A NONPERTURBATIVE CALCULATION OF BASIC CHIRAL QCD PARAMETERS WITHIN ZERO MODES ENHANCEMENT MODEL OF THE QCD VACUUM. I

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Abstract

A new zero modes enhancement (ZME) model of true vacuum of QCD is breifly described. It makes possible to analytically investigate and calculate numerically low-energy QCD structure from first principles. Expressions of basic chiral QCD parameters (the pion decay constant, quark and gluon condensates, the dynamically generated quark mass, etc) as well as the vacuum energy density, suitable for numerical calculations, have been derived. Solution to the Schwinger-Dyson (SD) equation for the quark propagator in the infrared (IR) domain on the basis of the ZME effect in QCD was used for this purpose. There are only two independent quantities (free parameters) by means of which calculations should be done within our approach. The first one is the constant of integration of the above mentioned quark SD equation of motion. The second one is a scale at which nonperturbative effects begin to play a dominant role.

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I. THE ZERO MODES ENHACEMENT MODEL OF QUARK CONFINEMENT AND DCSB

Today there are no doubts that the dynamical mechanisms of quark confinement [1] and dynamical chiral symmetry breaking (DCSB) [2, 3] are closely related to the complicated topological structure of the QCD nonperturbative vacuum. For this reason any correct nonperturbative model of quark confinement and DCSB necessary becomes a model of a true QCD vacuum. Also, it becomes clear that nonperturbative infrared (IR) divergenses are closely related, on the one hand, to the above mentioned nontrivial vacuum structure. On the other hand, they play an important role in the large scale behaviour of QCD [4-6]. If it is true that QCD is an IR unstable theory (i.e., has no IR stable fixed point) then the low-frequency modes of the Yang-Mills fields should be enhanced due to nonperturbative IR divergences. So the full gluon propagator can diverge faster than the free one at small momentum, in accordance with [7, 8]

$$D_{\mu\nu}(q) \sim (q^2)^{-2}, \qquad q^2 \to 0 \tag{1.1}$$

which describes the zero modes enhancement (ZME) effect in QCD. If the low-frequency components of the virtual fields in the true vacuum have a larger amplitude indeed than those of the bare (perturbative) vacuum [5] then the Green function for a single quark should be reconstructed on the basis of this effect. It is important to understand that a possible effect of the ZME (1.1) is our primary dynamical assumption. We will consider this effect as a rather realistic confining ansatz for the full gluon propagator in order to use it as input information for the quark SD equation. Such singular behaviour of the full gluon propagator requires the introduction of a small IR regulation parameter ϵ in order to define the initial SD equations and Slavnov-Taylor (ST) identities in the IR region. Because of this the quark propagator and other Green's functions become dependent, in general, on this IR regulation parameter ϵ which is to be set to zero at the end of computation $\epsilon \to 0^+$.

Let us consider the exact, unrenormalized SD equation for the quark propagator in momentum space

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F \int \frac{d^n q}{(2\pi)^n} \Gamma_\mu(p,q) S(p-q) \gamma_\nu D_{\mu\nu}(q), \qquad (1.2)$$

where C_F is the eigenvalue of the quadratic Casimir operator in the fundamental representation. Other notions are the usual ones. Let us only note that our parametrization of the full quark propagator is as follows

$$-iS(p) = \hat{p} A(-p^2) + B(-p^2). \qquad (1.3)$$

The full gluon propagator in the arbitrary covariant gauge is

$$D_{\mu\nu}(q) = -i \left\{ \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \frac{1}{q^2} d(-q^2, a) + a \frac{q_\mu q_\nu}{q^4} \right\}, \qquad (1.4)$$

where a is a gauge fixing parameter ($a = 0$ for Landau gauge).

Assuming that in the IR region

$$d(-q^2, a) = \left(\frac{\mu^2}{-q^2} \right) + \beta(a) + O(q^2), \qquad q^2 \to 0, \qquad (1.5)$$

where μ is the appropriate mass scale parameter, we obtain the above mentioned generally accepted form of the IR singular asymptotics for the full gluon propagator (1.1) [4-5, 7-9] (enhancement of the zero modes).

In order to actually define an initial SD equation (1.2) in the IR region (at small momenta) let us apply the gauge-invariant dimensional regularization method of 't Hooft and Veltman [10] in the limit $n = 4 + 2\epsilon, \quad \epsilon \to 0^+$, where ϵ is the above mentioned a small IR regulation parameter. We consider the SD equations and the corresponding quark-gluon ST identity in Euclidean space ($d^n q \to i d^n q_E, \quad q^2 \to -q_E^2, \quad p^2 \to -p_E^2,$ but for simplicity the Euclidean subscript will be omitted). Let us use, in the sense of the distribution theory, the relation [11]

$$(q^2)^{-2+\epsilon} = \frac{\pi^2}{\epsilon} \delta^4(q) + (q^2)_+^{-2} + O(\epsilon), \qquad \epsilon \to 0^+, \qquad (1.6)$$

where $(q^2)_+^{-2}$ is a functional acting on the main (test) functions according to the so-called "plus prescription" standard formulae [11]. Substituting (1.4-1.6) into the quark SD equation (1.2) on account of the above mentioned "plus prescription" formulae and expanding in powers of q and keeping the terms of order q^{-2} (the Coulomb order terms), in agreement with (1.5), one finally arrives at the quark propagator expansion in the IR region for unrenormalized quantities [12] (in four-dimensional Euclidean space).

As mentioned above, all Green's functions become dependent generally on the IR regularization parameter ϵ. In order to extract the finite Green's functions in the IR region, we introduce the renormalized (IR finite) quark-gluon vertex function at zero momentum transfer and the quark propagator as follows $\Gamma_\mu(p,0) = Z_1(\epsilon)\bar{\Gamma}_\mu(p,0)$ and $S(p) = Z_2(\epsilon)\bar{S}(p)$ at $\epsilon \to 0^+$, respectively. Here $Z_i(\epsilon)$ $(i = 1, 2)$ are the corresponding IR renormalization constants. The ϵ-parameter dependence is indicated explicitly to distinguish them from the usual UV renormalization constants. $\bar{\Gamma}_\mu(p,0)$ and $\bar{S}(p)$ are the renormalized (IR finite) Green's functions and therefore do not depend on ϵ in the $\epsilon \to 0^+$ limit, i.e. they exist as $\epsilon \to 0^+$. The correct treatment of such strong singularity (1.6) within distribution theory [11] enabled us to extract the required class of test functions in the renormalized quark SD equation. The test functions do consist of the quark propagator and the corresponding quark-gluon vertex function. By the renormalization program we have found the regular solutions for the quark propagator (see below). For that very reason relation (1.6) is justified, it is multiplied by the appropriate smooth test functions. Due to a quark convergence condition, a cancellation of nonperturbative IR divergences takes place. Because of this condition the explicitly gauge-dependent terms (the so-called next-to-leading terms) in the above mentioned quark propagator expansion become ϵ - order terms. For this reason these noninvariant terms vanish in the $\epsilon \to 0^+$ limit.

Absolutely in the same way should be reconstructed the ghost self-energy and the corresponding ST identity for the quark-gluon vertex [12]. We develop a method for the extraction of the IR-finite Green's functions in QCD which means that they do not depend on the IR regulation parameter ϵ as $\epsilon \to 0^+$. For this purpose we have worked out a renormalization program in order to cancel all the IR nonperturbative divergences which makes it possible to obtain a close set of the SD equations and the corresponding ST identity in the quark sector. By completing our renormalization program, we explicitly show that only multiplication by the quark IR renormalization constant will remove all nonperturbative IR divergences from the theory on a general ground. We have shown [12] that for the covariant gauges the complications due to ghost contributions can be considerable in our approach.

The closed set of equations, obtained in our paper [12], which will be used for numerical calculation of basic chiral QCD parameters as well as the vacuum energy density, should read

$$S^{-1}(p) = S_0^{-1}(p) + \tilde{g}^2 \bar{\Gamma}_\mu(p,0) S(p) \gamma_\mu, \tag{1.7}$$

$$\frac{1}{2}\bar{b}(0)\bar{\Gamma}_\mu(p,0) = i\partial_\mu S^{-1}(p) - \frac{1}{2}\bar{b}(0) S(p) \bar{\Gamma}_\mu(p,0) S^{-1}(p), \tag{1.8}$$

where $S_0^{-1}(p)$ is the free quark propagator and $\tilde{g}^2$ includes the mass scale parameter μ^2, determining the validity of the above mentioned deep IR singular asymptotic behaviour of the full gluon propagator (1.5). Let us note that the IR finite quark renormalization constant, explicitly not shown here, which should multiply free quark propagator in Eq.(1.7) is to be set to unity without losing generality (multiplicative renormalizability) [12]. It is worth noting also that Eq.(1.7) and Eq.(1.8) describe the leading terms of the corresponding expansions of the quark SD equation and ST identity in the IR region, respectively [12].

In order to solve the system (1.7-1.8), it is convenient to represent the quark-gluon vertex function at zero momentum transfer as follows

$$\bar{\Gamma}_\mu(p,0) = F_1(p^2)\gamma_\mu + F_2(p^2)p_\mu + F_3(p^2)p_\mu \hat{p} + F_4(p^2)\hat{p}\gamma_\mu. \tag{1.9}$$

Substituting this representation into the ST identity (1.8), one obtains

$$\begin{aligned}
F_1(p^2) &= -\overline{A}(p^2), \\
F_2(p^2) &= -2\overline{B}'(p^2) - F_4(p^2), \\
F_3(p^2) &= 2\overline{A}'(p^2), \\
F_4(p^2) &= \frac{A^2(p^2)B^{-1}(p^2)}{E(p^2)}.
\end{aligned} \tag{1.10}$$

Here the prime denotes differentiation with respect to the Euclidean momentum variable p^2 and $\overline{A}(p^2) = A(p^2)E^{-1}(p^2)$, $\overline{B}(p^2) = B(p^2)E^{-1}(p^2)$ with $E(p^2) = p^2A^2(p^2) + B^2(p^2)$. For the sake of convenience, the ghost self-energy at zero point $\bar{b} \equiv \bar{b}(0)$ is included into the definition of a new coupling constant $\lambda = g^2[\bar{b}(0)]^{-1}(2\pi)^{-2}$.

Proceeding now to the dimensionless variables by $p^2 = \mu^2 t = \mu^2 \frac{\lambda}{2} z$ and parameters $\frac{2}{\lambda} t_0 = z_0, t_0 = \frac{k_0^2}{\mu^2}$, introducing then the following notations

$$A(p^2) = \mu^{-2} A(t) = -\mu^{-2} \frac{2}{\lambda} g(z), \ B^2(p^2) = \mu^{-2} B^2(t) = \mu^{-2} \frac{2}{\lambda} B^2(z_0, z), \tag{1.11}$$

and doing some algebra, the initial system (1.7-1.8) can be rewritten as follows (normal form)

$$g'(z) = -[\frac{2}{z} + 1] g(z) + \frac{1}{z} + \tilde{m}_0 B(z) \tag{1.12}$$

$$B'(z) = -\frac{3}{2} g^2(z) B^{-1}(z) - [\tilde{m}_0 g(z) + B(z)], \tag{1.13}$$

where $\tilde{m}_0 = m_0 (\frac{2}{\lambda})^{1/2}$. It is easy to check that solutions to this system in the chiral limit $m_0 = 0$ are:

$$g(z) = z^{-2} [\exp(-z) - 1 + z] \tag{1.14}$$

and

$$B^2(z_0, z) = 3 \exp(-2z) \int_z^{z_0} \exp(2z') g^2(z')\, dz'. \tag{1.15}$$

The exact solutions (1.14) for $A(z)$ and (1.15) for dynamically generated quark mass function $B(z_0, z)$ are not entire functions. Functions $A(z)$ and $B(z_0, z)$ have removable singularities at zero. In addition, the dynamically generated quark mass function $B(z_0, z)$ also has algebraic branch points at $z = z_0$ and at infinity. Apparently, these unphysical singularities are due to ghost contributions. The quark propagator may or may not be an entire function but in either cases the pole-type singularities should disappear. This is a general feature of quark confinement and holds in any gauge.

In order to reproduce automatically the correct behaviour of the dynamically generated quark mass function at infinity, it is necessary to put $z_0 = \infty$ in (1.15) from the very beginning. Obviously, in this case solution (1.15) cannot be accepted at zero $z = 0$, so one needs to keep the constant of integration z_0 arbitrary but finite in order to obtain a regular, finite solution at the zero point.

The region $z_0 > z$ can be considered as nonperturbative, whereas the region $z_0 \leq z$ can be considered as perturbative. By approximating the full gluon propagator by its deep IR asymptotics such as $(q^2)^{-2}$ in the whole range $[0, \infty)$, we nevertheless obtain that our solution for the dynamical quark mass function $B(z_0, z)$ manifests the existence of the boundary value momentum (dimensionless) z_0 which separates the IR (nonperturbative) region from the intermediate and UV (perturbative) regions. If QCD confines then a characteristic scale, at which confinement and other nonperturbative effects become essential, must exist. On the other hand, because of this one can eliminate the influence of the above mentioned unphysical singularities, coming from the solutions to the quark SD equations (due to necessary ghost contributions), on the S-matrix elements reproducing physical quantities.

Thus, within our approach to QCD at large distances in order to obtain numerical values of any physical quantity, e.g. the pion decay constant (see below), the integration over the whole range $[0, \ \infty]$ reduces to the integration over the nonperturbative region $[0, \ z_0]$, which determines the range of validity of the deep IR asymptotics (1.1) of the full gluon propagator and consequently the range of validity of the corresponding solutions (1.14) and (1.15) for the IR piece of the full quark propagator.

Let us make now the main conclusions. First, if the enhancement of the zero modes of the vacuum fluctuations (1.1) takes place indeed then the quark Green's function, reconstructed on the basis of this effect, has no poles. In other words, the enhancement of the zero modes at the expense of the virtual gluons alone removes nevertheless a single quark from the mass shell (quark confinement theorem of Ref. 12). Second, a chiral symmetry violating part of the quark propagator in this case is automatically generated. From the obtained system (1.12-1.13) it explicitly follows that a chiral symmetry preserving solution ($m_0 = B(z) = 0, A(z) \neq 0$) *is forbidden*. So a chiral symmetry violating solution ($m_0 = 0, B(z) \neq 0, A(z) \neq 0$) for the quark SD equation *is required*. Thus the enhancement of the zero modes automatically leads to quark confinement and DCSB at the fundamental quark level and they are in close connection with each other.

II. THE VACUUM ENERGY DENSITY AND GLUON CONDENSATE

The effective potential method for composite operators [13] makes it possible to investigate the vacuum of QCD since in the absence of the external sources the effective potential is nothing but the vacuum energy density, the main characteristic of the vacuum. In this Section we will evaluate the vacuum energy density within the ZME effect in QCD (1.1) as described in the preceding Section. The effective potential at one-loop level is [13]

$$\begin{aligned} V(S,D) &= V(S) + V(D) = \\ &- i \int \frac{d^n p}{(2\pi)^n} Tr\{\ln(S_0^{-1}S) - (S_0^{-1}S) + 1\} \\ &+ i\frac{1}{2} \int \frac{d^n p}{(2\pi)^n} Tr\{\ln(D_0^{-1}D) - (D_0^{-1}D) + 1\}, \end{aligned} \tag{2.1}$$

where $S(p)$ (1.3), $S_0(p)$ and $D(p)$ (1.4), $D_0(p)$ are the full, free quark and gluon propagators, respectively. The trace over space-time and color group indices is assumed but they are suppressed for simplicity in this equation. Let us recall that the free gluon propagator can be obtained from (1.4) by setting simply $d(-q^2, a) = 1$.

Evidently the effective potential is normalized as follows $V(S_0, D_0) = V(S_0) = V(D_0) = 0$. Because of this normalization the vacuum energy density now should be defined as follows $\epsilon = V(S_0, D_0) - V(S,D) = -V(S,D)$. In fact, this is the difference between perturbative (normalized to zero) and nonperturbative vacuums. This means that $\epsilon = \epsilon_q + \epsilon_g$ with $\epsilon_q = -V(S)$, $\epsilon_g = -V(D)$, where $V(S)$ and $V(D)$ are given by (2.1).

Going to Euclidean space ($d^4p \to id^np$, $\quad p^2 \to -p^2$) and dimensionless variables and parameters (2.10), we finally obtain after some algebra ($n = 4$)

$$\epsilon_q = -\frac{3}{8\pi^2} k_0^4 z_0^{-2} \int_0^{z_0} dz\, z \left\{\ln z \left[zg^2(z) + B^2(z_0, z)\right] - 2zg(z) + 2\right\}, \tag{2.2}$$

where we introduced the UV cutoff which should be identified with the arbitrary constant of integration z_0 as was discussed in previous Section. The explicit expressions for $g(z)$ and

$B^2(z_0, z)$ are given by (1.14) and (1.15) respectively. Within our approach this expression will be used for the numerical calculation of the vacuum energy density due to confining quarks with dynamically generated masses.

The vacuum energy density due to nonperturbative gluon contributions in the same variables is

$$\epsilon_g = \frac{1}{\pi^2} k_0^4 z_0^{-2} \int_0^{z_0} dz\, z\, \{\ln(1 + \frac{6}{z}) - \frac{3}{2z} + b\} \tag{2.3}$$

Here one important remark is in order. In fact, vacuum energy density ϵ_g does not vanish at $z_0 \to \infty$ as it should because of the above mentioned normalization. Thus, it needs an additional regularization at this limit. From (2.3), it follows that the term containing the constant b should be substracted from this expression. So regularized vacuum energy density should be calculated through the relation (2.3) which becomes

$$\epsilon_g = -\frac{1}{\pi^2} k_0^4 z_0^{-2} \times \left[18 \ln(1 + \frac{z_0}{6}) - \frac{1}{2} z_0^2 \ln(1 + \frac{6}{z_0}) - \frac{3}{2} z_0 \right]. \tag{2.4}$$

Precisely this expression will be evaluated numerically. The vacuum energy density due to confining quarks (2.2) automatically disappears at $z_0 \to \infty$, so it does not requires any additional regularization.

The vacuum energy density is important on its own right as the main characteristics of the nonperturbative vacuum of QCD. On the other hand, it makes possible to estimate such important phenomenogical parameter as the gluon condensate introduced within the QCD sum rules approach to resonance physics [14]. Indeed, through the vacuum energy density ϵ it can be expressed as follows

$$\langle 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle = -\frac{32}{9} \epsilon = -\frac{32}{9} (\epsilon_q + \epsilon_g). \tag{2.5}$$

For the derivation of this formula see Ref. 14. The weakness of this derivation is, of course, that it was obtained on the basis of the perturbative calculation of the $\beta(\alpha_s)$-function. In any case, it would be instructive to estimate the gluon condensate with the help of (2.5).

III. BASIC CHIRAL QCD PARAMETERS

Because of its especially small mass, the pion is the most striking example of the Goldstone realization of chiral symmetry $SU(2)_L \times SU(2)_R$. Beside the quark condensate and the dynamically generated quark mass it is one of the three important chiral QCD parameters that determine the scale of chiral dynamics.

I. The pion decay constant F_π is defined in the current algebra (CA) as

$$\langle 0|J^i_{5\mu}(0)|\pi^j(q)\rangle = iF_\pi q_\mu \delta^{ij}. \tag{3.1}$$

(The normalization $F_\pi = 92.42\ MeV$ is used [15, 16]). Clearly, this matrix element can be written in terms of the pion- quark-antiquark proper vertex and quark propagators as

$$iF_\pi q_\mu \delta^{ij} = \int \frac{d^4p}{(2\pi)^4} Tr\{\left(\frac{\lambda^i}{2}\right)\gamma_5\gamma_\mu S(p+q)G^j_5(p+q,p)S(p)\}. \tag{3.2}$$

Here and below, the trace is understood over the Dirac and colour indices. To get expression for F_π one has to differentiate Eq.(3.2) with respect to the external momentum q_ν and then set $q = 0$.

Information on the BS pion wave function up to terms of order q can be obtained from the corresponding axial-vector vertex. Indeed, in our paper [17] it has been found that this vertex can be decomposed in a self-consistent way into pole (dynamical) and regular parts as follows

$$\Gamma^i_{5\mu}(p+q,p) = -\frac{q_\mu}{q^2}G^i_5(p+q,p) + \Gamma^{iR}_{5\mu}(p+q,p), \tag{3.3}$$

where the BS bound-state amplitude is

$$G^i_5(p+q,p) = -\frac{1}{F_\pi}\left(\frac{\lambda^i}{2}\right)\gamma_5 G(p+q,p), \tag{3.4}$$

with

$$G(p+q,p) = 2\overline{B}(-p^2) + \hat{q}R_6(-p^2) + \hat{p}\hat{q}R_{11}(-p^2) \tag{3.5}$$

and arbitrary form factors are the residues of the corresponding form factors, entering the vertex from the very beginning [17]. The regular part at zero momentum transfer $q = 0$ is determined as

$$\Gamma^{iR}_{5\mu}(p,p) = (\frac{\lambda^i}{2})\gamma_5\{\gamma_\mu G_1 + p_\mu G_2 + p_\mu \hat{p} G_3 + \hat{p}\gamma_\mu G_4\}, \tag{3.6}$$

where in the Euclidean space

$$\begin{aligned} G_1(p^2) &= -\overline{A}(p^2) - R_6(p^2) \\ G_2(p^2) &= 2\overline{B}'(p^2) \\ G_3(p^2) &= 2\overline{A}'(p^2) \\ G_4(p^2) &= -R_{11}(p^2). \end{aligned} \tag{3.7}$$

This system is nothing else but the conditions for the cancellation of the dynamical singularities at $q = 0$ for the corresponding form factors [17]. The regular part at $q = 0$ also depends on the same form factors R_6 and R_{11} as the BS bound-state amplitude up to terms of order q.

Then by taking into account the BS pion wave function up to terms of order q given by (3.4-3.5) and (1.3) with the substitution $p \to p + q$ and expanding in powers of q, the expression (3.2) can easily be evaluated. Going over to the Euclidean space ($d^4p \to id^4p,\ p^2 \to -p^2$) and using dimensional variables (1.11), one finally obtains

$$\begin{aligned} F^2_{CA} = \frac{12\pi^2}{(2\pi)^4}\mu^2 \int_0^\infty dt\, t\{-\overline{B}(t)[AB + \frac{1}{2}t(A'B - AB')] \\ -\frac{3}{4}tABR_{11}(t) + \frac{1}{4}R_6(t)(E - 3B^2)\}, \end{aligned} \tag{3.8}$$

Here and below the primes denote differentiation with respect to the dimensionless Euclidean momentum variable t, $A = A(t), B = B(t)$ and quantities with overline are shown after Eqs. (1.10). Here and in what follows we denote the pion decay constant F_π in the chiral limit of the CA representation by F_{CA}.

The main problem now is to find a good nonperturbative ansatz for both arbitrary form factors $R_j(t)(j = 6, 11)$ in the IR region. In nonperturbative calculations these terms cannot be ignored by saying formally they are of order g^2, as it was done in the perturbative treatments [3, 18-19]. In connection with this let us point out that the difference between the vector and axial-vector currents disappears in the chiral limit. For this reason let us assume [20] that the IR finite quark-gluon vector vertex function at zero momentum transfer (1.9-1.10) is a good approximation to the regular piece of the axial-vector vertex at zero momentum transfer in the chiral limit (3.6-3.7). A fortunate feature that admits to exploit partial analogy between vector and axial-vector currents in the chiral limit for the flavor non-singlet channel is that the contribution to the pion decay constant in the CA representation (3.8) does not depend on the form factor $G_2(p^2)$ at all. In this case the analogy between (1.10) and (3.7) becomes complete, and one obtains

$$R_6(p^2) = 0, \qquad R_{11}(p^2) = -\frac{A^2(p^2)B^{-1}(p^2)}{E(p^2)}. \tag{3.9}$$

Of course, we cannot prove these relations but it will be shown later (part II) that this dynamical assumption (nonperturbative ansatz) leads to very good numerical results for all chiral QCD parameters thereby justifying it once more.

In terms of the new parameters and variables (1.11) and on account of (3.9) we finally recast (3.8) as follows

$$F^2_{CA} = \frac{3}{8\pi^2} k_0^2 z_0^{-1} \int_0^{z_0} dz \, \frac{zB^2(z_0, z)}{\{zg^2(z) + B^2(z_0, z)\}}. \tag{3.10}$$

This expression will be used for numerical calculation of the pion decay constant in the CA representation.

II. The second important chiral QCD parameter is the dynamically generated quark mass m_d, defined as the inverse of the full quark propagator (1.3) in the chiral limit at zero point [12, 17, 21]

$$m_d = [iS_{ch}(0)]^{-1}, \tag{3.11}$$

where $S_{ch}(0)$ denotes the full quark propagator in the chiral limit $m_0 = 0$. Obviously, this definition assumes also regularity at the zero point. Though the dynamical quark mass m_d

is not an experimentally observable quantity, by all means, it is desirable to find such kind of solutions to the quark SD equations in which dependence on a gauge-fixing parameter disappears. In this sense m_d defined by (3.11) becomes gauge-invariant. As it was briefly discussed in Section 1, exactly such a nonperturbative quark propagator has been found within our approach to QCD at large distances [12].

Using the standard decomposition of the quark propagator (1.3) and its inverse, dynamical chiral symmetry breaking (DCSB) at the fundamental (microscopic) quark level can be implemented by the following condition

$$\{S^{-1}(p), \gamma_5\}_+ = i\gamma_5 2\overline{B}(-p^2) \neq 0, \tag{3.12}$$

so that the γ_5 invariance of the quark propagator is broken and the measure of this breakdown is the double of the dynamically generated quark mass function $2\overline{B}(-p^2)$. Precisely this quantity at zero $2\overline{B}(0)$ can be defined as a scale of DCSB at the fundamental quark level [17]. In accordance with (3.11), let us denote it by

$$\Lambda_{CSBq} = 2\overline{B}(0) = 2m_d, \tag{3.13}$$

The definitions m_d and Λ_{CSBq} have now direct physical sense within the above mentioned solutions to the quark SD equation.

Let us write down the final result for the dynamically generated nonperturbative quark mass (3.11) too, expressed in terms of the new parameters and variables (1.11)

$$m_d = k_0\{z_0 B^2(z_0, 0)\}^{-1/2}, \tag{3.14}$$

where $B^2(z_0, 0)$ is given by (1.15) at zero point.

III. As it is well known, the order parameter of DCSB - quark condensate can also be expressed in terms of the quark propagator scalar function $B(-p^2)$ (1.3). Its definition is

$$\langle \overline{q}q \rangle = \langle 0|\overline{q}q|0\rangle = -\int \frac{d^4p}{(2\pi)^4} TrS(p). \tag{3.15}$$

The final result expressed in terms of new variables and parameters (1.11) is as follows

$$\langle \overline{q}q \rangle_0 = -\frac{3}{4\pi^2} k_0^3 z_0^{-3/2} \int_0^{z_0} dz\, z B(z_0, z), \tag{3.16}$$

and as a function of m_d it can be expressed on account of (3.14). Thus there are only two independent (free) quantities by means of which all calculations should be done in our approach. The first one is the constant of integration z_0 of the above mentioned quark SD equation of motion. The second quantity is a scale k_0 at which nonperturbative effects begin to play a dominant role.

The ZME model of a true QCD vacuum enables us to describe quark confinement and DCSB on a general ground. We begin Part II with numerical investigation of the low energy QCD structure at the chiral limit. At low energies QCD is under control of $SU_L(N_f) \times SU_R(N_f)$ chiral symmetry (N_f is the number of different flavors) and its dynamical breakdown in the vacuum to the corresponding vectorial subgroup. So to correctly calculate basic low energy QCD parameters in the chiral limit means to corectly understand the dynamical structure of QCD at low energies. That is why it is important to start from the chiral limit.

REFERENCES

[1] K.B.Wilson, Phys.Rev., **D10** (1974) 2445;
A.Casher, J.Kogut and L.Susskind, Phys.Rev., **D10** (1974) 732

[2] R.Jackiw and K.Johnson, Phys.Rev., **D8** (1973) 2386;
J.M.Cornwall and R.F.Norton, Phys.Rev., **D10** (1973) 3338

[3] A.Barducci, R.Casalbuoni, S.De Curtis, D.Dominici and R.Gatto, Phys.Rev., **D38** (1988) 238;
K.Higashijima, Prog.Theor.Phys.Suppl., **104** (1991) 1-69

[4] W.Marciano and H.Pagels, Phys.Rep., **C36** (1978) 139

[5] S.Mandelstam, Phys.Rev., **D20** (1979) 3223

[6] J.L.Gervais and A.Neveu, Phys.Rep., **C23** (1976) 240;
L.Susskind and J.Kogut, Phys.Rep., **C23** (1976) 348

[7] H.Pagels, Phys.Rev., **D15** (1977) 2991

[8] M.Baker, J.S.Ball and F.Zachariasen, Nucl.Phys., **B186** (1981) 531, 560;
N.Brown and M.R.Pennington, Phys.Rev., **D39** (1989) 2723;
D.Atkinson, H.Boelens, S.J.Hiemastra, P.W.Johnson, W.E.Schoenmaker and K.Stam, Jour.Math.Phys., **25** (1984) 2095

[9] V.Sh.Gogohia, Phys.Rev., **D40** (1989) 4157; Phys.Rev., **D41** (1990) 3279

[10] G.'t Hooft and M.Veltman, Nucl.Phys., **B44** (1972) 189

[11] J.N.Gelfand and G.E.Shilov, Generalized Functions (AP, NY, 1964)

[12] V.Sh.Gogohia, Int.Jour.Mod.Phys., **A9** (1994) 759

[13] J.M.Cornwall, R.Jakiw and E.Tomboulis, Phys.Rev., **D10** (1974) 2428;
R.W.Haymaker, Riv.Nuovo Cim., **14** (1991) 1-89

[14] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl.Phys., **B147** (1979) 385, 448

[15] B.R.Holstein, Phys.Lett., **B244** (1990) 83

[16] W.J.Marciano and A.Sirlin, Phys.Rev.Lett., **71** (1993) 3629

[17] V.Sh.Gogohia, Int.Jour.Mod.Phys., **A9** (1994) 605

[18] H.Pagels, Phys.Rev., **D19** (1979) 3080

[19] H.Pagels and S.Stokar, Phys.Rev., **D20** (1979) 2947;
P.Langacker and H.Pegels, Phys.Rev., **D9** (1974) 3413

[20] V.Sh.Gogohia, Gy.Kluge and B.A.Magradze, Phys.Lett., **B244** (1990) 68

[21] J.M.Cornwall, Phys.Rev., **D22** (1980) 1452