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Longitudinal/Goldstone Boson Equivalence and Phenomenology of Probing the Electroweak Symmetry Breaking [*]

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Abstract

We formulate the Equivalence between the longitudinal weak-boson and the Goldstone boson as a criterion for sensitively probing the electroweak symmetry breaking mechanism and develop a precise power counting rule for chiral Lagrangian formulated electroweak theories. With these we semi-quantitatively analyze the sensitivities to various effective operators related to electroweak symmetry breaking via weak-boson scatterings at the CERN Large Hadron Collider (LHC).

Recent LEP/SLC experiments can test the electroweak (EW) theory to the accuracy of one-loop corrections, and support the spontaneously broken $SU(2) \times U(1)$ gauge theory as the correct theory of the EW interactions. However, light Higgs boson has not been found, and the current experiments are insensitive to the spontaneous symmetry breaking (SSB) sector of the theory, compatible with a wide range of the Higgs boson mass 60GeV$\leq m_H \leq$ 1TeV. So the SSB mechanism in the EW theory is still a mystery, and it is thus important to probe *all possible* SSB mechanisms: either weakly or strongly interacting.

We know that only the longitudinal component V_L^a of the weak-boson V^a ($W^{\pm}$,Z^0) (arising from "eating" the would-be Goldstone boson (GB)) is sensitive to the SSB

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sector, while the transverse component V_T^a is not. The physical V_L^a scattering amplitude is quantitatively related to the corresponding GB amplitude by the electroweak Equivalence Theorem (ET)$^{1\sim3}$ which comes from the following ET identity$^{2\sim3}$

$$T[V_L^{a_1},\cdots,V_L^{a_n};\Phi_\alpha] = C\cdot T[-i\pi^{a_1},\cdots,-i\pi^{a_n};\Phi_\alpha] + B \ , \tag{1}$$

$$\begin{aligned} C \ &\equiv C_{mod}^{a_1}\cdots C_{mod}^{a_n} \ , \\ B \ &\equiv \textstyle\sum_{l=1}^{n}\big(\ C_{mod}^{a_{l+1}}\cdots C_{mod}^{a_n} T[v^{a_1},\cdots,v^{a_l},-i\pi^{a_{l+1}},\cdots,-i\pi^{a_n};\Phi_\alpha] \\ &\qquad\qquad + \text{permutations of } v\text{'s and } \pi\text{'s}\ \big) \ , \\ v^a \ &\equiv v^\mu V_\mu^a \ , \quad v^\mu \ \equiv \epsilon_L^\mu - k^\mu/M_a = O(M_a/E) \ , \quad (M_a = M_W, M_Z) \ , \end{aligned} \tag{2}$$

where π^a's are GB fields, and Φ_α denotes other possible physical in/out states. The renormalization scheme-dependent constant modification factor C_{mod}^a has been generally studied in Ref.2-3, which can be exactly simplified as $C_{mod}^a = 1$ in certain convenient renormalization schemes$^{3\sim4}$.

For strongly interacting SSB models, the V_L^a-amplitude on the L.H.S. of (1) is experimentally measurable, while the GB-amplitude on the R.H. S. of (1), though not directly measurable, carries the information about the SSB mechanism. Similar to V_T^a, the B-term in (1) is not sensitive to the SSB mechanism. If, under certain conditions, the B-term can be neglected, (1) reveals the *equivalence* between the V_L^a-amplitude and the GB-amplitude. In this case the V_L^a-scattering experiments can be used to sensitively and unambiguously probe the SSB mechanism. When B is not negligible, measurements of the V_L^a, V_T^a and B amplitudes with higher precision will be required for probing the SSB mechanism, and those expreriments at LHC will be harder.

The conditions for neglecting the B-term in (1), i.e. the condition for the validity of the ET, is actually subtle. We first note that the spin-0 GB's are invariant under the proper Lorentz transformations, while, on the contrary, V_L, V_T and B are Lorentz non-invariant. Therefore the ratio of the B-magnitude relative to the GB-amplitude in (1) is Lorentz frame dependent. So neglecting B makes sense only if the Lorentz frame belongs to a group of frames whithin which Lorentz transformation does not significantly enhance B. We call such frames *safe frames.* The condition for a Lorentz frame to be *safe* is given in Ref.3, which is

$$E_j \sim k_j \gg M_W, \qquad\qquad (\ j = 1,2,\cdots,n\ ) \ ,$$

where E_j is the energy of the j-th external V_L^a-line. For a given process, E_j can be easily obtained from the kinematics. So this condition is a convenient criterion for judging whether the experimental center of mass frame is *safe* or not for a given process, i.e. it can discriminate processes which are not sensitive for probing the SSB mechanism.[a] With this consideration, the ET can be precisely formulated as^3

$$T[V_L^{a_1},\cdots,V_L^{a_n};\Phi_\alpha] = C\cdot T[-i\pi^{a_1},\cdots,-i\pi^{a_n};\Phi_\alpha] + O(M_W/E_j\text{—suppressed}), \tag{3a}$$

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[a] See the example given in Ref.3.

$$E_j \sim k_j \gg M_W, \qquad (j = 1, 2, \cdots, n) \ ; \tag{3b}$$

$$B \ll C \cdot T[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] \ . \tag{3c}$$

(3b) and (3c) are the conditions for neglecting the B-term in (1) (for the validity of the ET), or the conditions for sensitively probing the SSB mechanism via V_L^a-scattering experiments. Here we see the *profound physical content of the ET*, i.e. ET is not merely a tool for simplifying calculations.

The next thing is to realize the quantitative meaning of the condition (3c). To a given order N in a perturbative expansion, the amplitude T can be written as $T = \sum_{\ell=0}^{N} T_\ell$ with $T_0 > T_1, \cdots, T_N$. Let $T_{\rm min} = \{T_0, \cdots, T_N\}_{\rm min}$. Then, to the precision of $T_{\rm min}$, condition (3c) precisely implies[3]

$$\begin{aligned} B \ &\approx O(\tfrac{M_W^2}{E_j^2})\, T_0[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] + O(\tfrac{M_W}{E_j})\, T_0[V_{T_j}^{a_{r_1}}, -i\pi^{a_{r_2}}, \cdots, -i\pi^{a_{r_n}}; \Phi_\alpha] \\ &\ll T_{\rm min}[-i\pi^{a_1}, \cdots, -i\pi^{a_n}; \Phi_\alpha] \ . \end{aligned} \tag{4}$$

In the chiral Lagrangian formulated EW theory (CLEWT), the $O(E^2)$ leading amplitude T_0 is model-independent. Thus, for probing the SSB mechanism, we should take into account the next-to-leading $O(E^4)$ amplitude T_1, i.e. $T_{\rm min} = T_1$. By means of our power counting rule (6), we can estimate that for leading contributions, $T_1 = O(\frac{E^4}{f_\pi^2 \Lambda^2} f_\pi^{4-n})$ and $B = O(g^2 f_\pi^{4-n})$ [b]. Thus condition (4) requires $\frac{M_W^2}{E^2} \ll \frac{1}{4}\frac{E^2}{\Lambda^2}$, or $(0.7{\rm TeV}/E)^4 \ll 1$. So the probe is generally sensitive when $E \geq 1$ TeV which is possible at the LHC.

In the CLEWT, the Lagrangian can be written in the following form[6]

$$\mathcal{L}_{eff} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)} + \mathcal{L}^{(2)\prime} + \sum_{n=1}^{14} \mathcal{L}_n \ = \sum_n \ell_n \frac{f_\pi^{\ r_n}}{\Lambda^{a_n}} \mathcal{O}_n(W_{\mu\nu}, B_{\mu\nu}, DU, U, f, \bar{f}), \tag{5}$$

where $\mathcal{L}_G$,$\mathcal{L}_F$ are the kinetic terms of the gauge fields and fermions. The explicit formula for $\mathcal{L}_{eff}$ is given in Ref.5~6, in which $\mathcal{L}^{(2)}, \mathcal{L}^{(2)\prime}, \mathcal{L}_{1\sim 11}$ are CP conserving, and $\mathcal{L}_{12\sim 14}$ are CP violating. Here, the dimensionless coefficients ℓ_n's can be naturally regarded as of $O(1)$ [7]. In Ref.5, we developed the following power counting rule in the CLEWT for the S-matrix element T

$$\begin{aligned} &T = c_T f_\pi^{D_T} \left(\frac{f_\pi}{\Lambda}\right)^{N_\mathcal{O}} \left(\frac{E}{f_\pi}\right)^{D_{E0}} \left(\frac{E}{\Lambda}\right)^{D_{EL}} \left(\frac{M_W}{E}\right)^{e_v} H(\ln E/\mu) \ , \\ &N_\mathcal{O} = \sum_n a_n \ , \quad D_{E0} = 2 + \sum_n \mathcal{V}_n (d_n - 2) + (i_F + \frac{1}{2} e_F) \ , \quad D_{EL} = 2L \ , \end{aligned} \tag{6}$$

where the dimensionless coefficient c_T contains possible powers of gauge couplings (g, g') and Yukawa couplings (y_f) from the vertices in T , which can be easily

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[b] In the CLEWT, $f_\pi = 246 GeV$ and the effective cut-off $\Lambda \simeq 4\pi f_\pi \simeq 3.1 TeV$.

Table 1. Contributions of the model-dependent operators to the $W^{\pm}W^{\pm}\rightarrow W^{\pm}W^{\pm}$ amplitudes

Operators	$T_1[4\pi]$	$T_1[3\pi, W_T]$	$T_1[2\pi, 2W_T]$	$T_1[\pi, 3W_T]$	$T_1[4W_T]$
$\mathcal{L}^{(2)\prime}$	$\ell_0\ \frac{E^2}{\Lambda^2}$	$\ell_0\ g\frac{f_\pi E}{\Lambda^2}$	$\ell_0\ g^2\frac{f_\pi^2}{\Lambda^2}$	$\ell_0\ g^3\frac{f_\pi^3}{E\Lambda^2}$	/
$\mathcal{L}_{1,13}$	/	$\ell_{1,13}\ e^2g\frac{f_\pi E}{\Lambda^2}$	$\ell_{1,13}\ e^4\frac{f_\pi^2}{\Lambda^2}$	$\ell_{1,13}\ e^2g\frac{f_\pi E}{\Lambda^2}$	$\ell_{1,13}\ e^2g^2\frac{f_\pi^2}{\Lambda^2}$
$\mathcal{L}_2$	$\ell_2\ e^2\frac{E^2}{\Lambda^2}$	$\ell_2\ e^2g\frac{f_\pi E}{\Lambda^2}$	$\ell_2\ e^2\frac{E^2}{\Lambda^2}$	$\ell_2\ e^2g\frac{f_\pi E}{\Lambda^2}$	$\ell_2\ e^2g^2\frac{f_\pi^2}{\Lambda^2}$
$\mathcal{L}_3$	$\ell_3\ g^2\frac{E^2}{\Lambda^2}$	$\ell_3\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_3\ g^2\frac{E^2}{\Lambda^2}$	$\ell_3\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_3\ g^4\frac{f_\pi^2}{\Lambda^2}$
$\mathcal{L}_{4,5}$	$\ell_{4,5}\ \frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2}$	$\ell_{4,5}\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_{4,5}\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{4,5}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{4,5}\ g^4\frac{f_\pi^2}{\Lambda^2}$
$\mathcal{L}_{6,7,10}$	/	/	/	/	/
$\mathcal{L}_{8,14}$	/	$\ell_{8,14}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{8,14}\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{8,14}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{8,14}\ g^4\frac{f_\pi^2}{\Lambda^2}$
$\mathcal{L}_9$	$\ell_9\ g^2\frac{E^2}{\Lambda^2}$	$\ell_9\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_9\ g^2\frac{E^2}{\Lambda^2}$	$\ell_9\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_9\ g^4\frac{f_\pi^2}{\Lambda^2}$
$\mathcal{L}_{11,12}$	/	$\ell_{11,12}\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_{11,12}\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{11,12}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{11,12}\ g^4\frac{f_\pi^2}{\Lambda^2}$

determined from the vertices. H is a function of $\ln(E/\mu)$ insensitive to E, where μ denotes the relevant renormalization scale. d_n is the number of derivatives in the type-n vertex, $\mathcal{V}_n$ is the number of type-n vertices in T, i_F and e_F are numbers of internal and external fermion lines, respectively.

With this counting rule, we can estimate the sensitivities to probing specific operators in (5) via various W-W scattering amplitudes. In Table-1, we list the results in the important $W^{\pm}W^{\pm}$ channel as a typical example. We first see that $\mathcal{L}_{6,7,10}$ do not contribute to this channel. Table-1 then shows that the $4W_L^{\pm}$ channel can probe $\mathcal{L}_{4,5}$ most sensitively, while the contributions of $\mathcal{L}^{(2)\prime}$, $\mathcal{L}_{2,3,9}$ to this channel lose E-power dependence by a factor-2. This channel cannot probe $\mathcal{L}_{1,8,11\sim 14}$. $\mathcal{L}_{1,8,11\sim 14}$ can only be probed via channels with $W_T^{\pm}$('s), among which $\mathcal{L}_{11,12}$ are most dominant though they are still suppressed by a factor gf_π/E relative to the leading contributions to the $4W_L^{\pm}$ channel. $\mathcal{L}_{1,8,13,14}$ are generally suppressed by higher powers of the factor gf_π/E and are thus less sensitive. For a more complete classification, see Table-3 in Ref.5.

We have further calculated the number of events per $[100\text{fb}^{-1}\cdot\text{GeV}]$ at the LHC from our counting rule (6) combined with the effective-W approximation. We have compared them with the corresponding available explicit calculations in Ref.8 for a few typical examples. The comparison shows that the deviations are reasonably within a factor of $2\sim 3$ which is of the same order as the uncertainty of the effective-W approximation. Therefore our power counting rule does give correct semi-quantitative results and is thus very useful and convenient for making a systematical analysis for the sensitivities to probing the SSB mechanism at the LHC and future linear colliders. In the typical case with $\ell_n \sim O(1)$, the number of the LHC events for the W^+W^+ channel are shown in Fig.1. By comparing with the events from B, we see that the probe of $\mathcal{L}_{4,5}$ are most sensitive, that of $\mathcal{L}_{3,9,11,12}$ are marginal, and that of $\mathcal{L}^{(2)\prime}$, $\mathcal{L}_{1,2,8,13,14}$ are insensitive. More of the details are given in Ref.5.

Figure 1: Sensitivities of operators $\mathcal{L}^{(2)\prime}, \mathcal{L}_{1\sim 14}$ whith $\ell_n \sim O(1)$, at the 14 TeV LHC

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