# Addendum to the Test of CP Violation in Tau Decay * 

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#### Abstract

We discuss the test of CP and CPT violation in $\tau$ decay without using the polarized electron beam by comparing partial fractions of $\tau^{-}$and $\tau^{+}$decay into channels with strong final state interactions. For example, $\Gamma\left(\tau^{-} \rightarrow \pi^{-}+\pi^{0}+\nu\right) \neq$ $\Gamma\left(\tau^{+} \rightarrow \pi^{+}+\pi^{0}+\nu\right)$ signifies violation of CP. The optimum energy to investigatge CP violation in $\tau$ decay is discussed. We conclude that this energy is a few MeV below $\psi(2 s)$ in order to avoid the charm contribution and over abundance of hadrons at the $\psi(2 s)$ peak.


Submitted to the Proceedings of the
Workshop on the Tau-Charm Factory
21-23 June 1995
Argonne National Laboratory, Argonne, Illinois

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## 1 Introduction

Understanding CP violation in the elementary particle system is a fascinating subject in itself. It is also a key to understanding the preponderance of matter over antimatter in our universe. Up to now the only evidence of CP violation on the elementary particle level is the decay of the $k_{L}$ system and this is too meager to construct a credible standard theory for CP violation for all particles. In this paper we discuss measurement of CP violation in $\tau$ decay. This is an interesting subject because $\tau$ is the heaviest lepton and thus if a charged Higgs boson is responsible for CP violation we would most likely see the effect here among all the leptons. Also the Kobayashi-Maskawa theory [1] says that CP violation should not occur in the leptonic sector because the gauge eigenstate and mass eigenstate are identical in the lepton sector due to zero neutrino masses in the Standard Model. These basic assumptions of Kobayashi-Maskawa must be tested. CP violation in $\tau$ has been investigated previously mainly in the production of $\tau$ pair coming from the possible existence of the electric dipole moment [2] of $\tau$. However since the electric dipole moment of $\tau$ is induced by weak or semiweak corrections to the electromagnetic vertex of $\tau$ its effect is expected to be less than $\left(m_{\tau} / m_{w}\right)^{2} \alpha=3 \times 10^{-6}$ and thus impossible to detect even with $10^{8} \tau$ pairs available in the Tau-Charm Factory. Similarly the interference between CP violating neutral Higgs boson exchange and the one photon exchange diagrams is also completely negligible $[3,4]$. Thus CP can be assumed to be conserved in the production the $\tau$ pair; we need to consider only CP violation in the decay of $\tau$.

Since the decay of $\tau$ is a weak interaction, if CP violation in $\tau$ is weak, then its effect should be of order 1 whereas if it is milliweak its effect should be of order $10^{-3}$ and detectable with $10^{8} \tau$ pairs available at the Tau-Charm Factory.

In my previous papers [3, 4] I have discussed how to use the polarized electron beam to investigate CP violation in $\tau$ decay by constructing rotationally invariant quantities such as $\vec{w}_{i} \cdot \vec{a},\left(\vec{w}_{i} \times \vec{a}\right) \cdot \vec{b},\left(\vec{w}_{i} \times \vec{\mu}\right) \cdot \vec{w}_{\mu}$ where $\vec{a}$ and $\vec{b}$ are momenta of hadrons in the semileptonic decay of $\tau ; \vec{\mu}$ and $\vec{w}_{\mu}$ are momentum and the polarizaiton of muon in the decay $\tau^{-} \rightarrow \mu^{-}+\nu_{\tau}+\bar{\nu}_{\mu}$ or its charge conjugate; $\vec{w}_{i}$ is the initial beam polarization

$$
\begin{equation*}
\vec{w}_{i}=\frac{w_{1}+w_{2}}{1+w_{1} w_{2}} \hat{e}_{z} \tag{1}
\end{equation*}
$$

where $\hat{e}_{z}$ is the direction of the incident electron and $w_{1}$ and $w_{2}$ are polarization of the electron and positron in the $z$ direction.

In section 2 I point out that $\Gamma\left(\tau^{-} \rightarrow \nu_{\tau}+a+b\right) \neq \Gamma\left(\tau^{+} \rightarrow \bar{\nu}_{\tau}+\bar{a}+\bar{b}\right)$ also signifies CP violation. We give the physical reason for it. We also compare the merits of this kind of measurement with those using polarized beams. In section 3 we discuss the optimum energy to do $\tau$ physics at the Tau-Charm Factory.

## 2 CP Violation in $\tau$ Decay using Branching Fractions

CPT conservation says that the total widths of $\tau^{-}$and $\tau^{+}$must be equal. Also partial widths into those channels without final state interactions, such as $\tau^{-} \rightarrow \mu^{-}+\bar{\nu}_{\mu}+\nu_{\tau}$, $\tau^{-} \rightarrow e^{-}+\bar{\nu}_{e}+\nu_{\tau}, \tau^{-} \rightarrow \pi^{-}+\nu_{\tau}$, and $\tau^{-} \rightarrow k^{-}+\nu_{\tau}$, must be the same as the corresponding channels for $\tau^{+}$decay [5]. However for decay channels that contain final state interactions, such as $\tau^{-} \rightarrow \pi^{-}+\pi^{0}+\nu_{\tau}, \pi^{-}+\pi^{-}+\pi^{+}+\nu_{\tau}, \pi^{-}+\pi^{0}+\pi^{0}+\nu_{\tau}, \pi^{-}+k^{0}+\nu_{\tau}$, and $\pi^{0}+k^{-}+\nu_{\tau}$, the CP violation can show up as the inequality in partial widths for charge conjugate decay modes. For example, $\Gamma\left(\tau^{-} \rightarrow \pi^{-}+\pi^{0}+\nu\right) \neq \Gamma\left(\tau^{+} \rightarrow \pi^{+}+\pi^{0}+\bar{\nu}\right)$ signifies violation of CP , but $\Gamma\left(\tau^{-} \rightarrow \mu^{-}+2 \nu\right) \neq \Gamma\left(\tau^{+} \rightarrow \mu^{+}+2 \nu\right)$ or $\Gamma\left(\tau^{-} \rightarrow\right.$ all $) \neq$ $\Gamma\left(\tau^{+} \rightarrow\right.$ all $)$ will indicate that CPT is violated. The polarization vector $\vec{w}_{i}$ defined in Eq. (1) can be used to construct many rotationally invariant products to investigate T , CP , CVC, and charged Higgs boson exchange in leptonic [4] and semileptonic [3] decays of $\tau$. The polarization dependent quantities will yield information on structure of CP violations whereas the polarization independent quantities such as the difference in partial widths between $\tau^{-} \rightarrow \nu_{\tau}+\pi^{-}+\pi^{0}$ and $\tau^{+} \rightarrow \bar{\nu}_{\tau}+\pi^{+}+\pi^{0}$ will merely indicate the existence and magnitude of the CP violation. As pointed out in Ref. [3] this difference in partial widths is due to the combined effects of CP violation and the inelastic final state interaction such as $2 \pi$ going into $4 \pi$ and vice versa. In the absence of CP violation the probabilities of $2 \pi$ going into $4 \pi$ and vice versa in the $\tau^{-}$decay are equal to those in the $\tau^{+}$decay. However in the presence of CP violation the amplitudes for the decay is proportional to $\exp \left(i \delta_{w}+i \delta_{s}\right)$ for $\tau^{-}$and $\exp \left(-i \delta_{w}+i \delta_{s}\right)$ for $\tau^{+}$and thus they become different.

## 3 Optimum Energy to do $\tau$ Physics

The energy of the machine should be set below charm threshold; i.e. $E_{\mathrm{cm}}=1869.3 \mathrm{MeV}$ for each beam. Near the threshold of $\tau$ pair production, $\tau$ pair events can uniquely be identified by $e$-hadron, $e-\mu, \mu$-hadron events. Above the charm threshold charm events produce the unwelcome leptonic background [6]. The best energy to run is either at $\psi(2 s, 3,685 \mathrm{MeV})$ or slightly below it. The total cross section for $e^{+} e^{-} \rightarrow \psi(3.685) \rightarrow \tau^{+} \tau^{-}$can be written as [7]

$$
\begin{equation*}
\sigma_{r}(w)=12 \pi \frac{\Gamma(\psi \rightarrow 2 e) \Gamma(\psi \rightarrow 2 \tau)}{\left(w^{2}-M_{R}^{2}\right)^{2}+\Gamma_{t}^{2} M_{R}^{2}} \tag{2}
\end{equation*}
$$

where $w=2 E, M_{R}=3.685 \mathrm{GeV}, \Gamma_{t}=243 \mathrm{keV}, \Gamma(\psi \rightarrow 2 e)=21.4 \mathrm{keV}$, and $\Gamma(\psi \rightarrow 2 \tau)=$ $\Gamma(\psi \rightarrow 2 e)\left(\beta\left(3-\beta^{2}\right) / 2\right)$, with $\beta=\sqrt{1-\left(\left(2 M_{\tau}\right)^{2} / w^{2}\right)} \approx 0.26426$, and $\beta\left(2-\beta^{2}\right) / 2 \approx 0.38717$. At the peak of the resonance we have

$$
\begin{equation*}
\sigma_{r}\left(M_{R}\right)=\frac{12 \pi}{M_{R}^{2}} B^{2}(\psi \rightarrow 2 e) 0.38717=32.40 \times 10^{-33} \mathrm{~cm}^{2} \tag{3}
\end{equation*}
$$

The peak cross section of $e^{+} e^{-} \rightarrow \tau^{+} \tau^{-}$at continuum which occurs at $2 E=4.174 \mathrm{GeV}$ is $[3,4]$

$$
\begin{equation*}
\sigma_{c}(4.174)=\frac{\pi \alpha^{2}}{6} \times 1.036 \frac{1}{M_{\tau}^{2}}=3.562 \times 10^{-33} \mathrm{~cm}^{2} \tag{4}
\end{equation*}
$$

Thus

$$
\begin{equation*}
\frac{\sigma_{r}(3.685)}{\sigma_{c}(4.174)}=9.096 \tag{5}
\end{equation*}
$$

This number must be reduced because the machine width is much wider than the resonance width and the radiative corrections further broaden the effective machine width. This problem was first solved [7] by the author in 1974 immediately after the discovery of $J / \psi$. The most comprehensive account was given in Ref. [8] which we follow here. Qualitatively if the machine width is $\Delta$ and the resonance width is $\Gamma_{t}$, then only the fraction $\Gamma_{t} / \Delta$ of the beam is effective in producing the resonance peak if $\Delta \gg \Gamma_{t}$. The effect of radiative corrections can be estimated by the change in the height of the Gaussian peak of the machine energy by the radiative corrections because only the peak height matters when the resonance is narrower than the beam width. The result is [8]

$$
\begin{equation*}
\sigma_{\exp }(3.685)=\sigma_{r}(3.685)\left[\sqrt{\frac{\pi}{8}} \frac{\Gamma_{t}}{\Delta}\right]\left[\left(\frac{\sqrt{8} \Delta}{3.685}\right)^{T} \Gamma\left(\frac{T}{2}+1\right)\right]+\sigma_{c}(3.685) \tag{6}
\end{equation*}
$$

where $\sigma_{c}(3.685)=2.476 \times 10^{-33} \mathrm{~cm}$ is the continuum cross section, $\Delta$ is the Gaussian beam width defined by

$$
\begin{equation*}
G\left(w, w^{\prime}\right)=\frac{1}{\sqrt{2 \pi} \Delta} \exp \left[-\frac{\left(w-w^{\prime}\right)^{2}}{2 \Delta^{2}}\right] \tag{7}
\end{equation*}
$$

and is related to the full width at a half maximum (FWHM) by

$$
\begin{equation*}
\Delta=\frac{(F W H M)}{2.3848} \tag{8}
\end{equation*}
$$

$T$ is called the equivalent radiator thickness defined by

$$
\begin{equation*}
T=\frac{2 \alpha}{\pi}\left[\ln \frac{M_{R}^{2}}{m_{e}^{2}}-1\right]=0.14229 \tag{9}
\end{equation*}
$$

$\Gamma$ is the Gamma function and its value is

$$
\begin{equation*}
\Gamma\left(1+\frac{T}{2}\right)=0.96365 \tag{10}
\end{equation*}
$$

The first square bracket shows that only a fraction of the incoming beam, $\Gamma_{t} / \Delta$, is effective in producing the resonance. The factor $\sqrt{\pi / 8}$ comes form the fact that $\Gamma_{t}$ is the FWHM of the Breit-Wigner formula whereas FWHM of the Gaussian beam profile is given by Eq. (8). The second square bracket represents the reduction of the Gaussian peak height due to
the photon emission whose effective cutoff is $\Delta E=\sqrt{8} \Delta$. The Gamma function, Eq. (10), comes from the folding of the Gaussian function with the photon straggling function [8]. At the Beijing Electron-Positron Collider $\Delta=1.4 \mathrm{MeV}$ and thus from Eq. (6) we have

$$
\begin{equation*}
\sigma_{\exp }(2.685, \Delta=1.4 \mathrm{MeV})=0.0411 \sigma_{r}(3.685)+\sigma_{c}(3.685) \tag{11}
\end{equation*}
$$

there is a scheme [9] to make $\Delta$ as small as 0.14 MeV using a monochromatizer; we have then

$$
\begin{equation*}
\sigma_{\exp }(3.685, \Delta=0.14 \mathrm{MeV})=0.286 \sigma_{r}(3.685)+\sigma_{c}(3.685) \tag{12}
\end{equation*}
$$

Since the branching fraction to $\tau$ pair is $0.34 \%$ in $\sigma_{r}(3.685)$ there are several hundred $\pi$ 's for each $\tau$ pair produced by $\sigma_{r}(3.685)$.

The BES Collaboration [6] has successfully carried out $\tau$ experiments using $\psi^{\prime}$ under the conditions shown in Eq. (11), where the first term is about 0.48 of the last term. For their experiment the hadron background did not cause any problem for four reasons: (1) most of the hadron backgrounds are multiprong events whereas $\tau$ events are mostly twoprong events. This fact can be used to eliminate the background. (2) They did not use the monochromatizer. (3) Particle ID has about $10^{-3}$ efficiency. (4) Accuracy of $10^{-2}$ is good enough for them, whereas CP experiment needs $10^{-3}$ accuracy.

An alternative to use Eq. (11) or (12) is to avoid $\psi^{\prime}$ all together and run the machine at a slightly lower energy, say at 3.680 GeV . From the consideration of background this is probably the ideal energy to run the Tau-Charm Factory. At $W=3.680 \mathrm{GeV}$ the component of polarization of $\tau^{ \pm}$in the beam direction averaged over the production angle is slightly improved:

$$
\bar{w}_{z}=\int_{-1}^{1} w_{z} \frac{d \sigma}{d \cos \theta} d \cos \theta / \sigma=\frac{w_{1}+w_{2}}{1+w_{1} w_{2}} \frac{1+2 a}{2+a^{2}} \equiv \frac{w_{1}+w_{2}}{1+w_{1} w_{2}} F(a),
$$

where $a=2 M_{\tau} / W$. At $w=4.174 \mathrm{GeV}$ we have $F=0.992$, but at $w=3.680 \mathrm{GeV}$ we have $F=0.9996$. The cross section is reduced from $\sigma_{c}(4.174)=3.562 \times 10^{-33} \mathrm{~cm}^{2}$ to $\sigma_{c}(3.680)=2.44 \times 10^{-33} \mathrm{~cm}^{2}$. This energy is preferred in order to avoid both the charm background and overabundance of hadrons in the $\psi(2 s)$ peak.

## Acknowledgments

The author wishes to thank Bill Dunwoodie, W.K.H. Panofsky, and Martin Perl for discussions on the optimum energy for doing $\tau$ physics at the Tau-Charm Factory. I would also like to thank Karl Brown for explaining to me how the beam monochromatizer works.

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[^0]:    *Work supported by the Department of Energy, contract DE-AC03-76SF00515.

