodynamics

### 2.1. Equations of Hydrodynamics

The problem of structure formation in a stellar wind is in general arbitrarily three-dimensional. Rotation imposes a latitudinal dependence on both the photosphere and the wind, and large-scale variations at the stellar surface can impose an arbitrary latitudinal or azimuthal dependence on the wind. However, to study how a line-driven wind reponds to rotationally induced variations within a more tractable, *two-dimensional* model, we confine the simulations here to the *equatorial plane,* where rotation has the strongest impact, and where the flow can be constrained to a surface of constant colatitude, $\theta = \pi/2$. (See Pizzo 1982 for discussion of similar approximations in modeling the solar wind.) This assumption naturally suppresses the centrifugal wind compression effect of Bjorkman & Cassinelli (1993), which is only of minor importance for O stars. The induced "spot" variations thus only have an azimuthal extent, and assumptions about latitudinal structure are only required when computing observational diagnostics (§ 4), not the inherent dynamics.

We use a time-dependent numerical hydrodynamics code to evolve a model of a radiatively-driven wind from a rotating star toward an equilibrium corotating steady state. The code, VH-1, was developed by J. M. Blondin and colleagues at the University of Virginia, and uses the piecewise parabolic method (PPM) algorithm developed by Collela & Woodward (1984). VH-1 solves the Lagrangian forms of the equations of hydrodynamics in the fluid rest frame, and remaps conserved quantities onto an Eulerian grid at each time step. The equations to be numerically integrated, written in Eulerian form using spherical polar coordinates, include the conservation of mass,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho v_r r^2\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(\rho v_\phi\right) \;=\; 0 \quad , \tag{1}$$

and the conservation of the $r$ (radial) and $\phi$ (azimuthal) components of momentum,

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} \;=\; \frac{v_\phi^2}{r} - \frac{1}{\rho}\frac{\partial P}{\partial r} + g_r^{\rm ext} \quad , \tag{2}$$

$$\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} \;=\; -\frac{v_r v_\phi}{r} - \frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi} \quad , \tag{3}$$

where $\rho$ is the mass density, $v_r$ and $v_\phi$ are the $r$ and $\phi$ components of the velocity, and $t$ is the time. The $\theta$ (latitudinal) component of the momentum conservation equation is assumed satisfied in the equatorial plane by the trivial solution $v_\theta(\theta = \pi/2) = 0$, i.e. no latitudinal flow into or out of the computational domain, with all partial derivatives in the $\theta$ direction considered negligible. The code also includes an equation for the conservation of energy, but in all models presented here this is dominated by rapid radiative processes, which keep the gas very nearly isothermal with a constant wind temperature $T$ equal to the stellar effective temperature $T_{\rm eff}$. We use a perfect gas law equation of state to evaluate the pressure $P$.

As in Owocki, Cranmer, & Blondin (1994), the external radial acceleration here includes gravity and radiative driving by line scattering, and

$$g_r^{\rm ext} \;=\; -\frac{GM_*(1-\Gamma)}{r^2} + g_r^{\rm lines} \quad . \tag{4}$$

Here $G$ and $M_*$ are the gravitational constant and stellar mass, and $\Gamma$ $(= \kappa_e L_*/4\pi G M_* c)$ is the Eddington factor that accounts for the reduction in effective gravity by outward radiation pressure on electrons. We evaluate the line-scattering acceleration $g_r^{\rm lines}$ in the local Sobolev (1960) approximation. This suppresses the wind's strong line-driven instability (Owocki & Rybicki 1984), as it is not currently feasible to incorporate this inherently nonlocal effect in multidimensional hydrodynamic models.

For one-dimensional winds, the Sobolev line force per unit mass can be parameterized as

$$g_r^{\rm lines} \,=\, kf \left( \frac{1}{\kappa_e v_{\rm th} \rho} \left| \frac{\partial v_r}{\partial r} \right| \right)^{\alpha} \frac{GM_* \Gamma}{r^2} \quad , \tag{5}$$

where the Castor, Abbott, & Klein (1975, hereafter CAK) parameters $\alpha$ and $k$ are related to the slope and normalization of the assumed power-law ensemble of lines. The constant $k$ is defined in terms of the electron scattering coefficient $\kappa_e$ and a fiducial ion thermal speed $v_{\rm th}$ (Abbott 1982). The stellar radiation field is modeled here by a spherical star with no limb darkening, and $f$ is the finite-disk correction factor used by Friend & Abbott (1986) and Pauldrach, Puls, & Kudritzki (1986). We ignore the effects of rotational oblateness and gravity darkening on the full *vector* radiative force (Cranmer & Owocki 1995), which we take to be purely radial. For simplicity, we also do not use Abbott's (1982) added ionization-balance parameterization of the line force, and set his exponent $\delta = 0$.

The Sobolev approximation assumes a monotonically accelerating velocity field, but this is not guaranteed in the time-dependent simulations presented below. Winds with nonmonotonic velocities have *nonlocal* line forces, since multiple resonance surfaces can create additional attenuation of the stellar flux (Rybicki & Hummer 1978; POF). We do not treat this nonlocal coupling directly, but we have compared CIR models using an upper limit (unattenuated) and a lower limit (strongly attenuated) for the line force in multiply-resonant regions of the wind; we find the dynamics to be quite similar in both limits. The upper limit, which we use in all models presented below, involves taking the absolute value of the radial velocity gradient $|\partial v_r / \partial r|$ in equation (5) and in the finite disk factor $f$. The lower limit assumes decelerating flows ($\partial v_r / \partial r < 0$) receive the same small force contribution from nonradial rays as a flow that is not accelerating at all ($\partial v_r / \partial r = 0$).

### 2.2. Local Radiative Force Enhancement

We induce azimuthal structure in our models by varying the Sobolev line force over a localized "star spot" in the lower wind. Since this force is directly proportional to the stellar flux ($L_* / 4\pi r^2$), increasing or decreasing $g_r^{\rm lines}$ over a small area is operationally equivalent to assuming a bright or dark region of the photosphere. Note, however, that by modulating the radiative force in this manner we do *not* mean to literally propose the existence of strong flux-varying spots on early-type stars. We merely use this simple method to perturb the wind in lieu of more definite knowledge about the physical cause(s) of surface inhomogeneities. Because the line driving grows weaker as one moves deeper into the subsonic wind and photosphere, the force enhancement is essentially confined to the transonic and supersonic wind, obviating the need to model a perturbed stellar atmosphere.

The induced variation in the line force is assumed to have a specified radial and azimuthal dependence which remains fixed to the stellar surface, and thus rotates through the computational domain with the star. The force enhancement is a function of radius $r$ and a corotating azimuthal angle $\psi$,

$$\psi \,=\, \phi - \Omega t \quad , \tag{6}$$

with $\Omega \equiv V_{\rm eq} / R_*$ the star's constant rotational angular velocity. The perturbed line force has the form

$$\begin{aligned} g_r^{\rm lines}(r,\psi) &= g_0(r) + \delta g(r,\psi) \\ &= g_0(r) \left\{ 1 + A\,\gamma(r) \exp\left[ -\left(\psi - \psi_0\right)^2 / \sigma^2 \right] \right\} \quad , \end{aligned} \tag{7}$$

where $g_0$ is the unperturbed Sobolev force (eq. [5]), $\psi_0$ is the azimuthal position of the center of the spot, and $A$ is its dimensionless amplitude. We will refer to "dark spots" as those with $-1 < A < 0$ and "bright spots" as those with $A > 0$. The azimuthal variation of the force is here assumed to be Gaussian about $\psi_0$, with full width at half maximum (FWHM) given by $\Phi \equiv 2\sigma\sqrt{\ln 2}$. The radial modulation $\gamma(r)$ is constrained by the geometrical extent of the spot. Close to the star, the spot is all that can be seen, and so $\gamma(r \to R_*) \to 1$; far from the star, the spot only represents a fraction of the observed stellar disk, and so $\gamma(r)$ approaches a small, but constant value as $r \to \infty$.

For a field point directly over ($\psi = \psi_0$) a *circular* flux enhancement with angular diameter $\Phi$, the radial function $\gamma(r)$ can be derived analytically from the the normalized residual flux,

$$\frac{\mathcal{F} - \mathcal{F}_0}{\mathcal{F}_0} \;=\; A\frac{r^2}{R_*^2} \int\!\!\int D(r, \mu', \phi')\, \mu' d\mu' d\phi' \;=\; A\gamma(r) \quad , \tag{8}$$

where $\mathcal{F}$ and $\mathcal{F}_0$ are the total and unperturbed fluxes. The amplitude $A$ takes into account the relative magnitude of the spot's "residual effective temperature," and is equivalent to $(T^4_{\rm spot} - T^4_0)/T^4_0$. The area integral is taken over a solid angle centered about the $z$-axis, with angles $\theta' = \cos^{-1}\mu'$ and $\phi'$ measured from the field point in the wind at radius $r$, and the residual limb darkening function $D$ set to zero for rays not intercepting the spot. Thus, for simple linear limb darkening,

$$\gamma(r) \;=\; \frac{2\pi r^2}{R_*^2} \int_{\mu_0(r)}^{1} \frac{1}{4\pi} \left( 2 + 3\sqrt{\frac{\mu'^2 - \mu_*^2}{1 - \mu_*^2}} \right) \mu' d\mu' \quad , \tag{9}$$

where $\mu_* \equiv [1 - (R_*^2/r^2)]^{1/2}$ defines the stellar limb, and

$$\mu_0(r) \;=\; \begin{cases} \mu_*, & r\cos(\Phi/2) \leq R_* \\ \sqrt{1 - R_*^2 \sin^2(\Phi/2)/S^2}, & r\cos(\Phi/2) > R_* \end{cases} \tag{10}$$

defines the visible edge of the spot. The distance $S$ from the field point to the edge of the spot is $[r^2 + R_*^2 - 2rR_*\cos(\Phi/2)]^{1/2}$. We thus adopt the radial modulation function given by the analytic integral of (9),

$$\gamma(r) \;=\; \frac{1}{2}\left[ 1 + \left( \frac{1 - \mu_0^2}{1 - \mu_*^2} \right) - \left( \frac{\mu_0^2 - \mu_*^2}{1 - \mu_*^2} \right)^{3/2} \right] \quad , \tag{11}$$

which approaches unity as $r \to R_*$ and approaches a constant value of $[1 + \sin^2(\Phi/2) - \cos^3(\Phi/2)]/2$ as $r \to \infty$. Although this residual flux integral is also able to provide oblique ($\psi \neq \psi_0$) and nonradial components of the flux enhancement of a star spot, we restrict our present models to the explicit *radial* perturbation given by equations (7) and (11). Again we emphasize that we do not mean to model in detail an actual star spot, but are only using the spot-like force enhancement as a convenient way to perturb the wind base.

Figure 1 shows contours of the force enhancement in the equatorial plane for a spot with $\Phi = 20°$, as well as wind "streaklines" for various stellar rotation speeds, as discussed further below (§ 3.3). Note that the spot significantly affects only a relatively small area of the wind: an azimuthal extent of $\sim 2\Phi$ and a radial extent of about a stellar radius. This allows several spots to be superposed on the stellar surface without any appreciable overlap in their force enhancements. Following the empirical arguments of Kaper & Henrichs (1994), who suggest a variable dipole magnetic field as the seed of large-scale wind structure, we place *two* spots separated by 180° on our model stars.

Before examining how a rotating wind responds to the localized force enhancement, it is instructive to see how a non-rotating wind is affected. Figure 2 shows the radial velocity and density (directly over the spot) at a reference radius of $10R_*$, well beyond the region of significant direct force enhancement, as a function of amplitude $A$. The general trend is for a "bright" spot to increase the local mass loss and thus increase the density of the wind near the star. Further out in the wind, where the effect of the spot drops off, the radiative force cannot accelerate the higher density material as strongly, so it approaches a *lower* terminal speed than in the unperturbed wind. Conversely, "dark" spots decrease the mass loss in the surrounding wind and thus allow the less dense material to be accelerated more strongly, leading to a much *higher* terminal speed. The three sets of data in Figure 2 correspond to: (1) one-dimensional, finite-disk, "modified CAK" (mCAK) solutions with a realistic critical point analysis and numerical integration, (2) two-dimensional hydrodynamical models using VH-1, and (3) a simple analytic fit to the data. The one-dimensional mCAK models contain the radial spot modulation $\gamma(r)$, but no information about neighboring streamline divergence or convergence. For spots with $A \gtrsim 0.6$, the one-dimensional models cease to have steady-state solutions that reach to infinity because too much mass is driven off the star to be accelerated beyond its gravitational escape velocity. The two-dimensional models can drive more mass to infinity because the density is reduced by a slight azimuthal expansion, which leads to a faster-than-radial divergence of flow tubes (see, e.g., MacGregor 1988).

The simple fit to the velocity and density variation in Figure 2 depends on only one free parameter, and makes use of the approximate dependence of the CAK mass loss rate on an arbitrary force multiplier (see Cranmer & Owocki 1995, eq. 24). Directly over the spot,

$$\dot{M} \approx \dot{M}^{(0)}(1+A)^{1/\alpha} \ , \tag{12}$$

where $\dot{M}^{(0)}$ is the unperturbed mass loss rate, and we assume that the CAK critical point $r_c$, where the mass flux is determined, is close enough to the star that $\gamma(r_c) \approx 1$. In a one-dimensional steady state, the mass conservation equation is integrated in the usual way to obtain $\dot{M} = 4\pi\rho v_r r^2$, and this provides a relation between the velocity and density at a given radius. We thus define fitting functions which obey this multiplicative constraint:

$$\rho \approx \rho^{(0)}(1+A)^{s/\alpha} \ , \tag{13}$$

$$v_r \approx v_r^{(0)}(1+A)^{(1-s)/\alpha} \ , \tag{14}$$

where we have found the best fit value of $s = 1.77$ for our dashed-line fits in Figure 2.

### 2.3. Numerical Specifications

Let us next describe some of the details of our numerical discretization, boundary conditions, and initial conditions. We specify flow variables on a fixed two-dimensional spatial mesh in radius $r$ and azimuthal angle $\phi$. In our standard models we use 200 radial zones, from $R_*$ to $30R_*$, with the zone spacing concentrated near the stellar base where the flow gradients and spot enhancements are strongest. The radial spacing starts at the lower boundary with $\Delta r = 0.002R_*$, then increases by 3% per zone out to a maximum of $\Delta r = 0.82R_*$ at the outer boundary. The azimuthal mesh contains 160 constantly-spaced zones, ranging from $0°$ to $180°$ with a spacing of $1\overset{\circ}{.}125$. Limited test runs with double the resolution in radius and azimuth showed some correspondingly greater detail in wind fine structure (e.g., shocks and radiative-acoustic waves), but overall the results were qualitatively similar to those for the standard resolution.

We specify the boundary conditions in our numerical method in two phantom zones beyond each edge of the grid. The azimuthal boundaries at $\phi = 0°, 180°$ are periodic, ensuring symmetry in the full equatorial plane. At the outer radial boundary, the wind is invariably supersonic outward, and so we set the flow variables in the outer phantom zones by a simple constant-gradient extrapolation. The lower radial boundary of the wind is somewhat more problematic, and we use the boundary conditions described by Owocki et al. (1994): constant-slope extrapolation for $v_r$, rigid rotation for $v_\phi$, and a fixed base density $\rho_B$. Because the mass loss rates of line-driven winds are determined from the equations of motion alone, we are able to specify an appropriate "photospheric" density that yields a stable, subsonic boundary outflow (see also Owocki, Castor, & Rybicki 1988).

The time-dependent hydrodynamical method requires a reasonable initial condition to be specified over the entire grid at time $t = 0$. For this we relax an analytically derived mCAK model to a steady state on a one-dimensional numerical grid, and then copy this onto the full two-dimensional mesh. This ensures that any time dependence results only from the induced force perturbations. The models are stepped forward in time at a fixed fraction (0.25) of the standard Courant-Friedrichs-Lewy time step. Because the radiative force enhancement, switched on at $t = 0$, only varies in time by corotating with the stellar surface, the wind responds by forming an outwardly moving "front," behind which the wind has settled to a rotating steady state. The dynamical flow time for gas to radially cross the computational grid is approximately $2 \times 10^5\,$s for our unperturbed initial state wind. Typically we find that models perturbed by star spots settle to a corotating steady state within two dynamical flow times, and so we plot all models at $t = 4 \times 10^5\,$s.

## 3. Numerical Results

Because the DAC phenomenon is primarily observed in O-star winds, we choose to center our study on a standard model of the O4f supergiant $\zeta$ Puppis. Specifically, we take $M_* = 60 M_\odot$, $R_* = 19 R_\odot$, $L_* = 8 \times 10^5 L_\odot$, and $T_{\rm eff} = 42,000\,$K (see, e.g., Howarth & Prinja 1989; Kudritzki et al. 1992). The measured rotational $V_{\rm eq} \sin i$ for $\zeta$ Puppis is $230\,{\rm km\,s^{-1}}$, which we take for the equatorial rotation velocity of our standard model. We neglect the small ($\sim$7%) oblateness induced by this degree of rotation, which corresponds to a Roche equipotential surface rotating at 63% of its critical angular velocity. We assume an isothermal wind of temperature $T_{\rm eff}$, corresponding to a sound speed $a = 24\,{\rm km\,s^{-1}}$, and use the line-driving constants $\alpha = 0.60$ and $k = 0.15$ (see eq. [5]). In one-dimensional mCAK models, these result in a terminal velocity $v_\infty = 2580\,{\rm km\,s^{-1}}$ and a mass flux $\dot{M} = 3.28 \times 10^{-6}\,M_\odot/{\rm yr}$. The base density for our subsonic lower boundary condition is $\rho_B = 6 \times 10^{-11}\,{\rm g\,cm^{-3}}$.

The localized "star spot" radiative force enhancement described above depends primarily on two quantities: the amplitude $A$ and the azimuthal full width $\Phi$. These, together with the equatorial rotation velocity $V_{\rm eq}$, are the three free parameters we vary in our study of non-axisymmetric structure formation. Table 1 outlines the input parameters and several output quantities (to be discussed below) for the models we computed. Models 1 and 2 are standard "bright" and "dark" spot models, with $A = +0.5$ and $-0.5$, and they represent a basis to explain the general hydrodynamical phenomenon of stream interaction. The subsequent models in Table 1 are intended to confirm our understanding of the physics of CIR formation, and are discussed below in a more limited fashion.

### 3.1. Standard Bright Spot: Model 1

Figure 3 shows gray-scale plots for the density, radial velocity, azimuthal velocity, and radial Sobolev optical depth in Model 1, *normalized to the unperturbed wind.* To ease comparison with other models, the azimuthal coordinates here have been incremented by a constant factor to align the peak of the spot ($\psi_0$)

with the center-line or $x$-axis of the diagram. Note the expected tendency for a bright spot to create higher density and lower radial velocity. However, the *azimuthal* velocity only differs by a small subsonic amount from the unperturbed angular-momentum-conserving form $v_\phi^{(0)}(r) = V_{\rm eq} R_*/r$. This demonstrates the almost purely *radial* effect of the spot enhancement.

Most of the corotating structure from the spot settles onto nearly constant spiral "streaklines" in the wind. In the present models, streaklines are equivalent to flow streamlines in the star's rotating frame of reference. Figure 4 compares streamlines and streaklines computed for Model 1. By numerically integrating the kinematic relation

$$\frac{r\,d\phi}{dr} = \frac{v_\phi(r,\phi) - r\Omega_F}{v_r(r,\phi)} \quad , \tag{15}$$

from a locus of points on the stellar surface spaced evenly in $\phi$, one can alternatively compute either streamlines in the inertial reference frame, with $\Omega_F = 0$, or streaklines in the rotating reference frame, with $\Omega_F = \Omega$. Areas with a higher (lower) concentration of streaklines correspond to regions of relative compression (rarefaction), though not all density variations are reflected in the streaklines. The dashed lines in Figure 4 show the streamline and streakline originating directly over the spot. The streamline appears nearly radial, but careful inspection shows that it has a modest (18°) prograde deflection, resulting from corotation of the relatively slow wind outflow near the surface. Through most of the wind, however, the streamlines are close to radial, and this allows one to qualitatively interpret the azimuthal coordinate as a *time* dimension. For any streamline at a fixed value of $\phi$, which is intercepted by different streaklines as the (corotating steady state) system sweeps by, the two-dimensional hydrodynamics becomes effectively one-dimensional, but now truly time dependent. This concept allows us to understand the "spread out" CIR structure in terms of a simpler model of radial wave or shock propagation. Hundhausen (1973) modeled solar-wind CIR formation and evolution in one dimension using this approximation.

We can disentangle the actual patterns of high density, low velocity, and more optically thick CIR structure by examining several causally-connected regions of this model:

**I. Direct Enhancement:** Close to the stellar surface, the Gaussian-shaped spot increases the mass flux and wind density over a limited ($r \lesssim 2R_*$) region near the star. This enhanced-density patch is slightly deformed by rotation from the contours shown in Figure 1, but is essentially equivalent. The density increases over the spot by a maximum factor of ~2.6, only slightly smaller than that predicted by the non-rotating analysis (see eq. [13]). This region also shows considerable azimuthal spreading in $v_\phi$ as the wind begins to adjust to the presence of the spot.

**II. Prograde Precursor:** Just ahead of the spot ($\psi > \psi_0$), a small fraction of the enhanced higher-density wind is able to "leak out" azimuthally and settle onto a set of relatively unperturbed streaklines. The density in this feature is only enhanced by a factor of ~1.2, indicating that it comes from material in the prograde tail of the Gaussian distribution. It becomes isolated from the direct spot enhancement at a relatively large distance from the star ($r \approx 6.4R_*$) after the CIR rarefaction (IV) has appeared between it and the CIR shock (III).

**III. CIR Compression:** The low radial velocity wind from the center of the spot curls around on more tightly-wound streaklines than the surrounding unperturbed wind, and these streams begin to interact at a finite radius from the star ($r \approx 1.6R_*$). Alternately, in the above one-dimensional interpretation, the slow stream can be considered equivalent to a radially-extended Gaussian "wave packet" which nonlinearly steepens as the fast mean wind begins to overtake it. The result of this collision of fast and slow streams is a corotating weak shock compression (the CIR) which, because it is driven by ram pressure from the mainly unperturbed wind, propagates out at very near the unperturbed wind velocity. Because the flow is isothermal, we do not see a separation into a distinct forward and reverse shock pair, as is observed in the more nearly adiabatic solar wind.

**IV. CIR Rarefaction:** Ahead of the nonlinear shock the streaklines fan out and form a lower-density rarefied region. The formation of this rarefaction is mandated by mass flux conservation, and the radial velocity correspondingly peaks slightly above its unperturbed value here. Because the density in this feature never dips too far below the unperturbed density ($\min[\rho/\rho^{(0)}] \approx 0.82$), the rarefaction propagates out at nearly the same velocity as the CIR compression.

**V. Radiative-Acoustic "Kink:"** In a purely hydrodynamical wind, the radial CIR shock structure is the sole result of the nonlinear steepening of the initial enhancement. Any nondissipative signals propagating in the rest frame of the wind (at characteristic speeds $\pm a$) are limited to the relatively undisturbed lateral (nonradial) direction. In a line-driven wind, however, Abbott (1980) and Rybicki et al. (1990) found that large spatial-scale linear perturbations propagate in the radial direction at modified "radiative-acoustic" characteristic speeds. Abbott (1980) derived

$$C_{\pm} \; = \; -\tfrac{1}{2}U \pm \sqrt{\left(\tfrac{1}{2}U\right)^2 + a^2} \tag{16}$$

for radial modes, where $U \equiv \partial(g_r^{\rm lines})/\partial(\partial v_r/\partial r)$ and $C_{\pm}$ reduces to the purely acoustic case if $U = 0$. In most of the wind, though, $U \gg a$, and the outward (positive root) solutions are subsonic, and the inward (negative root) solutions are supersonic. In Model 1 we see both an acoustic lateral mode (spreading in $v_\phi$ at large radii; Figure 3c) and a nonlinear analog of the inward radiative-acoustic mode, which propagates slowly outward in the star's frame ($0 < v_r + C_- < v_r$) as a weak discontinuity, or "kink," in the radial velocity gradient. Because of its slow propagation (more tightly-wound streaklines) this feature eventually collides with the CIR rarefaction from the other spot at a radius of $\sim$13.5 $R_*$ and ceases to exist.

We trace these five features in Figure 5, which is a close-up of the density gray-scale shown in Figure 3a. Unique tracks were found by searching for local extrema (in radius) of various quantities, and following contiguous patterns around in azimuth. The direct spot enhancement (I) appears at the stellar surface as a local maximum in the normalized density $\rho/\rho^{(0)}$, and collides with the CIR/kink pair of features (III and V) at a radius of $\sim 3.9 R_*$. These "bifurcated" extrema are found by tracking local minima and maxima in the radial velocity, as shown in Figure 6 below. They appear at a relatively small radius $r_L$ (see Table 1), where the spot perturbation is still linear. The remaining precursor/rarefaction pair of features (II and IV) correspond to other local minima and maxima in the normalized density, and they appear further out (at a larger radius $r_{NL}$) where the disturbance has definitely steepened into a nonlinear shock. It is interesting that the CIR compression and rarefaction do not form together at the same point, but this is understandable, since the latter can be considered an effect or response of the former.

To get an indication of which wind structures should yield the most prominent signatures in observed line profile variations, let us examine the radial Sobolev optical depth,

$$\tau_r(r,\phi) \; \equiv \; \frac{\kappa_L \, \rho(r,\phi) \, v_{\rm th}}{|\partial v_r(r,\phi)/\partial r|} \; . \tag{17}$$

The gray-scale plot in Figure 3d shows the changes in the optical depth relative to the mean, unperturbed wind. Since both the line absorption coefficient $\kappa_L$ and the ion thermal speed $v_{\rm th}$ are assumed to have the same constant values in both the mean and perturbed flow, all variations here stem from changes in the ratio of density to velocity gradient. Somewhat surprisingly, the regions of strongest optical depth enhancement occur not within the dense CIR compression (feature III), but rather within the relatively shallow velocity gradient region after the Abbott kink (feature V).

Figure 6 plots the radial variation of velocity and density from selected slices of constant azimuthal angle $\phi$. For this corotating steady state, the changing features in these line plots also indicate the time evolution of structure at fixed azimuths. This allows us to follow the outward propagation of both the CIR density enhancement (local minima in velocity) and the trailing radiative-acoustic kink (local maxima in velocity). In the high-density CIR, the wind is either strongly accelerating or decelerating, so both the numerator and the denominator in the Sobolev optical depth (eq. [17]) are enhanced, resulting in very little net increase. Just outward from the kink, however, the density is nearly unperturbed, while the velocity gradient is much shallower, implying a large increase in the Sobolev optical depth. In synthetic line profiles (see § 4.2 and Figure 11a), this near plateau produces a distinct absorption feature quite similar to slowly evolving DACs. As the (corotating) steady-state structure rotates in front of the observer's line of sight, material flowing *through* the kink appears at the velocities of the local maxima in Figure 6, but the evolution of the feature is governed by the radiative-acoustic mode propagation, which leads to an apparent acceleration that is much slower than the actual acceleration of the wind material.

Ahead of the CIR, between the compressive density maximum and the rarefied minimum, there is a region of high acceleration that arises from the prograde edge ($\psi > \psi_0$) of the Gaussian spot, which steepens into a monotonic sawtooth structure connecting the shock with the lower-density unperturbed wind. This region contains a lower net optical depth than the unperturbed wind, implying a relative *lack* of absorption in synthetic line profiles. This effect is only slightly weaker than the enhanced absorption due to the plateau, suggesting that isolated patches of extra absorption (DACs) may be difficult to model theoretically without a corresponding lower optical depth feature (with apparent relative "emission").

### 3.2. Standard Dark Spot: Model 2

A wind perturbed by a locally decreased radiative force ($A = -0.5$) produces a lower-density, high-speed stream, and thus settles to a steady state on the computational grid faster than a model with slow streams. Figure 7 shows gray-scale plots for the density, radial velocity, azimuthal velocity, and radial Sobolev optical depth for Model 2, normalized in the same way as in Figure 3. Although the dark spot produces an extremely rarefied wind, a high-density CIR forms (on the leading edge of the perturbation) where the high velocity stream collides with the slower unperturbed wind. The corotating structure present in Model 2 is qualitatively simpler than that in Model 1. The CIR/kink pair of features initially appears at $r_L \approx 2.2 R_*$, and advects smoothly throughout the wind. There is no analog to the second pair of features (starting further out at $r_{NL}$) in this model.

For slices of constant $\phi$, Figure 8 plots the radial dependence of the velocity and density. The contrast with the slow structure in Figure 6 is apparent. Note that the back-propagating radiative-acoustic kink is also present in this model, comprising the left edge of the flat-topped velocity peaks. However, since the kink here is a reaction to the forward-steepened shock, it forms within the fast and rarefied upstream region, and thus does not contribute strongly to the Sobolev optical depth of the corotating feature. The high density CIR also does not have an enhanced opacity because of the steep velocity gradients near the shock, as in Model 1. Indeed, Figure 7d shows that most of the highest optical depth material comes from the unperturbed wind, and that the high-speed CIR should be mainly a source of *decreased* absorption. The isolated "clumps" of highest optical depth in Figure 7d (and in Figure 3d) are artifacts of the finite differencing used to compute the radial velocity gradient, and do not significantly affect the volume-integrated quantities used in constructing line profile diagnostics.

The CIR structure in Model 2 appears very similar to that seen in hydrodynamic models of the solar wind, e.g., note the resemblance between Figure 8 and Figures 2 and 3 of Hundhausen (1973). The two major differences between solar wind high-speed streams and those in our model are: (1) the distinct forward and reverse adiabatic shocks in the former and (2) the back-propagating radiative-acoustic kink in the latter. Despite this similarity, all subsequent models in our parameter study use the bright spot of Model 1, which promises to simulate better the slow DACs in early-type stellar winds.

### 3.3. Variation of Spot Amplitude, Width, and Stellar Rotation Velocity

Let us now examine the effect of changing various spot and wind parameters. To provide a basis for understanding the full hydrodynamical calculations, we can estimate the expected effects in terms of a simple wind streakline picture (see Figure 4). Note that the shape of these streaklines can be well approximated by neglecting both the wind's acceleration and the angular-momentum-conserving azimuthal velocity, which nearly "cancel each other out" when computing streakline deflection. Thus, the angular deflection is given in this approximation by the Archimedean spiral relation,

$$\phi - \phi_0 \approx -\frac{\Omega}{v_\infty}(r - r_0) \quad . \tag{18}$$

Mullan (1984a) used this to estimate the interaction radius $r_i$ between a fast and slow stream initially set apart on the stellar surface by a given azimuthal separation $\Delta\phi$,

$$\frac{r_i}{R_*} = 1 + \frac{\Delta\phi}{V_{\rm eq}} \left( \frac{v_\infty^{(f)} \, v_\infty^{(s)}}{v_\infty^{(f)} - v_\infty^{(s)}} \right) \quad . \tag{19}$$

Here $v_\infty^{(f)}$ and $v_\infty^{(s)}$ are the terminal velocities of the fast and slow streams. Table 1 contains this simple prediction for $r_i$ for all the hydrodynamical models in our parameter study. We assume $\Delta\phi = \Phi$, and for bright-spot models, we take the fast stream to be the unperturbed wind, and the slow stream to have $v_\infty^{(s)}$ given by equation (14). Conversely, for the dark-spot model the fast stream is given by equation (14) and the slow stream is unperturbed wind.

Models 3A, 3B, and 3C vary the spot amplitude $A$, and thus the direct enhancement in density and velocity over the spot (see Table 1). The resulting structure looks qualitatively similar to that of Model 1, but the fast/slow stream interaction takes place at different radii. Figure 9a traces the corresponding CIR compression and radiative-acoustic kink features for these models. As expected, CIRs form further out when there is a smaller discrepancy between the fast and slow stream speeds, but the linear minima and maxima in velocity first appear (at $r_L$) much closer to the star than the simple interaction analysis above predicts. The "nonlinear" radius $r_{NL}$, where the CIR rarefaction branches off from the prograde dense precursor, is relatively *constant* with $A$, indicating a qualitatively different formation mechanism from the compression/kink features.

Figure 10 shows how the velocity law of the kink and CIR shock varies with spot amplitude, with each model rotated in azimuth to line up similar features. For the smallest values of $A$ (0.01, 0.1), no shock has yet formed, so both features appear "kink-like," propagating nearer to the slow radiative-acoustic mode speed than to the mean wind speed (see Figure 9a). The shock steepening is evident for larger values of $A$, and the most extreme model ($A = 2.50$) shows considerable slowing of the dense CIR resulting from the inverse dependence of the radiative force with density (eq. [5]). Our canonical Model 1 CIR, then, seems just on the verge between both "slowing" mechanisms, as well has having a shape between the low-$A$ weak discontinuities and the high-$A$ shocks. In all cases, however, the decelerating plateau-region ahead of the kink has the same characteristic (negative) acceleration, indicating that the enhanced optical depth at this radius may not vary strongly with spot amplitude.

Models 4A, 4B, and 4C vary the azimuthal spot width $\Phi$, and Figure 9b shows their CIR compression and kink features. Increasing or decreasing the full-width $\Phi$ simply alters the spatial scale of the interactions, and, in the limit of very small spots (where the star's sphericity can be neglected) the CIR structures seem self-similar with respect to an overall expansion factor. Table 1 indicates an inverse relationship between $\Phi$ and the maximum CIR density, and this can be understood heuristically by the fact that when a given spot amplitude is spread over a larger area, the collision of fast and slow streams is more diluted, and the resulting shocks are not as strong. Model 4A, with the smallest width $\Phi = 10°$, shows an apparent reversal in this trend, and we suspect that some of the detailed shock structure is under-sampled in our relatively coarse (in this case) azimuthal grid. Note that Mullan's (1984a) interaction radius $r_i$ agrees well with the location of the nonlinear rarefaction/precursor feature $r_{NL}$ for these models.

Observations (Prinja 1988; Henrichs et al. 1988) suggest that the recurrence and acceleration times of DACs tend to vary inversely with the projected equatorial rotation speed, $V_{\rm eq} \sin i$. To examine how well our dynamical models might reproduce these observed trends, Models 5A and 5B vary the equatorial stellar rotation velocity $V_{\rm eq}$. Figure 9c shows their CIR compression and kink features. The dominant effects are the overall variation of the spiral streaklines (see, e.g., eq. [18]), and the inverse centrifugal dependence of $v_\infty$ on the rotation rate (Friend & Abbott 1986). The strength of the CIR shock decreases for more slowly rotating stars, and we believe that this results from the variation of the streaklines with respect to the star-spot enhancement (Figure 1). Perturbed gas which rapidly advects out of the region of direct enhancement receives less of a "boost" of extra density, and thus does not form as strong a compression when interacting with the ambient wind.

## 4. Synthetic Observational Diagnostics

### 4.1. SEI Line Profile Construction

Ultraviolet P Cygni lines are sensitive probes of the wind structure of hot luminous stars, and it is important to model accurately observed variations in their profile shape. Here we use a multidimensional extension of the SEI (Sobolev with Exact Integration) method of Lamers, Cerruti-Sola, & Perinotto (1987) to compute synthetic line profiles. In this method, the source function is calculated locally in the wind using the Sobolev escape probability approximation, and the emergent flux profile is computed by numerically integrating the formal solution to the transfer equation. Bjorkman et al. (1994) discuss a two-dimensional extension of the basic SEI algorithm, and our computational approach is similar in that we do not yet treat the general case of nonlocal (or line doublet) resonance coupling. Our method, however, efficiently computes line profiles from an arbitrary three-dimensional distribution of density and velocity, for observers at arbitrary vantage points.

Following the notation of POF, we parameterize the opacity of a model pure-scattering resonance line by defining a dimensionless line-strength

$$k_L \equiv \left( \frac{\dot{M} v_{\rm th}}{4\pi R_* v_\infty^2} \right) \kappa_L \quad , \tag{20}$$

where $v_\infty$ and $\dot{M}$ are taken from the unperturbed model wind. We assume the mass absorption coefficient $\kappa_L$ is constant in radius, which is valid for lines of interest in the dominant ionization stage of the wind, and at least allows qualitative comparison for other lines. We consider two representative cases: a moderate unsaturated line ($k_L = 1$) and a strong saturated line ($k_L = 100$).

The local two-dimensional (solid angle) integrals required to obtain the escape probability and core-penetration probability for the Sobolev source function are computed using Romberg's successive refinement algorithm. The explicit form of these integrals is found in Lamers et al. (1987). POF compare the use of the local Sobolev formalism with a self-consistent multiple-resonance technique in structured wind models, and find significant disagreement in the resulting line profiles. However, our CIR model winds are much less structured than the one-dimensional instability models used by POF, with at most only two zones of nonmonotonic velocity variation in the entire wind. Further, here we concentrate on *residual* line profile variability, which should be less susceptible to consistency errors in the source function than the actual line profile shape.

We perform the "exact integration" for the line flux using a cylindrical $(p', \phi', z')$ coordinate system with the observer oriented along the positive $z'$-axis at an infinite distance from the origin. The equation of radiative transfer is evaluated in differential form along rays parallel to this axis, and along each ray the specific intensity is integrated using second order implicit Euler differencing. The resulting emergent intensities at the outer boundary of the computational grid are then integrated by nested Romberg quadrature in $p'$ and $\phi'$ to form the flux, and this process is repeated for each frequency point in the total line profile. We refer the reader to Lamers et al. (1987) and POF, who summarize these integrals, but only apply them in the spherically symmetric ($\phi'$ independent) case.

We make two major approximations in our line profile construction technique:

1. The three-dimensional wind structure is formed by interpolating in latitude between the two-dimensional equatorial plane models and the one-dimensional unperturbed polar wind. For simplicity we assume the same Gaussian structure of the "star spots" in latitude as in longitude, and apply it to the entire wind:

$$\begin{aligned} \rho(r,\theta,\phi) &= \rho^{(0)}(r)\,[1-E(\theta)] + \rho^{(2{\rm D})}(r,\phi)\,E(\theta) \\ v_r(r,\theta,\phi) &= v_r^{(0)}(r)\,[1-E(\theta)] + v_r^{(2{\rm D})}(r,\phi)\,E(\theta) \\ v_\phi(r,\theta,\phi) &= v_\phi^{(0)}(r)\,[1-E(\theta)]\,\sin\theta + v_r^{(2{\rm D})}(r,\phi)\,E(\theta)\,\sin\theta \quad , \end{aligned} \tag{21}$$

   where $E(\theta) \equiv \exp[-(\pi/2-\theta)^2/\sigma^2]$ (see eq. [7]). As in the two-dimensional models, we retain our assumption that $v_\theta = 0$. Note that the azimuthal velocity has an extra factor of $\sin\theta$ included to preserve angular momentum conservation out of the equatorial plane, and this provides a latitudinal wind variation even for *unperturbed* models.

2. The total Doppler width of the Gaussian line profile contains both thermal and microturbulent contributions,

$$v_D \equiv \sqrt{v_{\rm th}^2 + v_{\rm turb}^2} \quad , \tag{22}$$

with the thermal speed set by $v_{\rm th} = 0.28a$, as appropriate for CNO driving ions. For simplicity, we assume a constant microturbulent velocity $v_{\rm turb} = 100\,{\rm km\,s^{-1}}$, though better line-profile fits have been obtained by assuming this varies in proportion to the mean wind velocity (Haser et al. 1995). Though phenomenological, this use of a microturbulent velocity allows realistic line-profile synthesis with a minimum of free parameters. It also compensates for the suppression here of the small-scale instability, which one-dimensional, nonlocal simulations have shown to result in many of the same observational signatures as microturbulence (POF; Owocki 1994).

### 4.2. Time Variability in Dynamical Models

We produce time-variable P Cygni line spectra by positioning an "observer" in the equatorial plane at successive azimuthal angles $\phi$ with respect to the two-dimensional models. For the standard rotation velocity used in Models 1, 2, 3, and 4, the line profile variability repeats with a period

$$\Pi = \frac{\pi R_*}{V_{\rm eq}} = 2.091\,{\rm days}\ . \tag{23}$$

Of course, since spiral streakline structures in the models often subtend more then 180° of azimuth, continuous DAC-like signatures can exist for times longer than this period. We arbitrarily define $t = 0$ when the observer is positioned directly over the center of one of the photospheric spot perturbations.

As emphasized by Lamers (1994), we find that most of the line profile variability occurs in the absorption column of the wind ($0 \le p' \le R_*$), with comparatively little variation in the larger emission volume ($p' > R_*$). Because the flux integration over this emission volume dominates the CPU time in our SEI line-profile synthesis, we only computed full, variable emission-volume profiles for a few selected test cases, namely for observers at four equally-spaced azimuthal angles ($\phi = 0°, 45°, 90°, 135°$) for Model 1, as well as at one arbitrary azimuthal angle for an unperturbed $\phi$-independent wind model. For an unsaturated ($k_L = 1$) line, we found that the perturbations in the absorption-column flux reach as high as 47% of the continuum level, whereas those in emission-volume flux never exceed 1.9%. Since these latter variations would only be marginally observable with IUE signal-to-noise ratios of 20 to 40, we neglect the perturbed emission volume when computing subsequent line profiles on a finer time- and velocity-sampled grid.

Figures 11a and 11b show the absorption-column line profile variability for Model 1, repeated over three data periods (1.5 rotation periods) to emphasize the rotationally-modulated structure. Following the standard observational convention, the gray-scale is normalized by a "minimum absorption" (maximum flux) template, constructed independently at each line velocity. This choice contains the implicit bias that the variability takes the form of extra *absorption,* which is only partially appropriate for Model 1. The unsaturated ($k_L = 1$) line exhibits definite DACs that apparently accelerate through the profile on ~3.9-day time scales, even though their recurrence time is shorter. The saturated ($k_L = 100$) line exhibits blue-edge variability on the same time scale. Figures 11a and 11b also contain the time-averaged line profiles for Model 1 and the minimum and maximum absorption templates, which show the extent of the absorption variability at each velocity. We also plot the standard deviation of the data, allowing a qualitative comparison to observed temporal variance spectra.

Figure 12 shows the absorption-column line profile variability for Model 2. The accelerating features in this unsaturated ($k_L = 1$) line differ from those in Figure 11a in two apparent ways. First, because of the high-velocity stream induced by the dark spot, the variability extends out to nearly 5000 km s$^{-1}$, almost twice the unperturbed wind terminal speed. Second, the enhanced absorption at lower line velocities

($v \lesssim 2000\,\mathrm{km\,s^{-1}}$) represents the mean state, and the isolated accelerating features appear as a lack of absorption. Although we anticipated this trend in § 3.2, it is surprising to note the overall *similarity* between the bright and dark spot profiles at higher line velocities ($2000\,\mathrm{km\,s^{-1}} \lesssim v \lesssim 2400\,\mathrm{km\,s^{-1}}$). The primary difference here is that the strongest absorption feature trails, in Model 1, and leads, in Model 2, the overall region of enhanced absorption. Far from the star, the near-terminal-speed wind is beginning to laterally homogenize the bright or dark spot perturbations into a simpler pattern of alternating large-scale compressions and rarefactions.

In order to interpret more clearly the evolution of DACs in our models, we utilize the observational fitting technique of Henrichs et al. (1983) and Kaper (1993), which represents the DACs as dense, plane-parallel slabs of gas in the observer's line of sight. For each component, the dependence of the quotient flux (normalized by the minimum absorption template) on the line-of-sight velocity $v$ is fit by

$$I(v) \,=\, \exp\left\{ -\tau_c \exp\left[ -\left( \frac{v - v_c}{v_t} \right)^2 \right] \right\} \quad , \tag{24}$$

where $\tau_c$ is a representative central optical depth, $v_c$ is the line velocity of the center of the DAC, and $v_t$ is related to its width in velocity space. These three parameters are varied and fit to each feature in the time series using Marquardt's $\chi^2$ method (Bevington 1969). Although many of our synthetic DACs are asymmetric about $v_c$, with slightly more absorption on the low-velocity side of the feature, the fits always reproduce well the overall line shape. One additional useful quantity, the column density of the DAC, is given for our model lines by

$$N_{col} \,=\, \frac{\sqrt{\pi}}{\kappa_L v_{\rm th} \langle m \rangle} \frac{\tau_c v_t}{(1 + v_c/c)} \quad , \tag{25}$$

where $\langle m \rangle$ is the mean mass of gas atoms and ions (see POF and Kaper 1993).

In Figure 13 we plot the resulting fit parameters ($v_c$, $v_t$, $N_{col}$) as a function of time for the bright-spot, unsaturated-line models in the parameter study. The inherent "overlap" in the time series (i.e., multiple DACs at a given time) has been removed to more clearly show the evolution of the individual DAC. As is often seen in observed line profile variability, the feature accelerates through the line profile while growing progressively narrower, its column density increasing to a maximum value, then decreasing as the DAC nears its terminal velocity. In our models, $v_c$ often reaches or exceeds the wind's unperturbed $v_\infty$, but the rapidly dropping values of $N_{col}$ (which also is related to the equivalent width of the DAC) might preclude actual observation of this final period of evolution. In fact, most of the DACs we track seem to approach an initial "pseudo terminal speed" ($\sim$ 0.8-0.9 $v_\infty$) while the column density is at its peak, then accelerate further to the wind's terminal speed as the column density decreases.

As expected, the DACs produced by radiative-acoustic kinks accelerate quite slowly. Figure 13c compares the acceleration of DACs from Models 1, 5A, and 5B with several analytic "beta" velocity laws. We compute $v(t)$ by numerically integrating the kinematic relation

$$t \,=\, t(r[v]) \,=\, \int_{R_*}^{r} \frac{dr'}{v_0 + (v_\infty - v_0)(1 - R_*/r')^\beta} \quad , \tag{26}$$

where we take $v_0 = a$ and $v_\infty = 2580\,\mathrm{km\,s^{-1}}$. The slow acceleration of the DACs is equivalent to $\beta \approx 2 - 4$, which agrees with the observations of Prinja et al. (1992) and Prinja (1994). Note, however, that an estimation of a single characteristic $\beta$ for a DAC is problematic, since (1) its terminal speed is not clearly defined, and (2) $v_c(t)$ experiences several minor acceleration and deceleration episodes superimposed on the overall DAC acceleration. Figure 14 plots acceleration versus velocity for the DAC of Model 1, and compares it to the beta laws defined above. Note the similarity in both magnitude and nonmonotonic behavior between this theoretical acceleration and that found by Prinja et al. (1992) for DACs in the wind of $\zeta$ Puppis.

## 5. Summary, Conclusions, and Future Work

We have carried out two-dimensional hydrodynamical simulations of an azimuthally inhomogeneous radiation-driven wind from a rotating O star. The wind responds to a photospheric radiative force enhancement by forming large-scale corotating structures extending far beyond the region of direct perturbation. Although classical CIR compressions and rarefactions are often seen, the most important structures observationally are slowly-propagating radiative-acoustic kinks or plateaus which have a large Sobolev optical depth. These plateaus show up as strong DACs when formed behind streams resulting from *enhanced mass loss,* and they accelerate at a slower rate than the wind passes through them. Preliminary P Cygni line profile synthesis has shown several important trends in the emergent DACs from these structures:

1. The slow acceleration of DAC features is fit reasonably well by $\beta \approx 2-4$, and we find no significant correlation between their acceleration time scale and the star's rotation velocity. Of course, since the CIRs in our models are linked to the rotating surface, there is a definite correlation between the *recurrence* time scale of DACs and $V_{\rm eq}$.

2. Despite minor variations, the DAC parameters $v_c$ and $v_t$ do not depend strongly on the amplitude $A$ or full width $\Phi$ of the model spot perturbation. The primary exception is the high-amplitude Model 3C ($A = 2.50$) which accelerates much more slowly due to the higher density in the CIR shock.

3. The optical depth $\tau_c$ and column density $N_{col}$ seem to be sensitive probes of the amplitude of the initial surface perturbation (see Figure 13a, bottom panel). The concavity of $N_{col}$ during the strongest period of DAC evolution may be able to provide information about the azimuthal size of the perturbation, but this is not as clearly observable an effect (see Figure 13b, bottom panel).

Let us now compare our model results with actual observations of large-scale wind structure. In many cases both (1) slow and quasi-episodic DACs and (2) faster periodic modulations are seen simultaneously in OB-star winds (Kaper 1993; Massa et al. 1995). As seen above, the first type of variation can be readily reproduced by the CIR and radiative-acoustic plateau that results from an azimuthally localized mass-loss enhancement. Though our models assume strict rotational periodicity, such DACs could also be readily caused by *transient* CIRs for which the surface mass loss "eruption" lasts long enough to make structure that covers a substantial portion of the stellar disk. The second type of structure observed in OB winds (fast, near-sinusoidal flux variations accelerating *with* the wind) is more difficult to produce with a CIR model, because the required optical depth or density variations disrupt the mean wind velocity streaklines. To explain these faster modulations, we intend to investigate models in which wind structure is induced by nonradial pulsation (NRP) of the underlying star (Owocki, Cranmer, & Fullerton 1995).

The success here in reproducing realistic DACs suggests that the CIR model warrants further study and development. One important extension will be to use a more complete radiative force that incorporates the line-driven instability. The resulting stochastic variations may disrupt the large-scale CIR structure, but slowly evolving kink-like plateaus have been seen to survive and propagate in various one-dimensional instability simulations (Fullerton & Owocki 1992; Owocki, Fullerton, and Puls 1994). In addition, extending the present models to three dimensions may shed light on the variability of wind-compressed disks and other latitudinally-varying structures in, e.g., Be-star winds. Finally, it will be important to develop techniques to synthesize other observational diagnostics, such as subordinate-level UV and optical lines, infrared photometry, and continuum and line polarization, which will allow further constraints on models of wind variability. The resulting phenomenological "atlas" would provide a solid basis for interpreting the great diversity of OB-star wind variability observations in terms of fundamental, dynamical models of time-dependent wind structure.

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Fig. 1.— Contours of the star-spot force enhancement for a spot with full width at half maximum $\Phi = 20°$. The contour levels shown range from $0.1A$ to $0.9A$ in intervals of $0.1A$. Overplotted are streaklines (obtained by integrating eq. [15]) of unperturbed wind models from stars rotating at 0, 130, 230, and 330 km $s^{-1}$.

Fig. 2.— Variation with spot amplitude (for $\Phi = 20°$) of (a) radial velocity and (b) density at $r = 10R_*$, for non-rotating winds. Solid lines show one-dimensional mCAK solutions, filled circles show two-dimensional hydrodynamical solutions, and dashed lines show a simple one-parameter fit to both sets of data.

Fig. 3.— CIR structure for Model 1, settled to a steady state. Shown are the (a) density, (b) radial velocity, (c) azimuthal velocity, and (d) radial Sobolev optical depth, all normalized to the unperturbed wind initial condition.

Fig. 4.— Streamlines (left) and streaklines (right) for Model 1, integrated from 72 equally-spaced points on the star, separated by 5° intervals. Dashed lines indicate initial points situated at the center of the spot enhancement.

Fig. 5.— Normalized density gray-scale for Model 1, as in Figure 3a. Overplotted are dashed lines which trace the: (I) direct spot enhancement, (II) prograde precursor, (III) CIR compression, (IV) CIR rarefaction, and (V) radiative-acoustic Abbott kink.

Fig. 6.— Line plots for Model 1 of the radial variation of (a) radial velocity and (b) density in the equatorial plane at 16 equally-spaced azimuthal angles, 11.°25 apart.

Fig. 7.— As in Figure 3, except for Model 2.

Fig. 8.— As in Figure 6, except for Model 2.

Fig. 9a.— Velocity minima and maxima corresponding to CIR compression and radiative-acoustic kink features, shown for Model 1 and variable amplitude Models 3A, 3B, 3C. The heavy dashed line is the streakline for the unperturbed one-dimensional standard model wind.

Fig. 9b.— As in Figure 9a, except for Model 1 and variable spot-width Models 4A, 4B, 4C.

Fig. 9c.— As in Figure 9a, except for Model 1 and variable rotation Models 5A, 5B.

Fig. 10.— Radial velocity showing the the radiative-acoustic kink and CIR shock for models with varying spot amplitude. Each $\phi$ slice was chosen independently to align the initial kink deceleration at the same radius.

Fig. 11a.— SEI absorption-column line variability for Model 1, computed for an unsaturated line, $k_L = 1$. The gray-scale values range from white (maximum flux) to black (minimum flux), measured relative to the dashed-line templates in the middle panel. The solid lines show the time-averaged line profile, in the middle panel, and the standard deviation, or square root of the variance, in the bottom panel.

Fig. 11b.— As in Figure 11a, except for a saturated line, $k_L = 100$.

Fig. 12.— As in Figure 11a, except for Model 2.

Fig. 13a.— Best-fit DAC features $v_c$ (central line velocity), $v_t$ (characteristic width), and $N_{col}$ (slab-model column depth), shown for Model 1 and variable amplitude Models 3A, 3B, 3C. The line styles for each set of models are identical to those in Figure 9.

Fig. 13b.— As in Figure 13a, except for Model 1 and variable spot-width Models 4A, 4B, 4C.

| Model | $A$ | $\Phi$ | $V_{\rm eq}$ (km s$^{-1}$) | max $(\rho/\rho^{(0)})$ | $\Delta v_r$ [a](km s$^{-1}$) | $r_L/R_*$ [b] | $r_{NL}/R_*$ [c] | $r_i/R_*$ [d] |
|---|---|---|---|---|---|---|---|---|
| 1 | +0.50 | 20° | 230 | 3.747 | -629.2 | 1.607 | 6.417 | 6.736 |
| 2 | -0.50 | 20° | 230 | 5.071 | +2272. | 2.188 | — | 7.646 |
| 3A | +0.01 | 20° | 230 | 1.023 | -29.30 | 8.169 | 6.005 | 306.0 |
| 3B | +0.10 | 20° | 230 | 1.243 | -193.0 | 2.782 | 6.157 | 31.10 |
| 3C | +2.50 | 20° | 230 | 39.95 | -1285. | 1.210 | 6.157 | 1.981 |
| 4A | +0.50 | 10° | 230 | 2.840 | -598.3 | 1.163 | 3.736 | 3.868 |
| 4B | +0.50 | 40° | 230 | 2.206 | -493.3 | 2.526 | 10.38 | 12.47 |
| 4C | +0.50 | 80° | 230 | 1.977 | -238.8 | 4.383 | >30 | 23.94 |
| 5A | +0.50 | 20° | 130 | 3.969 | -783.4 | 1.892 | 9.324 | 11.90 |
| 5B | +0.50 | 20° | 330 | 2.485 | -482.9 | 1.492 | 4.934 | 4.556 |

Table 1: Summary of Model Parameter Study

[a]We define $\Delta v_r$ as the maximum absolute perturbation $(v_r - v_r^{(0)})$, either positive or negative.

[b]The CIR compression (feature III) and the Abbott kink (feature V) branch out together at the linear bifurcation radius $r_L$.

[c]The prograde dense precursor (feature II) and the CIR rarefaction (feature IV) branch out together at the nonlinear bifurcation radius $r_{NL}$.

[d]See equation (19).

Fig. 13c.— As in Figure 13a, except for Model 1 and variable rotation Models 5A, 5B.

Fig. 14.— DAC acceleration versus velocity for Model 1 (dashed line) and analytic "beta" velocity laws (solid lines).