Abelian dominance in chiral symmetry breaking *

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Calculations of the chiral condensate $\langle\bar\psi\psi\rangle$ on the lattice using staggered fermions and the Lanczos algorithm are presented. Three gauge fields are considered: the quenched non-Abelian field, the Abelian field projected in the maximal Abelian gauge, and the monopole field further decomposed from the Abelian field. The results show that the Abelian monopoles largely reproduce the chiral condensate values of the full non-Abelian theory, both in SU(2) and in SU(3).

1. Introduction

Since the Abelian monopole mechanism for confinement in QCD was conjectured [1,2], there have been extensive studies using lattice gauge theory in the pure gauge sector [3–9]. Only a small amount of work has been done in the quark sector [10–13].

In the present work, we do a systematic study of the role of Abelian projection and Abelian monopoles in chiral symmetry breaking in the confined phase at zero temperature. We use a different approach from that used in [10], namely, the eigenvalue method using the Lanczos algorithm [14], which allows us to work at zero mass. In addition to working in SU(2) gauge theory, we also perform calculations in SU(3) for which there has been very little study of the Abelian monopole mechanism.

2. Chiral Condensate $\langle\bar\psi\psi\rangle$ on the lattice

The spontaneous breakdown of chiral symmetry is signaled by the non-vanishing of the order parameter (chiral condensate) $\langle\bar\psi\psi\rangle$. On a lattice of volume V, it is given by

$$\langle\bar\psi\psi(m,V)\rangle = -\frac{1}{V}\langle \mathrm{Tr}\left[\frac{1}{\not{D}+m}\right]\rangle \qquad (1)$$

where the angular brakets denote the gauge field configuration average. This can be expressed in terms of the eigenvalues of the Dirac operator $\not{D}$

$$\begin{aligned}\langle\bar\chi\chi(m,V)\rangle &= \frac{-1}{V}\sum_{n=1}^{N}\frac{1}{i\lambda_n+m}\\ &= \frac{-1}{V}\sum_{\lambda_n\geq 0}\frac{2m}{\lambda_n^2+m^2}. \qquad (2)\end{aligned}$$

Here $\bar\chi$, χ are the single-component staggered fermion fields and $\langle\bar\psi\psi\rangle = \frac{1}{4}\langle\bar\chi\chi\rangle$. The eigenvalues are calculated using the well-established Lanczos algorithm [14]. To truly probe the physics of spontaneous chiral symmetry breaking, one should attempt to work in the limit of zero quark mass and infinite volume. The chiral limit $m\to 0$ should be taken after $V\to\infty$

$$\begin{aligned}\langle\bar\chi\chi\rangle &= -\lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\sum_{\lambda_n\geq 0}\frac{2m}{\lambda_n^2+m^2}\\ &= -\lim_{m\to 0}\int_0^\infty d\lambda\frac{2m\rho(\lambda)}{\lambda^2+m^2} = -\pi\rho(0) \qquad (3)\end{aligned}$$

where $\rho(\lambda)=\frac{1}{V}dn/d\lambda$ is called the spectral density function and is normalized to $\int_0^\infty d\lambda\rho(\lambda) = N_c$, the number of colors. Eq. (3) relates chiral symmetry breaking to the small modes in the eigenvalue spectrum. So the task is reduced to finding the small eigenvalues, rather than the entire spectrum of the fermion matrix.

3. Abelian Projection on the Lattice

The lattice formulation of Abelian projection was developed in [4,5]. The idea is to fix the gauge

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of a SU(N) theory so that a residual gauge freedom $U(1)^{N-1}$ remains. The Abelian degrees of freedom are extracted by a subsequent projection (Abelian projection): $U(x,\mu) = c(x,\mu)\ u(x,\mu)$ where $u(x,\mu)$ is the diagonal *abelian-projected field* and $c(x,\mu)$ the nondiagonal matter (gluon) field. In general the gauge condition can be realized by making some adjoint operator $\mathcal{R}$ diagonal:

$$\tilde{\mathcal{R}}(x) = G(x)\mathcal{R}(x)G^{-1}(x) = \text{diagonal}. \qquad (4)$$

Several gauge conditions have been studied and it has been found that the so-called Maximal Abelian gauge [5] most readily captures the long-distance features of the vacuum. It is realized by maximizing the quantity, in SU(2),

$$R = \sum_{x,\mu} \text{tr}\left[\sigma_3 \tilde{U}(x,\mu)\sigma_3 \tilde{U}^\dagger(x,\mu)\right] \qquad (5)$$

or in SU(3)

$$R = \sum_{x,\mu}\sum_{i=1}^{3} |\tilde{U}_{ii}(x,\mu)|^2 \qquad (6)$$

where $\tilde{U}(x,\mu) = G(x)U(x,\mu)G^{-1}(x+\mu)$. In practice, R is maximized iteratively by solving for $G(x)$ repeatedly until some criterion is satisfied. It is well-known that there exist monopoles in a compact U(1) field as topological fluctuations. The Abelian-projected field $u(x,\mu)$ can be further decomposed into monopole plus photon contributions [15].

4. Numerical results

4.1. Results for SU(2)

Gauge fixing was done with the help of overrelaxation [16] which reduced the number of iterations by a factor of 3 to 5. We used the stopping criterion that $\delta = 1 - 1/2\text{Tr}G(x) = 1 - r_0$ converges to 10^{-6} which requires about 500 iterations with overrelaxation. Fig. 1 shows the raw data obtained for $\rho(\lambda)$. To extract a value at $\lambda = 0$, we fit the distributions by a straight line $\rho(\lambda) = a + b\lambda$ in an interval $[\lambda_{min}, \lambda_{max}]$ which excludes eigenvalues strongly influenced by finite volume effects [18]. Fig. 2 shows the results of such a fit for the interval [0.015,0.05]. The full and Abelian values for the extracted $\langle\bar{\chi}\chi\rangle$ are consistent with those obtained in Ref. [10] which used a quite different approach. The interesting feature here is that using the monopole contribution brings the values even closer to those of the full theory, while the photon configurations give almost no effects. We found either no or very few small eigenvalues for the photon configurations. For purposes of comparison, we also performed Abelian projection at $\beta = 2.5$ using a different gauge-fixing condition: the Polyakov gauge which diagonalizes the Polyakov loop according to Eq. (4). The result is that the Abelian and the monopole spectral density functions are almost an order of magnitude larger than those of the full theory. In Ref. [10], a similar result is found using the field-strength gauge.

4.2. Results for SU(3)

The gauge field configurations were generated using the Cabibbo-Marinari [17] pseudo-heat-bath method on a 8^4 lattice at $\beta = 5.7$ and a 10^4 lattice at $\beta = 5.9$. Configurations are selected after 4000 thermalization sweeps from a cold start, and every 500 sweeps thereafter. Fig. 3 shows the results obtained for 150 configurations on the 8^4 lattice. Fig. 4 shows the results obtained for 100 configurations on the 10^4 lattice. One can see that a similar pattern emerges in SU(3): the Abelian and the monopole contributions give values that are close to those of the full theory. It was also confirmed that the photon configurations give negligible contribution.

5. Conclusion

We have calculated chiral symmetry breaking on the lattice in the quenched approximation using the Lanczos algorithm which allows calculations to be done directly at zero quark mass. The results show that Abelian projected fields and Abelian monopoles in the Maximal Abelian gauge can largely reproduce the values of the full theory, both in SU(2) and in SU(3). These results extend the idea of Abelian dominance and provide some evidence that Abelian monopoles can

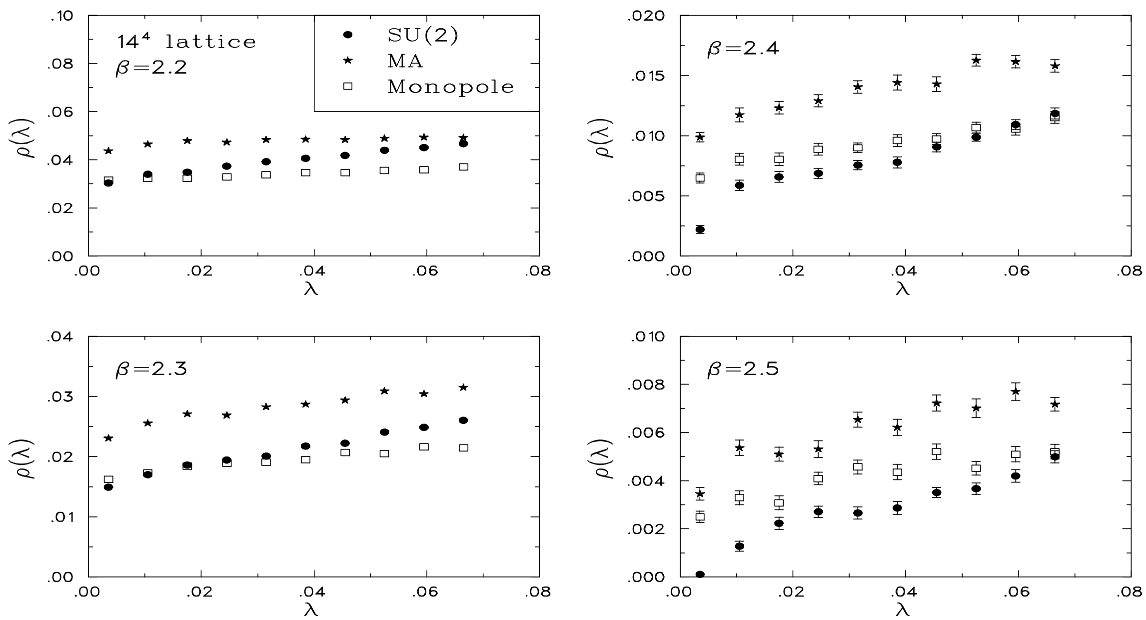

Figure 1. Raw data for the spectral density functions in SU(2).

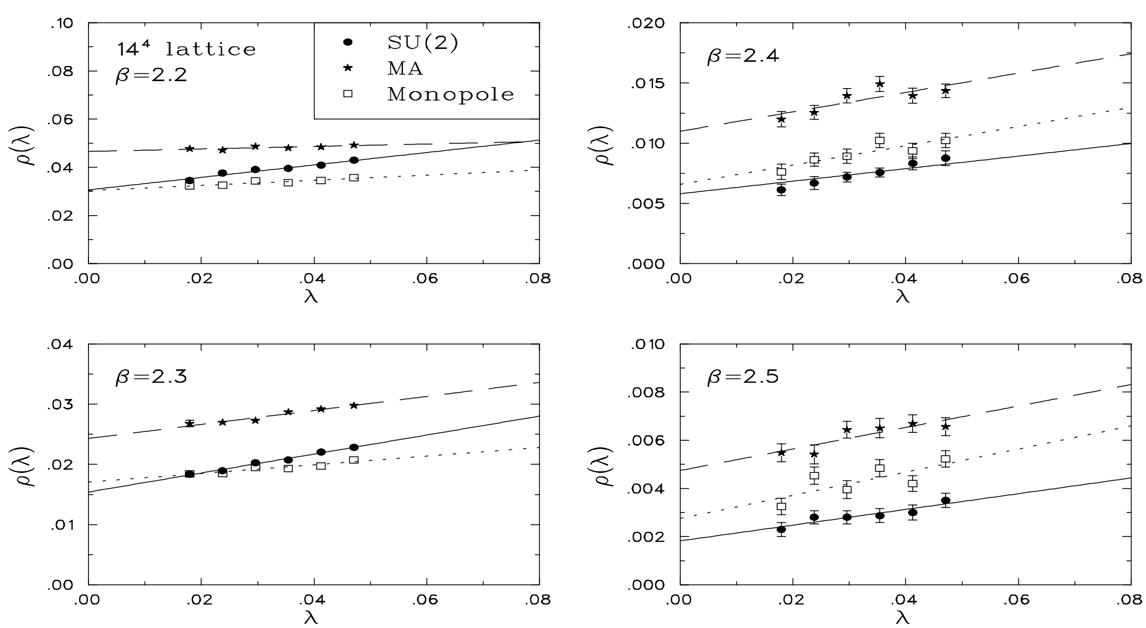

Figure 2. Fitted spectral density functions in the eigenvalue interval [0.015,0.05] in SU(2).

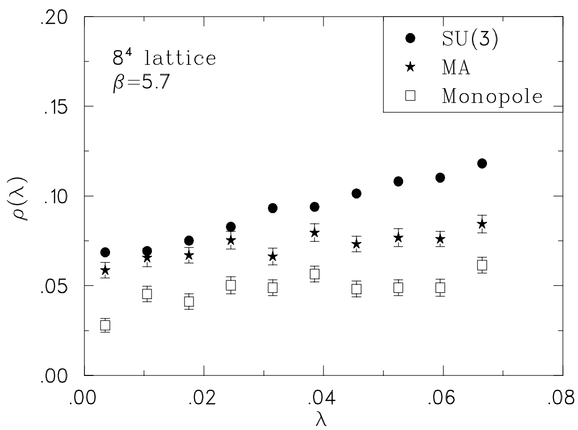

Figure 3. Spectral density function in SU(3) on the 8^4 lattice at β=5.7.

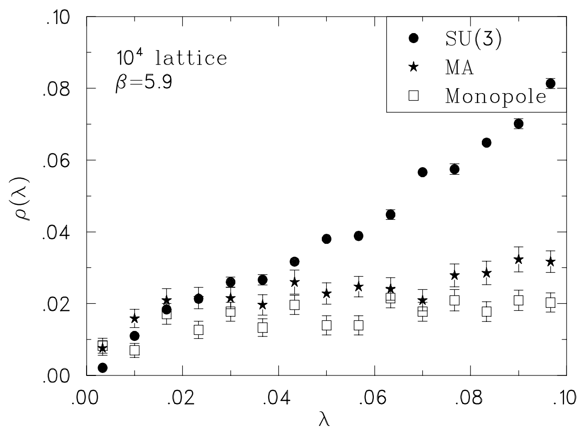

Figure 4. Spectral density function in SU(3) on the 10^4 lattice at β=5.9.

describe the long-distance physics of light quarks in QCD.

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