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The moduli space metric for tetrahedrally symmetric 4-monopoles

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Abstract

The metric on the moduli space of SU(2) charge four BPS monopoles with tetrahedral symmetry is calculated using numerical methods. In the asymptotic region, in which the four monopoles are located on the vertices of a large tetrahedron, the metric is in excellent agreement with the point particle metric. We find that the four monopoles are accelerated through the cubic monopole configuration and compute the time advance. Numerical evidence is presented for a remarkable equivalence between a proper distance in the 4-monopole moduli space and a related proper distance in the point particle moduli space. This equivalence implies that the approximation to the time advance (and WKB quantum phase shift) calculated using the point particle derived metric is exact.

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1 Introduction

The dynamics of SU(2) BPS monopoles may be approximated by the time evolution of a finite number of collective coordinates [1, 2]. In this moduli space approach the dynamics of k monopoles is approximated by geodesic motion on the k-monopole moduli space $\mathcal{M}_k$, which is a $4k$-dimensional manifold. To study monopole dynamics therefore requires the construction of the metric on $\mathcal{M}_k$, which is determined by the kinetic part of the field theory action. In the case $k = 2$, Atiyah and Hitchin [3] were able to calculate the metric using indirect methods and making use of its hyperkähler property. However, for $k > 2$ the problem is a more difficult one and no metrics have yet been calculated. Recently, Gibbons and Manton [4] have calculated, for general k, the asymptotic metric on regions of $\mathcal{M}_k$ which describe well-separated monopoles. This asymptotic metric is obtained by treating the monopoles as point particles and is of a generalized Taub-NUT form.

The moduli space $\mathcal{N}$, of tetrahedrally symmetric 4-monopoles, is a one-dimensional totally geodesic submanifold of $\mathcal{M}_4$, and the associated four-monopole scattering process has been investigated in some detail [5]. The fact that $\mathcal{N}$ is one-dimensional allows the monopole trajectories to be determined even though the metric is not known. In this paper we construct the metric on $\mathcal{N}$ by working with Nahm data and using numerical methods. In the asymptotic region, in which the four monopoles are located on the vertices of a large tetrahedron, the metric is in excellent agreement with the point particle metric. We use this metric to calculate the time advance/delay. We also provide numerical evidence for the following rather remarkable equivalence. Let l be a good global coordinate on $\mathcal{N}$, such that $l = 0$ represents coincident monopoles and large l represents well-separated monopoles. Then the proper distance from the point $l = 0$ to a point with l large is equal to the proper distance from the singularity to the same point l in the generalized Taub-NUT space. This equivalence implies that the approximation to the time advance (and quantum phase shift) calculated using the point particle derived metric is exact. This is similar to a previous numerical result found in the 2-monopole case [6], and suggests that it may be a general feature of the point particle metric.

2 Four monopoles with tetrahedral symmetry

In this section we recall the results [5] on tetrahedrally symmetric charge four monopoles that we shall require later. Monopoles are equivalent to various other kinds of mathematical creatures, and here we shall use two of these; namely, spectral curves and Nahm data. Spectral curves [7] are algebraic curves in the holomorphic tangent bundle to the Riemann sphere. Let ζ be the standard inhomogeneous coordinate on the base space and η the fibre coordinate. Then a 4-monopole with tetrahedral symmetry has a spectral curve

$$\eta^4 + i36a\kappa^3\eta\zeta(\zeta^4 - 1) + 3\kappa^4(\zeta^8 + 14\zeta^4 + 1) = 0. \tag{2.1}$$

where $a \in (-a_c, a_c)$, with $a_c = 3^{-5/4}\sqrt{2}$, and κ is half the real period of the elliptic curve

$$y^2 = 4(x^3 - x + 3a^2). \tag{2.2}$$

Hence there is a one-parameter family of tetrahedrally symmetric 4-monopoles. Since this family of monopoles is singled out from the general 4-monopole configuration by the imposition of a symmetry, this implies that the corresponding submanifold $\mathcal{N} \subset \mathcal{M}_4$ is totally geodesic. The associated monopole dynamics has been studied in detail and describes the scattering of four monopoles which are initially well-separated and positioned on the vertices of a contracting regular tetrahedron. As the monopoles merge they scatter instantaneously through a configuration with cubic symmetry and emerge on the vertices of an expanding tetrahedron dual to the incoming one. For the purposes of constructing the metric on $\mathcal{N}$ we require a good global coordinate which we can identify with the distance of each monopole from the origin, at least when the monopoles are well-separated.

Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$ be the four points, each a distance $|l|$ from the origin, given by

$$\begin{aligned} \mathbf{x}_1 &= (-l,-l,-l)\tfrac{1}{\sqrt{3}} \\ \mathbf{x}_2 &= (-l,+l,+l)\tfrac{1}{\sqrt{3}} \\ \mathbf{x}_3 &= (+l,+l,-l)\tfrac{1}{\sqrt{3}} \\ \mathbf{x}_4 &= (+l,-l,+l)\tfrac{1}{\sqrt{3}}. \end{aligned} \tag{2.3}$$

They are the vertices of the tetrahedron on which the monopoles are located when they are well-separated. For well-separated monopoles the asymptotic spectral curve can be obtained as a product of the individual monopole's spectral curves. If $|l|$ is large and we take the four monopoles to have positions $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$, then we obtain the asymptotic spectral curve

$$\eta^4 + i\frac{16}{3^{3/2}}l^3\eta\zeta(\zeta^4-1) + \frac{4}{9}l^4(\zeta^8+14\zeta^4+1) = 0. \tag{2.4}$$

The spectral curve (2.1) has this form in the limit $a \to a_c$, upon which $\kappa \to \infty$. By comparing (2.1) and (2.4) we see that we can make the identification

$$l = \Lambda a^{1/3}\kappa, \qquad \text{where} \qquad \Lambda = 3^{7/6}2^{-2/3}. \tag{2.5}$$

At $a = 0$ (2.1) is the spectral curve of the cubic 4-monopole [8, 5], which has all four Higgs zeros at the origin. If we define, in the usual way, the positions of the monopoles to be given by the zeros of the Higgs field, then $l = 0$ is when all four monopoles have zero distance from the origin. Hence $l \in \mathbb{R}$ is a good global coordinate on $\mathcal{N}$ with a natural interpretation as the distance of each monopole from the origin.

We have used the spectral curve approach to monopoles to identify a convenient coordinate on $\mathcal{N}$, but in order to discuss the metric we now need to turn to the ADHMN formulation [9, 10]. This is usually presented in terms of Nahm data consisting of three Nahm matrices (T_1, T_2, T_3), but in order to discuss the metric we must, following Donaldson [11], introduce a fourth Nahm matrix T_0. Then we have that charge k monopoles are equivalent to Nahm data (T_0, T_1, T_2, T_3), which are four $k \times k$ matrices which depend on a real parameter $s \in [0, 2]$ and satisfy the following;
(i) Nahm's equation

$$\frac{dT_i}{ds} + [T_0, T_i] = \frac{1}{2}\epsilon_{ijk}[T_j, T_k] \qquad i = 1, 2, 3 \tag{2.6}$$

(ii) T_0 is regular for $s \in [0,2]$. $T_i(s)$, $i=1,2,3$, is regular for $s \in (0,2)$ and has simple poles at $s=0$ and $s=2$,

(iii) the matrix residues of (T_1,T_2,T_3) at each pole form the irreducible k-dimensional representation of SU(2),

(iv) $T_i(s) = -T_i^\dagger(s), \qquad i=0,1,2,3,$

(v) $T_i(s) = T_i^t(2-s), \qquad i=0,1,2,3.$

Let G be the group of analytic $su(k)$-valued functions $h(s)$, for $s \in [0,2]$, which are the identity at $s=0$ and $s=2$, and satisfy $h^t(2-s) = h^{-1}(s)$. Then gauge transformations $h \in G$ act on Nahm data as

$$T_0 \to hT_0h^{-1} - \frac{dh}{ds}h^{-1} \tag{2.7}$$

$$T_i \to hT_ih^{-1} \quad i=1,2,3. \tag{2.8}$$

Note that the gauge $T_0 = 0$ may always be chosen, which is why this fourth Nahm matrix is usually not introduced. However, when discussing the metric on Nahm data we need to consider the action of the gauge group and so this extra Nahm matrix needs to be kept, at least temporarily.

In the gauge $T_0 = 0$ the Nahm data corresponding to a tetrahedrally symmetric 4-monopole, whose spectral curve we have discussed above, is given by [5]

$$T_i(s) = x(s)X_i + y(s)Y_i + z(s)Z_i \qquad i=1,2,3 \tag{2.9}$$

where x,y,z are the real functions

$$x(s) = \frac{\kappa}{5}\left(-2\sqrt{\wp(\kappa s)} + \frac{1}{4}\frac{\wp'(\kappa s)}{\wp(\kappa s)}\right) \tag{2.10}$$

$$y(s) = \frac{\kappa}{20}\left(\sqrt{\wp(\kappa s)} + \frac{1}{2}\frac{\wp'(\kappa s)}{\wp(\kappa s)}\right) \tag{2.11}$$

$$z(s) = \frac{a\kappa}{2\wp(\kappa s)}. \tag{2.12}$$

Here $\wp$ is the Weierstrass function satisfying

$$\wp'^2 = 4\wp^3 - 4\wp + 12a^2 \tag{2.13}$$

with prime denoting differentiation with respect to the argument. The tetrahedrally symmetric Nahm triplets are

$$(X_1,X_2,X_3) = \left(\begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & 0 & 2 & 0 \\ 0 & -2 & 0 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & 0 \end{bmatrix}, \begin{bmatrix} 0 & i\sqrt{3} & 0 & 0 \\ i\sqrt{3} & 0 & 2i & 0 \\ 0 & 2i & 0 & i\sqrt{3} \\ 0 & 0 & i\sqrt{3} & 0 \end{bmatrix}, \begin{bmatrix} 3i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -3i \end{bmatrix}\right)$$

$$
(Y_1,Y_2,Y_3)=2\left(\begin{bmatrix}0&-\sqrt{3}&0&-5\\ \sqrt{3}&0&3&0\\ 0&-3&0&-\sqrt{3}\\ 5&0&\sqrt{3}&0\end{bmatrix},\begin{bmatrix}0&-i\sqrt{3}&0&5i\\ -\sqrt{3}i&0&3i&0\\ 0&3i&0&-\sqrt{3}i\\ 5i&0&-\sqrt{3}i&0\end{bmatrix},\begin{bmatrix}2i&0&0&0\\ 0&-6i&0&0\\ 0&0&6i&0\\ 0&0&0&-2i\end{bmatrix}\right)
$$

$$
(Z_1,Z_2,Z_3)=\sqrt{3}\left(\begin{bmatrix}0&i&0&0\\ i&0&0&0\\ 0&0&0&-i\\ 0&0&-i&0\end{bmatrix},\begin{bmatrix}0&1&0&0\\ -1&0&0&0\\ 0&0&0&-1\\ 0&0&1&0\end{bmatrix},\begin{bmatrix}0&0&1&0\\ 0&0&0&1\\ -1&0&0&0\\ 0&-1&0&0\end{bmatrix}\right)
$$

Although this Nahm data does not satisfy property (v) this can be achieved by a suitable change of basis.

In the next section we shall use this Nahm data to calculate the metric on $\mathcal{N}$.

3 Calculation of the metric

It is known that the transformation between the monopole moduli space metric and the metric on Nahm data is an isometry [12, 13]. We can therefore calculate the metric, $g(l)$, on $\mathcal{N}$ by computing the metric on the Nahm data given in the previous section. This requires the computation of the tangent vector (V_0, V_1, V_2, V_3) corresponding to the point with Nahm data (T_0, T_1, T_2, T_3). In principal, since the Nahm data is explicitly known, this could be achieved by direct differentiation,

$$
V_i = \frac{dT_i}{dl}. \tag{3.1}
$$

However, this is not a practical way to proceed since the Weierstrass function (2.13), in terms of which the Nahm data is given, itself depends on l (through its dependence on a). Instead we calculate the tangent space to $\mathcal{N}$ by solving the linearized Nahm equation

$$
\dot{V}_i + [V_0, T_i] + [T_0, V_i] = \epsilon_{ijk}[T_j, V_k] \qquad i = 1, 2, 3 \tag{3.2}
$$

and

$$
\dot{V}_0 + \sum_{i=0}^{3}[T_i, V_i] = 0 \tag{3.3}
$$

where $V_i,\ i = 0, 1, 2, 3,$ is an analytic $su(4)$-valued function of $s \in [0, 2]$. Dot denotes differentiation with respect to s. The metric on Nahm data is then given by

$$
g(l) = -\Omega \int_0^2 \sum_{i=0}^{4} \mathrm{tr}(V_i^2)\ ds \tag{3.4}
$$

where tr denotes trace and Ω is a normalization constant.

From now on we use the gauge freedom to set $T_0 = 0$. Equation (3.3) is the background gauge constraint which ensures that the tangent vectors we compute are horizontal *ie* that the tangent vectors are orthogonal to the gauge orbits. The tangent vectors are tetrahedrally symmetric so we may write

$$V_i = q_1 X_i + q_2 Y_i + q_3 Z_i \qquad i = 1, 2, 3 \tag{3.5}$$

where $\mathbf{q} = (q_1, q_2, q_3)^t$ is an analytic real 3-vector function of $s \in [0, 2]$. It is easily checked that the tetrahedral symmetry of the Nahm triplets implies the following identities

$$\sum_{i=1}^{3} [X_i, Y_i] = \sum_{i=1}^{3} [X_i, Z_i] = \sum_{i=1}^{3} [Y_i, Z_i] = 0. \tag{3.6}$$

Substituting (2.9) and (3.5) into the background gauge equation (3.3) and using the identities (3.6) gives the solution $V_0 = 0$. The remaining equations (3.2) become the following equation for the 3-vector $\mathbf{q}$

$$\dot{\mathbf{q}} = M\mathbf{q} \qquad \text{where} \quad M = \begin{bmatrix} 4x & -96y & -12z/5 \\ -6y & -16y - 6x & -6z/5 \\ -4z & -32z & -4x - 32y \end{bmatrix}. \tag{3.7}$$

Substituting (3.5) into (3.4) gives the metric in terms of $\mathbf{q}$ as

$$g = 12\Omega \int_0^2 (5q_1^2 + 80q_2^2 + 3q_3^2) \; ds. \tag{3.8}$$

The ordinary differential equation (3.7) has regular-singular points at $s = 0$ and $s = 2$, since the functions appearing in M have first order poles at these points. Analysis of the initial value problem at $s = 0$ reveals that there is a two-dimensional family of solutions to (3.7) which are normalizable for $s \in [0, 2)$. They are given by the two-parameter, α_1, α_2, family of initial conditions

$$\mathbf{q} \sim (0, \alpha_1 s^3, \alpha_2 s^2)^t \quad \text{as} \quad s \sim 0. \tag{3.9}$$

Repeating the analysis for the initial value problem at $s = 2$ gives a two-parameter, β_1, β_2, family of normalizable solutions for $s \in (0, 2]$, with initial conditions

$$\mathbf{q} \sim (16\beta_1(2-s)^3, 3\beta_1(2-s)^3, \beta_2(2-s)^2)^t \quad \text{as} \quad s \sim 2. \tag{3.10}$$

The solution we require is the one-parameter family which is normalizable in the closed interval $s \in [0, 2]$. For later convenience we take α_2 to be the free parameter which describes this family of solutions. To compute these solutions by solving an initial value problem would be a difficult shooting problem if it were not for the fact that equation (3.7) is linear. This reduces the task to a simple problem in linear algebra which we implement as follows. Given a value for α_2, say α, let $\mathbf{p}_1(s)$ denote the solution $\mathbf{q}(s)$ of (3.7) corresponding to the initial conditions (3.9) with $(\alpha_1, \alpha_2) = (0, \alpha)$. This solution is calculated for $s \in [0, 1]$. Numerically we compute this solution using a fourth order Runge-Kutta method. Let $\mathbf{p}_2(s)$ denote a second solution, but this time with initial conditions $(\alpha_1, \alpha_2) = (1, 0)$. Similarly, let $\mathbf{p}_3(s)$ and $\mathbf{p}_4(s)$ be the solutions calculated for $s \in [1,2]$ obtained from the initial conditions (3.10) with parameter values $(\beta_1, \beta_2) = (1, 0)$ and $(\beta_1, \beta_2) = (0, 1)$ respectively. Next form the 3×4 matrix

$$U = \begin{bmatrix} | & | & | & | \\ \mathbf{p}_1(1) & \mathbf{p}_2(1) & \mathbf{p}_3(1) & \mathbf{p}_4(1) \\ | & | & | & | \end{bmatrix} \tag{3.11}$$

and find the unique solution of the linear matrix equation

$$U\mathbf{w} = \mathbf{0} \tag{3.12}$$

for $\mathbf{w} = (1, w_2, w_3, w_4)^t$. Numerically this is performed by row reduction of the matrix U followed by back substitution. Then the required solution $\mathbf{q}(s)$ is given by

$$\mathbf{q}(s) = \begin{cases} \mathbf{p}_1(s) + w_2\mathbf{p}_2(s) & \text{if } 0 \le s \le 1 \\ \\ -w_3\mathbf{p}_3(s) - w_4\mathbf{p}_4(s) & \text{if } 1 < s \le 2 \end{cases} \tag{3.13}$$

To summarize, the above procedure consists in integrating (3.7) twice from each end of the interval $[0, 2]$ to the centre and then finding a linear combination of these solutions which match at the centre.

We note that in the special case $l = 0$ (*ie* $a = 0$), which corresponds to the cubic monopole, the tangent vector may be calculated explicitly in closed form. In this case the third component of $\mathbf{q}$ decouples from the other two and we have the solution $\mathbf{q} = (0, 0, q_3)^t$ with

$$q_3 = \frac{\alpha_2}{\kappa^2 \wp(\kappa s)} \tag{3.14}$$

where κ and $\wp$ take their values corresponding to $a = 0$.

The next issue we confront is to ensure that the tangent vector we compute is dual to the coordinate l. This requires the determination of the correct l-dependent normalization factor α_2. In terms of $\mathbf{q}$ the equation (3.1) becomes $(q_1, q_2, q_3) = (dx/dl, dy/dl, dz/dl)$. We calculate the correct normalization factor by considering the third component, $q_3 = dz/dl$, in the limit $s \to 0$. Substituting the asymptotic behaviour of $\wp(\kappa s)$ as $s \sim 0$ into the expression (2.12) for z and comparing with the definition of α_2 given by (3.9) we obtain

$$\alpha_2 s^2 = \frac{d}{dl}(\frac{1}{2} a \kappa^3 s^2)$$

which gives

$$\alpha_2 = \frac{3l^2}{2\Lambda^3}. \tag{3.15}$$

There is now only one constant left to determine, which is the overall metric factor Ω. This is fixed by the requirement that the metric tends to the sum of the monopole masses in the limit of infinite separation *ie* $g(l) \to 16\pi$ as $l \to \infty$.

We now apply the above numerical scheme to calculate the metric. The integral in (3.8) is calculated using a standard composite Simpsons rule. The result is displayed in Fig. 1 (solid curve) for $0 \leq l \leq 18$. We see that the metric is a monotonic increasing function of l, which implies that the monopoles speed up as they approach each other and are accelerated through the cubic monopole configuration $l = 0$.

Recently, Gibbons and Manton [4] have calculated the asymptotic metric, on regions of $\mathcal{M}_k$ which describe well-separated monopoles, by treating the monopoles as point particles. In our case of interest the imposed tetrahedral symmetry implies that all four monopoles have the same internal phase. This implies that there are no electric charge differences between the monopoles so that we can consider pure monopoles not dyons. For k well-separated monopoles with positions $\mathbf{x}_i$, $i = 1, .., k$, the point particle lagrangian is [4]

$$\mathcal{L} = 2\pi \sum_{i=1}^{k} \dot{\mathbf{x}}_i^2 - 2\pi \sum_{1\leq i<j\leq k} \frac{(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j)^2}{|\mathbf{x}_i - \mathbf{x}_j|}. \tag{3.16}$$

For our case of interest $k = 4$ and the positions are the vertices of the tetrahedron given in equation (2.3). Then the above lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\widetilde{g}(l)\dot{l}^2 \quad \text{where} \quad \widetilde{g}(l) = 16\pi(1 - \frac{\sqrt{6}}{l}). \tag{3.17}$$

The point particle metric $\widetilde{g}(l)$ is the approximation to the true metric $g(l)$, and the two should agree in the large l limit. In Fig. 1 we plot the point particle metric (dashed curve) for comparison with the true metric. We see that indeed the two metrics are in excellent agreement in the asymptotic limit. This is a useful check not only on the numerics used in this paper but also on the point particle approximation applied in [4]. Note that the metric $\widetilde{g}$ has a singularity (*ie* a point at which its determinant changes sign) at $l = \sqrt{6}$.

Given monopoles which are initially well-separated with $l = L$, the time taken, t, for the monopoles to scatter and reach this separation again may be computed as

$$t = \Delta\sqrt{\frac{2}{\mathcal{T}}}$$

where $\mathcal{T}$ is the total kinetic energy in the system and Δ is the proper distance from L to the origin

$$\Delta = \int_0^L \sqrt{g(l)}\, dl. \tag{3.18}$$

The time advance δt, due to the acceleration of the monopoles, is related to the proper distance via

$$\delta t = \sqrt{\frac{2}{\mathcal{T}}}(4L\sqrt{\pi} - \Delta). \tag{3.19}$$

Note that the asymptotic behaviour of the metric (which is the point particle metric (3.17)) means that the time advance has a logarithmic divergence in the limit $L \to \infty$.

It is interesting to compare the true time advance with that obtained in the point particle approximation. In the case of 2-monopoles the point particle metric is the Taub-NUT metric with a negative mass parameter [14]. This also has a singularity, at a finite value r_0 of the radial distance r between the monopoles. Given a point in $\mathcal{M}_2$ corresponding to large r, the Atiyah-Hitchin metric can be used to calculate the proper distance of this point to the bolt (which describes coincident monopoles). The proper distance from the same point r in Taub-NUT space to the singularity can also be calculated and the two compared. It is curious that numerically these two distances are found to agree [6]. This equivalence, for which there is at present no explanation, implies that the approximation to the time advance calculated using the Taub-NUT metric is exact.

Given that we have the metric g and its point particle approximation $\widetilde{g}$ we can investigate the possibility that a similar result to that above exists in the 4-monopole case. The proper distance from the singularity in the generalized Taub-NUT space is

$$\begin{aligned} \widetilde{\Delta} &= \int_{\sqrt{6}}^{L} \sqrt{\widetilde{g}(l)}\, dl = 4\sqrt{\pi}(\sqrt{L(L-\sqrt{6})} + \sqrt{\frac{3}{2}}\log(\frac{\sqrt{L}-\sqrt{L-\sqrt{6}}}{\sqrt{L}+\sqrt{L-\sqrt{6}}})) \\ &\sim 4\sqrt{\pi}(L - \sqrt{\frac{3}{2}}(\log(\frac{4L}{\sqrt{6}}) + 1) \quad \text{as} \quad L \to \infty. \end{aligned} \tag{3.20}$$

It is this which we wish to compare with Δ in the large L limit. Numerically we have computed the metric $g(l)$ at values up to $l = 18$. It can be seen from Fig. 1 that this is a reasonably large value since the metrics are very similar for $l > 10$. Setting $L = 18$ in (3.20) gives the result $\widetilde{\Delta} = 89.7$. The integration to calculate Δ at $L = 18$ from the numerical values for $\sqrt{g}$ is performed with the use of the numerical routines FITPACK. We fit a spline under tension to the data values and integrate the resulting spline to obtain the result $\Delta = 89.2$. So, to within the numerical accuracy of the calculation, we find that the two answers agree. This implies that there is no relative WKB quantum phase shift from the point particle approximation in the quantized dynamics of the above classical motion.

The result in the two monopole case and the numerical evidence presented here suggests that perhaps this feature of the point particle metric is a general one. At present there is little explanation for this possible equivalence, but perhaps the answer lies in some global geometrical properties of the metrics and the fact that the point particle metric inherits the hyperkähler property of the true metric on $\mathcal{M}_k$.

4 Conclusion

We have introduced a numerical scheme to calculate the monopole moduli space metric from Nahm data. This scheme has been used to calculate the metric on a totally geodesic submanifold of the 4-monopole moduli space, corresponding to tetrahedrally symmetric monopoles. The results compare well with the asymptotic point particle metric and we have presented evidence for a curious exact result using an approximate metric. The scheme can be applied to calculate the metric on submanifolds of $\mathcal{M}_k$ for which the Nahm data is known. A suitable candidate is the submanifold of $\mathcal{M}_3$ obtained by imposing a twisted line symmetry on three monopoles [15]. In this scattering process it appears that the zeros of the Higgs field stick at the origin for a finite time interval. A calculation of the metric would reveal the time scale over which this sticking takes place.

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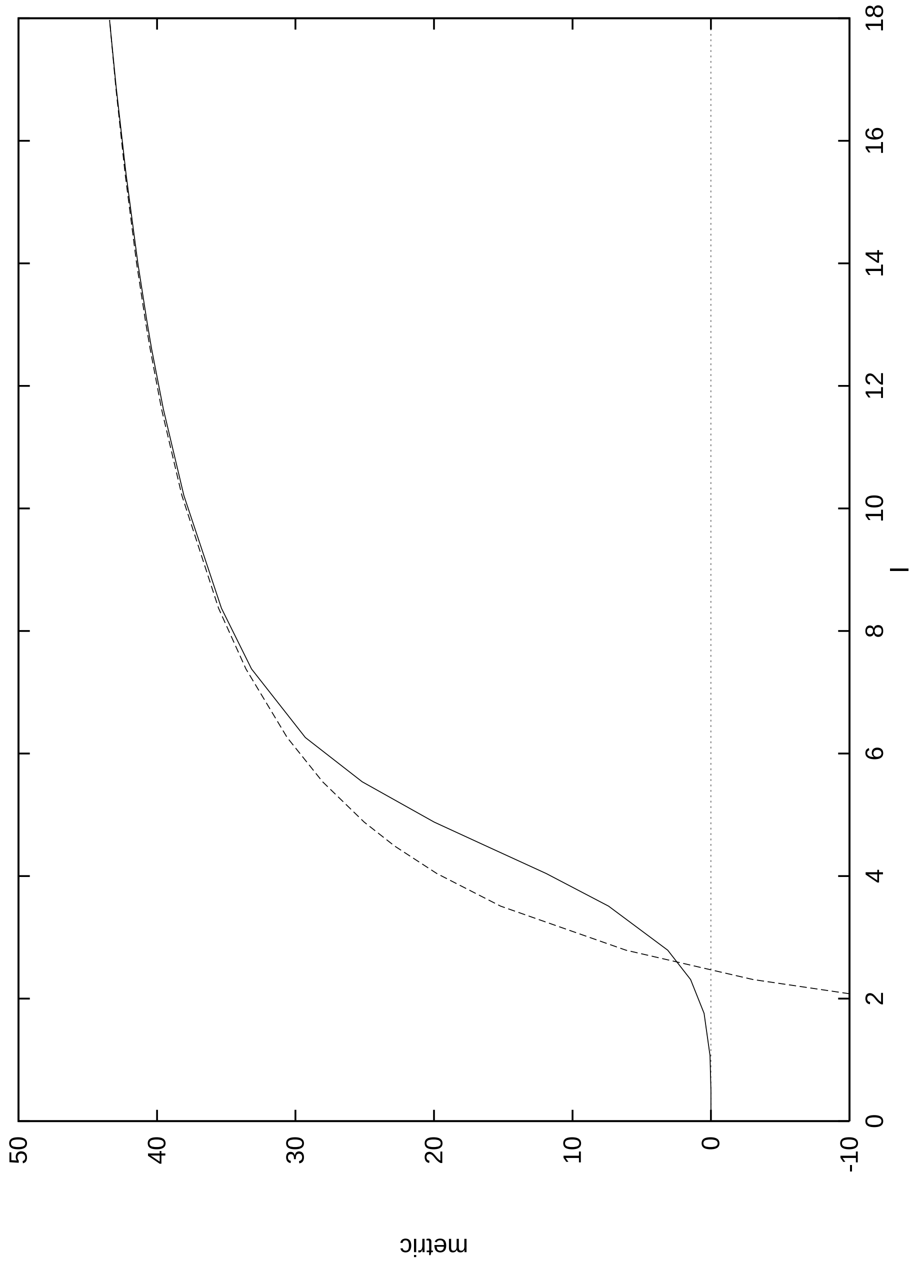

Figure 1: The metric $g(l)$ (solid curve) and the point particle metric $\widetilde{g}(l)$ (dashed curve).