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One-loop corrections to five-parton amplitudes with external photons ¹

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Abstract

Recently the calculations of all five-parton one-loop QCD amplitudes have been completed. In this letter we describe how to get the corresponding amplitudes with one gluon replaced by a photon and we give the explicit results for the process $0 \rightarrow 2g2Q1\gamma$.

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1 Introduction

The study of photoproduction of 3+1 jets at HERA, as well as the study of direct photon production in connection with two jets at hadron colliders, requires the knowledge of five-parton amplitudes with external photons. Since by now all one-loop five-parton QCD amplitudes are known [1, 2, 3], it is natural to exploit these results in order to get the desired amplitudes.

For amplitudes with no external quarks the photon can couple through a quark loop only. As a result, the corresponding tree-level amplitudes vanish. We therefore concentrate on the process with external quarks.

After the setup of the notation we describe how to get the amplitudes for $0 \rightarrow 2q2g1\gamma$ and $0 \rightarrow 2q1g2\gamma$ out of $0 \rightarrow 2q3g$ and we give the complete results for $0 \rightarrow 2q2Q1\gamma$. Of course, it is also possible to get the amplitudes for $0 \rightarrow 2q3\gamma$ and the pure QED processes out of the corresponding QCD amplitudes [3, 4], but these processes are phenomenologically less important. The construction of photonic one-loop amplitudes for one external quark pair has also been discussed in [3].

The quarks (antiquarks) are denoted by q and Q (\bar{q} and \bar{Q}), the photon by γ and the gluons are labeled from 1 to n . These labels may be used for the color index or the momentum of the corresponding particle. The dependence on the helicities h_i of the particles is often suppressed

Throughout we use the normalization $\text{Tr}(T^a T^b) = \delta^{ab}$, where T^a are the $SU(3)$ generators in the fundamental representation. Furthermore, we use the following compact notation for the color part:

$$(i \dots j)_{q\bar{q}} \equiv (T^{g_i} \dots T^{g_j})_{q\bar{q}} \quad (1)$$

$$(i \dots j) \equiv \text{Tr}(T^{g_i} \dots T^{g_j}) \quad (2)$$

After replacing the structure constants f^{abc} with $-i/\sqrt{2}([a, b]c)$ and using the Fierz identity, the color part of every Feynman diagram can be written as a linear combination of terms of the form given in eqs. (1) and (2). The coefficient of an amplitude proportional to such a color factor is called a subamplitude.

For the presentation of the (unrenormalized) amplitudes we follow closely the conventions in ref. [2]. The amplitudes will be expressed as functions of dot products, spinor inner products and a set of auxiliary functions ($\left\{ \begin{smallmatrix} ij \\ kl \end{smallmatrix} \right\}_x$, $\langle ijkl \rangle$, \mathcal{P}_{ij} and $\mathcal{F}(i, j, k)$). All these functions as well as the ubiquitous prefactor c_F are defined in ref. [2].

2 One external quark pair

Consider the process $0 \rightarrow 2q3g$ where one gluon, say g_3 , is replaced by a photon. The only differences of the two processes are the coupling constant and the color part. First of all we have to replace $g \rightarrow e_q$, where e_q is the electric charge of the quark to which the photon couples, and secondly we have to make the transition $T^{g_3} \rightarrow \sqrt{2}$.¹

¹The factor $\sqrt{2}$ appears because of the chosen normalization of the T^i

Although both steps seem to be rather trivial, they may cause some complications. The former, because the photon may couple to quarks with different electric charge and the latter, because we have to calculate additional color subamplitudes which belong to color structures involving a trace $\text{Tr}(T^{g_3})$. Contrary to the gluonic process this trace is not vanishing any longer after the transition $T^{g_3} \rightarrow \sqrt{2}$.

As long as we have only one external quark and the corresponding antiquark, the difficulty concerning the replacement $g \rightarrow e_q$ can be handled as follows: First of all, for diagrams without a fermion loop there is no problem, we always have $g \rightarrow e_q$, where e_q is the electric charge of the external quark. For diagrams with a fermion loop we have to separate the diagrams where the photon couples to an internal fermion loop (called loop-coupled diagrams) from those where the photon couples to the external quark (externally-coupled diagrams). In the former case we have to replace $g \rightarrow \sum e_Q \equiv [e]$ where the sum extends over all fermions Q in the loop whereas in the latter case we have the usual replacement $g \rightarrow e_q$.

Since the subamplitudes $0 \rightarrow 2q3g$ are needed for the presentation of the results we review the color structure of the helicity amplitudes of this process. At tree level it is given by

$$\mathcal{A}^{(0)}(q, 1, 2, 3, \bar{q}) = g^3 \sum_{P(123)} (123)_{q\bar{q}} b_{123}^{(0)}(q, 1, 2, 3, \bar{q}) \quad (3)$$

where $P(1\dots n)$ denotes all permutations of the elements $1\dots n$. As mentioned above, we suppress the helicity dependence of the amplitudes in the notation. At one loop the decomposition reads:

$$\begin{aligned} \mathcal{A}^{(1)}(q, 1, 2, 3, \bar{q}) = g^3 \left(\frac{g}{4\pi} \right)^2 & \left[\sum_{P(1,2,3)} (123)_{q\bar{q}} b_{123}^{(1)}(q, 1, 2, 3, \bar{q}) \right. \\ & + \sum_{i=1,2,3} \left((i)_{q\bar{q}}(kl) b_i^{(1)}(q, 1, 2, 3, \bar{q}) + (i) (kl)_{q\bar{q}} b_{i,kl}^{(1)}(q, 1, 2, 3, \bar{q}) \right) \\ & \left. + (q\bar{q}) (123) b_q^{(1)}(q, 1, 2, 3, \bar{q}) + (q\bar{q}) (321) b_{\bar{q}}^{(1)}(q, 1, 2, 3, \bar{q}) \right] \end{aligned} \quad (4)$$

In this equation k and l are the two indices from the set $\{1, 2, 3\} \setminus i$ for a given $i \in \{1, 2, 3\}$. The subamplitudes $b_{i,kl}^{(1)}$ are not needed for $0 \rightarrow 2q3g$ since the corresponding color part vanishes. However, as we shall see, these subamplitudes have to be known for the construction of the photonic processes. All subamplitudes can either be extracted from ref. [3] or they are explicitly given in ref. [4].

If we replace $g_3 \rightarrow \gamma$, the color decomposition at tree level is

$$\mathcal{A}^{(0)}(q, 1, 2, \gamma, \bar{q}) = g^2 \sqrt{2} e_q \sum_{P(1,2)} (12)_{q\bar{q}} b_{12}^{1\gamma,(0)}(q, 1, 2, \gamma, \bar{q}) \quad (5)$$

while at one-loop we have

$$\begin{aligned} \mathcal{A}^{(1)}(q, 1, 2, \gamma, \bar{q}) = & \\ g^2 \sqrt{2} e_q \left(\frac{g}{4\pi} \right)^2 & \left[\sum_{P(1,2)} (12)_{q\bar{q}} b_{12}^{1\gamma,(1)}(q, 1, 2, \gamma, \bar{q}) + (q\bar{q})(12) b_{\text{tr}}^{1\gamma,(1)}(q, 1, 2, \gamma, \bar{q}) \right] \end{aligned} \quad (6)$$

Note that according to the discussion above the subamplitudes may contain terms proportional to $[e]/e_q$.

The tree-level results can be obtained immediately with

$$b_{12}^{1\gamma,(0)}(q, 1, 2, \gamma, \bar{q}) = b_{123}^{(0)}(q, 1, 2, \gamma, \bar{q}) + b_{132}^{(0)}(q, 1, 2, \gamma, \bar{q}) + b_{312}^{(0)}(q, 1, 2, \gamma, \bar{q}) \quad (7)$$

For the subamplitude $b_{\text{tr}}^{1\gamma,(1)}$ the externally-coupled diagrams cancel each other, therefore, we have no contributions proportional to $e_q N_F$. This fact enables us to get the subamplitudes $b_{\text{tr}}^{1\gamma,(1)}$ out of those of the process $0 \rightarrow 2q3g$. To do so we merely have to list all color factors of $0 \rightarrow 2q3g$ which, after the transition $g_3 \rightarrow \gamma$, yield the color part (12). These are (123), (321) and (3) $_{\bar{q}q}$ (12). Adding up the corresponding subamplitudes yields the desired result

$$b_{\text{tr}}^{1\gamma,(1)}(q, 1, 2, \gamma, \bar{q}) = \left(b_q^{(1)}(q, 1, 2, \gamma, \bar{q}) + b_{\bar{q}}^{(1)}(q, 1, 2, \gamma, \bar{q}) + b_3^{(1)}(q, 1, 2, \gamma, \bar{q}) \right) \Big|_{N_F \rightarrow ([e]/e_q)} \quad (8)$$

In order to present the results for the subamplitudes $b_{12}^{1\gamma,(1)}$ it is convenient to decompose them further.

$$b_{12}^{1\gamma,(1)}(q, 1, 2, \gamma, \bar{q}) = ic_\Gamma \mathcal{B}_{12}^{1\gamma}(q, 1, 2, \gamma, \bar{q}) + ic_\Gamma N_F \mathcal{B}_{12}^{1\gamma,\text{ext}}(q, 1, 2, \gamma, \bar{q}) + ic_\Gamma \frac{[e]}{e_q} \mathcal{B}_{12}^{1\gamma,\text{int}}(q, 1, 2, \gamma, \bar{q}) \quad (9)$$

In eq. (9) we put the loop-coupled diagrams in $\mathcal{B}_{12}^{1\gamma,\text{int}}$, whereas the externally-coupled diagrams are contained in $\mathcal{B}_{12}^{1\gamma,\text{ext}}$. Diagrams without a fermion loop are collected in $\mathcal{B}_{12}^{1\gamma}$. Note that the subamplitude $b_{21}^{1\gamma,(1)}$ can be obtained from $b_{12}^{1\gamma,(1)}$ by simple relabeling [3, 4].

Neglecting for the moment the problems of the coupling of the photon to quarks of different electric charge, the subamplitude $b_{12}^{1\gamma,(1)}(q, 1, 2, \gamma, \bar{q})$ can be constructed in the same manner as $b_{\text{tr}}^{1\gamma,(1)}$. We find

$$ic_\Gamma \mathcal{B}_{12}^{1\gamma}(q, 1, 2, \gamma, \bar{q}) = \left(b_{123}^{(1)}(q, 1, 2, \gamma, \bar{q}) + b_{132}^{(1)}(q, 1, 2, \gamma, \bar{q}) + b_{312}^{(1)}(q, 1, 2, \gamma, \bar{q}) + N_c b_{3,12}^{(1)}(q, 1, 2, \gamma, \bar{q}) \right) \Big|_{N_F \rightarrow 0} \quad (10)$$

For the remaining needed functions $\mathcal{B}_{12}^{1\gamma,\text{ext}}$ and $\mathcal{B}_{12}^{1\gamma,\text{int}}$ we will give the explicit results for the different helicity configurations, using the notation:

$$\mathcal{B}(h_1, h_2, h_3) \equiv \mathcal{B}(q, +, 1, h_1, 2, h_2, 3, h_3, \bar{q}, -) \quad (11)$$

First of all we remark that it is sufficient to give the subamplitudes for the helicity configurations² $(-, +, +)$, $(+, -, +)$, $(+, +, -)$ and $(+, +, +)$. The results for all other helicity configurations can be obtained with the help of discrete symmetries [3, 4].

For the loop-coupled diagrams we find for all helicity configurations

$$\mathcal{B}_{12}^{1\gamma,\text{int}}(h_1, h_2, h_3) = -\frac{1}{2} \mathcal{B}_{\text{tr}}^{1\gamma, N_F}(h_1, h_2, h_3) \quad (12)$$

²Since the tree level results vanish for the helicity configuration $(+, +, +)$, the corresponding one-loop amplitudes are not really needed. We just give them for completeness

where $\mathcal{B}_{\text{tr}}^{1\gamma, N_F}$ denotes the part of $b_{\text{tr}}^{1\gamma, (1)}$ which is proportional to $[e]/e_q$. Furthermore we get

$$\mathcal{B}_{12}^{1\gamma, \text{ext}}(-, +, +) = \mathcal{B}_{12}^{1\gamma, \text{ext}}(+, -, +) = 0 \quad (13)$$

For the other two helicity configurations the externally-coupled diagrams do not cancel and we obtain

$$\mathcal{B}_{12}^{1\gamma, \text{ext}}(+, +, -) = \frac{[q1][q2]}{3 \langle 12 \rangle [\bar{q}3] [q3]} \quad (14)$$

and

$$\mathcal{B}_{12}^{1\gamma, \text{ext}}(+, +, +) = \frac{\langle \bar{q}1 \rangle \langle \bar{q}2 \rangle [12]}{3 \langle \bar{q}3 \rangle \langle q3 \rangle \langle 12 \rangle^2} \quad (15)$$

With eq.(10) and eqs.(12 – 15) all ingredients of eq.(9) are given.

We turn now to the process $0 \rightarrow 2q1g2\gamma$. If we start from the process $0 \rightarrow 2q3g$ and say gluon 2 and gluon 3 become photons, there is only one color structure left.

$$\mathcal{A}^{(i)}(q, 1, \gamma_2, \gamma_3, \bar{q}) = g \left(\sqrt{2}e_q \right)^2 \left(\frac{g}{4\pi} \right)^{2i} (1)_{q\bar{q}} b^{2\gamma, (i)}(q, 1, \gamma_2, \gamma_3, \bar{q}) \quad (16)$$

The diagrams of this process are again decomposed into three classes. The first contains diagrams with no quark loop (i.e. we have to replace $g^2 \rightarrow e_q^2$), the second consists of diagrams where one photon couples to the quark loop and the other photon couples to the external quark ($g^2 \rightarrow [e]e_q$), and finally in the third class of diagrams both photons couple to the quark loop ($g^2 \rightarrow [e^2]$). Here $[e^2]$ denotes $\sum e_Q^2$. All diagrams of this process which contain a quark loop and two photons coupled to the external quark line, are self-energy insertions on external lines and vanish therefore in dimensional regularization.

Since the diagrams of the second class cancel each other for all helicity configurations it is easy to get the helicity amplitudes.

$$\begin{aligned} b^{2\gamma, (1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) &= \left[\sum_{P(1,2,3)} b_{123}^{(1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) + N_c b_1^{(1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) \right. \\ &+ N_c b_{3;12}^{(1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) + N_c b_{3;21}^{(1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) \\ &\left. + N_c b_{2;13}^{(1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) + N_c b_{2;31}^{(1)}(q, 1, \gamma_2, \gamma_3, \bar{q}) \right] \Big|_{N_F \rightarrow \frac{[e^2]}{e_q^2}} \quad (17) \end{aligned}$$

Obviously, the analytic expression obtained in this way is rather massive and further simplification is possible, but we do not pursue this any longer.

3 Two external quark pairs

For processes with external quarks which have different electric charge, the situation concerning the replacement $g \rightarrow e_q$ is more complicated. Since the gluon couples in the same way to the different quarks, but the photon does not, it is not possible to get the amplitudes for the photonic process without additional work.

We denote the charge of q and Q by e_q and e_Q respectively. The color decomposition at one loop ($i = 1$) and at tree level ($i = 0$) reads:

$$\begin{aligned} \mathcal{A}^{(i)}(\bar{q}, \bar{Q}; Q, q; \gamma) = & \quad (18) \\ & g^2 \sqrt{2} \left(\frac{g}{4\pi} \right)^{2i} \left[\delta_{q\bar{Q}} \delta_{Q\bar{q}} e_q u_2^{(i)}(\bar{q}, \bar{Q}; Q, q; \gamma) + \delta_{q\bar{Q}} \delta_{Q\bar{q}} e_Q d_2^{(i)}(\bar{q}, \bar{Q}; Q, q; \gamma) \right. \\ & \left. - \frac{1}{N} \delta_{q\bar{q}} \delta_{Q\bar{Q}} e_q u_1^{(i)}(\bar{q}, \bar{Q}; Q, q; \gamma) - \frac{1}{N} \delta_{q\bar{q}} \delta_{Q\bar{Q}} e_Q d_1^{(i)}(\bar{q}, \bar{Q}; Q, q; \gamma) \right] \end{aligned}$$

Since the two loop-coupled diagrams cancel each other, the subamplitudes $u^{(1)}$ and $d^{(1)}$ do not contain terms proportional to $[e]/e_q$.

We write the one-loop subamplitudes in the following form:

$$\begin{aligned} u_1^{(1)}(\bar{q}, \bar{Q}, Q, q, \gamma) = & \quad (19) \\ & ic_\Gamma u_1^l(\bar{q}, \bar{Q}, Q, q, \gamma) + \frac{ic_\Gamma}{N_c} u_1^s(\bar{q}, \bar{Q}, Q, q, \gamma) - ic_\Gamma \frac{N_F}{N_c} u^{N_F}(\bar{q}, \bar{Q}, Q, q, \gamma) \end{aligned}$$

$$\begin{aligned} u_2^{(1)}(\bar{q}, \bar{Q}, Q, q, \gamma) = & \quad (20) \\ & ic_\Gamma N_c u_2^l(\bar{q}, \bar{Q}, Q, q, \gamma) + \frac{ic_\Gamma}{N_c} u_2^s(\bar{q}, \bar{Q}, Q, q, \gamma) + ic_\Gamma N_F u^{N_F}(\bar{q}, \bar{Q}, Q, q, \gamma) \end{aligned}$$

The subamplitudes $d_1^{(1)}$ and $d_2^{(1)}$ are decomposed in the same way.

Since we can use parity to change all helicities, it is sufficient to consider only helicity configurations with positive photon helicity. Furthermore, we have the relations

$$d_i^{(i)}(\bar{q}, h_{\bar{q}}, \bar{Q}, h_{\bar{Q}}, Q, h_Q, q, h_q, \gamma, h_\gamma) = u_i^{(i)}(\bar{Q}, h_{\bar{Q}}, \bar{q}, h_{\bar{q}}, q, h_q, Q, h_Q, \gamma, h_\gamma) \quad (21)$$

For the remainder of this section we use the notation

$$u^{(i)}(h_q, h_Q, h_\gamma) \equiv u^{(i)}(\bar{q}, h_{\bar{q}}, \bar{Q}, h_{\bar{Q}}, Q, h_Q, q, h_q, \gamma, h_\gamma) \quad (22)$$

We will give the subamplitudes $u^{(i)}(+, +, +)$ and $u^{(i)}(+, -, +)$ below. The remaining subamplitudes can be obtained either with parity, eq. (21) or

$$u^{(i)}(-, +, +) = -u^{(i)}(+, -, +)|_{q \leftrightarrow \bar{q}, Q \leftrightarrow \bar{Q}} \quad (23)$$

$$u^{(i)}(-, -, +) = -u^{(i)}(+, +, +)|_{q \leftrightarrow \bar{q}, Q \leftrightarrow \bar{Q}} \quad (24)$$

First we recall the tree level results, which can be obtained easily from the tree level results of $0 \rightarrow 4q1g$

$$u_1^{(0)}(+, +, +) = u_2^{(0)}(+, +, +) = -i \frac{\langle \bar{q}\bar{Q} \rangle^2}{\langle \bar{Q}Q \rangle \langle \bar{q}\gamma \rangle \langle q\gamma \rangle} \quad (25)$$

$$u_1^{(0)}(+, -, +) = u_2^{(0)}(+, -, +) = i \frac{\langle \bar{q}Q \rangle^2}{\langle \bar{Q}Q \rangle \langle \bar{q}\gamma \rangle \langle q\gamma \rangle} \quad (26)$$

In order to present the results, it is convenient to decompose the one-loop subamplitudes in the following way:

$$u_1^s(h_q, h_Q, h_\gamma) = u_A(h_q, h_Q, h_\gamma) + u_B(h_q, h_Q, h_\gamma) \quad (27)$$

$$u_2^s(h_q, h_Q, h_\gamma) = -2 u_A(h_q, h_Q, h_\gamma) - u_B(h_q, h_Q, h_\gamma) \quad (28)$$

$$u_2^l(h_q, h_Q, h_\gamma) = u_A(h_q, h_Q, h_\gamma) - u_1^l(h_q, h_Q, h_\gamma) \quad (29)$$

So we have to give the results for u_1^l, u_A, u_B and u^{N_F} for the two helicity configurations $(+, +, +)$ and $(+, -, +)$. The term proportional to N_F is the simplest.

$$u^{N_F}(h_q, h_Q, h_\gamma) = iu_1^{(0)}(h_q, h_Q, h_\gamma) \left(\frac{2}{3} \frac{\mathcal{P}_{\bar{Q}Q}}{\varepsilon} + \frac{10}{9} \right) \quad (30)$$

For the leading terms of u_1 we get

$$\begin{aligned} u_1^l(+, +, +) &= -iu_1^{(0)}(+, +, +) \left(\frac{\mathcal{P}_{\bar{q}\bar{Q}}}{\varepsilon^2} + \frac{\mathcal{P}_{qQ}}{\varepsilon^2} - \frac{2}{3} \frac{\mathcal{P}_{\bar{Q}Q}}{\varepsilon} - \frac{29}{18} \right) \\ &+ \frac{\langle \bar{q}\gamma \rangle \langle q\bar{Q} \rangle^2 [q\gamma]^2}{2 \langle q\gamma \rangle \langle \bar{Q}Q \rangle} \left\{ \frac{\bar{q}\gamma}{\bar{Q}Q} \right\}_2 - \frac{\langle \bar{q}\bar{Q} \rangle \langle q\bar{Q} \rangle [q\gamma]}{\langle q\gamma \rangle \langle \bar{Q}Q \rangle} \left\{ \frac{\bar{q}\gamma}{\bar{Q}Q} \right\}_1 \\ &+ \frac{\langle \bar{q}q \rangle \langle \bar{q}\bar{Q} \rangle [qQ]}{2 \langle \bar{q}\gamma \rangle \langle q\gamma \rangle s_{\bar{Q}Q}} + \frac{\langle \bar{q}\bar{Q} \rangle \langle q\bar{Q} \rangle [q\gamma]}{2 \langle q\gamma \rangle \langle \bar{Q}Q \rangle s_{\bar{Q}Q}} \\ &- iu_1^{(0)}(+, +, +) \left(\mathcal{F}(\bar{q}, \bar{Q}, Q) + \mathcal{F}(q, Q, \bar{Q}) + \mathcal{F}(\bar{Q}, \bar{q}, \gamma) + \mathcal{F}(Q, q, \gamma) \right) \end{aligned} \quad (31)$$

and

$$\begin{aligned} u_1^l(+, -, +) &= -iu_1^{(0)}(+, -, +) \left(\frac{\mathcal{P}_{\bar{q}\bar{Q}}}{\varepsilon^2} + \frac{\mathcal{P}_{qQ}}{\varepsilon^2} - \frac{2}{3} \frac{\mathcal{P}_{\bar{Q}Q}}{\varepsilon} - \frac{29}{18} \right) \\ &- \frac{\langle \bar{q}\gamma \rangle \langle qQ \rangle^2 [q\gamma]^2}{2 \langle q\gamma \rangle \langle \bar{Q}Q \rangle} \left\{ \frac{\bar{q}\gamma}{\bar{Q}Q} \right\}_2 - \frac{\langle \bar{q}q \rangle \langle \bar{q}\bar{Q} \rangle \langle qQ \rangle [q\bar{Q}]}{\langle \bar{q}\gamma \rangle \langle q\bar{Q} \rangle \langle q\gamma \rangle} \left\{ \frac{\bar{q}\gamma}{\bar{Q}Q} \right\}_1 - \frac{\langle \bar{q}\bar{Q} \rangle^2}{\langle \bar{q}\gamma \rangle \langle q\gamma \rangle \langle \bar{Q}Q \rangle} \left\{ \frac{\bar{q}\gamma}{\bar{Q}Q} \right\}_0 \\ &- \frac{\langle \bar{q}\bar{Q} \rangle \langle \bar{Q}Q \rangle [\bar{Q}\gamma]}{\langle q\bar{Q} \rangle \langle \bar{Q}\gamma \rangle} \left\{ \frac{\bar{q}\gamma}{qQ} \right\}_1 - \frac{\langle \bar{q}q \rangle \langle \bar{q}\bar{Q} \rangle}{\langle \bar{q}\gamma \rangle \langle q\bar{Q} \rangle \langle q\gamma \rangle} \left\{ \frac{\bar{q}\gamma}{qQ} \right\}_0 + \frac{\langle \bar{q}\bar{Q} \rangle \langle Q\gamma \rangle [\bar{Q}\gamma]}{\langle q\gamma \rangle \langle \bar{Q}\gamma \rangle} \left\{ \frac{\bar{q}\bar{Q}}{qQ} \right\}_1 \\ &+ \frac{\langle \bar{q}q \rangle \langle \bar{q}\bar{Q} \rangle [q\bar{Q}]}{2 \langle \bar{q}\gamma \rangle \langle q\gamma \rangle s_{\bar{Q}Q}} - \frac{\langle \bar{q}\bar{Q} \rangle \langle qQ \rangle [q\gamma]}{2 \langle q\gamma \rangle \langle \bar{Q}Q \rangle s_{\bar{Q}Q}} \\ &- iu_1^{(0)}(+, -, +) \left(\mathcal{F}(\bar{q}, \bar{Q}, Q) + \langle \bar{q}q\bar{Q}Q \rangle^2 \mathcal{F}(q, Q, \bar{Q}) \right. \\ &\quad \left. + \langle \bar{q}\gamma\bar{Q}Q \rangle^2 \mathcal{F}(\bar{Q}, \bar{q}, \gamma) + \mathcal{F}(Q, q, \gamma) \right) \end{aligned} \quad (32)$$

With the help of the relations

$$u_B(+, -, +) = u_B(+, +, +)|_{Q \leftrightarrow \bar{Q}} \quad (33)$$

$$u_A(+, -, +) = -u_A(+, +, +)|_{Q \leftrightarrow \bar{Q}} \quad (34)$$

and the functions

$$\begin{aligned} u_B(+, +, +) &= iu_1^{(0)}(+, +, +) \left(\frac{\mathcal{P}_{\bar{q}q}}{\varepsilon^2} + \frac{\mathcal{P}_{\bar{Q}Q}}{\varepsilon^2} + \frac{3}{\varepsilon} \frac{\mathcal{P}_{\bar{Q}Q}}{\varepsilon} + \frac{13}{2} \right) \\ &- \frac{\langle \bar{q}q \rangle^2 \langle \bar{Q}Q \rangle [qQ]^2}{2 \langle \bar{q}\gamma \rangle \langle q\gamma \rangle} \left\{ \frac{\bar{Q}Q}{\bar{q}\gamma} \right\}_2 - \frac{\langle \bar{q}q \rangle \langle \bar{Q}\gamma \rangle [q\gamma] [Q\gamma]}{\langle q\gamma \rangle} \left\{ \frac{\bar{q}q}{\bar{Q}Q} \right\}_2 \\ &+ \frac{\langle \bar{q}q \rangle \langle q\bar{Q} \rangle \langle \bar{Q}\gamma \rangle [q\gamma]}{\langle q\gamma \rangle^2 \langle \bar{Q}Q \rangle} \left(\left\{ \frac{\bar{Q}Q}{\bar{q}q} \right\}_1 + \left\{ \frac{\bar{Q}Q}{\bar{q}\gamma} \right\}_1 \right) \\ &+ \frac{\langle \bar{q}\bar{Q} \rangle^2}{2 \langle \bar{q}\gamma \rangle \langle q\gamma \rangle \langle \bar{Q}Q \rangle} \left\{ \frac{\bar{Q}Q}{\bar{q}\gamma} \right\}_0 + \frac{\langle \bar{q}q \rangle^2 [qQ]^2}{2 \langle \bar{q}\gamma \rangle \langle q\gamma \rangle [\bar{Q}Q] s_{\bar{q}\gamma}} - \frac{\langle \bar{q}\bar{Q} \rangle [Q\gamma]}{\langle q\gamma \rangle s_{\bar{Q}Q}} \\ &+ iu_1^{(0)}(+, +, +) \left(\mathcal{F}(\bar{q}, q, \gamma) + \langle \bar{Q}q\gamma\bar{q} \rangle^2 \mathcal{F}(q, \bar{q}, \gamma) \right) \end{aligned} \quad (35)$$

and

$$\begin{aligned}
u_A(+, +, +) &= -iu_1^{(0)}(+, +, +) \left(\frac{\mathcal{P}_{\bar{q}\bar{Q}}}{\varepsilon^2} + \frac{\mathcal{P}_{qQ}}{\varepsilon^2} - \frac{\mathcal{P}_{\bar{q}Q}}{\varepsilon^2} - \frac{\mathcal{P}_{q\bar{Q}}}{\varepsilon^2} \right) \\
&- \frac{\langle \bar{q}q \rangle \langle q\bar{Q} \rangle [q\gamma]}{\langle qQ \rangle \langle q\gamma \rangle} \left\{ \bar{Q}Q \right\}_1^{\bar{q}\gamma} - \frac{\langle \bar{q}Q \rangle \langle \bar{Q}Q \rangle [Q\gamma]}{\langle qQ \rangle \langle Q\gamma \rangle} \left\{ \bar{q}\gamma \right\}_1^{q\bar{Q}} \\
&- \frac{\langle \bar{q}Q \rangle \langle \bar{Q}\gamma \rangle [Q\gamma]}{\langle q\gamma \rangle \langle Q\gamma \rangle} \left\{ \bar{q}Q \right\}_1^{q\bar{Q}} + \frac{\langle \bar{q}q \rangle \langle \bar{q}\bar{Q} \rangle}{\langle \bar{q}\gamma \rangle \langle qQ \rangle \langle q\gamma \rangle} \left\{ \bar{Q}Q \right\}_0^{q\bar{Q}} \\
&- iu_1^{(0)}(+, +, +) \left(\mathcal{F}(\bar{q}, \bar{Q}, Q) - \langle \bar{Q}Qq\bar{q} \rangle^2 \mathcal{F}(q, \bar{Q}, Q) + \mathcal{F}(\bar{Q}, \bar{q}, \gamma) \right. \\
&\quad - \mathcal{F}(\bar{Q}, q, \gamma) + \mathcal{F}(\bar{Q}, Q, q) - \mathcal{F}(\bar{Q}, Q, \bar{q}) \\
&\quad \left. + \mathcal{F}(Q, q, \gamma) - \langle \bar{Q}Q\gamma\bar{q} \rangle^2 \mathcal{F}(Q, \bar{q}, \gamma) \right)
\end{aligned} \tag{36}$$

we obtain all the needed functions.

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