Exact Results And Soft Breaking Masses In Supersymmetric Gauge Theory

Nick Evans,[*] Stephen D.H. Hsu,[†]
Myckola Schwetz,[‡] and Stephen B. Selipsky[§]

Department of Physics
Yale University
New Haven, CT 06520-8120

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Abstract

We give an explicit formalism connecting softly broken supersymmetric gauge theories (with QCD as one limit) to $N = 2$ and $N = 1$ supersymmetric theories possessing exact solutions, using spurion fields to embed these models in an enlarged $N = 1$ model. The functional forms of effective Lagrangian terms resulting from soft supersymmetry breaking are constrained by the symmetries of the enlarged model, although not well enough to fully determine the vacuum structure of generic softly broken models. Nevertheless by perturbing the exact $N = 1$ model results with sufficiently small soft breaking masses, we show that there exist nonsupersymmetric models that exhibit monopole condensation and confinement in the same modes as the $N = 1$ case.

1 Introduction

Remarkable, exact results on the vacuum structure of four dimensional $N = 2$ supersymmetric quantum field theories with an $SU(2)$ gauge symmetry and matter fields in the fundamental representation have recently been obtained [1], with generalizations to $SU(N_c)$

[*]nick@zen.physics.yale.edu
[†]hsu@hsunext.physics.yale.edu
[‡]ms@genesis2.physics.yale.edu
[§]stephen@genesis1.physics.yale.edu

gauge groups [2]. In these models, an $N = 1$ supersymmetry preserving mass perturbation for the adjoint superfield leads to monopole condensation, confinement and chiral symmetry breaking. There has been much interest in how such exact supersymmetric results generalize to nonsupersymmetric gauge theories [3].

In this paper we show how $N = 2$, $N = 1$ and softly broken supersymmetric models can all be embedded in a single enlarged $N = 1$ model in which coupling constants are treated as "spurion" fields. The effective superpotential of the enlarged model can be exactly determined from its symmetries and the $N = 2$ limit. In the $N = 2$ limit the D-terms are also exactly determined, but when supersymmetry is broken the potential and superpotential each receive contributions from additional D-terms of the enlarged model, which vanish in the $N = 2$ limit. Although constrained by the symmetries of the model, these cannot be fully determined. If the $N = 2$ symmetry is broken solely by soft breaking masses, these unknown functions typically are necessary to determine details of the vacuum structure. The exact superpotential does however include all the lowest-dimension gauge interaction terms, and continues to have singularity structure[1] consistent with points at which monopoles become massless. We can thus confirm that when the soft breaking masses are perturbations to an $N = 1$ preserving mass, the vacuum remains essentially that of the $N = 1$ model.

An important step in obtaining the exact results of [1] is to treat the couplings in the $N = 2$ models as "spurion" chiral superfields in an enlarged $N = 1$ model, which fail to propagate if their kinetic term coefficients are taken to infinity [4]. Their scalar component vacuum expectation values (vev's) can thus be frozen at any chosen values. The F-terms of the Wilsonian effective Lagrangian of a supersymmetric model [4, 5, 6] must be holomorphic in the fields as well as invariant under the gauge and global symmetries of the model (assuming a supersymmetric regularization scheme). The superpotential of the enlarged model is therefore holomorphic in the couplings. A recent paper [7] showed that soft supersymmetry

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[1]Strictly speaking there is no longer a moduli space after breaking $N = 2$ supersymmetry. However, we can still discuss monodromies in the configuration space of the theory, where an external source is used to traverse a closed loop.

breaking interactions and mass terms can be introduced in supersymmetric models without altering the holomorphic constraint on the F-terms. The spurion fields can be coupled to a sector that generates supersymmetry breaking expectation values for the spurion field f-components[2]. Freezing out the spurions generates soft supersymmetry breaking masses and interactions in the embedded model.

For large vevs of the adjoint scalar field a_Φ in the models we study, the $SU(N_c)$ gauge symmetry is broken to a subgroup of weakly interacting $U(1)$ gauge symmetries. It has been argued [1] that the $N = 2$ theory remains in the Coulomb phase even when the scalar vev is small and the model therefore strongly coupled. One might have expected that the $SU(N_c)$ dynamics would be unimpeded when the scalar vev was much smaller than the strong interaction scale Λ, and that the low energy theory would exist in a confining phase with all the gauge bosons confined within glueballs. However, the solutions' self-consistency [1] suggests that for the special choice of couplings that gives the $N = 2$ model, the quantum effects that cause condensation and confinement exhibit special cancellations, leading to the surprising low energy dynamics of a strongly coupled Coulomb phase with massless matter in the form of fermions, monopoles and dyons.

The solutions' consistency with our expectations for how these theories ought to behave helps confirm that higher dimension terms do not qualitatively change the solutions. Following the analysis [1], which does not determine operators with more than two derivatives ($\mathcal{O}(p^2)$) or four fermions, we will assume that the gauge particle dynamics are essentially controlled by the effective operators of lowest dimension, at sufficiently low energy scales. The model's symmetries permit higher dimension operators containing for example powers of $|WW/\Lambda^3|^2$, and (by $N = 2$ supersymmetry) also containing superderivatives and powers of Φ/Λ. Since the scalar component of Φ is pinned at $\langle a_\Phi \rangle \sim \Lambda$ when the supersymmetry is broken to $N = 1$, these terms are not at first sight negligible. However, since they must

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[2]Our superfield notation [8] is $\Phi = a_\Phi + \sqrt{2}\,\theta\psi_\Phi + \theta\theta f_\Phi$, with $\Phi^\dagger\Phi\big|_D \equiv \int d^2\theta\, d^2\bar{\theta}\, \Phi^\dagger\Phi = |f_\Phi|^2$, $\Phi\big|_F \equiv \int d^2\theta\, \Phi = f_\Phi$, $\Phi\big|_A \equiv a_\Phi$, etc.

be related by $N = 2$ supersymmetry to the corresponding higher derivative W terms, they should also be suppressed in the infrared regime. Thus despite these caveats, a tractable electric–magnetic duality is useful in determining the theory's vacuum structure.

We expect that introducing $N = 2$ breaking masses destroys cancellations in the dynamics below the scale of the breaking mass, and confines the $U(1)$ gauge interaction below that scale (via the Higgs mechanism in the dual theory). Provided that the breaking mass is sufficiently small relative to the scale Λ of the strong interactions, the theory will continue to exist as a $U(1)$ gauge theory between the $SU(2)$ breaking scale parameterized by $\langle a_\Phi \rangle$, which is pinned at $\sim \Lambda$, and the breaking mass scale. Softly broken $N = 1$ and $N = 0$ models, when close in parameter space to the $N = 2$ model, are therefore anticipated to occupy the same phase.

The paper is organized as follows. Section 2 explicitly describes the class of models we study. We allow the models' coupling constants to be chiral superfield spurions, whose values are eventually frozen in all of spacetime [4, 7]. We then review the analysis that leads to the Seiberg–Witten ansatz for the vacuum structure for $SU(2)$ gauge symmetry, and monopole condensation in $N = 1$ models close in parameter space to the $N = 2$ model. Allowing soft supersymmetry breaking terms in our Lagrangian via nonvanishing spurion field f-components, we show that to lowest order in the soft breakings, gauge kinetic terms are unchanged by the introduction of soft masses. However, the potential minimum in these models typically depends upon insufficiently constrained contributions from D-terms. Section 3 discusses how similar results can be obtained in $SU(N_c)$ gauge theories and with matter fields in the fundamental representation, and how QCD may be reached via a continuous interpolation from models of this type.

2 Supersymmetric $SU(2)$

2.1 An enlarged model

Consider an $N = 1$ supersymmetric $SU(2)$ gauge theory, containing one matter chiral superfield in the adjoint representation. Promoting the coupling constants into chiral superfields (including a D-term normalization, K, which we shall use to generate a squark mass) [7] yields the Lagrangian

$$\mathcal{L} = \frac{1}{8\pi} Im \left(\frac{1}{2}\tau_0 W_\alpha W^\alpha \Big|_F + (\tau_0 + K^\dagger K)\Phi^\dagger e^V \Phi \Big|_D \right) + m\Phi^2 \Big|_F + \text{h.c.}$$

$$+ \Lambda_m^2 \left(m^\dagger m \Big|_D + \beta_m m \Big|_F + \text{h.c.} \right) + \Lambda_\tau^2 \left(\tau_0^\dagger \tau_0 \Big|_D + \beta_\tau \tau_0 \Big|_F + \text{h.c.} \right) \tag{1}$$

$$+ \Lambda_K^2 \left(K^\dagger K \Big|_D + \beta_K K \Big|_F + \text{h.c.} \right)$$

(note that Λ_m and K are dimensionless). The spurion fields $i = \{m, \tau_0, K\}$ do not propagate in the limit $\Lambda_i \to \infty$; we can fix their scalar components a_i at any expectation values we choose, and replace $f_i = -\beta_i^\dagger + \mathcal{O}(1/\Lambda_i^2)$. An equivalent viewpoint starts with conventional normalization of the spurion D-terms, placing the Λ_i suppression on the spurion-to-physical-field coupling operators, for example writing $m^\dagger m \Big|_D + (m/\Lambda_m)\Phi^2 \Big|_F$. The spurion vevs are then taken proportional to Λ_i and their classical and quantum evolutions are still relatively suppressed.

In any case, if we choose $\langle a_{\tau_0} \rangle \neq 0$ and $(\langle a_m \rangle, \langle a_K \rangle, \langle f_i \rangle) = 0$, the model reduces to $N = 2$ supersymmetric $SU(2)$, studied in [1], whose bare Lagrangian is

$$\mathcal{L}_{N=2} = \frac{1}{4\pi} Im\langle a_{\tau_0} \rangle \left(\frac{1}{2}W^\alpha W_\alpha \Big|_F + \Phi^\dagger e^V \Phi \Big|_D \right) . \tag{2}$$

In this notation, we will review the derivation [1] of the $N = 2$ model effective potential and its generalization to the $N = 1$ case of $\langle a_m \rangle \neq 0$. Nonzero values of $\langle f_{\tau_0} \rangle$ and $\langle f_K \rangle$ generate supersymmetry breaking mass terms for gauginos and squarks respectively. In the limit $\langle f_{\tau_0,K} \rangle \to \infty$ the gauginos and squarks decouple from the low energy theory, leaving only $SU(2)$ gauge bosons and adjoint fermions.

The model has an anomaly-free $U(1)_R$ symmetry, whose charge assignments

$$
\begin{array}{cc}
\theta & +1 \\
W_\alpha & +1 \\
\tau_0 & 0 \\
\Phi & 0 \\
m & +2 \\
K & \text{arbitrary}
\end{array}
\tag{3}
$$

significantly constrain the terms which can appear in the low energy effective Lagrangian. Allowable D-terms have net R-charge of zero, whereas F-terms must have net R-charge of $+2$. The anomaly breaks a second $SU(2)_R$ symmetry of the embedded $N = 2$ model to a discrete symmetry; in the $SU(2)$ gauge case this gives the $\mathbf{Z_2}$ symmetry.

A further constraint applies to the spurion sources β_i, which may be treated as decoupled chiral superfields in their own right. Coupled only linearly to the f-components of m, τ and K, they do not contribute perturbatively to 1PI diagrams. Moreover, since scalar components a_{β_i} do not couple at all to other fields in the bare potential, they will only appear suppressed by powers of Λ_i in induced terms in the effective Lagrangian. The only dependence in the effective theory on β_i vevs thus occurs indirectly, through the f-component vevs f_i of coupling-field spurions, induced by a_{β_i}.

2.2 The low energy effective theory in the $N = 2$ limit

To fix our notation we briefly review the analysis [1] relevant to the $N = 2$ model in (2). The form of the $\mathcal{O}(p^2)$ effective Lagrangian's superpotential can be deduced from its holomorphic properties as described below. In the $N = 2$ limit the effective Lagrangian's D-terms are then determined by $N = 2$ supersymmetry, giving the effective Lagrangian as

$$
\frac{1}{4\pi} Im \left\{ \left. \frac{\partial \mathcal{F}(A)}{\partial A} \bar{A} \right|_D + \left. \frac{1}{2} \frac{\partial^2 \mathcal{F}(A)}{\partial A^2} W^\alpha W^\alpha \right|_F \right\} ,
\tag{4}
$$

where A is the $N = 1$ chiral multiplet and $\mathcal{F}$ the "prepotential" [1, 9]. The exact solution for the effective Lagrangian of the $N = 2$ model determines the effective Lagrangian of the enlarged model, and hence of the models with soft supersymmetry breaking masses, up to

terms compatible with the symmetries of the enlarged model that vanish in the $N = 2$ limit. We shall discuss these extra terms in the following sections.

In the $N = 2$ model the classical potential for a_Φ, $V_{cl} = Tr([a_\Phi, a_\Phi^\dagger]^2)$, is minimized (and vanishes) when a_Φ has any diagonal complex vev. This breaks $SU(2)$ down to $U(1)$ and yields a classical moduli space, parameterized by the gauge invariant superfield $U \equiv Tr[\Phi^2]$. We assume that at low energies the quantum theory also remains in the Coulomb phase, so that the particle content is restricted to the $U(1)$ gauge superfield W_α and the massless matter fields.

At arbitrarily low scales in the Coulomb phase where we need consider only the lowest dimension terms in the Lagrangian a duality is assured [1]. With the effective action simply quadratic in the gauge field, the duality transformation is conveniently derived [1] by considering a functional integral over the gauge superfield W. Imposing the condition $Im(\mathcal{D}W) = 0$ via a Lagrange multiplier V, incorporated by a new dual gauge field $W_D = i\mathcal{D}V$, we can complete the square and perform the Gaussian integral, to obtain an equivalent theory for the dual field with $\tau_D = -1/\tau$:

$$\mathcal{Z} \sim \int \mathcal{D}[W_D] \, \exp\left(\frac{1}{8\pi}Im \int \tau_D W_D^\alpha W_{D\alpha}\Big|_F\right) . \tag{5}$$

In order to maintain the explicit $N = 2$ supersymmetry of (4) in the dual theory we must rewrite the matter D-terms

$$\frac{\partial \mathcal{F}(A)}{\partial A}\bar{A} = \frac{\partial \mathcal{F}_D(A_D)}{\partial A_D}\bar{A}_D , \tag{6}$$

where $\tau = \partial^2 \mathcal{F}(A)/\partial A^2$ and $\tau_D = \partial^2 \mathcal{F}_D(A_D)/\partial A_D^2$. We find

$$A_D = \frac{\partial \mathcal{F}(\mathcal{A})}{\partial A} , \qquad \tau = \frac{\partial A_D}{\partial A} . \tag{7}$$

This manipulation is independent of whether τ is a coupling constant or, as in our enlarged model, a chiral superfield itself. Duality therefore continues to hold at low energies even after supersymmetry is broken by $\langle f_\tau \rangle \neq 0$, with supersymmetry breaking in the dual theory driven by $\langle f_{\tau_D} \rangle$.

The theory is, in addition, invariant to shifts by $n\pi$ in the real part of τ, with n an integer, since these correspond to unphysical shifts in the θ angle. The duality and θ angle transformations are generated by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tau \\ 1 \end{pmatrix} \tag{8}$$

where we have written τ as the ratio of the two components of the vector. The transformations therefore belong to the group $SL(2, Z)$.

Now consider the effective theory for scales much less than U. Its F-terms must be holomorphic and invariant under the global $U(1)_R$ symmetry of (3), and therefore of the form

$$\mathcal{L}_{gauge} = \frac{1}{8\pi} Im \left(\tau(\tau_0, U) W_\alpha W^\alpha \big|_F \right) + \text{h.c.} \tag{9}$$

If τ were everywhere analytic in U, $Im(\tau)$ would be a harmonic function, which would be unbounded below, resulting in the gauge theory having an imaginary coupling. This is avoidable only if τ has extra singularity structure at finite U, associated with bound states becoming massless. At large U, the $U(1)$ coupling is perturbative and we can use the perturbative β function above the scale U to write $\tau \sim (iM/2\pi) \ln U$. This has a monodromy at large U: when $U \to Ue^{2\pi i}$, $\tau \to \tau - M$. We can write it as an $SL(2, Z)$ transformation acting on $(\tau, 1)$:

$$M_\infty = \begin{pmatrix} 1 & -M \\ 0 & 1 \end{pmatrix} . \tag{10}$$

Any new massless bound states will be produced in the strong coupling regime and give rise to singularities in τ at the values of U where they become massless. The singularity structure therefore occurs in the weakly coupled regime of the dual theory. Close to U_i the dual theory has additional terms

$$\Delta\mathcal{L} = M^\dagger e^V M \big|_D + \tilde{M}^\dagger e^V \tilde{M} \big|_D + \sqrt{2} A_D M \tilde{M} \big|_F + \text{h.c.} \tag{11}$$

compatible with the symmetries, where M and $\tilde{M}$ are the two $N = 1$ chiral multiplets of an $N = 2$ matter multiplet, and $A_D \sim i(U - U_i)/\sqrt{U_i}$ gives the mass of the bound state. Around

these points the monodromies in τ must be consistent with its monodromy at infinity. The simplest possibility, consistent with the $\mathbf{Z_2}$ symmetry $U \leftrightarrow -U$, is a pair of singular points at each of which a single dyon becomes massless.

Assuming that at one of the points, U_1, the massless state is an $(n_m, n_e) = (1, 0)$ magnetic monopole, the perturbative β function in the dual theory (in which the monopole is an electrically charged particle) gives $\tau_D \sim \frac{ik}{2\pi} \ln(U - U_1)$ and monodromy $M_1 = \begin{pmatrix} 1 & -k \\ 0 & 1 \end{pmatrix}$. Consistency with the monodromy at large U allows us to then calculate the monodromy M_2 of τ_D around the other singularity at U_2:

$$M_2 = (SM_1S^{-1})^{-1}M_\infty = \begin{pmatrix} 1 & -M \\ -k & kM+1 \end{pmatrix} \tag{12}$$

which is compatible with a $(1, -1)$ dyon becoming massless at U_2. The charge is of course redefinable by circuiting infinity and inducing a monodromy, $(n_m, n_e) \to (-n_m, -n_e - 2n_m)$. Furthermore, changing the θ angle of τ by π simply switches the labelling of monopoles and dyons, and thus the two singular points U_i are physically equivalent, as the $\mathbf{Z_2}$ symmetry requires. An $SL(2, Z)$ invariant function with two singularities is determined, up to scalings, by the monodromy structure, and can be interpreted as the modular parameter of a torus.

A torus with modular parameter τ (the ratio of two periods of the torus) corresponds to a general cubic elliptic curve with roots e_1, e_2, e_3,

$$y^2 = 4(x - e_1)(x - e_2)(x - e_3) . \tag{13}$$

Up to $SL(2, Z)$ transformations, τ can be written

$$\tau = \frac{\partial A_D}{\partial A} = \frac{\int_{e_1}^{e_2} dx/y}{\int_{e_2}^{e_3} dx/y} . \tag{14}$$

A and A_D are thus defined by the curve only up to an unknown function of x and U. They can be determined by the boundary conditions $A \to \sqrt{2}\Phi$ as $U \to \infty$, and $A_D \sim i(U - U_i)/\sqrt{U_i}$ as $U \to U_i$, expanding the hypergeometric functions for small and large arguments [10]. The elliptic curve appropriate to the pure gauge $N = 2$ $SU(2)$ model is then

$$y^2 = (x - U_i)(x + U_i)(x - U) , \tag{15}$$

where $U_2 = -U_1$ by the $\mathbf{Z_2}$ symmetry. The period integrals are

$$A \quad = \quad \frac{\sqrt{2}}{\pi} \int_{-U_i}^{U_i} \frac{dx\sqrt{x-U}}{\sqrt{x^2 - U_i^2}} \tag{16}$$

$$A_D \quad = \quad \frac{\sqrt{2}}{\pi} \int_{U_i}^{U} \frac{dx\sqrt{x-U}}{\sqrt{x^2 - U_i^2}} \ . \tag{17}$$

The singular point U_i can be obtained by matching to the one-loop beta function result, in the perturbative large-U regime, and up to threshold corrections represented by a constant c of order one, is

$$a_{U_i} \approx \Lambda^2 \equiv c\mu^2 \exp\left(i\pi\tau_0\right) , \tag{18}$$

where μ^2 is the Wilsonian scale associated with the "bare" Lagrangian. The scalar potential in the dual theory, in the vicinity of either singular point where the dual theory is weakly coupled, is thus (to order $|a_D|^2$)

$$V \approx \frac{-2\pi^2}{(1 + \ln|a_D/16\Lambda|)}|a_M a_{\tilde{M}}|^2 + \frac{|a_D|^2}{2}\left(|a_M|^2 + |a_{\tilde{M}}|^2\right) - \frac{2\pi^2}{\ln|a_D/16\Lambda|}\left(|a_M|^2 - |a_{\tilde{M}}|^2\right)^2 . \tag{19}$$

This is minimized by $a_M = a_{\tilde{M}} = 0$, leaving a_D as a quantum moduli space. At $a_D = 0$ the monopoles are massless.

Before introducing $N = 2$ supersymmetry breaking mass terms into the model, it is worth discussing the region of validity of the effective theory discussed above. We first require μ^2 to stay below the scale U where the $SU(2)$ gauge group is broken to the $U(1)$ subgroup. Furthermore, the monopole fields must have masses in excess of μ, since we have cut off the monopole contribution to the τ function at their mass of approximately $(U - U_i)/\sqrt{U_i}$. (If instead the monopole mass were less than μ, then some of the gauge coupling's running would be due to loops computed in the low energy effective theory with internal momenta $< \mu$, and the τ function would not depend on $(U - U_i)/\sqrt{U_i}$.) The vacuum structure of the theories can be obtained from the limit $\mu \to 0$, so the effective theory is useful even close to the point $a_D = 0$ where the monopoles are light.

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If we are to extend the $N = 2$ model's effective theory to models with $N = 2$ breaking masses these models must continue to exhibit an energy range with a $U(1)$ gauge symmetry. As the $N = 2$ breaking masses grow we expect the theories' mass gap to increase (since the cancellations preventing confinement of the $SU(2)$ gluons will break down). For an energy range with a $U(1)$ gauge symmetry to exist the confinement scale of the $U(1)$ gauge boson (and therefore the $N = 2$ breaking masses) must be less than the $SU(2)$ breaking scale U. The resulting tractable models occupy a "ball" in parameter space around the $N = 2$ model. At tree level these models possess an $SU(2)$ gauge symmetry; a light adjoint of fermions; and a light adjoint of scalars, not necessarily degenerate with the fermions. We cannot rule out the possibility that the models' qualitative properties change dramatically as the soft breakings are taken larger than Λ (necessary to recover the QCD limit), although for small soft breakings we find that the behavior is smooth.

Even though introducing $N = 2$ breaking terms lifts the degeneracy of the moduli space, we can still discuss monodromies of the effective τ function in what was the moduli space by coupling a chiral superfield source $J(x)$ to the adjoint field $\Phi(x)$. The additional term

$$\mathcal{L}_J \;=\; \int d^2\theta \; J\Phi \;+\; \text{h.c.} \tag{20}$$

shifts $\langle a_\Phi \rangle$ away from its ($J = 0$) vacuum value when $\langle f_J \rangle \neq 0$. The additional interaction in (20) can of course induce new J-dependent terms in the effective Lagrangian $\mathcal{L}_{eff}^J$. However, for small $N = 2$ breaking masses a background source with size of order the small breaking mass is sufficient to explore the monodromies around the original $N = 2$ singular points. In the case of the gauge kinetic term, the lowest order term induced by J,

$$\left(\; J^\dagger J \; (DW)^2 / \Lambda^6 \; \right)\Big|_D \;\sim\; |F_J|^2 / \Lambda^6 \; F_{\mu\nu}^2 \;, \tag{21}$$

is clearly a subleading contribution near the $N = 2$ limit, leaving the monodromies and singularities of the broken model still controlled by the $N = 2$ τ function. We therefore expect to find light monopoles or dyons in the effective theory for $\langle a_\Phi \rangle$ near the $N = 2$ singular points.

2.3   Breaking to $N = 1$ supersymmetry

The $N = 2$ supersymmetric model discussed in section 2.2 can be broken to an $N = 1$ model by introducing a mass term for the adjoint matter fields [1, 11], corresponding in the enlarged model to allowing $\langle a_m \rangle \neq 0$. The effective Lagrangian is thus that of the $N = 2$ model plus terms, invariant under the $N = 1$ supersymmetry and gauged and global $U(1)$ symmetries of the model, that go to zero as the adjoint mass is turned off. The effective superpotential can therefore receive corrections of the form

$$\Delta W = m f(U/\Lambda, \tau_0)\big|_F \tag{22}$$

where f is an unknown function. This unknown mass renormalization arises from interactions of A with $SU(2)$ gauge bosons at scales above U.

The introduction of this mass term can also give rise to additional D-terms of the form

$$\Delta\mathcal{L}_D = m^\dagger m \, G\left(\frac{A_D}{\Lambda}, \frac{A_D^\dagger}{\Lambda}, \tau_0, \tau_0^\dagger, \frac{MM^\dagger}{\Lambda^2}, \frac{\tilde{M}M^\dagger}{\Lambda^2}, \frac{\tilde{M}\tilde{M}^\dagger}{\Lambda^2}\right)\bigg|_D \tag{23}$$

plus higher dimension terms in the gauge fields. In general G is some complicated unknown function[3]. Note that the corrections in (23) to the kinetic energy terms are suppressed by $\mathcal{O}(m^2/\Lambda^2)$, and for small m we can consider them $\mathcal{O}(p^4)$. In this sense these corrections do not destroy the $\mathcal{O}(p^2)$ exactness of the $N = 2$ solution. Upon eliminating f_{A_D} from such terms and the canonical D-term for A_D, the potential only receives contributions of the form

$$\frac{1}{1 + G'|_A}\left|\frac{dW}{df_{A_D}}\right|^2 \tag{24}$$

with G' a function related to G. Now, supersymmetric vacua satisfy $V = 0$, and G is nonsingular unless the effective theory completely breaks down. The extrema of W thus coincide with the minimum of the potential, and we have

$$\sqrt{2}\, a_M a_{\tilde{M}} + m\,(da_U/da_D) = 0$$
$$a_D a_M = a_D a_{\tilde{M}} = 0 \,. \tag{25}$$

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[3]Functions like G induced by supersymmetry breaking appear in D-terms of the effective Lagrangian, here and below. While there are some constraints on these functions, for example from weak coupling behavior and nonsingularity of the Kahler metric, these are usually not sufficient to globally determine the behavior.

As Seiberg and Witten found, the potential is minimized for $\langle a_D \rangle = 0$ and the monopoles condense with

$$a_M = a_{\tilde{M}} = \left(-a_m a_U'(0)/\sqrt{2} \right)^{1/2} . \tag{26}$$

The theory possesses a mass gap, since the magnetic $U(1)$ gauge boson acquires a mass by the Higgs mechanism, and electric charges are confined.

2.4   Breaking to $N = 0$ with nonzero f_m

As a first example of soft supersymmetry breaking consider giving a nonzero vacuum expectation value to the f-component of m in the $N = 1$ model. At tree level this generates an extra $N = 1$ breaking mass term for a_Φ

$$2Re(f_m Tr(a_\Phi^2)) , \tag{27}$$

which is renormalized by gauge interactions, through the function f in (22), but remains in the effective theory. In addition the D-terms in (23) may generate additional terms

$$|f_m|^2 \ G\left(\frac{A_D}{\Lambda}, \frac{A_D^\dagger}{\Lambda}, \tau_0, \tau_0^\dagger, \frac{MM^\dagger}{\Lambda^2}, \frac{\tilde{M}M^\dagger}{\Lambda^2}, \frac{\tilde{M}\tilde{M}^\dagger}{\Lambda^2} \right)\bigg|_A + \ a_m f_m G\big|_F + \ \text{h.c.} \tag{28}$$

Firstly, we note that the unconstrained terms are small ($\mathcal{O}(p^2)$ or higher) when $m \ll \Lambda$ and $f_m \ll \Lambda^2$. Secondly, introducing the soft breaking mass leaves the lowest dimension gauge kinetic term unchanged from the $N = 2$ model (ignoring not only the higher dimension terms mentioned above, but also suppressed terms such as $m^\dagger m (DW)^2/\Lambda^4|_D \sim |f_m^2/\Lambda^4|(F_{\mu\nu})^2)$. We conclude from the singularity structure of τ that even in the softly broken model there remain two points at which monopole bound states become massless. The soft breaking may generate a_D interactions of unknown sign and similarly contribute to the masses of the scalar components of the monopole fields.

When f_m is the only $N = 2$ breaking parameter, these terms are suppressed by f_m/Λ relative to the bare scalar mass term, but unfortunately the tree level potential is then unbounded below as $\langle a_U \rangle \to -\infty$. Higher dimension D-terms, contributing for example $|a_U|$

terms to the potential, determine whether it is truly unbounded or is instead minimized at some finite a_U. Such unknown terms may however be small compared to an additional $N = 2$ breaking mass generated by $\langle a_m \rangle \neq 0$. In the limit $f_m/\Lambda^2 \ll a_m/\Lambda \ll 1$, the bare masses dominate the corrections to the pure $N = 2$ model and we obtain the potential

$$
\begin{aligned}
V \approx {} & \frac{-2\pi^2}{(1 + \ln|a_D/16\Lambda|)} \left| a_M a_{\tilde{M}} - \frac{i\Lambda a_m}{\sqrt{2}} \right|^2 + \frac{|a_D|^2}{2} \left(|a_M|^2 + |a_{\tilde{M}}|^2 \right) \\
& - \frac{2\pi^2}{\ln|a_D/16\Lambda|} \left(|a_M|^2 - |a_{\tilde{M}}|^2 \right)^2 + 2Im(f_m \Lambda a_D)
\end{aligned}
\tag{29}
$$

which is minimized (via (26), up to terms of order f_m^2 and a_m^2) by $a_M = a_{\tilde{M}} = (i\Lambda a_m/\sqrt{2})^{1/2}$ and $a_D = i\sqrt{2} f_m^*/a_m$. We thus obtain an $\mathcal{O}(p^0)$ solution of a model with $N = 0$ supersymmetry.

We conclude that up to small corrections the vacuum structure of a model with both a small $N = 1$ preserving mass and a small soft supersymmetry breaking mass is equivalent to that of the pure $N = 1$ model. There thus exist $N = 0$ models which are close in parameter space to Seiberg and Witten's $N = 1$ model that exhibit monopole condensation and confinement. Unfortunately the terms induced in the potential by soft breaking from D-terms are not determined by the super-, gauge or global symmetries of the model and thus no conclusions can be drawn as to the vacuum structure when $\langle a_m \rangle \to 0$. This theme will recur in the discussion below.

2.5   Breaking to $N = 0$ with a gaugino mass

The $N = 2$ model can be perturbed directly to an $N = 0$, softly broken supersymmetric model by including a gaugino mass term. In the enlarged model such a mass can arise from setting $\beta_\tau \neq 0$ and hence $\langle f_{\tau_0} \rangle \neq 0$. The effective Lagrangian is again that of the $N = 2$ model plus, potentially, all additional terms consistent with the symmetries of the model that vanish as the gaugino mass is switched off. The lowest dimension gauge kinetic term is given by the τ function of $N = 2$ and hence the singularities indicating the presence of massless monopole fields are unchanged by the soft supersymmetry breaking. There are

however extra terms generated in the dual theory close to U_i, arising from the terms in the $N = 2$ effective Lagrangian when $\langle f_{\tau_0} \rangle \neq 0$. In particular, f_{τ_0} enters through U_i's dependence (18) on τ_0,

$$
\begin{aligned}
V \ \approx \ & \frac{-1}{4(1 + \ln|a_D/16\Lambda|)} \left| a_D f_{\tau_0} \ln|a_D/16\Lambda| + \frac{3}{2} a_D f_{\tau_0} - 2\sqrt{2}\pi a_M a_{\tilde{M}} \right|^2 \\
& - \pi\sqrt{2} Re(a_D f_{\tau_0} a_M a_{\tilde{M}}) + \frac{1}{2}|a_D|^2 \left(|a_M|^2 + |a_{\tilde{M}}|^2 \right) \\
& - \frac{2\pi^2}{\ln|a_D/16\Lambda|} \left(|a_M|^2 - |a_{\tilde{M}}|^2 \right)^2 - \frac{1}{2}|a_D|^2|f_{\tau_0}|^2 \left(1 + \frac{1}{2}\ln|a_D/16\Lambda| \right) .
\end{aligned}
\tag{30}
$$

There may in addition be D-terms that vanish in the $N = 2$ limit and give contributions to the superpotential of the form

$$
H(\tau_0^\dagger, \tau_0, A)\Big|_D .
\tag{31}
$$

Such terms may induce a scalar monopole mass contribution, for example

$$
\tau_0^\dagger A_D\Big|_D \ = \ f_{\tau_0}^* f_{A_D} ,
\tag{32}
$$

which on eliminating f_{A_D} provide a shift proportional to f_{τ_0} within $|f_{A_D}|^2$ in (30). The resulting cross terms with the monopole fields induce a scalar monopole mass. Its sign and magnitude, which would signify whether they condense, is therefore undetermined by the symmetries.

These terms perturb, in an unknown fashion, the potential (30) which was minimized at $(a_D = 0, a_M = 0)$. One can however read off the gaugino mass in the effective theory, arising from the f-component of the τ function, as

$$
m_{\tilde{\gamma}_D} \ = \ \frac{m_{\tilde{\gamma}}}{a_\tau} \ \sim \ \tau'_D(a_{\tau_0}) f_{\tau_0} \ \sim \ -f_{\tau_0} \frac{\Lambda^2}{a_D} .
\tag{33}
$$

The gaugino and dual gaugino masses diverge as $a_D \to 0$, which we may interpret as the decoupling of the massive gaugino fields from the effective theory, as the decreasing monopole mass drives its region of validity to zero. If the theory behaves without singularity as $\langle f_{\tau_0} \rangle \to 0$, the contribution of (31) to the potential must cause the vacuum expectation $\langle a_D \rangle$

to become nonzero and proportional to $\langle f_{\tau_0}\rangle^l$ with $l < 1$, so that $m_{\tilde{\gamma}_D}$ smoothly approaches zero in the $N = 2$ limit.

As before if the gaugino mass is introduced as a perturbation to the $N = 1$ model ($\langle f_{\tau_0}\rangle \ll \langle a_m\rangle$), then the corrections to the potential from the gaugino mass will be small and the minima will be that of the $N = 1$ model with monopole condensation.

2.6   Breaking to $N = 0$ with a squark mass

Allowing $\langle f_K\rangle \neq 0$ in (1) generates a soft supersymmetry breaking squark mass, induced at tree level by the term $|f_K|^2|a_\Phi|^2$. Since K has arbitrary $U(1)$ charge it could not appear in the F-terms of the effective theory, but it does appear in D-terms of the form

$$
\begin{aligned}
\mathcal{L}_K \quad &= \quad \Bigg[K^\dagger K\ \Phi^\dagger \Phi\ G_1\left(\tau_0, \tau_0^\dagger, \frac{\Phi^\dagger \Phi}{\Lambda^2}, \frac{\Phi^2}{\Lambda^2}, \frac{M^\dagger M}{\Lambda^2}, \frac{\tilde{M}^\dagger \tilde{M}}{\Lambda^2}\right) \\
&\quad + \ K^\dagger K\ \Phi^2\ G_2\left(\tau_0, \tau_0^\dagger, \frac{\Phi^\dagger \Phi}{\Lambda^2}, \frac{\Phi^2}{\Lambda^2}, \frac{M^\dagger M}{\Lambda^2}, \frac{\tilde{M}^\dagger \tilde{M}}{\Lambda^2}\right)\Bigg]_D \quad ,
\end{aligned} \tag{34}
$$

with G_1 and G_2 unknown functions. The bare squark mass is preserved up to gauge renormalization, but the induced monopole mass is undetermined and hence so is the vacuum structure. The τ function of the $N = 2$ model is again preserved, indicating the presence of massless monopole states at $a_D = 0$. The potential minima when the $N = 1$ model is perturbed will still lie close (for small $\langle f_K\rangle$) to $a_D = 0$ and a_M given by (26), not altering the vacuum structure from the pure $N = 1$ case.

3 $SU(N_c)$ Gauge Symmetry And Matter Fields

The exact results for $N = 2$ supersymmetric QCD with an $SU(2)$ gauge group have been generalized to the case of an $SU(N_c)$ gauge group [2]. The potential for the adjoint scalars is minimized by a vev that classically breaks $SU(N_c)$ to $U(1)^{N_c}$. For large scalar vev the $U(1)$ couplings can again be calculated in perturbation theory and shown to possess a monodromy in U. The monodromy is consistent with the existence of N points on the quantum moduli

space at which $N - 1$ dyons become massless. The function τ is again exactly specified (up to scaling) by the monodromy structure.

Seiberg and Witten have also demonstrated how $N = 2$ matter multiplets can be included in the $SU(2)$ model. For heavy matter fields, the $SU(2)$ dynamics are invariant at low energies, except for the addition of an extra singularity at large $U = m/\sqrt{2}$ corresponding to where a matter field mass vanishes due to its Yukawa coupling to the scalar vev. As the mass decreases, the singularity moves towards the origin of the moduli space and merges with the pure glue singularities as dictated by the requirement that the dyons fill out multiplets of the relevant flavor symmetries. Again the τ function is exactly determined by the monodromy structure. These results can also be generalized to $SU(N_c)$ gauge models [12].

In each of these cases the exact form of the τ function in the effective theory is known (up to higher dimensional contributions). We can break the models to $N = 0$ models with squark and gaugino masses by promoting the bare couplings τ_0 and K to the status of chiral superfields and allowing them to acquire a nonzero f-component vev. As for the $SU(2)$ theory, the D-terms will generate unknown contributions to the potential, but the gauge multiplet's kinetic term is given exactly by τ. If the soft breaking masses are small relative to an $N = 1$ preserving adjoint matter mass then the models are pinned near the singularities and the monopoles condense.

Among these models is the particularly interesting case of an $SU(3)$ gauge symmetry and $N = 1$ matter multiplets in the fundamental representation. We may choose to introduce an $N = 1$ supersymmetry preserving mass for the adjoint matter field and soft supersymmetry breaking masses for the gauginos and fundamental representation scalars. At tree level these masses can be raised until the fields are integrated from the theory, leaving QCD with quarks. Although we cannot explicitly take this limit in the effective theory, since the light degrees of freedom there are not those appropriate to QCD, we may begin the interpolation from the $N = 2$ result towards QCD. The behavior of the theory is that monopoles condense and the

quarks are confined. This picture's consistency with the t'Hooft–Mandlestam picture [13] of quark confinement in QCD suggests that the models are smoothly connected.

4 Discussion

We have embedded the $N = 2$ and $N = 1$ supersymmetric models possessing exactly calculable results within a larger $N = 1$ model, which in certain limits induces soft supersymmetry-breaking masses in the original theories. This procedure generates new contributions to the effective Lagrangian, which unfortunately holomorphy and $U(1)$ symmetries do not completely determine. Thus the information we are able to extract about the behavior of corresponding $N = 0$ models is limited. However, at leading order in the breaking masses the gauge kinetic terms are still completely determined, leaving unchanged the τ function of the $N = 2$ and $N = 1$ models. Since the singular behavior of τ is consistent with the original ansatz that a monopole becomes massless at each of the singular points, we expect that these are still the only singularities in the low-energy effective Lagrangian of the nonsupersymmetric models. If the soft supersymmetry breaking masses are induced as sufficiently small perturbations to the $N = 1$ preserving model, then monopole condensation and confinement are preserved and seen not to depend on the presence of exactly massless gauginos or squarks, nor on exact supersymmetry degeneracies. While we have encountered some limitations in applying Seiberg and Witten's techniques to nonsupersymmetric models, the analysis nevertheless reveals some details of condensation and confinement. Our results are consistent with the supersymmetric models being smoothly connected to nonsupersymmetric QCD-like models.

Acknowledgements

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