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PHYSICAL PARAMETERS AND RENORMALIZATION OF $U(1)^a \times U(1)^b$ MODELS

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ABSTRACT

We analize the structure of models with unbroken and spontaneously broken $U(1)^a \times U(1)^b$ gauge symmetry. We show that the quantum corrections to the $2N$ gauge charges, with N = number of fermions + number of scalars, can be absorbed in the redefinition of *three* independent gauge couplings (g^a, g^b, and g^{ab}). We establish the (one-loop) conditions on the matter content for $g^{ab} = 0$ (a value usually assumed in the literature) and we show that in the minimal extensions of the Standard Model with an extra $U(1)$ symmetry the choice $g^{ab} = 0$ is not stable under radiative corrections induced by the standard Higgs fields. Moreover, $g^{ab} = 0$ to all orders seems to require an exact symmetry. The spontaneous breaking of the gauge symmetry induces further mixing between the two gauge bosons and introduces a fourth independent physical parameter. A consequence of our analysis is that the usual tree-level description with only three physical parameters (*i.e.*, two gauge couplings and one gauge boson mixing angle) is not in general a justified zero order limit of the treatment including radiative corrections.

1. Introduction.

The extensions of the Standard Model (SM) with an extra $U(1)$ gauge symmetry have been extensively studied during the last years [1]. They appear as a possible low-energy limit in many grand unified scenarios [2], and they are not banished to very high energies by present data [3]. As a matter of fact, precision experiments at LEP as well as direct searches at large hadron colliders (TEVATRON) set (stringent) limits on new gauge interactions, but do not exclude their discovery at future colliders (LHC or NLC) [4].

Usually these analyses assume definite models with few free parameters. In this way, the fits (which often depend also on few independent observables) are simplified. Beyond tree level, however, the number of free parameters is related to the number of independent renormalized parameters. Hence, if a parameter is not let to vary, one must make sure that the constrained model is stable under quantum corrections. In the case of extra gauge interactions this is a delicate point [5]. In this paper we study at the one-loop level models which include a sector with $U(1)^a \times U(1)^b$ gauge symmetry. From our analysis it follows that a generic extension of the SM with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ has four new free parameters: the mass $M_{Z'}$ of the extra vector boson Z'; the mixing angle ϕ between the Z (mass eigenstate) and Z'^0 (Y'-current eigenstate) vector bosons; the overall strength g_2 of the new Y' current; and the mixing g_{12} [6] of the Y' current with the standard hypercharge Y. In particular, we prove that g_{12} is a free parameter in these models: it is physical (to be determined experimentally) and necessary to absorb the *infinities* when calculating quantum corrections. g_{12} can be consistently ignored if an extra symmetry is present in the theory but this is not the case in many popular models. We do not claim that the effects due to a non-vanishing g_{12} are always sizeable and they could not be neglected in a tree-level analysis. However, it is worth to emphasize that:

• When obtaining experimental bounds on Z' models, the nonstandard (tree-level) contributions are often calculated assuming $g_{12} = 0$ and varying the extra

Y' charge (for example, considering different combinations of the two nonstandard $U(1)$ subalgebras in E_6). It seems more systematic, however, to stick to a particular model (which corresponds to a definite Y' charge), and to let all its free parameters including g_{12} vary, rather than constraining the whole class of models with a particular choice of one free parameter.

• The three gauge couplings g_1, g_2, and g_{12} cover the whole parameter space of a model with a gauge symmetry subgroup $U(1)_Y \times U(1)_{Y'}$. For example, the $U(1)_Y \times U(1)_{Y'} \subset SU(2)_R \times U(1)_{B-L}$ sector of left-right symmetric models (resulting from $SO(10)$) is a particular case with the two new gauge couplings related:

$$g_1 = \frac{g_R g_{B-L}}{\sqrt{\frac{2}{5}g_R^2 + \frac{3}{5}g_{B-L}^2}},\ g_2 = \sqrt{\frac{2}{5}g_R^2 + \frac{3}{5}g_{B-L}^2},\ g_{12} = \frac{g_R^2 - g_{B-L}^2}{\sqrt{\frac{5}{3}g_R^2 + \frac{5}{2}g_{B-L}^2}} \tag{1}$$

(this model is known as the χ model in the literature (see Ref. [1] for definitions)). In general, since the three parameters get renormalized and *run* with the scale, they bring information of other (larger) scales (which may point out to grand unification, left-right symmetry at higher scales, etc). In particular, if $g_1,\ g_2$ and g_{12} satisfy Eq. (1) at some scale, it indicates that there is left-right symmetry restoration at that scale.

• At any rate, in the absence of extra symmetries, a fully consistent one-loop analysis of precision data including a relatively light Z' requires considering g_{12} as a physical parameter.

As a first step to analyse Z' extensions of the SM at one loop, we study in this paper the structure of models with $U(1)^a \times U(1)^b$ gauge symmetry [7]. In Section 2 we discuss the tree-level Lagrangian. In Section 3 we fix the choice of renormalized parameters and introduce our renormalization (on-shell) scheme. The one-loop renormalization of the $U(1)^a \times U(1)^b$ model is worked out in detail, emphasizing the need of an exact extra symmetry to guarantee that g_{12} can be neglected to all orders. (In the Abelian case discussed here we denote this gauge coupling g^{ab}.) We assume throughout the paper that the theory is vectorlike, although our examples are based on realistic extensions of the SM. Thus, it must not create any confusion when we refer to $SO(10)$ or left-right models to specify $U(1)^a \times U(1)^b$ matter contents (models). The results which we illustrate with these examples apply in both cases, except for simple modifications (factors). In Section 4 we present the renormalization of the model with spontaneously broken symmetry. In this case three renormalized parameters in the Higgs potential are replaced by the two heavy gauge boson masses and their mixing angle. Section 5 is devoted to conclusions.

2. Classical Lagrangian and physical parameters.

The classical Lagrangian for n fermions f_i and m scalars ϕ_i with $U(1)^a \times U(1)^b$ gauge symmetry reads

$$\mathcal{L}^{class} = -\frac{1}{4} F^T_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{n} \overline{f}_i (i \not{D} - m_i) f_i + \sum_{i=1}^{m} (D_\mu \phi_i)^\dagger (D^\mu \phi_i) - V(\phi_i) \; , \tag{2}$$

where the antisymmetric tensor

$$F_{\mu\nu} = \begin{pmatrix} F^a_{\mu\nu} \\ F^b_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \\ \partial_\mu A^b_\nu - \partial_\nu A^b_\mu \end{pmatrix} \tag{3}$$

and the covariant derivatives

$$\begin{aligned} D_\mu f_i &= \partial_\mu f_i + i (\tilde{q}^a_i \; \tilde{q}^b_i) \begin{pmatrix} A^a_\mu \\ A^b_\mu \end{pmatrix} f_i \; , \\ D_\mu \phi_i &= \partial_\mu \phi_i + i (\tilde{Q}^a_i \; \tilde{Q}^b_i) \begin{pmatrix} A^a_\mu \\ A^b_\mu \end{pmatrix} \phi_i \; . \end{aligned} \tag{4}$$

$A^{a,b}$ are the two gauge boson fields, and $\tilde{q}^{a,b}_i$ and $\tilde{Q}^{a,b}_i$ the $2N$ $(N = n + m)$ fermion and scalar charges, respectively, whereas $V(\phi_i)$ is a polinomial of at most fourth order preserving the $U(1)^a \times U(1)^b$ gauge symmetry. $\mathcal{L}$ is the most general Lagrangian renormalizable by power counting and invariant under the transformations

$$\begin{aligned} f_i &\rightarrow \exp\{-i (\tilde{q}^a_i \; \tilde{q}^b_i) \begin{pmatrix} \theta^a \\ \theta^b \end{pmatrix} \} \; f_i \; ; \\ \phi_i &\rightarrow \exp\{-i (\tilde{Q}^a_i \; \tilde{Q}^b_i) \begin{pmatrix} \theta^a \\ \theta^b \end{pmatrix} \} \; \phi_i \; ; \\ \begin{pmatrix} A^a_\mu \\ A^b_\mu \end{pmatrix} &\rightarrow \begin{pmatrix} A^a_\mu \\ A^b_\mu \end{pmatrix} + \begin{pmatrix} \partial_\mu \theta^a \\ \partial_\mu \theta^b \end{pmatrix} \; , \end{aligned} \tag{5}$$

with $\theta^{a,b}$ the two gauge parameters.

The invariance of $F_{\mu\nu}$ under gauge transformations also allows for a gauge kinetic term of the form $F^T_{\mu\nu} K F^{\mu\nu}$, with K an arbitrary 2×2 symmetric matrix. K can be absorbed, however, into a vector boson field redefinition (note that a redefinition of A_μ also redefines the $2N$ charges of the matter fields in Eq. (4)). Without loss of generality we can then assume $K = -\frac{1}{4} I$, still leaving the arbitrarity of rotating the two gauge fields. Then only $2N-1$ charges are physical, since the rotation left, which is related to the impossibility of distinguishing on physical grounds between the two degenerate (massless) gauge bosons, can be used to fix one of the $2N$ charges to zero. Hence the $2N$ charges are determined fixing one charge conventionally and fitting $2N-1$ independent experiments. We assume fermion fields with masses m_i, whereas $V(\phi_i)$ includes scalar masses and couplings. We also assume that the Yukawa couplings are forbidden by some symmetry, for they are not important for our discussion.

In summary, $\mathcal{L}$ in Eq. (1) is a generic (classical) Lagrangian of at most dimension four with $U(1)^a \times U(1)^b$ gauge symmetry. This is not altered at the quantum level: the theory is renormalizable [8] and gauge invariance does not allow for any other term. Among the physical parameters of the model, however, quantum corrections can be used to distinguish between those which are *renormalized* (and in this sense are free) from those which are *constants*. For example, in QED with just a $U(1)$ gauge symmetry and N matter fields there is one free parameter, the electric charge e usually identified with the charge of the proton, and $N-1$ constants, the ratios of the remaining charges to e. As we shall show, the gauge sector in a model with $U(1)^a \times U(1)^b$ symmetry and N matter fields depends on *three* free parameters and $2N-4$ constants: the charges $(\tilde{q}^a_i \ \tilde{q}^b_i)$ can be splitted into

$$(\tilde{q}^a_i \ \tilde{q}^b_i) \equiv (q^a_i \ q^b_i) \begin{pmatrix} g^a & g^{ab} \\ 0 & g^b \end{pmatrix} , \qquad (6)$$

where $(q^a_i \ q^b_i)$ are constant charges (four of them fixed arbitrarily) and g^a, g^{ab}, and g^b three parameters (gauge couplings) which will absorb all the quantum corrections. (We use only one superscript, a, b, for diagonal terms g^a, g^b.) In general, beyond tree level, just two gauge couplings g^a and g^b (one for each $U(1)$ subgroup) are not enough to renormalize the theory.

Obviously for $N = 1$ there is only one free parameter (independent charge), since in this case the gauge fields can be rotated to decouple completely one gauge boson. For $N > 1$, three experiments involving two matter fields with independent charges (let say $\tilde{q}^a_{1,2}\ \tilde{q}^b_{1,2}$) can be used to fix g^a, g^{ab} and g^b (once fixed one $\tilde{q}$ charge and the 4 charges $(q^a_{1,2}\ q^b_{1,2})$ conventionally); the remaining charges would then be fixed after determining $(\tilde{q}^a_i\ \tilde{q}^b_i)$, $i = 3, ..., N$ from $2N-4$ independent experiments:

$$(q^a_i\ q^b_i) = (\tilde{q}^a_i\ \tilde{q}^b_i) \begin{pmatrix} \frac{1}{g^a} & -\frac{g^{ab}}{g^a g^b} \\ 0 & \frac{1}{g^b} \end{pmatrix} , \quad (i = 3, ..., N) \ . \tag{7}$$

In spontaneously broken theories the former discussion applies but the gauge boson mass eigenstate bases are fixed and there is no freedom to rotate them. Hence, in the broken case there are $2N$ physical charges and $2N$ independent experiments are needed to fix them. Then Eq. (6) remains general, $g'^{ba} \neq 0$ (we use a prime to denote the couplings to mass eigenstates),

$$(\tilde{q}'^a_i\ \tilde{q}'^b_i) \equiv (q^a_i\ q^b_i) \begin{pmatrix} g'^a & g'^{ab} \\ g'^{ba} & g'^b \end{pmatrix} \equiv (q^a_i\ q^b_i) \begin{pmatrix} g^a & g^{ab} \\ 0 & g^b \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \ . \tag{8}$$

ϕ is the angle rotating from the gauge boson basis triangularizing the renormalized gauge coupling matrix to the renormalized gauge boson mass eigenstate basis.

3. Renormalization of $U(1)^a \times U(1)^b$: unbroken case.

In this Section we study the renormalization of a theory with unbroken gauge symmetry $U(1)^a \times U(1)^b$. We discuss the parametrization of the gauge couplings (valid to all orders) and work out in detail their renormalization in the on-shell scheme at one loop. We show that even the popular (minimal) extensions of the standard model with one extra $U(1)$ require two new gauge couplings in order to cancel the divergent contribution of the Higgs fields. Moreover, even if the models are enlarged adding extra matter in order to fulfil the one-loop conditions for consistently neglecting the second gauge coupling g^{ab}, there is no guarantee for the cancellation of infinities at two loops. As a matter of fact, in the examples we have looked at the cancellation of infinities to all orders requires an exact extra symmetry: a more general gauge invariance or its discrete remnant.

The renormalized Lagrangian in terms of bare quantities has the same expression as the classical Lagrangian in Eq. (2)

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F^{0\ T}_{\mu\nu}F^{0\mu\nu} + \sum_{i=1}^{n}\overline{f}^0_i(i\ \not{D}\ - m^0_i)f^0_i \\ & + \sum_{i=1}^{m}[(D_\mu\phi^0_i)^\dagger(D^\mu\phi^0_i) - \mu^{0\ 2}_i\phi^{0\dagger}_i\phi^0_i] \\ & - V^{(>2)}(\phi^0_i) - \frac{1}{2}\partial^\mu A^{0\ T}_\mu\xi^{0\ -1}\partial^\nu A^0_\nu\ ,\end{aligned} \tag{9}$$

where μ^0_i are the scalar masses and $V^{(>2)}$ contains the terms of dimension 3 and 4. A covariant gauge fixing term has been added. In this gauge the ghosts decouple. Both ultraviolet and infrared divergences are regularized using dimensional regularization. Renormalized fields and couplings are related to these bare quantities (we denote fermion and scalar charges by little q when referring to both)

$$\begin{aligned}& A^0_\mu = Z^{\frac{1}{2}}_A A_\mu\ ;\ \ f^0_i = Z^{\frac{1}{2}}_{f_i} f_i\ ;\ \ \phi^0_i = Z^{\frac{1}{2}}_{\phi_i}\phi_i\ ; \\ & \tilde{q}^{0\ T}_i = \tilde{q}^T_i Z_{\tilde{q}_i}\ ;\ \ m^0_i = m_i + \delta m_i\ ;\ \ \mu^{0\ 2}_i = \mu^2_i + \delta\mu^2_i\ ;\end{aligned} \tag{10}$$

and analogously for the couplings in $V^{(>2)}$ and for $\xi^{0\ -1}$. $A^{(0)}_\mu$ and $\tilde{q}^{(0)}_i$ are 2 dimensional vectors and $\xi^{(0)}, Z^{\frac{1}{2}}_A$ and $Z_{\tilde{q}_i}$ are 2×2 matrices. The non-diagonal terms generate counterterms which will be needed to cancel infinities. The gauge symmetry translates into Ward identities for Green functions. In particular for renormalized one-particle irreducible Green functions,

$$\partial_z{}^\mu\langle\bar{f}_i(x)f_i(y)A^r_\mu(z)\rangle^{\text{irre}} \ = \ i\tilde{q}^u_i Z^{uv}_{\tilde{q}_i} Z^{\frac{1}{2}\ vr}_A\langle\bar{f}_i(x)f_i(y)\rangle^{\text{irre}}(\delta(z-y)-\delta(z-x))\ , \tag{11}$$

and analogously for scalar fields. The finiteness of the other quantities in Eq. (11) implies that the product $Z_{\tilde{q}_i}Z^{\frac{1}{2}}_A$ is also finite. As a matter of fact

$$Z_{\tilde{q}_i}Z^{\frac{1}{2}}_A = 1 \tag{12}$$

in appropriate renormalization schemes such as minimal subtraction and on-shell (see the Appendix). Then $Z_{\tilde{q}_i}$ is independent of the matter field i and equal to $Z_A^{-\frac{1}{2}}$; and Eq. (11) for the gauge couplings reads [9]

$$(\tilde{q}_i^{0\ a}\ \tilde{q}_i^{0\ b}) = (\tilde{q}_i^{a}\ \tilde{q}_i^{b}) \begin{pmatrix} Z_A^{-\frac{1}{2}\ aa} & Z_A^{-\frac{1}{2}\ ab} \\ Z_A^{-\frac{1}{2}\ ba} & Z_A^{-\frac{1}{2}\ bb} \end{pmatrix} . \tag{13}$$

This is the generalization of the constancy to all orders of the charge ratios in QED to the case of $U(1)^a \times U(1)^b$. Splitting

$$(\tilde{q}_i^{a}\ \tilde{q}_i^{b}) \equiv (q_i^{a}\ q_i^{b}) \begin{pmatrix} g^a & g^{ab} \\ g^{ba} & g^b \end{pmatrix} , \tag{14}$$

and similarly for the bare couplings, Eq. (13) implies

$$\begin{aligned} &(q_i^{0\ a}\ q_i^{0\ b}) = (q_i^{a}\ q_i^{b}) ; \\ &\begin{pmatrix} g^{0\ a} & g^{0\ ab} \\ g^{0\ ba} & g^{0\ b} \end{pmatrix} = \begin{pmatrix} g^a & g^{ab} \\ g^{ba} & g^b \end{pmatrix} \begin{pmatrix} Z_A^{-\frac{1}{2}\ aa} & Z_A^{-\frac{1}{2}\ ab} \\ Z_A^{-\frac{1}{2}\ ba} & Z_A^{-\frac{1}{2}\ bb} \end{pmatrix} . \end{aligned} \tag{15}$$

Hence, it is possible also in this case to define charges which do not renormalize, $q_i^{(0)\ a,b}$, but to absorb all quantum corrections we must introduce a 2×2 matrix of gauge couplings, $\begin{pmatrix} g^{(0)\ a} & g^{(0)\ ab} \\ g^{(0)\ ba} & g^{(0)\ b} \end{pmatrix}$. To determine them, 4 charges defining a 2×2 invertible matrix, *e.g.*, $q_{1,2}^{a,b}$, must be fixed conventionally in Eq. (14). In the unbroken case with $U(1) \times U(1)$ gauge symmetry, however, Eq. (15) is too general. The freedom to define (rotate) the renormalized gauge fields in Eq. (10) allows to assume $g^{ba} = 0$ in Eqs. (14) and (15), and thus the matrix g triangular. Besides, the freedom to rotate the gauge bosons in Eq. (9) allows to assume $Z_A^{\frac{1}{2}\ ba} = 0$ in Eq. (10), and thus the matrix $Z_A^{\frac{1}{2}}$ (and its inverse $Z_A^{-\frac{1}{2}}$ in Eqs. (13) and (15)) triangular. Both minimal subtraction and on-shell schemes are compatible with this choice. Under this rotation the gauge fixing matrix $\xi^{0\ -1}$ also transforms, but it was arbitrary, although fixed. (The Ward identity for the gauge boson propagator implies that the gauge fixing term does not renormalize, *i. e.*, $\xi^{0\ -1} = Z_A^{-\frac{1}{2}\ T} \xi^{-1} Z_A^{-\frac{1}{2}}$). With the former choices the right-hand side of Eq. (15) gives a triangular g^0 matrix:

$$\begin{pmatrix} g^{0\ a} & g^{0\ ab} \\ 0 & g^{0\ b} \end{pmatrix} = \begin{pmatrix} g^a & g^{ab} \\ 0 & g^b \end{pmatrix} \begin{pmatrix} Z_A^{-\frac{1}{2}\ aa} & Z_A^{-\frac{1}{2}\ ab} \\ 0 & Z_A^{-\frac{1}{2}\ bb} \end{pmatrix} . \tag{16}$$

This is our main result: the renormalization of the gauge couplings in models with two abelian gauge symmetries requires three couplings $g^{(0)\ a}, g^{(0)\ ab}, g^{(0)\ b}$, satisfying Eq. (16). We have used the freedom existing in defining the degenerate (massless) gauge bosons.

Let us make explicit this analysis to one loop. Following the on-shell scheme prescription in the Appendix we evaluate the renormalized vector boson proper selfenergies. These can be written as the sum of transverse and longitudinal parts:

$$i\Pi^{rs}_{\mu\nu}(q^2) = i[\mathcal{A}^{rs}(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \mathcal{B}^{rs}(q^2)\frac{q_\mu q_\nu}{q^2}] \ . \tag{17}$$

Using the Feynman rules in Ref. [10] with $\xi = I$, we find from the diagrams in Fig. 1 (excluding the fourth diagram which only contributes in the broken case)

$$\begin{aligned}\begin{pmatrix} \mathcal{A}^{aa} & \mathcal{A}^{ab} \\ \mathcal{A}^{ba} & \mathcal{A}^{bb} \end{pmatrix} = & -q^2\frac{1}{16\pi^2}\{\sum_{i=1}^{n}[\frac{4}{3}C_{UV} - 8\int_0^1 dx\, x(1-x)\ln D_{f_i}]\begin{pmatrix} (\tilde{q}_i^a)^2 & \tilde{q}_i^a\tilde{q}_i^b \\ \tilde{q}_i^b\tilde{q}_i^a & (\tilde{q}_i^b)^2 \end{pmatrix} \\ & + \sum_{i=1}^{m}[\frac{1}{3}C_{UV} + \frac{1}{3} + \frac{2}{q^2}(\int_0^1 dx\, D_{\phi_i}\ln D_{\phi_i} - \mu_i^2\ln\mu_i^2)]\begin{pmatrix} (\tilde{Q}_i^a)^2 & \tilde{Q}_i^a\tilde{Q}_i^b \\ \tilde{Q}_i^b\tilde{Q}_i^a & (\tilde{Q}_i^b)^2 \end{pmatrix}\} \\ & -q^2\begin{pmatrix} 2(Z_A^{\frac{1}{2}\ aa} - 1) & Z_A^{\frac{1}{2}\ ba} + Z_A^{\frac{1}{2}\ ab} \\ Z_A^{\frac{1}{2}\ ab} + Z_A^{\frac{1}{2}\ ba} & 2(Z_A^{\frac{1}{2}\ bb} - 1) \end{pmatrix} ,\end{aligned} \tag{18}$$

with $C_{UV} = (\frac{1}{\epsilon} - \gamma + \ln 4\pi)$, $\epsilon = (4-d)$, $D_{f_i} = m_i^2 - q^2x(1-x)$, and $D_{\phi_i} = \mu_i^2 - q^2x(1-x)$. The last term in Eq. (18) stands for the one-loop counterterms. They result from expanding $Z_A = Z_A^{\frac{1}{2}\ T} Z_A^{\frac{1}{2}}$:

$$\begin{aligned}\begin{pmatrix} \sum\limits_{r=a,b} Z_A^{\frac{1}{2}\ ra} Z_A^{\frac{1}{2}\ ra} & \sum\limits_{r=a,b} Z_A^{\frac{1}{2}\ ra} Z_A^{\frac{1}{2}\ rb} \\ \sum\limits_{r=a,b} Z_A^{\frac{1}{2}\ rb} Z_A^{\frac{1}{2}\ ra} & \sum\limits_{r=a,b} Z_A^{\frac{1}{2}\ rb} Z_A^{\frac{1}{2}\ rb} \end{pmatrix} = & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \\ & \begin{pmatrix} 2(Z_A^{\frac{1}{2}\ aa} - 1) & Z_A^{\frac{1}{2}\ ba} + Z_A^{\frac{1}{2}\ ab} \\ Z_A^{\frac{1}{2}\ ab} + Z_A^{\frac{1}{2}\ ba} & 2(Z_A^{\frac{1}{2}\ bb} - 1) \end{pmatrix} + \dots .\end{aligned} \tag{19}$$

Eq. (19) makes apparent that $\mathcal{A}^{rs}$ (which is symmetric) in (18) and the corresponding on-shell conditions (gauge invariance assures $\mathcal{A}^{rs}(0) = 0$)

$$\frac{\partial\mathcal{A}^{rs}}{\partial q^2}|_{q^2=0} = 0 \ , \tag{20}$$

are independent of the choice (rotation) of the bare gauge field basis. At one loop this means that the three conditions in Eq. (20) fix $Z_A^{\frac{1}{2}\ aa}$, $Z_A^{\frac{1}{2}\ bb}$, and $Z_A^{\frac{1}{2}\ ab}+Z_A^{\frac{1}{2}\ ba}$. Hence, we can assume in agreement with Eq. (16) that $Z_A^{\frac{1}{2}\ ba}=0$ and use Eq. (20) to fix the remaining $Z_A^{\frac{1}{2}}$ matrix elements, and in turn the matrix elements of the inverse matrix $Z_A^{-\frac{1}{2}}$:

$$
\begin{aligned}
Z_A^{-\frac{1}{2}\ aa} &= 1+\frac{1}{32\pi^2}\{\sum_{i=1}^{n}\frac{4}{3}[C_{UV}-\ln m_i^2](\tilde{q}_i^a)^2+\sum_{i=1}^{m}\frac{1}{3}[C_{UV}-\ln \mu_i^2)](\tilde{Q}_i^a)^2\}\ , \\
Z_A^{-\frac{1}{2}\ ab} &= \frac{1}{16\pi^2}\{\sum_{i=1}^{n}\frac{4}{3}[C_{UV}-\ln m_i^2]\tilde{q}_i^a\tilde{q}_i^b+\sum_{i=1}^{m}\frac{1}{3}[C_{UV}-\ln \mu_i^2)]\tilde{Q}_i^a\tilde{Q}_i^b\}\ , \\
Z_A^{-\frac{1}{2}\ bb} &= 1+\frac{1}{32\pi^2}\{\sum_{i=1}^{n}\frac{4}{3}[C_{UV}-\ln m_i^2](\tilde{q}_i^b)^2+\sum_{i=1}^{m}\frac{1}{3}[C_{UV}-\ln \mu_i^2)](\tilde{Q}_i^b)^2\}\ .
\end{aligned}
\tag{21}
$$

Thus, in general $Z_A^{-\frac{1}{2}\ ab}$ is infinite and then $g^{0\ ab}$ too (see Eq. (16)).

The corresponding on-shell conditions on the fermion and the scalar selfenergies fix the field renormalization constants $Z_{f_i}^{\frac{1}{2}}$ and $Z_{\phi_i}^{\frac{1}{2}}$ and the mass countertems δm_i and $\delta\mu_i$, whereas the scalar three- and four-point functions are renormalized by the appropiate choice of renormalization constants.

It is interesting to know under which assumptions one can neglect g^{ab}, because it is convenient to have as few free parameters as possible when performing fits to experimental data. At any rate many existing bounds on gauge extensions of the standard model have been obtained fixing $g^{ab}=0$. (g^{ab} is a physical parameter and its experimental value can be compatible with zero accidentally.) The question is whether it renormalizes or not. Generically the answer depends on the renormalization scheme. What we really want to know is if a scheme (and a model) exists where $Z_A^{-\frac{1}{2}\ ab}=0$. In this case g^{ab}/g^b is constant under renormalization and the particular choice $g^{ab}=0$ is stable and consistent (although not necessary). At one loop the infinite part of $Z_A^{-\frac{1}{2}\ ab}$ cancels if (assuming $g^{ab}=0$)

$$
\frac{4}{3}\sum_{i=1}^{n}q_i^a q_i^b+\frac{1}{3}\sum_{i=1}^{m}Q_i^a Q_i^b=0 \tag{22}
$$

(see Eq. (21)). In chiral theories $\frac{4}{3}$ is replaced by $\frac{2}{3}$ and i runs over the 2-component spinors. Eq. (22) is fulfilled if the fermion and scalar fields define *complete* multiplets of a simple group containing one (or both) $U(1)$ factor(s). For example, this is the case if the matter contents of the $U(1)^a \times U(1)^b$ model defines complete multiplets of $SU(2)^a \times U(1)^b (\supset U(1)^a \times U(1)^b)$ or $SO(10)(\supset U(1)^a \times U(1)^b)$. We do not see, however, any necessity (based on anomaly cancellation, minimality, or grand unification) to assume this, specially in the scalar sector. In particular, consider the minimal model in Table 1 where $SO(10)$ is broken at very large scales ($\approx 10^{15}$ GeV) to $SU(3)_C \times SU(2)_L \times U(1)^a \times U(1)^b$, with $U(1)^a$ the hypercharge, Y, and $U(1)^b$ the extra $U(1)$ in $SO(10)$, usually denoted $U(1)_\chi$ [1]. If one assumes a minimal fermion content of 3 chiral families in the **16** representation, all of them *survive* the breaking of $SO(10)$ and the fermion contribution to $g^{Y\chi}$ is zero ($\sum_{i=1}^{n} q_i^Y q_i^\chi = 0$ for the fermions in Table 1). In the scalar sector, however, one usually accommodates the Higgs doublets in the **10** representation of $SO(10)$; when this group is broken there is no reason to keep the leptoquarks in the **10** light, with masses identical to those of the Higgs fields (on the contrary, it is phenomenologically preferred to give them large masses). The same argument applies in supersymmetric extensions of the SM with an extra $U(1)$. Then radiative corrections induced by the light Higgses generate a nonzero $g^{Y\chi}$ gauge coupling even in the minimal scenarios. $\sum_{i=1}^{m} Q_i^Y Q_i^\chi = 2\frac{1}{5}\sqrt{\frac{3}{2}}$ for the scalars in Table 1: one $SU(2)_L$ doublet and one singlet, together with their complex conjugated representations. One can insist in adding extra light matter (scalars) in order to satisfy Eq. (22) but this would not guarantee that $Z_A^{-\frac{1}{2}\ ab}$ is finite at two loops. For instance, the first diagram in Fig. 1 with a gauge boson crossing the fermion bubble vertically is proportional to $\sum_{x=a,b} \sum_{i=1}^{n} q_i^a q_i^x q_i^b q_i^x$ (and similarly for other diagrams). In contrast with the corresponding one-loop contribution this two-loop diagram is infinite for the χ model

$$\sum_{x=Y,\chi} \sum_{i=1}^{n} q_i^Y q_i^x q_i^\chi q_i^x = 3(-\frac{1}{10}\sqrt{\frac{1}{6}}). \tag{23}$$

And there is no reason for cancellations among diagrams. Hence one expects (although small) nonzero contributions to $g^{Y\chi}$ after renormalizing from the unification scale [9].

Let us compare this model with the LR model [1] in the same Table: $U(1)^a$ is the third component of $SU(2)_R$, T_3^R, and $U(1)^b$ is the baryon minus the lepton $(B-L)$ number, Q_{B-L}. This model is also contained in $SO(10)$ and if we also assume a minimal fermion contents of 3 chiral families in the **16** representation, $\sum_{i=1}^{n} q_i^R q_i^{B-L} = 0$ at one loop and $\sum_{x=R,B-L} \sum_{i=1}^{n} q_i^R q_i^x q_i^{B-L} q_i^x = 0$ at two loops. In fact $Z_A^{-\frac{1}{2}\ RB-L} = 0$ to all orders. This follows from the vanishing of $\mathcal{A}^{RB-L}$ in Eq. (18), what is guaranteed by an exact symmetry interchanging $u^c \leftrightarrow d^c, e^c \leftrightarrow \nu^c$ and changing the sign of the R gauge boson, $A^R \to -A^R$ but leaving unchanged the $B-L$ gauge boson, A^{B-L}. The Higgs sector has to be enlarged to maintain $\mathcal{A}^{RB-L} = 0$: at least one scalar must be added with the same quantum numbers as e^c in order to complete an $SU(2)_R$ doublet with N^c, and similarly for $\overline{N}^c$.

These two models illustrate the different cases:

- In general $Z_A^{-\frac{1}{2}\ ab}$ is infinite and g^{ab} is not only a physical parameter but a necessary one to absorb the infinities of the theory, as in the $U(1)_Y \times U(1)_\chi$ model.

- If as in the LR model $U(1)_R \times U(1)_{B-L}$ there is an exact symmetry requiring $Z_A^{-\frac{1}{2}\ ab} = 0$, g^{ab} can be consistently neglected. The exact symmetry in this model is a discrete remnant of the $SU(2)_R$ symmetry embedded in $SO(10)$.

- If a $U(1)$ factor is part of a non-abelian gauge group then gauge invariance guarantees the vanishing of $Z_A^{-\frac{1}{2}\ ab}$ and $g^{(0)\ ab}$. In the LR model this is guaranteed by $SU(2)_R \times U(1)_{B-L}$. (This is similar to the SM case where $Z_A^{-\frac{1}{2}\ LY} = 0$ is implied by $SU(2)_L \times U(1)_Y$.)

- In the χ model one can gauge $SU(5)$ (which contains the hypercharge), completing matter and vector boson representations, to guarantee $Z_A^{-\frac{1}{2}\ Y\chi}$ and $g^{(0)\ Y\chi}$ zero. Then $\sum_{x \in SU(5)\times U(1)_\chi} \sum_{i=1}^{n} q_i^Y q_i^x q_i^\chi q_i^x = 0$.

It is worth to emphasize that although $Y = \sqrt{\frac{3}{5}} T_3^R + \sqrt{\frac{2}{5}} Q_{B-L}$, $Q_\chi = \sqrt{\frac{2}{5}} T_3^R - \sqrt{\frac{3}{5}} Q_{B-L}$, the (generalized) χ and LR models are equivalent only if g^{ab} is included:

- $g^{RB-L} \neq 0$ violates the (discrete) symmetry and $Z_A^{-\frac{1}{2}\ RB-L}$ is infinite.
- If we *write* the LR model ($g^{RB-L} = 0$) in the Y, χ basis, $g^Y,\ g^\chi$ and $g^{Y\chi}$ (as well as $Z_A^{-\frac{1}{2}\ Y,\chi,Y\chi}$) are related (see Eq. (1)).

$Z_A^{-\frac{1}{2}}$ in the on-shell scheme (Eq. (21)) has also finite contributions. (In the minimal substraction scheme there are no such contributions.) They also cancel if there is an exact symmmetry distinguishing a and b and constraining the fermion and scalar masses, as in the LR model. If the masses violate the symmetry, one must expect that they will generate infinite $Z_A^{-\frac{1}{2}\ ab}$ contributions at higher orders, and a nonzero g^{ab}.

4. Renormalization of spontaneously broken $U(1)^a \times U(1)^b$.

The results of the unbroken case apply to the spontaneously broken phase [8]. It will be more convenient, however, to make a different choice of gauge fixing term in order to simplify real calculations and of renormalization conditions to improve the comparison with data in extended electroweak models.

In the broken phase the scalar fields in Eq. (9) with nonvanishing VEVs $v_i^0,\ i = 1, ..., l$, (that we assume to be real)

$$\phi_i^0 = \frac{1}{\sqrt{2}}(v_i^0 + h_i^0 + i\chi_i^0) \ , \tag{24}$$

are shifted. The term in the Lagrangian involving the covariant derivative of these scalars gives rise to the vector boson mass matrix:

$$\begin{aligned}\sum_{i=1}^{l}(D_\mu\phi_i^0)^\dagger(D^\mu\phi_i^0) = &\sum_{i=1}^{l}\{\frac{1}{2}(\partial_\mu h_i^0\partial^\mu h_i^0 + \partial_\mu\chi_i^0\partial^\mu\chi_i^0) + \chi_i^0\partial_\mu h_i^0\tilde{Q}_i^{0\ T}A^{0\ \mu} + \\ &(v_i^0 + h_i^0)\partial_\mu\chi_i^0\tilde{Q}_i^{0\ T}A^{0\ \mu} + \frac{1}{2}A_\mu^{0\ T}\tilde{Q}_i^0\tilde{Q}_i^{0\ T}A^{0\ \mu}(2v_i^0h_i^0 + h_i^0h_i^0 + \chi_i^0\chi_i^0)\} + \\ &\frac{1}{2}A_\mu^{0\ T}M^{0\ 2}A^{0\ \mu},\end{aligned} \tag{25}$$

with $\tilde{Q}_i^0 = \begin{pmatrix} g^{0\ a} & 0 \\ g^{0\ ab} & g^{0\ b}\end{pmatrix}\begin{pmatrix} Q_i^{0\ a} \\ Q_i^{0\ b}\end{pmatrix}$ and $M^{0\ 2} = \sum_{i=1}^{l}\tilde{Q}_i^0 v_i^{0\ 2}\tilde{Q}_i^{0\ T}$. This can be diagonalized rotating the gauge boson basis, $M^{0\ 2} = R_{\phi^0}^T M_d^{0\ 2} R_{\phi^0}$, where

$$M_d^{0\ 2} = \begin{pmatrix} M_a^{0\ 2} & 0 \\ 0 & M_b^{0\ 2}\end{pmatrix},\ R_{\phi^0} = \begin{pmatrix} \cos\phi^0 & \sin\phi^0 \\ -\sin\phi^0 & \cos\phi^0\end{pmatrix} \ , \tag{26}$$

and ϕ^0 is the angle defining the rotation from the basis where the gauge coupling matrix takes a triangular form (Eq. (16)) to the mass eigenstate basis, $A'^0_\mu = R_{\phi^0} A^0_\mu$. (Prime refers to gauge boson mass eigenstates.) In this basis

$$\begin{pmatrix} g'^{0\ a} & g'^{0\ ab} \\ g'^{0\ ba} & g'^{0\ b} \end{pmatrix} = \begin{pmatrix} g^{0\ a} & g^{0\ ab} \\ 0 & g^{0\ b} \end{pmatrix} R^T_{\phi^0} \ . \tag{27}$$

In order to simplify real computations we work in a R_ξ gauge [11] where the vector-scalar mixing in Eq. (25) cancels

$$\mathcal{L}^0_{GF} = -\frac{1}{2} \mathcal{F}^{0\ T} \xi^{0\ -1} \mathcal{F}^0 \ , \tag{28}$$

with $\xi^{0\ -1}$ a symmetric 2×2 matrix of gauge parameters and

$$\mathcal{F}^0 = \partial_\mu A^{0\ \mu} - \xi^0 \sum_{i=1}^{l} \tilde{Q}^0_i v^0_i \chi^0_i \ . \tag{29}$$

We must also add to the Lagrangian the corresponding Faddeev-Popov term for the ghosts $\binom{c^{0\ a}}{c^{0\ b}}$, which in this gauge do not decouple,

$$\mathcal{L}^0_{ghost} = -\overline{c}^{0\ T} [\partial^\alpha \partial_\alpha + \xi^0 \sum_{i=1}^{l} \tilde{Q}^0_i v^0_i (v^0_i + h^0_i) \tilde{Q}^{0\ T}_i] c^0 \ . \tag{30}$$

For electroweak precision tests it may be adequate to choose as free parameters the gauge boson masses, what motivates to use the on-shell scheme (see the Appendix). The set of independent parameters in the unbroken case, $2N-1$ gauge couplings, $\tilde{q}^{0\ r}_i$, the fermion masses and the scalar couplings (masses), is replaced in the broken case by the 2 gauge boson masses, $M^{0\ 2}_r$, $2N$ gauge couplings $\tilde{q}'^{0\ r}_i$, the fermion masses and the same but 3 scalar couplings (masses). This means trading three scalar couplings (masses) by the three parameters fixing the (symmetric) gauge boson mass matrix, the two mass eigenvalues and the rotation angle ϕ^0 in Eq. (26). This angle is included in the gauge coupling definition (see Eqs. (8) and (27))

$$(\tilde{q}'^{0\ a}_i \ \tilde{q}'^{0\ b}_i) = (\tilde{q}^{0\ a}_i \ \tilde{q}^{0\ b}_i)\ R^T_{\phi^0} = (q^{0\ a}_i \ q^{0\ b}_i) \begin{pmatrix} g^{0\ a} & g^{0\ ab} \\ 0 & g^{0\ b} \end{pmatrix} \begin{pmatrix} \cos\phi^0 & -\sin\phi^0 \\ \sin\phi^0 & \cos\phi^0 \end{pmatrix} \ . \tag{31}$$

In the symmetric case there are $2N-1$ independent charges because the gauge boson basis is defined up to a rotation, which is fixed conventionally. In the broken case the gauge boson basis (mass eigenstates) is fixed and the $2N$ charges (including the rotation angle) will be determined fitting $2N$ independent experiments. Two remarks are in order. In some models not all the parameters are independent. Since the scalar charges $\tilde{Q}_i^{0\ r}$ defining the mass matrix $M^{0\ 2}$ in Eq. (25) are free parameters, a general gauge boson mass matrix requires that at least three different (non-equivalent) scalars get a VEV ($l \geq 3$). (In order to break both $U(1)$'s, l must be ≥ 2.) Thus in the minimal χ model in Table 1, with h and N^c only, ϕ^0 is a function of the gauge boson masses [12]. Other parameters in the Higgs potential can be also replaced by some (of the remaining) VEVs (up to $l-3$). Otherwise, the VEVs v_i^0, which are determined minimizing the effective potential, are not independent parameters. They are considered as so, however, when fixing the corresponding counterterms to satisfy the vanishing tadpole conditions. All the other counterterms can be found following the standard procedure [13].

Introducing as before (Eq. (10)) renormalized fields and couplings (we concentrate on the vector boson parameters and fields)

$$A'^0_\mu = Z^{\frac{1}{2}}_{A'} A'_\mu \ ; \ \ \tilde{q}'^{0\ T}_i = \tilde{q}'^T_i Z_{\tilde{q}'_i} \ ; \ \ M^{0\ 2}_r = M^2_r + \delta M^2_r \ , \tag{32}$$

the theory can be multiplicatively renormalized. The Ward identity analogous to that in Eq. (11), but now involving also ghosts, implies that the product $Z_{\tilde{q}'_i} Z^{\frac{1}{2}}_{A'}$ is finite. Moreover, the on-shell scheme can be defined requiring $Z_{\tilde{q}'_i} Z^{\frac{1}{2}}_{A'} = 1$. Then, Eq. (13) also applies for $\tilde{q}'_i$,

$$\tilde{q}'^0_i = \tilde{q}'_i Z^{-\frac{1}{2}}_{A'} \ , \tag{33}$$

and splitting the $2N$ charges as in Eq. (31), we obtain

$$\begin{gathered}(q_i^{0\ a}\ q_i^{0\ b}) = (q_i^a\ q_i^b) \text{ and } \begin{pmatrix} g^{0\ a} & g^{0\ ab} \\ 0 & g^{0\ b} \end{pmatrix} \begin{pmatrix} \cos\phi^0 & -\sin\phi^0 \\ \sin\phi^0 & \cos\phi^0 \end{pmatrix} = \\ \begin{pmatrix} g^a & g^{ab} \\ 0 & g^b \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z^{-\frac{1}{2}\ aa}_{A'} & Z^{-\frac{1}{2}\ ab}_{A'} \\ Z^{-\frac{1}{2}\ ba}_{A'} & Z^{-\frac{1}{2}\ bb}_{A'} \end{pmatrix} \ ,\end{gathered} \tag{34}$$

for the broken case, too. This equation gives the renormalization of the gauge couplings and of the gauge mixing angle.

The one-loop expressions for the counterterms are obtained as before from $\mathcal{A}'^{rs}$ in Eq. (17). The vector boson proper selfenergies, $i\Pi'^{\alpha\beta}_{\mu\nu}(q^2)$, receive contributions from the diagrams in Fig. 1. (See Ref. [10] for the corresponding Feynman rules in the t'Hooft-Feynman gauge, $\xi = I$.) The divergent contribution to $\mathcal{A}'^{rs}$ coming from the first three diagrams is the same as in the unbroken case but exchanging $(\tilde{q}^a_i\ \tilde{q}^b_i)$ and $(\tilde{Q}^a_i\ \tilde{Q}^b_i)$ in Eq. (18) by the corresponding prime charges in Eq. (8). The fourth diagram contribution and the wave function and mass counterterms can be written:

$$\begin{aligned}\mathcal{A}'_{4+5} = &-\frac{1}{4\pi^2}\sum_{i=1}^{l}\sum_{r=a,b}[C_{UV} - \int_0^1 dx\ \ln D^r_i]\begin{pmatrix} (\tilde{Q}'^a_i)^2 & \tilde{Q}'^a_i\tilde{Q}'^b_i \\ \tilde{Q}'^b_i\tilde{Q}'^a_i & (\tilde{Q}'^b_i)^2 \end{pmatrix}(\tilde{Q}'^r_i)^2 v_i^2 \\ &-\begin{pmatrix} 2(Z^{\frac{1}{2}}_{A'}{}^{aa} - 1)(q^2 - M_a^2) - \delta M_a^2 & Z^{\frac{1}{2}}_{A'}{}^{ba}(q^2 - M_b^2) + Z^{\frac{1}{2}}_{A'}{}^{ab}(q^2 - M_a^2) \\ Z^{\frac{1}{2}}_{A'}{}^{ab}(q^2 - M_a^2) + Z^{\frac{1}{2}}_{A'}{}^{ba}(q^2 - M_b^2) & 2(Z^{\frac{1}{2}}_{A'}{}^{bb} - 1)(q^2 - M_b^2) - \delta M_b^2 \end{pmatrix},\end{aligned} \tag{35}$$

where $D^r_i = \mu_i^2 x + M_r^2(1-x) - q^2 x(1-x)$. In the broken phase the six on-shell conditions on $\mathcal{A}'^{rs}$,

$$\begin{aligned}\mathcal{A}'^{aa}(M_a^2) = \mathcal{A}'^{ab}(M_a^2) = 0\ ,\quad \frac{\partial\mathcal{A}'^{aa}}{\partial q^2}|_{q^2=M_a^2} = 0\ ; \\ \mathcal{A}'^{bb}(M_b^2) = \mathcal{A}'^{ab}(M_b^2) = 0\ ,\quad \frac{\partial\mathcal{A}'^{bb}}{\partial q^2}|_{q^2=M_b^2} = 0\ ;\end{aligned} \tag{36}$$

fix the counterterms $Z^{\frac{1}{2}}_{A'}{}^{rs}$, δM_r^2. We find (we write as in the unbroken case the

$Z_{A'}^{-\frac{1}{2}}$ renormalization constants)

$$Z_{A'}^{-\frac{1}{2}\ aa} = 1 + \frac{C_{UV}}{32\pi^2}\{\sum_{i=1}^{n}\frac{4}{3}(\tilde{q}_i'^a)^2 + \sum_{i=1}^{m}\frac{1}{3}(\tilde{Q}_i'^a)^2\} + \text{finite terms} \ ;$$

$$\begin{aligned} Z_{A'}^{-\frac{1}{2}\ ab} &= \frac{C_{UV}}{16\pi^2}\frac{1}{M_b^2 - M_a^2}\{M_b^2[\sum_{i=1}^{n}\frac{4}{3}\tilde{q}_i'^a\tilde{q}_i'^b + \sum_{i=1}^{m}\frac{1}{3}\tilde{Q}_i'^a\tilde{Q}_i'^b] \\ &\quad + 4\sum_{i=1}^{l}\sum_{r=a,b} v_i^2(\tilde{Q}_i'^r)^2\tilde{Q}_i'^a\tilde{Q}_i'^b\} + \text{finite terms} \ ; \end{aligned} \tag{37}$$

$$\begin{aligned} \delta M_a^2 &= \frac{C_{UV}}{16\pi^2}\{M_a^2[\sum_{i=1}^{n}\frac{4}{3}(\tilde{q}_i'^a)^2 + \sum_{i=1}^{m}\frac{1}{3}(\tilde{Q}_i'^a)^2] \\ &\quad + 4\sum_{i=1}^{l}\sum_{r=a,b} v_i^2(\tilde{Q}_i'^r)^2(\tilde{Q}_i'^a)^2\} + \text{finite terms} \ . \end{aligned}$$

The renormalization constants $Z_{A'}^{-\frac{1}{2}\ ba}$, $Z_{A'}^{-\frac{1}{2}\ bb}$ and δM_b^2 can be obtained from those in Eq (37) interchanging the indices $a \leftrightarrow b$. Note that in the spontaneously broken case there is no arbitrarity left in the gauge boson basis definition and then no ambiguity in the determination of the four independent elements of the $Z_{A'}^{-\frac{1}{2}}$ matrix. These four universal counterterms are absorbed in the 2×2 gauge coupling matrix $g' = gR_\phi$. The mixing angle ϕ provides a fourth independente coupling. All Green functions are then finite.

For later use we chose the on-shell renormalization scheme. All our results (except for the finite one-loop contributions to $Z_{A'}^{-\frac{1}{2}}$) also apply in the minimal subtraction scheme, which is simpler.

5. Summary and conclusions.

The extensions of the SM with an extra $U(1)$ symmetry are a possibility frecuently considered in the literature. The object of many of these analyses has been to estimate the (small) effects caused by this new physics on electroweak observables (ρ parameter, Z width, ...), and then to use precision data to constrain the independent parameters of the models (namely, the mass of the extra neutral boson Z' and its mixing with the standard gauge boson). Usually, the way to proceed has been to combine the SM predictions at one loop with the nonstandard effects estimated at tree level. In this framework, the aim of our study is to discuss the generic procedure to include (in the on-shell scheme) the full radiative corrections in models with two abelian gauge symmetries $U(1)^a \times U(1)^b$. This is not only important for consistency but for practical (numerical) reasons if the extra Z' is relatively light. We have proved that to absorb the *infinites* of the theory one needs *three* universal gauge couplings (g^a, g^b, and g^{ab}). In the general case these couplings are independent parameters to be fixed experimentally.

If grand unification at large scales is assumed (for example into $SO(10)$), then the gauge symmetry implies $g^{ab} = 0$ at the unification scale. Once the unified symmetry is broken, however, the order by order conditions on the matter fields to guarantee $g^{ab} = 0$ may not be satisfied. This is the case of the usual minimal models already at one loop, due to the Higgs (scalar) contributions. At two loops the fermion contributions do not fulfil the matter conditions either. Then radiative corrections generate a nonzero g^{ab} at low energies. In each particular model, this parameter can be estimated via the renormalization-group equations. The LR model is one exception because there is a discrete symmetry left, reminiscence of the left-right gauge symmetry, maintaining unmixed the two $U(1)$'s. $g^{RB-L} \neq 0$ violates (explicitly) this symmetry. Hence, we conclude that a complete analysis of Z' effects on precision electroweak data must contain the third gauge coupling g^{ab}. This is necessary not only to absorb the divergences of the theory when calculating beyond the tree level, but also because the low-energy renormalized value suggested by minimal unified scenarios (once the leading-log contributions are taken into account) is different from zero.

If $g^{ab} \neq 0$ all models with $U(1) \times U(1)$ charges which are linear combination of the $U(1)^a \times U(1)^b$ charges are equivalent [6].

In the broken case the mixing between the new and the standard gauge bosons, ϕ, redefines the currents and thus the gauge couplings. g^{ab} and ϕ are two independent parameters to be determined measuring the gauge boson currents (Eq. (8)).

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Appendix.

In this Appendix we establish the renormalization conditions in the complete on-shell scheme [13].

Two-point functions:

Massive case. The vector-vector propagator can be separated into transverse and longitudinal parts:

$$\Delta^{rs}_{\mu\nu} \equiv -i[T^{rs}(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + L^{rs}(q^2)\frac{q_\mu q_\nu}{q^2}] \ . \qquad (A1)$$

The on-shell conditions are fixed in such a way that when $\epsilon \equiv (q^2 - M_a^2) \to 0$ the transverse part has a pole in T^{aa} and the other components are regular (we neglect the finite width of the particles and consider only the real part of the propagators):

$$(T^{rs})|_{q^2\to M_a^2} = \begin{pmatrix} \frac{1}{\epsilon} & O(1) \\ O(1) & O(1) \end{pmatrix} \ . \qquad (A2)$$

The selfenergies are corrections to the inverse propagator. This can be written

$$\Delta^{-1\ rs}_{\mu\nu} \equiv i[T^{-1\ rs}(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + L^{-1\ rs}(q^2)\frac{q_\mu q_\nu}{q^2}] \ , \qquad (A3)$$

and for q^2 close to M_a^2

$$(T^{-1\ rs})|_{q^2\to M_a^2} = \begin{pmatrix} \epsilon + O(\epsilon^2) & O(\epsilon) \\ O(\epsilon) & O(1) \end{pmatrix} \ . \qquad (A4)$$

The behaviour for $q^2 \to M_b^2$ is analogous. The on-shell conditions on the inverse propagators read (T^{-1} is symmetric)

$$\begin{aligned} T^{-1\ aa}(M_a^2) = T^{-1\ ab}(M_a^2) = 0 \ , \quad \frac{\partial T^{-1\ aa}}{\partial q^2}|_{q^2=M_a^2} = 1 \ , \\ T^{-1\ bb}(M_b^2) = T^{-1\ ab}(M_b^2) = 0 \ , \quad \frac{\partial T^{-1\ bb}}{\partial q^2}|_{q^2=M_b^2} = 1 \ . \end{aligned} \qquad (A5)$$

Thus, the poles of the transverse gauge boson propagator coincide with the renormalized gauge boson masses and the propagator expressions at the poles are the asymptotic ones. The corresponding conditions on the selfenergies are given in the text (Eq. (35)).

Massless case. If $M_a^2 = M_b^2 = 0$, on-shell T^{rs} means

$$(T^{rs})|_{q^2\to 0} = \begin{pmatrix} \frac{1}{\epsilon} & O(1) \\ O(1) & \frac{1}{\epsilon} \end{pmatrix} , \tag{A6}$$

which implies

$$(T^{-1\ rs})|_{q^2\to 0} = \begin{pmatrix} \epsilon + O(\epsilon^2) & O(\epsilon^2) \\ O(\epsilon^2) & \epsilon + O(\epsilon^2) \end{pmatrix} , \tag{A7}$$

or

$$\begin{aligned} & T^{-1\ aa}(0) = T^{-1\ ab}(0) = T^{-1\ bb}(0) = 0 , \\ & \frac{\partial T^{-1\ aa}}{\partial q^2}|_{q^2=0} = \frac{\partial T^{-1\ bb}}{\partial q^2}|_{q^2=0} = 1 , \\ & \frac{\partial T^{-1\ ab}}{\partial q^2}|_{q^2=0} = 0 . \end{aligned} \tag{A8}$$

The conditions for the selfenergies are given in the text (Eq. (20)). The first three equations in Eq. (A8) and the corresponding conditions for the selfenergies are guaranteed by gauge invariance. The complete set of conditions is invariant under (renormalized) vector boson field rotations due to the gauge boson mass degeneracy. As a result the gauge field counterterms are fixed up to a rotation, which must be fixed conventionally (see the text).

Note that the on-shell conditions are imposed only on $T^{-1\ rs}(q^2)$. Since the theory is renormalizable, however, the counterterms for $L^{-1\ rs}(q^2)$ (already fixed by the above conditions on $T^{-1\ rs}(q^2)$) will cancel the longitudinal divergences and make finite the full renormalized inverse propagator. Another comment concerns the mixing between vector fields with Nambu-Goldstone scalars χ_i in models where the symmetry is spontaneously broken. Since the external vector fields satisfy the physical polarization condition $\partial_\mu V_r^\mu = 0$, the vector-scalar part of the propagator does not contribute to the S-matrix elements and does not affect the on-shell conditions.

Three-point functions:

Unbroken case. The on-shell conditions on the three-point functions $-i(\Gamma_i^{a\ \mu}\ \Gamma_i^{b\ \mu})$ (writing as a two component row vector the vertices of the two vector bosons) are

$$(\Gamma_i^{a\ \mu}\ \Gamma_i^{b\ \mu})|_{\substack{\not p_1=\not p_2=m_i \\ q^\mu=0}} = \gamma^\mu(\tilde{q}_i^a\ \tilde{q}_i^b)\ . \qquad (A9)$$

The charge renormalization constants $Z_{\tilde{q}_i}$ are fixed by the Ward identity in Eq. (11) (in momentum space)

$$q_\mu \Gamma_i^{r\ \mu}(p,q) = \tilde{q}_i^u Z_{\tilde{q}_i}^{uv} Z_A^{\frac{1}{2}\ vr}(S_i^{-1}(p+q) - S_i^{-1}(p)). \qquad (A10)$$

Differentiating with respect to q_ν, setting $q^\mu = 0$ and the external particles on-shell, and using (A9) and the on-shell conditions on the inverse fermion propagators

$$\frac{\partial S_i^{-1}(k)}{\partial \not k}|_{\not k=m_i} = 1, \qquad (A11)$$

we obtain the equality among renormalization constants

$$Z_{\tilde{q}_i} = Z_A^{-\frac{1}{2}}. \qquad (A12)$$

Spontaneously broken case. (A9) or the same condition with $q^2 = M_{a,b}^2$ lead to nonuniversal couplings, with corrections depending on the mass ratios $m_i^2/M_{a,b}^2$. However the corresponding Ward identities still guarantee $Z_{\tilde{q}'_i} Z_{A'}^{\frac{1}{2}}$ finite. Thus we can and do impose the renormalization condition $Z_{\tilde{q}'_i} = Z_{A'}^{-\frac{1}{2}}$ order by order. Although it seems somewhat artificial, this condition ensures that the radiative corrections are absorbed into the four universal couplings g'. Note that the three-point vertices for on-shell fields will have finite corrections depending on the fermion masses,

$$\Gamma_i^{r\ \mu}|_{\substack{\not p_1=\not p_2=m_i \\ q^2=M_r^2}} = \gamma^\mu q_i^s g'^{sr} + \text{finite higher order terms}\ . \qquad (A13)$$

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this rotation is scale independent due to the special matter content.

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Table Captions.

Table I: Charges for the fermions in the **16** representation of $SO(10)$ (upper-half): third component of $SU(2)_L$, T_3^L, normalized hypercharge, Y, and extra χ charge, Q_χ. The third component of $SU(2)_R$, $T_3^R = \sqrt{\frac{3}{5}}Y + \sqrt{\frac{2}{5}}Q_\chi$ and the baryon minus the lepton number $Q_{B-L} = \sqrt{\frac{2}{5}}Y - \sqrt{\frac{3}{5}}Q_\chi$ The bottom-half corresponds to the minimal Higgs contents in order to break both $U(1)$'s. Both h, N and $h', \bar{N}$ are needed in the supersymmetric case. h and h' complete a **(2,2)** representation of $SU(2)_L \times SU(2)_R$. N^c and its complex conjugated $\overline{N}^c$ are incomplete $SU(2)_R$ representations.

Figure Captions.

Figure 1: Diagrams contributing at one loop to the vector boson selfenergies. The fourth diagram only contributes in the broken case.

matter	T_3^L	$Y/\frac{1}{2}\sqrt{\frac{3}{5}}$	$Q_\chi/\frac{1}{2\sqrt{10}}$	T_3^R	$Q_{B-L}/\frac{1}{2}\sqrt{\frac{3}{2}}$
u	$\frac{1}{2}$	$\frac{1}{3}$	-1	0	$\frac{1}{3}$
d	$-\frac{1}{2}$	$\frac{1}{3}$	-1	0	$\frac{1}{3}$
u^c	0	$-\frac{4}{3}$	-1	$-\frac{1}{2}$	$-\frac{1}{3}$
d^c	0	$\frac{2}{3}$	3	$\frac{1}{2}$	$-\frac{1}{3}$
ν	$\frac{1}{2}$	-1	3	0	-1
e	$-\frac{1}{2}$	-1	3	0	-1
e^c	0	2	-1	$\frac{1}{2}$	1
ν^c	0	0	-5	$-\frac{1}{2}$	1
h^+	$\frac{1}{2}$	1	2	$\frac{1}{2}$	0
h^0	$-\frac{1}{2}$	1	2	$\frac{1}{2}$	0
N^c	0	0	-5	$-\frac{1}{2}$	1
h'^0	$\frac{1}{2}$	-1	-2	$-\frac{1}{2}$	0
h'^-	$-\frac{1}{2}$	-1	-2	$-\frac{1}{2}$	0
$\overline{N}^c$	0	0	5	$\frac{1}{2}$	-1

Table I