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ELECTROWEAK PHASE TRANSITION AND NUMERICAL SIMULATIONS IN THE SU(2) HIGGS MODEL

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ABSTRACT

Recent progress in non-perturbative investigations of the electroweak phase transition is reviewed, with special emphasis on numerical simulations in the four-dimensional SU(2) Higgs model.

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ABSTRACT

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1. Introduction

The symmetry restoration in the electroweak Standard Model at high temperature¹ is an interesting feature in the history of the early Universe. The anomalous baryon number violating processes are fast for temperatures above the electroweak phase transition of symmetry restoration, and are getting very slow soon after it. It is also possible that the baryonic excess was produced during the phase transition itself². The conditions for this are: a strong enough first order phase transition and a sufficiently strong CP-violation in the Standard Model. Concerning the strength of CP-violation, in spite of some optimism³, serious doubts have been raised⁴. In order to determine the properties of the electroweak phase transition one needs non-perturbative studies, which can be performed within the framework of lattice regularization. In this lecture a short review will be given about some recent developments in this field. The discussion will be centred around a recent series of numerical simulations performed in the four-dimensional SU(2) Higgs model⁵⁻⁸.

1.1. Dimensional reduction

The main difficulty in numerical simulations of the thermodynamics of the electroweak plasma on the lattice has been identified in earlier studies⁹: as a consequence of the weak gauge coupling, the observed first order phase transition gets very weak with increasing Higgs boson mass (M_H) already near $M_H \simeq M_W$ (with M_W the

W -boson mass). The screening masses (inverse correlation lengths) near the phase transition become much smaller than the temperature. It becomes very difficult to deal with this two-scale problem in a numerical simulation.

This suggests a reduction of the problem to a three-dimensional field theory by integrating out the heavy modes with non-zero Matsubara frequencies^{10–11}. The simplest possibility is to consider the three-dimensional fundamental Higgs model with couplings determined by a perturbative matching procedure¹². This amounts to choosing some perturbatively well calculable set of parameters of the four-dimensional continuum theory (usually derived from the effective potential in the phase with broken symmetry) and setting them equal to the corresponding parameters in the reduced three-dimensional continuum theory. Since the three-dimensional model is super-renormalizable, the correspondence of the renormalized parameters to the bare parameters in lattice regularization can be determined, near the continuum limit, in a two-loop perturbative calculation¹³. In this way the non-perturbative properties of the phase transition can be numerically studied in lattice simulations^{12,14}, which are less demanding than the direct lattice simulations in the original four-dimensional theory.

Originally, the three-dimensional reduced model contains also a Higgs field in the adjoint (triplet) representation, but its mass is governed by chromo-electric screening, and hence it is heavier than the Higgs doublet field and the three-dimensional gauge field, which gets its mass from chromo-magnetic screening. Therefore, at least in a first approximation, one can also neglect the adjoint Higgs field and stay with a simple three-dimensional fundamental SU(2) Higgs model. Further reduction, leading to scalar degrees of freedom alone, seems to give qualitatively different results¹⁵, and hence is not advisable.

The obvious question about reduction is the effect of the necessary approximations on the non-perturbative results. In order to discuss this, it is advantageous to think about the four-dimensional continuum theory as the continuum limit of a lattice-regularized model. Although the continuum limit of the SU(2) Higgs model (and also of the electroweak sector of the Standard Model) is believed to be trivial, the renormalized couplings go to zero only logarithmically with the lattice spacing. Therefore the continuum theory with non-zero renormalized couplings can be realized to a very good approximation along the *lines of constant physics* defined by the constant values of two dimensionless parameters: the mass ratio $R_{HW} \equiv M_H/M_W$ and the renormalized gauge coupling g_R^2 defined, for instance, from the screened (Yukawa) potential of external charges¹⁶. The points of the lines of constant physics can be parametrized by the logarithm of the inverse W -boson mass in lattice units: $\tau \equiv \log(aM_W)^{-1}$. Due to the triviality of the continuum limit, every line of constant physics with non-zero couplings ends on the boundary of the bare parameter space at a finite value of $\tau = \tau_{\max}$, but if the Higgs boson mass is not close to the triviality upper limit¹⁷ ($M_H \simeq 9M_W$), the value of τ_{\max} can be very large. In such cases, since lattice artefacts go to zero by powers of the lattice spacing, the continuum theory is

very well approximated.

In a given point of a line of constant physics, with large τ , the heavy and high-momentum modes can be integrated out by considering the effective action of averaged fields^{18,19} or by applying block-spin transformations²⁰. In fact, for the purpose of obtaining a three-dimensional lattice model it is simplest to take the block-spin approach and to perform so many block-spin transformations that in the imaginary time (or inverse temperature) direction only a single layer of points is left over. In this way a three-dimensional lattice action is obtained for the light and low-momentum modes, which is exactly equivalent to the original four-dimensional lattice action. (Here we assume the ideal situation of a *perfect action*, when no approximations are made in the blocking procedure.)

The three-dimensional lattice action contains, in general, many couplings. Besides the simplest nearest neighbour couplings, there are also less local terms, with non-locality at least as large as the inverse temperature represented by the periodicity in the fourth dimension of the original four-dimensional lattice. How well this multi-parameter action can be approximated by the simplest local action corresponding to the discretization of the simplest local three-dimensional continuum theory is an open question. The connection between the multi-parameter action and the simple local action giving the best approximation will, in general, also depend on the lattice spacing. This can be seen, for instance, if the integration of heavy degrees of freedom is done in two-loop perturbation theory with momentum cut-off²¹. In fact, the continuum limit in case of the three-dimensional block-spin action is not the same as the continuum limit for the simplest local three-dimensional action. Starting from different points of some four-dimensional line of constant physics with increasing scale parameter τ , the lattice spacing for the three-dimensional block-spin action is not changing at all. (The couplings in the action are, of course, changing.) One can also say that one should not take the continuum limit of an effective field theory beyond the scale where “new physics” appears.

The danger in taking only the simplest local three-dimensional action is that the approximation given by it may be of different quality in different parts of the parameter space: the action optimized in the perturbative region, deeply inside the Higgs phase, can deviate more from the block-spin action near the phase transition and in the symmetric phase. In fact, it is plausible that in the symmetric phase, which is similar to the high-temperature phase of a non-Abelian pure gauge theory, the rôle of non-locality is more important.

This shows that the numerical simulations of the electroweak phase transition in its unreduced four-dimensional form are important, even if finally the dimensional reduction, in some form, will turn out to be a good approximation.

1.2. Lattice action

The numerical simulations in the four-dimensional finite temperature theory have been restricted up to now to the SU(2) Higgs model. This is a good and suffi-

ciently simple theoretical laboratory, where the qualitative features of the symmetry restoring phase transition can be understood. The well known continuum action of this model is given in terms of the SU(2) gauge field $W_\mu^r(x)$ ($\mu = 1, 2, 3, 4$; $r = 1, 2, 3$) and the complex scalar doublet field $\phi_\alpha(x)$ ($\alpha = 1, 2$). The coupling parameters are: the (squared) gauge coupling g^2 and the quartic scalar self-coupling λ . In addition, there is a single mass parameter belonging to the scalar field, which can be represented in the Higgs phase by the vacuum expectation value v .

The Euclidean lattice action of the SU(2) Higgs model can be written as

$$S[U, \varphi] = \beta \sum_{pl} \left(1 - \frac{1}{2} \text{Tr} U_{pl} \right) + \sum_x \left\{ \frac{1}{2} \text{Tr} (\varphi_x^+ \varphi_x) + \lambda \left[\frac{1}{2} \text{Tr} (\varphi_x^+ \varphi_x) - 1 \right]^2 - \kappa \sum_{\mu=1}^4 \text{Tr} (\varphi_{x+\hat{\mu}}^+ U_{x\mu} \varphi_x) \right\}. \quad (1)$$

Here $U_{x\mu}$ denotes the SU(2) gauge link variable, U_{pl} is the product of four U 's around a plaquette, and φ_x is a complex $2 \otimes 2$ matrix in isospin space describing the Higgs scalar field. The bare parameters in the action are $\beta \equiv 4/g^2$ for the gauge coupling, λ for the scalar quartic coupling and κ for the scalar hopping parameter related to the bare mass square μ_0^2 by $\mu_0^2 = (1 - 2\lambda)\kappa^{-1} - 8$. In what follows we set the lattice spacing to 1 ($a = 1$), therefore all the masses and correlation lengths, etc., will always be given in lattice units, unless otherwise stated.

The large scale numerical simulations⁵⁻⁸, which will be discussed in the present lecture, have been performed mainly on the APE-Quadrics massive parallel computers at DESY, and in a smaller part also on the CRAY's of HLRZ-Jülich. The published results refer mainly to two Higgs boson masses: $M_H \simeq 20$ GeV and $\simeq 50$ GeV, always assuming that $M_W \equiv 80$ GeV. Recently, an intermediate Higgs boson mass $M_H \simeq 35$ GeV has also been considered⁸, mainly in order to be able to compare the results with past and future results of numerical simulations in the reduced three-dimensional models. This work is still going on and will be published soon²².

In the optimized updating procedure for the creation of the sample of field configurations, besides the algorithms described in detail previously⁶, also scalar field reflections²³ have been applied. In fact, this latter algorithm turned out to be very efficient in reducing the autocorrelations among field configurations, hence in reducing the necessary computer resources.

2. Latent heat

One of the most important physical quantities characterizing first order phase transitions is the latent heat, which is the discontinuity of the energy density $\Delta\epsilon$ across the phase transition. In the SU(2) Higgs model on the lattice its dimensionless

ratio with the fourth power of the transition temperature T_c can be obtained⁵ from

$$\frac{\Delta\epsilon}{T_c^4} = L_t^4 \left\langle \frac{\partial\kappa}{\partial\tau} \cdot 8\Delta L_{\varphi,x\mu} - \frac{\partial\lambda}{\partial\tau} \cdot \Delta Q_x - \frac{\partial\beta}{\partial\tau} \cdot 6\Delta P_{pl} \right\rangle. \quad (2)$$

Here L_t is the lattice extension in the inverse temperature direction, the partial derivatives of the bare couplings are taken along lines of constant physics, and

$$\begin{aligned} L_{\varphi,x\mu} &\equiv \frac{1}{2} \text{Tr} (\varphi_{x+\hat{\mu}}^+ U_{x\mu} \varphi_x), \\ Q_x &\equiv \left[\frac{1}{2} \text{Tr} (\varphi_x^+ \varphi_x) - 1 \right]^2, \\ P_{pl} &\equiv 1 - \frac{1}{2} \text{Tr} U_{pl}. \end{aligned} \quad (3)$$

The signs of the discontinuities such as $\Delta\epsilon$ etc. are usually defined as differences of values in the symmetric phase minus the Higgs phase.

For the numerical evaluation of the latent heat the knowledge of the lines of constant physics is required. Since in the Standard Model, for not very high Higgs boson masses, we are interested in weak couplings, estimates of $\lambda(\tau)$ and $\beta(\tau)$ can be obtained from the perturbative renormalization group equations. Similarly, $\kappa(\tau)$ can be estimated using one-loop lattice effective potentials^{5,6}. These estimates can then be checked in numerical simulations by determining R_{HW} and g_R^2 numerically. If necessary, one can also further tune the bare parameters in order to reach constant values of R_{HW} and g_R^2 . In practice, in the investigated parameter range, the perturbative estimates are usually sufficient. Assuming that the temperature is equal to T_c , the scale changes are determined by the lattice extensions in the fourth direction: $T_c = 1/L_t$. Taking $L_t = 2, 3, 4, \dots$, we get for the changes in the scale parameter $\Delta\tau = \log(3/2), \log(4/2), \dots$

The other ingredients in Eq. 2 are the discontinuities of the global quantities defined in Eq. 3. In finite spatial volumes the discontinuities are round-off by inverse volume $((L_x L_y L_z)^{-1})$ effects. These finite volume effects can be eliminated, in principle, by a cumbersome extrapolation to infinite volume. Another possibility is to use the metastability of the two phases in the vicinity of the phase transition. This implies that, in large enough volumes, during the Monte Carlo updating the field configuration remains in one metastable phase for a very long time. In this way one is able to determine expectation values characterizing single phases, and hence e. g. the required discontinuities. One has to have in mind, however, that in this way an independent precise determination of the location of the phase transition is necessary, because the metastability extends over a finite range of parameters. The uncertainty of the phase transition parameters appears in the errors of the discontinuities²².

First results^{5,6} on $\Delta\epsilon/T_c^4$ as a function of the Higgs boson mass are shown in Fig. 1, together with a comparison to two-loop resummed perturbation theory²⁴. As

Figure 1.

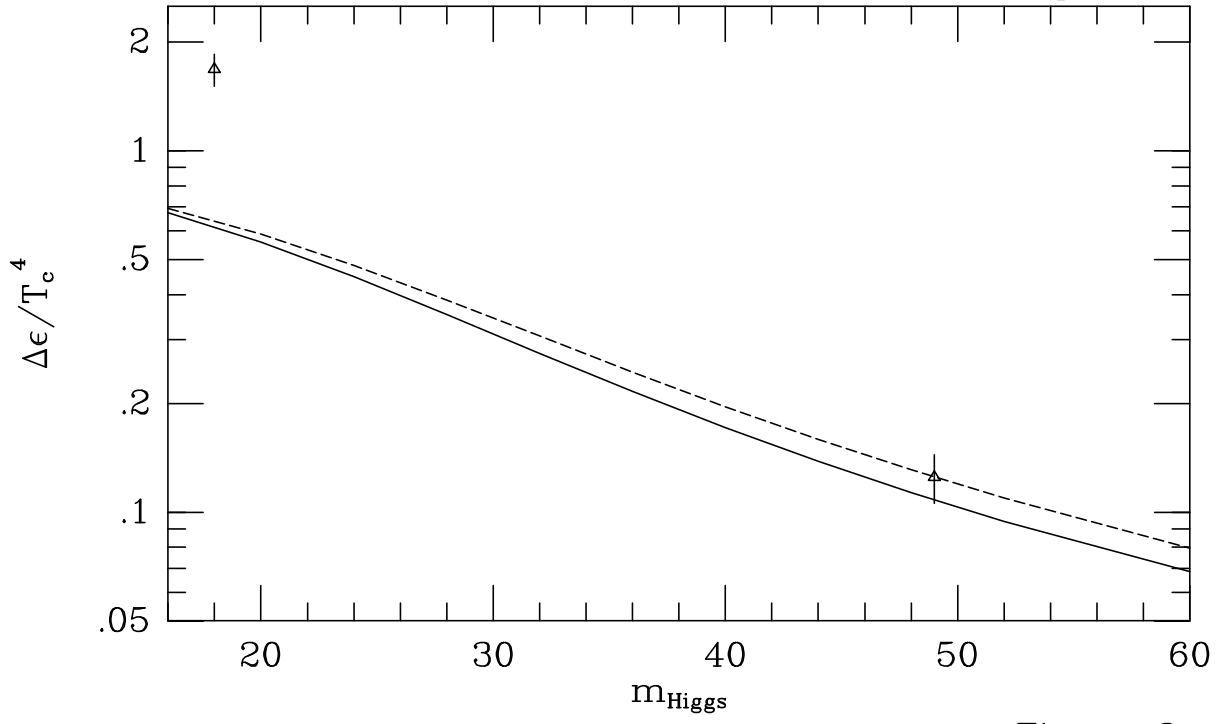
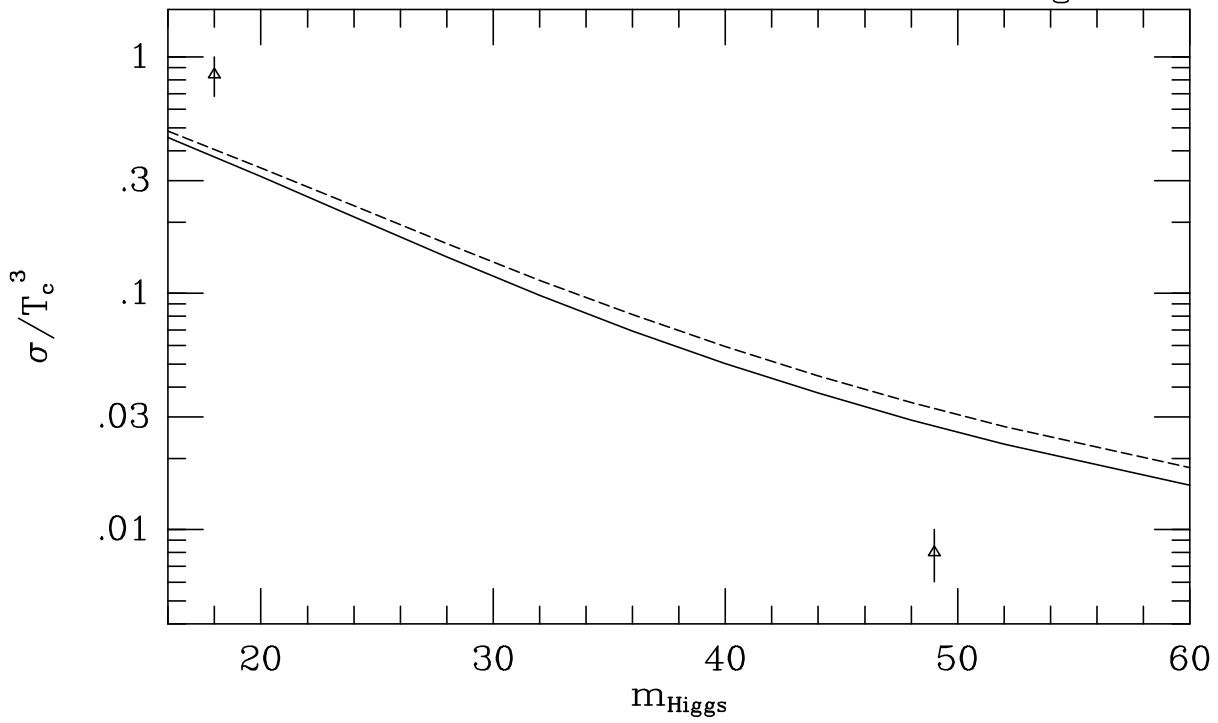


Figure 2.



can be seen, the results of the numerical simulation (shown by triangles) agree qualitatively well with perturbation theory. The latent heat characterizing the strength of the first order phase transition is decreasing considerably between $M_H \simeq 20$ GeV and $M_H \simeq 50$ GeV.

3. Interface tension

Besides the latent heat, another important physical quantity characterizing the first order phase transitions is the interface tension (σ): the free energy per unit area of the wall separating the two phases. Many characteristics of the nucleation dynamics are determined by these two quantities. The radius of the critical droplets is proportional to $\sigma/\Delta\epsilon$. The nucleation rate is approximately determined by

$$R_n \equiv \frac{\sigma^3}{(\Delta\epsilon)^2 T_c} = \frac{(\sigma/T_c)^3}{(\Delta\epsilon/T_c^4)^2}. \quad (4)$$

The typical droplet distance is roughly proportional to $R_n^{\frac{1}{2}}$. In fact, the quantity R_n defined in Eq. 4 can be taken as a good characterization of the strength of the first order phase transition: for $R_n = \mathcal{O}(1)$ we have a strong transition, whereas $R_n \ll 1$ corresponds to a weak one.

In numerical simulations one can exploit the tunnelling between the two nearly equal minima of the effective potential, which correspond to the two phases. On large enough lattices one can study the properties of the mixed phase when, as a result of tunnelling, both phases are simultaneously present in the system. For this purpose an elongated geometry of the lattice is advantageous. In a four-dimensional simulation, when the fourth dimension gives the inverse temperature ($T = L_t^{-1}$), one can take an extension L_z much larger than the other two “transverse” directions: $L_z \gg L_x = L_y$. In this case, for large enough transverse extensions when the mixed phase appears, there are transverse walls separating the two phases. For periodic boundary conditions, and not too large L_z , there are just two walls and two regions (one for each phase).

This situation can be used to determine the interface tension in several ways. For instance, one can determine σ from the small energy difference in the transfer matrix between the asymmetric and symmetric combinations of the two phases²⁵. Another possibility is to measure the doubly peaked distribution of some order parameter, when σ is given by the ratio of the maxima to the flat region between the two peaks. This can be effectively done by the “multicanonical” simulation method²⁶. In the “two-coupling” method²⁷ one divides the long lattice extension into two halves with different bare parameters: in one half slightly above ($\kappa > \kappa_c$) in the other half slightly below ($\kappa < \kappa_c$) of the phase transition. In the range $20 \text{ GeV} < M_H < 50 \text{ GeV}$ all three methods are applicable and give compatible results^{6,8}. In Fig. 2 the obtained

interface tension at $M_H \simeq 20$ GeV and $\simeq 50$ GeV is shown and compared with two-loop resummed perturbation theory²⁴. At $M_H \simeq 35$ GeV a recent result is⁸

$$\left. \frac{\sigma}{T_c^3} \right|_{35 \text{ GeV}} = 0.06 \pm 0.01 . \quad (5)$$

This shows that also the interface tension is a steeply decreasing function of the Higgs boson mass.

The combination introduced in Eq. 4, which is a good characterization for the strength of the phase transition, is decreasing from $R_n \simeq 0.23$ at $M_H \simeq 20$ GeV to $R_n \simeq 0.00003$ at $M_H \simeq 50$ GeV. That is, the phase transition at $M_H \simeq 50$ GeV becomes very weak indeed.

The interface tension, more precisely σ/T_c , plays a decisive rôle also in determining the minimal spatial lattice size required for an accurate numerical investigation of first order phase transitions. In order to determine discontinuities of global averages, screening lengths in separated metastable phases and other similar characteristics of the transition, one has to reach a mixed phase situation, which can be characterized by a well developed double peak structure of the order parameter distributions. The double peak structure appears when $L_x L_y \sigma/T_c = \mathcal{O}(1)$, with $L_x, L_y \leq L_z$ the transverse lattice extensions. The lattice in this volume should be fine enough to also resolve the interface structure, which becomes thinner for increasing M_H . These are physical requirements setting a minimum on the spatial lattice size, which are practically more important than the conditions imposed by the smallness of $\mathcal{O}(a)$ lattice artefacts. This implies that the requirements on spatial lattice sizes are essentially the same in dimensionally reduced three-dimensional simulations as in unreduced four-dimensional ones. The gain in computational resources by using the simplest local three-dimensional action is essentially equal to a factor L_t . The consequence is that, for instance, the extension of the present numerical studies to Higgs boson masses beyond $M_H \simeq 50$ GeV is not much easier in three dimensions than in four.

4. Discussion

The large scale numerical simulations discussed in the previous sections show that the symmetry restoring electroweak phase transition is of first order, with fast decreasing strength, in the Higgs boson mass range $20 \text{ GeV} < M_H < 50 \text{ GeV}$. This conclusion is in qualitative agreement with two-loop resummed perturbation theory²⁴. At $M_H \simeq 50$ GeV the strength of the phase transition is definitely not enough to produce a strong enough out-of-equilibrium situation in the early Universe for the creation of the observed baryon asymmetry. The observed difficulty of identifying the first order phase transition near $M_H \simeq 80$ GeV shows⁹ that in the range $50 \text{ GeV} < M_H < 100 \text{ GeV}$ presumably no surprises are to be expected. Nevertheless,

before a definite conclusion, the non-perturbative studies have to be extended towards higher Higgs boson masses, which are actually the experimentally allowed ones in the minimal Standard Model. This seems feasible in both unreduced and dimensionally reduced SU(2) Higgs models.

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