COSMIC STRINGS FROM
N= 2, D= 5 SUPERGRAVITY

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Abstract

Exact solutions of N=2 supergravity in five dimensions are found in the metric with cylindrical symmetry, a particular case corresponds to the exterior of a cosmic string.

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1   Introduction

In a recent work [1] the possibility of obtaining the cylindrically solutions to four-dimensional low-energy limit of string theory was investigated. In this work I consider the same problem for N=2, D=5 supergravity theory. During the last years there has been intensive studies of physical theories in more than four dimensions, among them gravity with torsion [2],strings, superstrings and supergravity. Recently Balbinot et al. [3,6] have considered cosmological solutions for the N=2 and D=5 supergravity theory [4,5] which contains the metric g_{MN}, a spin$-\frac{3}{2}$ field ψ_M^a (a =1, 2 is an internal index) and a 1-form B_M (M, N = 1, ..., 5 and $\mu\nu$ =1,..., 4). Looking for a "ground-state" configuration they have set the fermion field and the electromagnetic field in the metric g_{MN} equal to zero. They have assumed a local structure $V_4 \times S^1$, where V_4 is a four dimensional spacetime and they found exact solutions when V_4 is the flat Robertson-Walker spacetime. In a previous paper I have considered the Bianchi type-I spacetime. Here I want to consider this theory in the exterior of a cosmic string.

2   Field Equations

The Theory studied by Balbinot et al. [3] has the five-dimensional line element

$$dS^2 = g_{\mu\nu}(x^\mu)dx^\mu dx^\nu - \phi^2(x^\mu)(dx^5)^2 = ds^2 - \phi^2(x^\mu)(dx^5)^2. \qquad (1)$$

It is also assumed that the 1-form B_M is $B_M = (0,0,0,0,\psi(x^\mu)\ )$. With all these assumptions the theory is equivalent to one with a four dimensional lagrangian given by

$$\mathcal{L} = \frac{1}{4}\sqrt{g}\phi \left(R + 2\frac{D_\lambda\psi D^\lambda\psi}{\phi^2} \right). \qquad (2)$$

The field equations obtained from the variation of the above Lagrangian are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{3}{2\phi^2}(\psi_{,\mu}\psi_{,\nu} - \frac{1}{2}g_{\mu\nu}\psi_{,\lambda}\psi^{,\lambda}) + \frac{1}{\phi}(\phi_{;\mu\nu} - g_{\mu\nu}\Box\phi). \qquad (3)$$

$$\Box\phi + \frac{\psi_\lambda\psi^\lambda}{\phi} = 0, \qquad (4)$$

$$\Box\psi - \frac{\phi_\lambda\psi^\lambda}{\phi} = 0. \qquad (5)$$

In this section we set the field equations for a metric that corresponds to the exterior of a cosmic string. Here we are interested in a static infinite-lenght cosmic string, the fields must have static cylindrical symmetry. The spacetime must have three commuting Killing vectors fields, one of them timelike and the other two spacelike, one of them with closed orbits, such that any two are orthogonal to each other and each is hypersurface orthogonal. There is an axis where the Killing vector with closed orbits vanishes. The normalization is

chosen so that along any closed integral curve the parameter takes the values from 0 to 2π, and the norm of the spacelike and timelike Killing vectors is 1 and -1, respectively, on the axis. Using coordinates t,z,ρ and θ, where ρ is the geodesic distance from the axis in the direction orthogonal to the three Killing vector fields, $(\partial/\partial t)^a$ is the timelike Killing field , and $(\partial/\partial \theta)^a$is the Killing field that has closed orbits, and $(\partial/\partial z)^a$ is the Killing vector field along the axis. With all these assumptions the metric is

$$ds^2 = -e^{2A}dt^2 + e^{2B}dz^2 + e^{2C}d\theta^2 + d\rho^2, \tag{6}$$

where A, B, and C are functions of the radial distance ρ. The range of θ is $[0, 2\pi]$ and $\theta = 0$ and $\theta = 2\pi$ are identified. The boundary conditions on the axis of the cosmic strigs are

$$\lim_{\rho \to 0} A(\rho) \quad = \lim_{\rho \to 0} B(\rho) = 0, \tag{7}$$

$$\lim_{\rho \to 0} e^{2C}/\rho^2 \qquad = 1. \tag{8}$$

In order to make the algebra simpler we will use here the following metric,

$$ds^2 = -e^{2A}dt^2 + e^{2B}dz^2 + e^{2C}d\theta^2 + e^{2(A+B+C)}dr^2, \tag{9}$$

where A, B, and C are now functions of the new radial distance r, and the transformation is given by

$$d\rho = e^{A+B+C}dr. \tag{10}$$

The field equations for this particular spacetime are given by

$$B'' + C'' - A'B' - A'C' - B'C' + \frac{\phi''}{\phi} - A'\frac{\phi'}{\phi} + \frac{3}{4}\left(\frac{\psi'}{\phi}\right)^2 = 0 \tag{11}$$

$$(A' + B' + C')\frac{\phi'}{\phi} + A'B' + A'C' + B'C' - \frac{3}{4}\left(\frac{\psi'}{\phi}\right)^2 = 0 \tag{12}$$

$$A'' + C'' - A'B' - A'C' - B'C' + \frac{\phi''}{\phi} - B'\frac{\phi'}{\phi} + \frac{3}{4}\left(\frac{\psi'}{\phi}\right)^2 = 0 \tag{13}$$

$$A'' + B'' - A'B' - A'C' - B'C' + \frac{\phi''}{\phi} - C'\frac{\phi'}{\phi} + \frac{3}{4}\left(\frac{\psi'}{\phi}\right)^2 = 0 \tag{14}$$

and

$$\phi'' + \frac{\psi'^2}{\phi} = 0, \tag{15}$$

$$\psi'' - \frac{\psi'\phi'}{\phi} = 0, \tag{16}$$

here the prime derivative with respect to r. In the following section we look for exact solutions to the system of differential equations (7 -12).

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3 Exact Solutions

From Eqs.(15 and 16) it follows that

$$\phi = L\cos(\omega r + \delta_0), \tag{17}$$

$$\psi = \omega \int \phi = L\sin(\omega r + \delta_0). \tag{18}$$

where L, ω, and δ_0 are integration constants. Adding Eqs.(12 and 14) we have

$$\frac{\phi''}{\phi} + \frac{\phi'}{\phi}(A+B)' + (A+B)'' = 0 \tag{19}$$

and the solution is

$$A + B = q_1 \int \frac{dr'}{\phi(r')} - \log(\phi), \tag{20}$$

where q_1 is an integration constant and the other integration constant was set to zero without loss of generality. Adding Eqs.(12 and 13) and Eqs.(11 and 12) we obtain identical differential equations for A+C and B+C with the corresponding solutions,

$$A + C = q_2 \int \frac{dr'}{\phi(r')} - \log(\phi), \tag{21}$$

$$B + C = q_3 \int \frac{dr'}{\phi(r')} - \log(\phi), \tag{22}$$

From the above expressions we obtain the explicit solutions for A, B and C,

$$A = a_1 \log[\tan(\omega r + \delta_0) + \sec(\omega r + \delta))] - \frac{1}{2}\log[\cos(\omega r + \delta_0)] \tag{23}$$

$$B = b_1 \log[\tan(\omega r + \delta_0) + \sec(\omega r + \delta)] - \frac{1}{2}\log[\cos(\omega r + \delta_0)] \tag{24}$$

$$C = c_1 \log[\tan(\omega r + \delta_0) + \sec(\omega r + \delta_0)] - \frac{1}{2}\log[\cos(\omega r + \delta_0)] \tag{25}$$

where a_1, b_1, and c_1 are integration constants that satisfy the following relation

$$a_1 b_1 + a_1 c_1 + b_1 c_1 = \frac{3}{4}. \tag{26}$$

The local 'deficit angle' is of the form

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$$\delta(r) = 2\pi\left(1 - \frac{1}{\sqrt{g_{rr}}}\frac{\partial}{\partial r}\sqrt{g_{\theta\theta}}\right) = 2\pi\left[1 - \frac{\omega c_1 \sec(\omega r + \delta_0)}{\left(\tan(\omega r + \delta_0) + \sec(\omega r + \delta_0)\right)^{(a_1 + b_1)}}\right], \qquad (27)$$

that is non-vanishing in general. The case of the cosmic string corresponds to $A(r) = B(r)$, or $a_1 = b_1$; that is, the cosmic string exhibits explicit Lorentz invariance along its axis.

4   Acknowledgment

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