N=1 STRING DUALITY

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Abstract

We discuss duality between Type IIA string theory, eleven-dimensional supergravity, and heterotic string theory in four spacetime dimensions with $N = 1$ supersymmetry. We find theories whose infrared limit is trivial at enhanced symmetry points as well as theories with $N = 1$ supersymmetry but the field content of $N = 4$ theories which flow to the $N = 4$ fixed line in the infrared.

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1. Introduction

There has recently been dramatic progress in understanding non-perturbative aspects of string theory. Much of this progress centers around the idea of duality which allows one to study phenomena in a strongly coupled theory by relating these phenomena to weak coupling properties of a dual theory. At present the best understood dual pairs have $N = 4$ supersymmetry when reduced to $D = 4$ spacetime dimensions [1–8]. There is however increasing evidence that duality also extends to theories with $N = 2$ supersymmetry [9,10]. These theories have much richer dynamics than $N = 4$ theories as has been beautifully demonstrated in the global case [11]. Of course realistic chiral theories can have at most $N = 1$ supersymmetry and an understanding of the dynamics of such string compactifications is one of the most important unsolved problems in string theory. It is thus natural to ask whether string duality might be extended to theories with $N = 1$ supersymmetry and whether this duality can be used to study the dynamics of such theories. In this paper we will provide a partial answer to this question. We will construct two different types of dual pairs with $N = 1$ supersymmetry but we will see that both have rather simple dynamics at low-energies. The construction of more realistic $N = 1$ dual pairs remains an important open problem.

In this paper we will utilize the dictionary provided by the soliton string construction of [5] and [6] and the orbifold techniques developed in [10] to construct dual $N = 1$ pairs. We first review some of the essential elements of [10] and discuss the specific symmetries we will utilize in our orbifold constructions. We then construct two such dual pairs, one involving an asymmetric orbifold compactification of the IIA string with $(2, 1)$ world sheet supersymmetry which is dual to a heterotic compactification on a Calabi-Yau manifold and the other a duality between a compactification of eleven-dimensional supergravity (or strongly coupled IIA string theory) on a seven-manifold of G_2 holonomy and a Calabi-Yau compactification of the heterotic string. We will argue that both these pairs of theories preserve supersymmetry non-perturbatively and do not generate a superpotential. In the first example this is because there are no low-energy gauge groups except at enhanced symmetry points and the gauge groups at enhanced symmetry points are not asymptotically free. In the second example this is due to the fact that the low-energy theory has $N = 4$ field content at enhanced symmetry points much as in the construction of [10]. We end with some brief conclusions.

2. Orbifolds and Duality

Hull and Townsend [2] conjectured the existence of a duality relating the IIA theory compactified on $K3$ at weak coupling to the heterotic string on a four-torus at strong coupling. After compactification of both theories on a two-torus down to four dimensions we obtain a dual pair with $N = 4$ spacetime supersymmetry. We can then try to obtain further dual pairs by twisting. To do so we must understand how symmetries of one theory map onto symmetries of the other.

Let us start with the IIA theory on $K3 \times T^2$. We can twist this theory by a geometrical symmetry of $K3 \times T^2$. Such a symmetry acts on the cohomology of $K3$. In particular it acts on the 22 elements of $H^2(K3)$. The three self-dual elements of $H^2(K3)$ gives rise to three right-moving heterotic string coordinates while the nineteen anti-self-dual elements of $H^2(K3)$ yield left-moving heterotic string coordinates [6]. Combining these with the coordinates on T^2 which are common to both sides a geometrical action on $K3 \times T^2$ gives rise to an action on $(21, 5)$ (left,right) moving coordinates of the heterotic string.

The extra $(1, 1)$ (left,right) coordinate of the heterotic string, X_0, arises as a zero mode of the $U(1)$ Ramond-Ramond (RR) field in ten dimensions and is independent of $K3$. If we are to obtain $N = 1$ supersymmetry on the heterotic side however we must twist all six internal right-moving coordinates with the twists lying in a $SU(3)$ subgroup of $SO(6)$ (but not in an $SU(2)$ or smaller subgroup if we are to obtain only $N = 1$ supersymmetry). We are thus faced with an immediate problem since geometrical symmetries do not seem to act on X_0. We will consider two solutions to this problem. First, we can consider non-geometrical symmetries of the IIA theory. In particular, the IIA theory has a symmetry called $(-1)^{F_L}$ which acts as -1 on all states which have fermions arising from the left-moving sector of the theory [12,13]. In particular, $(-1)^{F_L}$ is -1 acting on all states in the RR sector. Since the $(20, 4)$ coordinates of the heterotic string on T^4 all arise from the RR sector of the theory $(-1)^{F_L}$ maps to a symmetry which is -1 acting on all these coordinates, including X_0. Our second solution involves recalling that the $U(1)$ RR gauge field of the IIA theory arises via the Kaluza-Klein mechanism from compactification of eleven-dimensional supergravity down to ten dimensions on an S^1. If we denote the S^1 coordinate by X_{11} then the geometrical action $X_{11} \to -X_{11}$ changes the sign of the $U(1)$ gauge field and thus induces the transformation $X_0 \to -X_0$ on the heterotic side. In the context of compactifications of d=11 supergravity one could also consider more general twists which mix X_{11} with the other compactified coordinates but we will not do so in this paper.

Since $K3$ plays a central role in duality it is reasonable to expect that symmetries of $K3$ surfaces will also play a central role in orbifold extensions of duality. At the moment the precise rules for constructing dual pairs via orbifolds are not understood. In particular, it is clear that there are subtleties associated with orbifolds constructed from symmetries which do not act freely (although these have been understood in special cases [7]). As a result we will restrict our attention in this paper to freely acting symmetries of $K3$ surfaces. It is known that $K3$ has at most a $Z_2 \times Z_2$ group of freely acting symmetries [14]. The first of these which we will call E can be taken to be the Enriques involution discussed in [10]. The second which we call A is an anti-holomorphic involution. A construction of E and A for a class of $K3$ surfaces was found using algebraic geometry in [14]. In a particular T^4/Z_2 orbifold limit of $K3$ we can construct E and A as follows. Let (z_1, z_2) be complex coordinates on a T^4 defined by the periodic identifications $z_i \sim z_1 + 1$, $z_i \sim z_i + i$ and define the Z_2 transformations

$$\Theta : \quad (z_1, z_2) \to (-z_1, -z_2)$$
$$E : \quad (z_1, z_2) \to (-z_1 + \tfrac{1}{2}, z_2 + \tfrac{1}{2}) \tag{2.1}$$
$$A : \quad (z_1, z_2) \to (\bar{z}_1 + \tfrac{1}{2}, \bar{z}_2 + \tfrac{i}{2}) \ .$$

Then dividing by the action of Θ gives an orbifold limit of $K3$ on which E and A act freely. Note that in real coordinates E and A have identical actions up to relabeling of coordinates but there is no basis in which both E and A act holomorphically. In the following sections we will use Θ, E, and A with some modifications to break the spacetime supersymmetry from $N = 8$ to $N = 4$, $N = 2$ and $N = 1$ respectively.

3. Type IIA - Heterotic duality

We can obtain a $N = 1$ compactification of the type IIA theory by proceeding in stages, first breaking $N = 4$ to $N = 2$ and then $N = 2$ to $N = 1$. To break to $N = 2$ we start with the construction in [10] of a Calabi-Yau space

$$X = \frac{K3 \times T^2}{Z_2^E} \ , \tag{3.1}$$

where Z_2^E acts as the Enriques involution E on $K3$ and as an involution on T^2. In the orbifold limit of $K3$ described earlier and with z_3 a complex coordinate on T^2 we can write this action as

$$Z_2^E : \quad (z_1, z_2, z_3) \to (-z_1 + \tfrac{1}{2}, z_2 + \tfrac{1}{2}, -z_3) \ , \tag{3.2}$$

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plus a RR gauge transformation which has no effect on perturbative IIA states but which is required for modular invariance on the heterotic side. Note that Z_2^E preserves the holomorphic three form $dz_1 \wedge dz_2 \wedge dz_3$ so X is Calabi-Yau.

Let us recall the mapping of this action to the heterotic side discussed in [10]. We decompose the Narain lattice for a six-dimensional toroidal compactification as $\Gamma^{22,6} = \Gamma^{(20,4)} \oplus \Gamma^{(2,2)}$ with the first factor associated with the original four-dimensional toroidal compactification and the second factor to the additional T^2. We denote an element of this lattice by $|p, q\rangle$ with $p \in \Gamma^{(20,4)}$ and $q \in \Gamma^{(2,2)}$. In order to obtain a lattice compatible with a $K3$ surface having an action of E we further decompose the first factor as

$$\Gamma^{(20,4)} = \Gamma^{(9,1)} \oplus \Gamma^{(9,1)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(1,1)} \ . \tag{3.3}$$

The action of Z_2^E on the heterotic side is given by interchange of the first two factors in (3.3), -1 on the third factor, and a Z_2 shift in the fourth factor. As discussed in [10], in the twisted sector the left and right-moving vacuum energies differ by $1/4$. Level matching thus requires a shift as described with the shift vector δ having length squared $\delta^2 = 1/2$. This shift corresponds to a RR gauge transformation in the dual IIA theory.

3.1. Twisting by $(-1)^{F_L}$

Now let us discuss the action of $(-1)^{F_L}$.* In the IIA theory a twist just by $(-1)^{F_L}$ simply takes the IIA theory to the IIB theory. This is because the twist kills the (R, NS) and (R, R) sectors in the IIA theory but then adds them back in with the opposite chirality for spinors arising from the left in the twisted sector. We can however obtain a non-trivial twist by combining the action of $(-1)^{F_L}$ with an order two shift on T^2. At a generic radius there are no massless states in the twisted sector and the supersymmetry is thus reduced by half.

On the heterotic side $(-1)^{F_L}$ maps to a twist which acts as $|p, q\rangle \rightarrow |-p, q\rangle$. This twist has $(20, 4)$ eigenvalues -1 on the (left,right). In the twisted sector the vacuum energies are $E_L = 1/4$ and $E_R = 0$. Thus, as in the previous example, the twist must be accompanied by a shift with $\delta^2 = 1/2$ in order to maintain modular invariance. Since only the lattice $\Gamma^{(2,2)}$ is left invariant we must put the shift in this factor.

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* The models of this and the following subsection were independently found and developed in somewhat more detail by Vafa and Witten [15].

Before combining these two actions let us first consider the dual pair of theories we obtain by modding out by the generalized action of $(-1)^{F_L}$ on both the heterotic and Type IIA sides. On the heterotic side we have seen the twist has 20 eigenvalues of -1 acting on the left and four eigenvalues -1 on the right. As a result the low-energy theory has 20 massless hypermultiplets and 4 massless vector multiplets arising from the 4 $U(1)$ gauge fields from T^2. At generic points in the moduli space this is the full low-energy gauge theory. This spectrum agrees with a similar analysis on the IIA side.

We can get larger gauge symmetry by going to enhanced symmetry points in the $\Gamma^{(2,2)}$ lattice. For example, we can go to a point with $SU(2) \times SU(2)$ symmetry by going to the self-dual radius in each S^1 of $\Gamma^{(2,2)}$. The projection by the shift vector leaves the adjoint representation invariant so in the low-energy theory we find a $N = 2$ theory with $SU(2) \times SU(2)$ gauge group (in perturbation theory) and no matter fields. Unlike the example in [10], this theory will have non-trivial quantum corrections to the vector multiplet moduli space [11]. This is certainly allowed in the heterotic theory since the dilaton is in a vector multiplet.

What is the interpretation of these corrections on the IIA side? In the $N = 2$ theories constructed in [10] the dilaton was in a hypermultiplet on the IIA side and such spacetime quantum corrections to the vector multiplet moduli space were forbidden on the IIA side, thus allowing a purely classical computation of the vector multiplet moduli space. In the example at hand this is no longer the case. The twist by $(-1)^{F_L}$ on the IIA side kills all the spacetime supersymmetries coming from the left and leaves invariant all those from the right. In worldsheet language this yields a $(4, 1)$ worldsheet theory. As in the heterotic string where all the supersymmetry comes from right-movers, this puts the dilaton in a vector multiplet on the IIA side as well. As a result the quantum corrections to the vector multiplet moduli space cannot be exactly determined by a classical computation in either theory. However the weakly coupled type IIA theory corresponds to the strongly coupled heterotic theory. Thus we should not expect to see the enhanced $SU(2)$ symmetry in the IIA perturbation theory because, once quantum effects are included, $SU(2)$ is not restored anywhere in the the moduli space of the pure gauge $N = 2$ theory [11]. This is indeed consistent with our construction: there are no enhanced symmetries in the type II theory at the self-dual radius of either S^1 of $\Gamma^{(2,2)}$.

3.2. An N=1 Example

We can now go on to further break the $N = 2$ supersymmetry down to $N = 1$ by combining the action of Z_2^E and $(-1)^{F_L}$. We have already constructed these two symmetries in both theories ensuring modular invariance on the heterotic side. This is not quite sufficient however to ensure a consistent action on the heterotic side. To see this consider the sector twisted by the product of the two symmetries. The rotation part of the product acts on lattice vectors as

$$|p_1, p_2, p_3, p_4, q\rangle \rightarrow |-p_2, -p_1, p_3, -p_4, -q\rangle \ , \tag{3.4}$$

and is accompanied by shifts in the last two factors. The p_i refer to momenta in the four lattices given in the decomposition (3.3). However a shift accompanied by a -1 rotation is equivalent to no shift at all as can be seen by redefining the coordinate in question. Thus the product acts without shifts. But since the eigenvalues of the product are the same as those of Z_2^E this is not consistent with modular invariance. However we can easily modify this by redefining the action of $(-1)^{F_L}$ on the heterotic side to include an additional shift by δ in the first $\Gamma^{(1,1)}$ factor. This has no effect in the sector twisted by $(-1)^{F_L}$ since the coordinate is inverted but cures the problem with modular invariance in the sector twisted by the product. It is not hard to see that the massless spectrum of these two $N = 1$ theories agrees at generic points in the moduli space. Unfortunately at generic points the low-energy spectrum does not include any gauge fields. However we can find low-energy gauge theories by going to the enhanced symmetry points discussed in [10] and then further projecting by the action of $(-1)^{F_L}$. One finds enhanced symmetry groups but with non-asymptotically free dynamics [15].

We now turn to a $N = 1$ example which has a different low-energy structure including a gauge group at generic points.

4. Eleven-dimensional Supergravity-Heterotic Duality

As discussed in section 2, we can obtain the action $X_0 \rightarrow -X_0$ on the heterotic side by viewing this on the IIA side as resulting from an inversion of the coordinate X_{11} in the Kaluza-Klein reduction of eleven-dimensional supergravity down to the IIA theory. In this section we will work directly with compactification of $d = 11$ supergravity on a seven-manifold to obtain $N = 1$ supersymmetry in $d = 4$. In order to obtain a dual pair

we can utilize the conjectured duality [2,4] between $d = 11$ supergravity on $K3$ and a T^3 compactification of the heterotic string. We can compactify both sides down to four dimensions on a further T^3 and then twist in order to obtain a dual pair. In doing this there are two important points we must keep in mind. First, this duality is at least at the moment under less control than string-string duality. In particular, it is not clear how one would deal with orbifolds having fixed points in $d = 11$ supergravity. Thus we will require that any symmetries we mod out by act freely on $K3 \times T^3$. Second, if we are to obtain only $N = 1$ supersymmetry in $d = 4$ then the seven-manifold we compactify on must have G_2 holonomy. In general a seven-manifold will have $SO(7)$ holonomy. The action of this on the 8_s spinor representation of $SO(7)$ will break all the supersymmetry. If we are to obtain precisely one supersymmetry then we must choose the holonomy in a subgroup of $SO(7)$ for which the 8_s has a single invariant component. This defines the embedding of G_2 in $SO(7)$. Such compactifications have been discussed previously in [16–19] and the construction of manifolds of G_2 holonomy has been described in [20].

We can construct a seven-manifold satisfying the above constraints by a generalization of the construction of the Calabi-Yau manifold X. Namely, we consider the quotient

$$Y = \frac{X \times S^1}{Z_2^A} \, , \tag{4.1}$$

where Z_2^A acts as the freely acting anti-holomorphic involution A on $K3$, as the anti-holomorphic involution $z_3 \to \bar{z}_3$ on the complex coordinate on the T^2 in the double cover of X, and as $X_{11} \to -X_{11}$ on the S^1 coordinate. Seven-dimensional spinors can be constructed as direct sums of positive and negative chirality six-dimensional spinors. The Calabi-Yau space X contains two opposite chirality covariantly constant six-dimensional spinors, η_+ and η_-, which are exchanged under complex conjugation. The sum of these, $\eta_+ + \eta_-$, is the unique covariantly constant seven-dimensional spinor on the quotient space Y. As described above, the existence of a single covariantly constant spinor implies that Y has G_2 holonomy.

In an orbifold limit of $K3$ we can construct Y as the quotient

$$\frac{T^7}{Z_2^\Theta \times Z_2^E \times Z_2^A} \, , \tag{4.2}$$

where

$$
\begin{aligned}
Z_2^\Theta : \quad & (z_1, z_2, z_3, X_{11}) \to (-z_1, -z_2, z_3, X_{11}) \\
Z_2^E : \quad & (z_1, z_2, z_3, X_{11}) \to (-z_1 + \tfrac{1}{2}, z_2 + \tfrac{1}{2}, -z_3, X_{11} + \tfrac{1}{2}) \\
Z_2^A : \quad & (z_1, z_2, z_3, X_{11}) \to (\bar{z}_1 + \tfrac{1}{2}, \bar{z}_2 + \tfrac{i}{2}, \bar{z}_3 + \tfrac{1}{2} + \tfrac{i}{2}, -X_{11}) \, .
\end{aligned}
\tag{4.3}
$$

The shift of X_{11} by one half in Z_2^E corresponds to the RR gauge transformation discussed below (3.2). With the z_3 shift included in Z_2^A, it is easily seen that Z_2^E, Z_2^A and $Z_2^E Z_2^A$ are all related by a change of basis and all act freely on both $K3$ and T^3.

Z_2^E and Z_2^A can actually be defined away from the orbifold limit as long as the four quadruplets of blown-up fixed points which are interchanged by the symmetries have been blown up in an identical manner. In particular the quotient Y can be constructed at the Aspinwall points [21] of $K3$ with an $SU(2)^{16}$ enhanced gauge symmetry. The quotient will then have $SU(2)^4$. In subsection 4.2 the dual heterotic theory at this point in the moduli space will be explicitly constructed.

The Betti numbers of Y are $b_1(Y) = b_6(Y) = 0$, $b_2(Y) = b_5(Y) = 4$ and $b_3(Y) = b_4(Y) = 19$. One way to check this is as follows. Of the 22 elements of $H^2(K3)$, 4 have eigenvalues $(1, 1)$ under (Z_2^E, Z_2^A), and 6 each have eigenvalues $(1, -1)$, $(-1, 1)$ and $(-1, -1)$. The 3 elements of $H^1(T^3)$ have eigenvalues $(1, -1)$, $(-1, 1)$ and $(-1, -1)$. Elements of $H^2(Y)$ arise from elements of $H^2(K3)$ or elements of $H^2(T^3)$ which are invariant under both Z_E^2 and Z_A^2. This gives $b_2(Y) = 4$. Elements of $H^3(Y)$ arise from the wedge product of two-forms on $K3$ and one forms on T^3 which are invariant and from the volume form on T^3. This gives $b_3(Y) = 18 + 1 = 19$. The remaining Betti numbers follow from (Hodge) duality.

Reduction of the three-form potential in $d = 11$ supergravity on Y gives b_2 $U(1)$ gauge fields in $d = 4$ and $b_3(Y)$ massless scalars. There are $b_3(Y)$ more massless scalars associated with deformations of the metric. The low-energy theory thus consists of $N = 1$ supergravity coupled to four vector supermultiplets and nineteen massless chiral multiplets, at least at generic points in the Y moduli space.

4.1. The Heterotic Dual

We can now map this compactification of $d = 11$ supergravity to a dual $N = 1$ compactification of the heterotic string. The starting point for the construction of the orbifold on the heterotic side is an even self-dual Lorentzian lattice $\Gamma^{(22,6)}$ which admits an appropriate $Z_2^E \times Z_2^A$ action. Examples of such lattices are constructed in the next section at enhanced symmetry points. A general lattice vector of such a lattice may be written in the following form

$$|p_1, p_2, p_3, p_4, q_1, q_2, q_3, r_1, r_2, r_3\rangle \in \Gamma^{(19,3)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(1,1)} \;, \qquad (4.4)$$

where p_i are four component left-moving vectors and q_i are vectors with one left-moving and one right-moving component, such that $|p, q\rangle \in \Gamma^{(19,3)}$. The r_i label points on the three $\Gamma^{(1,1)}$ factors corresponding to the torus T^3. The action of Z_2^E is

$$
\begin{aligned}
|p_1, p_2, p_3, p_4, & q_1, q_2, q_3, r_1, r_2, r_3\rangle \rightarrow \\
& e^{2\pi i \delta_E \cdot r_1} |p_3, p_4, p_1, p_2, -q_1, -q_2, q_3, r_1, -r_2, -r_3\rangle \; .
\end{aligned}
\tag{4.5}
$$

The shift satisfies $\delta_E^2 = 1/2$. This acts in the first $\Gamma^{(1,1)}$ corresponding to X_{11}. Likewise, the action of Z_2^A is

$$
\begin{aligned}
|p_1, p_2, p_3, p_4, & q_1, q_2, q_3, r_1, r_2, r_3\rangle \rightarrow \\
& e^{2\pi i (\delta_A^{(1)} \cdot r_2 + \delta_A^{(2)} \cdot r_3)} |p_2, p_1, p_4, p_3, q_1, -q_2, -q_3, -r_1, r_2, -r_3\rangle \; .
\end{aligned}
\tag{4.6}
$$

The shift $\delta_A^{(1)}$ satisfies $\delta_A^{(1)2} = 1/2$ and acts on the second $\Gamma^{(1,1)}$. Likewise the shift $\delta_A^{(2)}$ satisfies $\delta_A^{(2)2} = 1/2$ and acts on the third $\Gamma^{(1,1)}$. Z_2^E, Z_2^A and $Z_2^E \times Z_2^A$ then act freely. All these shifts are fixed by demanding level matching in the twisted sectors.

At a generic point on the lattice, the massless spectrum consists of four $U(1)$ vector supermultiplets together with nineteen complex chiral supermultiplets. Working in the RNS formalism, the four vector multiplets arise from the four left-moving bosonic states invariant under the $Z_2^E \times Z_2^A$ twists, combined with the two invariant bosonic right-moving vacuum states. Massless scalars arise from $\alpha_{-1}^I |0\rangle_L \otimes |i\rangle_R$ ($I = 1, \cdots, 22$, $i = 1, \cdots, 6$) projected onto invariant states. Six of these chiral multiplets correspond to deformations of the Calabi-Yau geometry. Twelve of the chiral multiplets arise from the chiral multiplet components of the four $N = 4$, $d = 4$ vector multiplets which survive the projection onto invariant states. The remaining states give rise to the gravitational multiplet and a single chiral multiplet containing the dilaton. There are generically no massless states in the twisted sectors, for the choices of shifts described above.

4.2. Enhanced Symmetry Points

We now construct heterotic string theories compactified on T^3 at a point of enhanced gauge symmetry which admit an action of $Z_2^E \times Z_2^A$ as described in the previous section. These will be dual to $d = 11$ supergravity compactified on a certain degenerate $K3$ surfaces. The starting point is the even, self-dual Lorentzian lattice $\Gamma^{(19,3)}$ of the form

$$
\Lambda = \Gamma^8 \oplus \Gamma^8 \oplus \Gamma^{(1,1)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(1,1)} \; ,
\tag{4.7}
$$

where Γ^8 is the root lattice of E_8. An even self-dual lattice with the desired properties may be obtained by orbifolding this lattice by a series of shifts. Further details of this general procedure may be found in [22].

We first shift by the vector $\delta = (1, 0^7; 1, 0^7; \frac{1}{2}, 0^2)(\frac{1}{2}, 0, 0)$, where the first bracket denotes a shift acting on the left, the second the shift acting on the right. Exponents denote repeated entries. This shift satisfies level-matching and generates a lattice which breaks the $E_8 \times E_8$ gauge group down to $SO(16) \times SO(16)$. To construct the lattice generated by this orbifold we proceed as follows. The lattice (4.7) can be decomposed as a $D_8 \times D_8 \times (D_1 \times D_1)^3$ lattice with conjugacy classes added as follows. We denote the singlet, vector, spinor and conjugate spinor conjugacy classes of D_n as $0, v, s$ and c respectively. Each $\Gamma^{(1,1)}$ corresponds to a $D_1 \times D_1$ factor with conjugacy classes $(0, 0) + (v, v) + (s, s) + (c, c)$, while each Γ^8 gives a D_8 factor with conjugacy classes (0) and (s). This lattice is then projected onto points $p \in \Lambda$ invariant under the action of $P = \exp(2\pi i p \cdot \delta)$ to yield the lattice Λ_0. The even self-dual lattice generated by the shift is then

$$\Lambda' = \Lambda_0 \cup (\delta + \Lambda_0) . \tag{4.8}$$

A further shift by $\delta' = ((\frac{1}{2})^4, 0^4; (\frac{1}{2})^4, 0^4; 0, \frac{1}{2}, 0)(0, \frac{1}{2}, 0)$ breaks the gauge group to $SO(8)^4$. To see this we decompose Λ_0 into a lattice with $D_4^4 \times (D_1 \times D_1)^3$ symmetry plus appropriate conjugacy classes. This lattice is then projected onto points invariant under $P' = \exp(2\pi i p \cdot \delta')$ to yield the lattice Λ_0'. The final even self-dual lattice generated by this pair of shifts is then

$$\Lambda'' = \Lambda_0' \cup (\delta + \Lambda_0') \cup (\delta' + \Lambda_0') \cup (\delta + \delta' + \Lambda_0') . \tag{4.9}$$

It may be checked that this lattice admits the $Z_2^E \times Z_2^A$ symmetry. Compactifying on a further T^3 and orbifolding with respect to this symmetry, as described in the preceding section, will yield a four-dimensional $N = 1$ heterotic theory with gauge group $SO(8)$. The associated worldsheet currents arise as invariant linear combinations of four level 1 currents on the original Narain lattice and so are at level 4. As in [10] each Z_2 leaves invariant an $N = 2$ hypermultiplet in the adjoint of $SO(8) \times SO(8)$ arising from Z_2 odd combinations of gauge currents combined with a Z_2 odd right-moving current. The combination of the two Z_2s leaves invariant three $N = 1$ chiral multiplets (two from an $N = 2$ hypermultiplet and one from the lower spin components of the $N = 2$ vector multiplet) in the adjoint of $SO(8)$. Altogether this comprises the field content of an $N = 4$ vector multiplet. (Of

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course the moduli and gravitational fields are in $N = 1$ representations.) The three chiral multiplets are permuted under the change of bases which permute the three non-trivial elements of $Z_2^A \times Z_2^E$, and there is a corresponding global $SU(3)$ flavor symmetry.

A lattice with $SU(2)^{16}$ symmetry may be obtained by shifting Λ'' with $\delta'' = ((\frac{1}{2})^2, 0^2, (\frac{1}{2})^2, 0^2; (\frac{1}{2})^2, 0^2, (\frac{1}{2})^2, 0^2; 0^2, \frac{1}{2})(0^2, \frac{1}{2})$. Now we decompose the lattice as $(D_2)^8 \times (D_1 \times D_1)^3$, plus conjugacy classes. As before we project onto points invariant under the shift, and add in twisted sectors. The resulting lattice admits the $Z_2^E \times Z_2^A$ symmetry. Upon further compactification and orbifolding as described above, we obtain a $N = 1$ heterotic theory in four dimensions with gauge group $SU(2)^4$ at level 4 and with $N = 4$ field content.

The two constructions described above give rise to rank 4 level 4 Kac-Moody algebras with $G = SO(8)$ and $G = SU(2)^4$. We presume that other rank 4 groups such as $SU(5)$ or $SU(3) \times SU(2) \times U(1)$ can also be obtained. Consistency of our picture requires that supersymmetry remains unbroken non-perturbatively. This is consistent with the fact that the field content of the low-energy gauge theory forms a finite $N = 4$ representation with a global $SU(3)$ symmetry. In the infrared this flows to a scale invariant $N = 4$ theory [23] which of course does not break supersymmetry on its own.

5. Conclusions

We have found examples of dual pairs of theories with $N = 1$ supersymmetry in four dimensions. We feel that this work along with earlier constructions [9,10] provides convincing evidence that string duality can be extended in a non-trivial way to backgrounds with $N = 2$ and $N = 1$ supersymmetry. However the $N = 1$ dual pairs constructed so far are rather simple and have non-generic low-energy behavior. In more realistic $N = 1$ theories we would expect to find spontaneous supersymmetry breaking, perhaps through gluino condensation. In analogy with the work of [9] and [24,25] it may be that more interesting pairs can be found by compactification of $d = 11$ supergravity on seven-manifolds of G_2 holonomy constructed as $K3$ fibrations of three-manifolds.

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