

The Role of Temperature in a Dimensional Approach to QCD_3 *

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Abstract

We analyze the role played by temperature in QCD_3 by means of a dimensional interpolating approach. Pure gauge QCD_3 is defined on a strip of finite width L , which acts as an interpolating parameter between two and three dimensions. A two-dimensional effective theory can be constructed for small enough widths giving the same longitudinal physics as QCD_3 . Explicit calculations of T-dependent QCD_3 observables can thus be performed. The generation of a deconfining phase transition, absent in QCD_2 , is proven through an exact calculation of the electric or *Debye* mass at high T. Low and high T behaviors of relevant thermodynamic functions are also worked out. An accurate estimate of the critical temperature is given and its evolution with L is studied in detail.

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1 Introduction

In the non-perturbative nature of non-abelian gauge theories in three and four dimensions stems the complexity of the confinement phenomenon. On the contrary confinement shows up as an outgrowth of one dimensional space kinematics in 2D non-abelian gauge theories. This is not a striking fact if one considers the importance of kinematical constraints in 2D Quantum Field Theories, e.g. in conformal field theories. It is therefore far from obvious what kind of connection links these confinement mechanisms in different dimensions, not even if such connection exists.

In a previous work [1] we have studied pure gauge QCD_3 on a strip of width L . This theory is defined on a 3D (2+1) space-time manifold having pure gauge QCD_2 ($L \rightarrow 0$) and QCD_3 ($L \rightarrow \infty$) as extreme limits. An important property of this dimensional approach is that it allows us to go beyond the pure QCD_2 results in a controlled way (the ϵ expansion, ϵ being the dimensionless parameter $\epsilon = g_3 L^{1/2}$). No matter how small ϵ is, one is dealing with a real 3D theory provided $\epsilon \neq 0$. This fact liberates the theory from the strong 2D constraints we previously mentioned, and at the same time provides us with a panoply of new physical phenomena. The explicit solution of the problem is obtained by the analysis of a 2D effective theory equivalent to the original 3D one in the regime of very small widths: the theory of heavy adjoint matter coupled to 2D gluons in a gauge invariant way. This theory possesses a complete glueball spectrum which can be exactly computed for small values of the strip width. Interesting results of various versions of this model can be found in recent literature [2].

In the dimensional approach the singularity and simplicity of the theory's line limit (QCD_2) arises due the absolute dominance of pure longitudinal gluons (2D gluons) when the width vanishes. All transverse effects are excluded (brute force!) by the requirement of pure 2D kinematics. As soon as we allow the strip to acquire a width, transverse gluons start to play a role. Their appearance transforms the physical content of the theory in a highly non trivial way. They are responsible for a rich new glueball spectrum having relevant consequences in the thermodynamics of the theory. As it will be shown throughout this paper, the new physics associated to transverse gluons is behind the existence of a

deconfining phase transition at high temperature.

In order to understand the connection between the transverse dynamics and temperature, we will calculate the free energy of two external static charges in pure gauge QCD_2 .

Let us begin by defining our fields in a 2D space-time with a compactified Euclidean time direction (as usual in the imaginary time formalism [3])

$$a_\alpha(x, t) = a_\alpha^0(x) + \sum_{n \neq 0} a_\alpha^n(x) e^{i2\pi nt/\beta} \quad (1)$$

We will work in the $a_1 = 0$ gauge, where the gluon self couplings disappear. The charges are represented as external static color sources, which we will take in the fundamental representation and forming a color singlet state (they behave like heavy quarks in a meson).

Since the sources are static only the gauge field zero mode will couple to them. Thus we just need to perform a Gaussian integral to obtain the free energy of the pair:

$$\begin{aligned} \exp \left[-\frac{1}{T} F_{Q\bar{Q}}(R) \right] &= \\ & \int [da_0][da_n] \exp \left\{ \frac{1}{T} \int dx \left[-\frac{1}{2} a_0^b (-\partial_x^2) a_0^b - \frac{1}{2} \sum_{n \neq 0} a_n^b (-\partial_x^2) a_n^b + i a_0^b j_0^b \right] \right\} = \\ & = \exp \left\{ -\frac{1}{T} \int dx \left[\frac{1}{2} j_0^b(x) \frac{1}{-\partial_x^2} j_0^b(x) \right] \right\} = \exp \left[-\frac{\sigma}{T} R \right] \end{aligned} \quad (2)$$

The external source corresponding to two static color charges in the singlet channel is given by $j_0^b = g_2 [Q^b \delta(x - x_Q) + \bar{Q}^b \delta(x - x_{\bar{Q}})]$ where $-Q^b \bar{Q}^b = C_N = (N^2 - 1)/2N$.

Therefore we obtain a linear growth of the energy with R , the string tension being $\sigma = (g_2^2/2)C_N$. Besides there is no temperature dependence in the free energy. This is an interesting feature that is worth analyzing in the framework of the strip formalism. In a strip with no width, in which there are no transverse degrees of freedom, the stringlike configuration joining both charges is the only one contributing to the free energy. The stringlike configuration is nothing but the classical solution of static pure gauge QCD_2 . Longitudinal perturbations around this classical solution are forbidden because of the presence of the two infinitely heavy sources. Transverse perturbations are not allowed by the kinematical constraint induced by the absence of a width. Therefore, as it is manifest in the free energy calculation (2), the system cannot jump to any other configuration at any temperature. The

probability of the system to be in the classical stringlike configuration is always one. In other words the entropy of the system is zero for all temperatures and thus the free energy becomes T-independent $S = -(\partial F/\partial T)_V = 0$. Dynamics is frozen and the system lies always in a confining phase.

The previous thermodynamic argument shows already why pure longitudinal gluon physics cannot “unfreeze” the 2D string joining the static quarks in the meson. It is clear now the relevance of transverse degrees of freedom in this picture. They are responsible of the thermodynamics of the quark-antiquark pair. The possibility of giving some small degree of transverse dynamics to the system in a controlled way becomes then crucial to understand the role of temperature in the previous scenario. This is precisely the purpose of our contribution. By using an effective 2D theory valid for narrow strips we will try to unveil the interesting connection between traversal dynamics, confinement and temperature in pure gauge QCD_3 .

2 Characterization of confinement

The v.e.v. of the Polyakov loop, defined as,

$$\langle 0 | \text{tr}_c g(0) | 0 \rangle_T \quad (3)$$

where

$$g(x) \equiv \exp[i(g_2/T)a_0^a(x)T^a] \quad (4)$$

is an order parameter of confinement depending on temperature. This is so because it is related to the free energy of an isolated static color pointlike charge through the expression [4],

$$e^{-(1/T)F_Q(T_Q;T)} = \langle 0 | \text{tr}_c g(0) | 0 \rangle_T \quad (5)$$

where T_Q is the color isospin of the static charge (we refer to $SU(2)$ -in general we would need the set of Casimirs characterizing the representation of $SU(N)$ -), and the matrix g is

taken in the representation labeled by T_Q . If the v.e.v. of the Polyakov is zero we are in a confining regime. The energy to bring the isolated charge from infinity to $x = 0$ would be infinite. On the contrary if it is not zero, we might liberate it (sending it to infinity) by supplying a finite amount of energy to the system: deconfinement occurs! The conventional wisdom about this order parameter relates it to the discrete symmetry associated to the center of the gauge group (Z_N). Since under gauge transformations the Polyakov operator $\text{tr}_c g$ is color invariant up to an element of this discrete group $\text{tr}_c g \rightarrow z_N \text{tr}_c g$, its v.e.v. is also an order parameter of the Z_N symmetry.

Nevertheless we are not going to adopt this approach to characterize confinement in our problem. Since we describe heavy sources with quark quantum numbers, the operator $g(x)$ is a $SU(N)$ matrix in the fundamental representation. As a $SU(N)$ matrix, one can perform not only local vector transformation upon it, but more general ones. In particular, we can distinguish left and right color indices and rotate them independently with $SU(N)$ matrices. In other words, we can perform $SU(N)_L \otimes SU(N)_R$ transformations on $g(x)$ in the following way,

$$\begin{aligned}
g(x) &\rightarrow U_L(x)g(x)U_R^\dagger(x) \\
U_L(x) &= e^{iT^a\alpha_L^a(x)} \\
U_R(x) &= e^{iT^a\alpha_R^a(x)}
\end{aligned} \tag{6}$$

If we choose $\alpha_L^a = \alpha_R^a$ then $U_L = U_R = U$ and we recover the usual color vector transformations under which $(\text{tr}_c g)$ is invariant (up to the center). However under *axial* color rotations, defined through the conditions $\alpha_L^a = -\alpha_R^a = \beta^a$ ($U_L = U_R^\dagger = U_A$), the Polyakov operator transforms non-trivially and its trace is no longer invariant ($\text{tr}_c g \rightarrow \text{tr}_c(U_A^2 g)$). Consequently, the v.e.v. of the Polyakov operator becomes also an order parameter of color axial symmetry breaking. Ordinary symmetry breaking analysis holds. If the vacuum preserves the color axial symmetry then $\langle \text{tr}_c g \rangle_T = 0$ and thus confinement occurs. If not, this v.e.v. will acquire a non-zero value and the system will be in a deconfining phase.

We investigate the confinement phase structure of pure gauge QCD_3 on the strip through the analysis of its behavior under such symmetry. The fundamental field in the study of the

Polyakov loop is the gauge field zero mode. We are concerned with its transformation rule under $U_A(N)$. For $SU(2)$ it takes a specially simple form. If $\pi(x)$ stands for the modulus of the gauge field zero mode and β for that of the axial angle, then the referred transformation reads,

$$\pi(x) \xrightarrow{U_A} \pi(x) + 2\beta \quad (7)$$

Color vector transformations keep the modulus unchanged. Axial ones shift it by a constant. Moreover if we perform a global vector rotation in such a way the gauge field zero mode at $x = 0$ gets aligned along the 3rd direction of color isospin, the above transformation is equivalent to a shift in $a_3(0)$, (from now on a_a stands for the zero mode field omitting the 0- subscript)

$$a_3(0) \xrightarrow{U_3} a_3(0) + 2\beta \quad (8)$$

This is possible because both the strip action and the operator $\text{tr}_c g$ are color singlets. Due to translation invariance, the v.e.v. of $\text{tr}_c g$ is independent of position. Thus Eq.(8) would apply to any value of x . Notice that these transformations constitute the maximal abelian subgroup of the axial color non-abelian group U_A .

We have found that we can characterize confinement by studying the behavior of the gauge field zero mode dynamics under global U_3 rotations, as defined by Eq.(8). The vacuum properties of the a -field effective theory become crucial to grasp the confinement mechanism.

The symmetry properties of the vacuum can be more easily understood in the Schrödinger formalism. To study the vacuum structure we need to know the behavior of the a field at low energies (long distances). The long distance action for the gauge field zero mode will be ruled by the terms with less number of derivatives,

$$S[a^a] = \int dx \left[\frac{1}{2}(1 + Z(a^2))(\partial_x a^a)^2 + V(a^2) + \dots \right] \quad (9)$$

where we choose $V(0) = 0$. Notice that Z and V depend only on the modulus of the gauge field due to color invariance.

The previous Euclidean action is one dimensional. Thus we are really dealing with a Quantum Mechanical (QM) problem in the Euclidean “time” x . If we focus in the Schrödinger equation for the ground state of the QM counterpart of Eq.(9), we will find ($a^a \rightarrow q^a$, $\partial_x a^a \rightarrow \frac{1}{1+Z} p^a$, $p^a = -i \frac{\partial}{\partial q^a}$)

$$\left[-\frac{1}{2(1+Z(q^2))} \nabla_q^2 + V(q^2) \right] \Psi_0[q^a] = E_0 \Psi_0[q^a] \quad (10)$$

The vacuum energy E_0 is the minimum of all the eigenvalues of the QM Hamiltonian operator of the above equation. The Euclidean action (9) is positive defined, thus $E_0 \geq 0$. Because of color invariance the wave functions of the QM problem depend only on the modulus of the gauge field zero mode $\Psi[q^2]$. Therefore they are invariant under vector rotations, $q \rightarrow U_V q U_V^\dagger$ (recall the gauge field zero mode is no longer a gauge degree of freedom). We are free to choose a vector transformation U_V rotating q in such a way the color isospin vector lies always along the third component axis. That is, $\Psi[q^a] = \Psi[(q^3)_{U_V}]$. Let us see what happens if besides we demand the wave function to be invariant under U_3 transformations. This would be precisely the case of the vacuum wave function of the strip action in the confining phase, as we have seen above. A U_3 symmetric wave function must satisfy,

$$\frac{\partial}{\partial q^3} \Psi[q^3] = 0 \Rightarrow \Psi = \text{const} \quad (11)$$

Therefore the wave function of the U_3 invariant vacuum is independent of the gauge field zero mode. If we plug the above constant vacuum wave function in the ground state equation Eq.(10), we find two new properties. First, a q dependent potential term is not compatible with such a solution. Thus $V = 0$ (recall we have chosen $V(0) = 0$). Second, the ground state energy is zero.

The U_3 invariant vacuum wave function (confining phase) is a constant whose energy eigenvalue is zero. Moreover the effective potential V cannot exist in this phase. The value of this constant defining the ground state wave function, Eq.(11), is completely arbitrary and so the wave function is. Thus the ground state becomes degenerate. This degeneracy is nothing but that induced by the U_3 symmetry.

Let us go into more detail at this point. A constant gauge field zero mode ($a = \text{const}$) minimizes the action (9) where there is no potential V for symmetry requirements. Therefore each ground state wave function is generated by a *constant* field a configuration, and vice versa. We can move from one zero energy configuration to another one just by changing the value of the constant field. This is certainly what a U_3 transformation does when acting on a constant a field¹.

However if the theory develops a non-zero effective potential for the gauge field zero mode a , the previous property is no longer true. The presence of an effective potential prevents the vacuum wave function to be a constant. The U_3 degeneracy is lost. In the broken phase the effective potential must be different from zero. Since spontaneous breaking of the U_3 symmetry cannot occur in such a low dimension (a dynamics is defined in $D = 1$) the only possibility for the vacuum to break it is through an explicit potential term (again this can be seen by studying the Schrödinger equation for the vacuum wave function Eq.(10)).

Therefore one can use the effective potential for the gauge field zero mode itself as an order parameter of confinement. A zero effective potential implies we are in a confining phase. A non-zero effective potential indicates deconfinement occurs.

We can check the previous pattern of confinement in pure gauge QCD_2 Eq.(2). In this simple example the effective action for the a_0 is automatically obtained just by removing the non-zero mode fields a_n . They do not couple either to the source nor to the zero mode, so they can be integrated out changing only the overall normalization constant. After integration the effective action for the gauge field zero mode contains only the kinetical term, which is certainly invariant under U_3 shifts, and no effective potential appears at any temperature. This agrees with the fact that pure gauge QCD_2 is always in a confining phase.

3 The generation of the deconfining phase

It has been proven [1] that a 2D effective action can be constructed to describe the physics of pure gauge QCD_3 on a very narrow strip. The effective action corresponds to adjoint

¹Notice that for a constant field a U_3 transformation is completely equivalent to a whole axial one U_A . We can always choose the gauge field to be aligned along the 3rd direction for all x .

scalar Chromodynamics, that is, to QCD_2 coupled to color adjoint matter,

$$\mathcal{L}_{\text{strip}} = \text{Tr}[\frac{1}{2}f_{\alpha\beta}^2 + (D_\alpha\phi)^2 + \mu_\phi^2(\epsilon)\phi^2] \quad (12)$$

ϵ being the dimensionless parameter $\epsilon = g_2 L$. The 2D coupling constant g_2 is related in turn to the 3D one, $g_2 = g_3^2/\epsilon$. In the regime where the above action is well-founded, $\epsilon \ll 1$, adjoint matter is heavy since

$$\mu_\phi^2 = \frac{g_2^2 N}{2\pi} \ln(1/\epsilon^2) = \frac{g_3^4 N}{2\pi\epsilon} \ln(1/\epsilon^2) \xrightarrow{\epsilon \rightarrow 0} \infty \quad (13)$$

Eventually it decouples, leaving pure gauge QCD_2 as the effective 2D theory of the widthless strip. Notice that Eq.(12) entitles us to go beyond QCD_2 dynamics in a well defined way. The ϕ field is nothing but the zero mode of the transverse component of the gluon field ($\phi \sim \int_0^L dx_2 A_2$). It represents then a 3D transverse gluon moving on a very narrow strip. The new dynamics introduced by this transverse degree of freedom changes radically that of pure gauge QCD_2 . 3D transverse gluons (in the effective form of 2D heavy scalar adjoint matter) bind themselves in color singlets by means of the confining interaction supplied by the presence of longitudinal gluons. They make up the glueball spectrum of QCD_3 on a narrow strip [1].

Apart of being interesting on its own, the existence of a non-trivial glueball spectrum has other consequences. At the very moment the system acquires new configurations to access to, thermodynamics ceases to be trivial. The partition function and all the thermodynamical properties derived from it become temperature dependent.

In a 3D language, the action (12) reproduces the dynamics of transverse fluctuations around the classical, and unique, solution for $\epsilon = 0$ ($A_\mu = \delta_{\mu 0} a_0^{cl} + \phi$). Unlike in pure gauge QCD_2 one may now excite kinematically the classical string configuration in the transverse direction by giving some energy to the system. Whether these thermodynamical fluctuations are able to break the longitudinal string confining the charges, or not, is a matter of the particular dynamics enclosed in the effective action (12).

In order to investigate the high temperature properties of QCD on the strip, we choose the $a_1 = 0$ gauge and select both the gauge and the scalar field to be periodic in the Euclidean

time coordinate. We can expand them in terms of their 1D Fourier modes (as in Eq.(1)) and obtain the following 1D strip action at temperature T ,

$$S(T) = \frac{1}{T} \int dx \left\{ \frac{1}{2} \phi_n^a \left[(-\partial_x^2 + \mu^2(\epsilon) + \omega_n^2) \delta_{n,-n'} \delta_{ab} + (M_1)_{ab}^{n,-n'} + (M_2)_{ab}^{n,-n'} \right] \phi_{n'}^b \right\} \quad (14)$$

where,

$$\begin{aligned} \omega_n &\equiv 2\pi nT \\ (M_1)_{ab}^{n,-n'} &\equiv -2ig_2 \epsilon^{abc} \omega_n a_{-n'-n}^c \\ (M_2)_{ab}^{n,-n'} &\equiv g_2^2 \epsilon^{abc} \epsilon^{bdf} a_{n''}^c a_{-n-n'-n''}^c \end{aligned} \quad (15)$$

The ϕ integration can be formally done yielding the effective action for the gauge field modes,

$$S_{eff}[a_n] = \frac{1}{T} \int dp \frac{1}{2} \text{Tr} \left[\ln(p^2 + \mu^2 + \omega^2 + M_1 + M_2) \right] \quad (16)$$

Now the trace runs over both color and the mode index n . The calculation of that trace is impossible in the most general case. However expression (16) is useful to determine the limits in which a well defined approximation can be established.

We are concerned with the possibility that the theory undergoes a deconfining transition at high temperature. According to what was discussed in the last section, we need to know whether the ϕ dynamics is able to generate a non-zero effective potential for a_0 or not. At zero T such a term is not allowed since a_0 is a truly gauge degree of freedom in that case. Local gauge invariance prevents its existence. At finite T the gauge field zero mode becomes a covariant object under gauge transformations. As a consequence we cannot longer resort to gauge invariance to protect the theory from getting a non-zero effective potential. This possibility becomes therefore a dynamical issue.

At very high T and for very narrow strips, for which adjoint matter is very heavy, the following inequalities hold,

$$T \gg \mu(\epsilon) \gg g_2$$

$$\omega_{n \neq 0}^2 + \mu^2(\epsilon) \gg \mu^2(\epsilon) \gg g_2^2 \quad (17)$$

The previous regime legitimates a perturbative expansion of the logarithm in the effective action in terms of the gauge field modes (16). The coupling constant dependence arises through the M_1 and M_2 operators Eq.(15). Their contributions are small when compared to those arising from the non-interaction terms.

After performing the gauge field expansion keeping only terms up to order g_2^2 , one obtains a typical finite T 1-loop calculation for the mass of the l -mode (see Fig.1),

$$m_l^2(\epsilon, T) = g_2^2 T \left\{ \sum_n \int \frac{dp}{2\pi} \frac{1}{p^2 + \mu^2 + \omega_n^2} - \sum_n \int \frac{dp}{2\pi} \frac{\omega_n^2 + \omega_n \omega_{l+n}}{(p^2 + \mu^2 + \omega_n^2)(p^2 + \mu^2 + \omega_{l+n}^2)} \right\} \quad (18)$$

The first, and very important, property of the above mass is that it is zero for all non-zero modes. Every non-zero mode is massless to this order of the approximation. We will point out the relevance of this feature in our conclusions.

On the contrary, the gauge field zero mode acquires a non-zero mass which is computed systematically under the conditions given by the inequalities in Eq.(17). Therefore at high temperatures the electric, or Debye, mass is given by

$$m_{el}^2(\epsilon, T) = g_2^2 \left\{ \frac{T}{2\mu} - \frac{1}{2\pi} + \frac{\mu^2}{8\pi^3 T^2} \zeta(3) - \frac{3\mu^4}{64\pi^5 T^4} \zeta(5) + \frac{15\mu^6}{1024\pi^7 T^6} \zeta(7) \right\} \\ + O\left[\left(\frac{\mu}{T}\right)^8, \left(\frac{g_2}{\mu}\right)^4\right] \quad (19)$$

Moreover one can prove that, within the regime determined by the conditions (17), there are no corrections to the derivative dependent part of the effective action (16) (they are suppressed as powers of g_2^2/μ^2). In this way we get an exact high temperature result for the zero mode effective action of QCD on the strip,

$$S_{eff}[a_0] = \frac{1}{T} \int dx \left[\frac{1}{2} a_0^b (-\partial_x^2) a_0^b + \frac{1}{2} m_{el}^2(\epsilon, T) a_0^b a_0^b \right] + O\left(\frac{g_2}{\mu}\right)^4 \quad (20)$$

Our question about whether transverse gluon fluctuations, even if small, would be able to break the tube flux structure at high enough temperature has now a clear answer. According

to our discussion in the previous section the appearance of the mass term in Eq.(20) is enough to ensure the breaking of the U_3 axial symmetry and, as a consequence, the generation of a non-vanishing confinement order parameter ($\langle \text{tr}_c g \rangle \neq 0$). We conclude then that QCD_3 defined on a strip experiences a deconfining phase transition at high temperature.

But we can go further, for we can explicitly check this result by calculating the Wilson loop ($W(T, R)$) in an analogous way as we did in Eq.(2). We only have to substitute the linear 1D propagator of pure gauge QCD_2 by the exponentially decreasing one derived from the action (20). The functional integration is immediate because is quadratic in a_0 ,

$$\begin{aligned} -\ln W(T, R) &= \frac{1}{T} F_{Q\bar{Q}}(T, R) \stackrel{T \gg \mu \gg g_2}{\approx} \frac{1}{T} \int dx \frac{1}{2} j_0^b(x) \left(\frac{1}{-\partial_x^2 + m_{el}^2} \right) j_0^b(x) \\ &= \frac{1}{T} \frac{\sigma}{m_{el}} (1 - e^{-m_{el} R}) \stackrel{R \gg g_2^{-1}}{\rightarrow} \frac{\sigma}{m_{el} T} \end{aligned} \quad (21)$$

In the $a_1 = 0$ gauge the Wilson loop can be easily related to the Polyakov operator, $W(T, R) = \langle \text{tr}_c [g(R)g^\dagger(0)] \rangle$. Because of cluster decomposition, our above result certainly implies,

$$W(T, R) \stackrel{R \gg g_2^{-1}}{\rightarrow} \langle \text{tr}_c g(0) \rangle^2 \Rightarrow \langle \text{tr}_c g \rangle = e^{-\frac{1}{2T} \frac{\sigma}{m_{el}}} \neq 0 \quad (22)$$

In agreement with the general arguments given in the previous section. Notice how the absence of a mass is the responsible of the vanishing of the v.e.v. of the Polyakov operator. The strong IR behavior of the gauge field zero mode turns the ordinary analytical expansion of the exponential operator into an essential singularity.

4 Low temperature glueballs

The theory at zero temperature exhibits a confining behavior. The Wilson loop fulfills an area law at long distances in the same way as pure gauge QCD_2 . The only difference appears in the exact value of the string tension, which is now renormalized by quantum transverse fluctuations (ϕ -loops) [5, 6]. The spectrum of the theory at small ϵ can be found in reference [1]. The low lying spectrum is calculated explicitly and is formed by heavy glueballs, made

up of two heavy constituent transverse gluons. The radial wave functions are proportional to Airy functions with energies given by,

$$M_r(\epsilon) = 2\mu_R(\epsilon) + \epsilon_r \left(\frac{\sigma^2}{\mu_R(\epsilon)} \right)^{1/3} \quad (23)$$

($-\epsilon_r$ are the zeroes of the Airy function and σ is the $\phi - \phi$ string tension). The *renormalized* scalar mass μ_R is obtained by using a self-consistent treatment of the IR divergences [7],

$$\mu_R^2(\epsilon) = \frac{g_2^2 N}{4\pi} \ln\left(\frac{1}{\epsilon^2}\right) \quad (24)$$

The low temperature behavior of the partition function will be ruled by the lowest mass states of the physical Fock space. They are eigenstates of the Hamiltonian operator at zero temperature. Because confinement occurs at zero temperature the only physical states contributing to the partition function are glueballs (singlet $\phi - \phi$ boundstates). In the most general case (even when vacuum excitations and non-planarity effects are suppressed, e.g. in the $N \rightarrow \infty$) the eigenstates of the full Hamiltonian will not coincide with those obtained by solving the bound state equation of the $\phi - \phi$ system Eq.(23). This is so because glueballs can interact among them in an effective way [8] and a further mass renormalization can occur.

In our case we are going to consider glueball states as almost-free particles, exposed only to contact interactions. Therefore our Hamiltonian eigenstates will be given by the solutions of the two body $\phi - \phi$ bound state equation. This approximation is completely justified for small enough ϵ since for very heavy glueballs, as it is the case, the glueball interaction can be neglected. The OBEP (One Boson Exchange Potential) is extremely short range ($\sim 1/M_{GB} \ll R_{GB}$, M_{GB} being the glueball mass and R_{GB} the glueball radius), as one can easily estimate using the strip results. The glueball radius can be computed using the known radial wave functions. If we call $R_0 \equiv (\mu_R \sigma)^{-1/3}$ then the glueball radius can be obtained from the *mean squared radius* of the two particle system,

$$R_{GB} \equiv \langle x^2 \rangle^{1/2} \simeq 1.707510 R_0 \quad (25)$$

Therefore in the small ϵ limit, the correlation length of the glueball exchange is negligible when compared to the typical radius of the glueball because

$$\frac{\xi_{GB}}{R_{GB}} \simeq 1/(M_{GB} R_0) \stackrel{\epsilon \ll 1}{\simeq} \left(\frac{\sigma}{\mu_R^2}\right)^{1/3} \xrightarrow{\epsilon \rightarrow 0} 0 \quad (26)$$

On the other hand the typical radius of a glueball R_{GB} decreases logarithmically as the strip width tends to zero, $R_{GB} \sim R_0 = (\mu_R \sigma)^{-1/3} \sim (\ln(1/\epsilon^2))^{-1/3} \xrightarrow{\epsilon \rightarrow 0} 0$. We conclude that glueballs are extended small objects in the low ϵ regime.

Consequently our system is made up of small quasi-free glueballs in the small width regime. Moreover since they are also very heavy, a non-relativistic approach (NR) is likewise appropriate at low temperature. For this reason the NR free glueball gas model becomes an excellent approximation to the real theory at low temperature. Since the dimensionless width ϵ is a free parameter, we can play with it in order to make our approach as good as possible. In this way the narrower the strip is the better the free gas approximation is.

The thermodynamics of the NR gas of glueballs is described by a partition function which factorizes in individual partition functions for each glueball state. At fixed volume, the pressure ($\equiv T \ln Z/V$) is then,

$$P_{GB}(T; \epsilon) = P_1(T; \epsilon) + P_2(T; \epsilon) + \dots + P_r(T; \epsilon) + \dots \quad (27)$$

The general term in Eq.(27) corresponds to the pressure of a gas of 2D NR scalar r -particles at temperature T with masses M_r given by Eq.(23). That is,

$$P_r(T; \epsilon) = \exp\left(-\frac{M_r(\epsilon)}{T}\right) \left(\frac{M_r(\epsilon)}{2\pi}\right)^{1/2} T^{3/2} \quad (28)$$

An expected consequence of the previous result is that low T pressures vanish when we shrink the width strip to the line limit (recall $M_r(\epsilon) \xrightarrow{\epsilon \rightarrow 0} \infty$). However that is not the whole story. We can go beyond our qualitative expectation and obtain the small ϵ behavior of the pressure exactly. If we make the ϵ dependence in Eq.(28) explicit by using Eqs. (23) and (13), we find a non-analytic form for its $\epsilon \rightarrow 0$ limit,

$$\ln Z_{GB}/V = P_{GB}(T; \epsilon) \approx e^{-\gamma \frac{(-\ln \epsilon)^{1/2}}{\epsilon}} (-\ln \epsilon)^{1/2} \xrightarrow{\epsilon \rightarrow 0} 0 \quad \gamma = \frac{g_3^2}{T} \left(\frac{N}{\pi}\right)^{1/2}$$

5 Near the phase transition

Pure gauge QCD on a narrow strip turns out to be a two phase theory. In section 3. we have studied its structure at very high T . We encountered a deconfining regime in which both transverse (ϕ -particles) and longitudinal gluons (2D gluons) are the physical degrees of freedom. In contrast to the high T behavior, the Fock space of the theory at low T is made up of color singlet glueballs. Thermodynamics of the “cold” phase has no much to do to that of the free gluon plasma. Thermodynamic functions of the former correspond to an admixture of infinite species of glueball gases characterized by their different masses Eq.(27). In the latter only transverse gluons, as a ϕ field gas, contribute to the partition function.

We are going to investigate the critical behavior of the system both in its gluon plasma phase and in its glueball phase. If we are able to establish accurately the thermodynamic properties of the two phases in the neighborhood of the phase transition point, we will be in an optimal position to estimate its critical temperature.

Let us begin analyzing the plasma phase. The starting point is the effective action for the gauge field modes Eq.(16), obtained after ϕ_n integration. The partition function is then just,

$$\begin{aligned} Z_{plasma}(T; \epsilon) &= Z_\phi(T; \epsilon) \int [da_n] e^{-S_{eff}[a_n]/T} \\ Z_\phi(T; \epsilon) &\equiv e^{-\frac{1}{T} \int dp \sum_n \frac{1}{2} \text{tr}_c [\ln(p^2 + \mu^2 + \omega_n^2)]} \end{aligned} \quad (30)$$

Certainly the latter operation is not feasible in the most general case. However we will not need to work it out in an exact manner. Only very general properties of the functional integration will be required to understand the problem under consideration. The existence of *two* independent dimensionless parameters is the main ingredient to see why a complete knowledge of the effective action is not necessary. Because the theory possesses three parameters with dimension of mass (g_2, μ, T), we can form the dimensionless ratios ($\mu/T, g_2/\mu$). Thus we can express any relevant function or functional in terms of them. For

example, the high T calculation in section 3. is specified by the conditions ($\mu/T \ll 1, g_2/\mu \ll 1$). However nothing forces us to choose *a priori* $\mu < T$, being both variables independent. We can take advantage of the freedom to select an arbitrarily large mass by working on a narrow enough strip. So that at any given T we can fulfill the conditions $\mu \gg g_2, \mu \gg T$ just by picking a small enough value of ϵ . The first inequality is understood in the same way as in the calculation of the spectrum at zero temperature. That is the interaction term is small when compared to the kinetic piece of the bound state equation (NR approach). In this approach higher order contributions of the gauge field are suppressed as powers of g_2/μ . Therefore at any T we expect a weak coupling expansion of the effective action Eq.(16) in the gauge field modes to be valid when $\epsilon \ll 1$.

The structure of $S_{eff}[a_n]$ in the weak coupling regime is easy to obtain from Eq.(16) by expanding the logarithm in powers of $(p^2 + \mu^2 + \omega^2)^{-1}(M_1 + M_2)$,

$$\begin{aligned}
S_{eff}[a_n] &= S_{der}[\partial_x a_n] + V_{eff}[a_n] \\
V_{eff}[a_n] &= \int dx \left\{ \frac{1}{2} m_0^2 (a_0^b)^2 + \sum_{n_1+n_2+n_3=0} \lambda_{n_1 n_2 n_3}^{abc} (a_{n_1}^a a_{n_2}^b a_{n_3}^c) + \dots \right\} \\
S_{der}[\partial_x a_n] &= \int dx \left\{ \frac{1}{2} \sum_n (1 + \lambda'_n) (\partial_x a_n^b)^2 + \dots \right\} \tag{31}
\end{aligned}$$

Although it is impossible to determine an explicit solution for all the coefficients, it is at our hand to establish their behavior for small ϵ . We were able to calculate the first term of the effective potential in section 3. Although we only gave its high T expansion, it is also possible to show its form for $\mu \gg T$. The frequency sum in Eq.(18) can be transformed in an integral over the zero component of the momentum p_0 [3]. The T-dependent part of this integral can be worked out, in the same way as the remaining p_1 integration when we work in the $\mu \gg T$ regime. The whole operation results in a exponentially decaying behavior for the electric mass, $\sim e^{-\mu/(2\pi T)} \xrightarrow{\epsilon \rightarrow 0} 0$. The exponential decay of the action coefficients is a general feature valid at every order for large enough masses. As a consequence all the coefficients of the different powers of a_n appearing in Eq.(31) vanish in the $\epsilon \rightarrow 0$ limit. Moreover one finds that coefficients of higher powers decay more strongly than those of the lower ones. The above characteristic of the effective action coefficients is completely understandable within a

3D framework. We know the 3D gauge theory tends to its 2D counterpart in the line limit. Thus all coefficients in the effective potential must disappear when $\epsilon \rightarrow 0$ leaving the pure QCD_2 action as the only remnant of color interaction.

In summary. We find that at any given T, *above* the critical temperature, we can reduce the contribution from the effective action to that of a 2D gluon gas provided we take the strip width in a convenient way. Indeed the $\epsilon \rightarrow 0$ limit is T-independent and therefore this limit is equivalent to write simultaneously ($\mu \gg g_2, \mu \gg T$) when T is taken at a fixed value.

Mathematically we can express the above property as follows, according to Eqs.(30) and (31),

$$\begin{aligned} Z_{plasma}(T; \epsilon) &\stackrel{\epsilon \ll 1}{\approx} Z_\phi(T; \epsilon) Z_{QCD_2} \\ Z_{QCD_2} &\equiv \int [da_n] e^{-\frac{1}{T} \int dx \sum_n [\frac{1}{2} a_n^b (-\partial_x^2) a_n^b]} \end{aligned} \quad (32)$$

That is, the partition function of the gluon plasma factorizes in two pieces. One corresponds to a free gas of transverse gluons (Z_ϕ) and the other one to a free gas of longitudinal gluons (Z_{QCD_2}). The QCD_2 gluon partition function is trivial. As mentioned in the introduction, it is T-independent and thus it has no thermodynamic significance as we will see in a moment. All the temperature dependence is produced by the quasi-free motion of transverse gluons. The plasma pressure is immediately obtained from Eq.(32) using standard finite T field theory techniques,

$$P_{plasma}(T; \epsilon) \stackrel{\epsilon \ll 1}{\approx} P_\phi(T; \epsilon) + P_\phi^{vac} = (N_c^2 - 1) e^{-\mu(\epsilon)/T} \left(\frac{\mu(\epsilon)}{2\pi} \right)^{1/2} T^{3/2} + P_\phi^{vac} \quad (33)$$

Now comes the time to find out the properties of the glueball phase above the critical temperature. In section 3. we discovered the way the glueball admixture started to react under a small increase of the heat bath temperature. Only the lowest mass glueballs produced a discernible effect on the gas pressure, as Eq.(27) shows. The contributions to the global pressure of gases made out of heavy glueballs are exponentially suppressed at low T. In general, at any T the pressure of a given class of glueball is always exponentially suppressed respect to that of lighter boundstates.

The transverse gluon ϕ is lighter than all its glueball boundstates. The pressure of the ϕ gas is always higher than any individual pressure of the glueball soup. Only the fact that we need to sum up a great amount of single contributions, from the infinite number of glueball states we can create from ϕ particles, can avoid that the total pressure of the glueball phase remains below the plasma pressure at any T. According to Gibbs criterion, if $P_{GB} < P_{plasma}$ at any T then the transition cannot occur.

The mass distribution of the states becomes then crucial. The particular form of the density of the states determines the real shape of the pressure and thus the possibility for the phase transition to happen. One can really appreciate this point when writing the total pressure as

$$P_{GB} = \int_0^\infty dm \rho(m) e^{-m/T} \left(\frac{m}{2\pi}\right)^{1/2} T^{3/2} \quad (34)$$

If the density of states increases rapidly for large masses ($m \gg T$) then it can compensate the thermodynamical suppression dictated by the Boltzmann factor and produce the necessary cut. The density of high energy states derived from the NR spectrum (23) generates the following high T-dependence of the glueball pressure,

$$\rho(m) \stackrel{m \gg \mu_R}{\sim} m^{1/2} \Rightarrow P_{GB} \sim T^{7/2} > P_{plasma} \sim T^{3/2} \quad \text{at large } T \quad (35)$$

As we can see, the transition occurs (see Fig.2b).

However although we can take for granted the existence of the deconfining transition, we are not describing properly the glueball phase near T_c . The reason is we cannot extrapolate the NR spectrum (23) to the highest energy boundstates. The accuracy of the calculation is based on the validity of the NR approach. For very energetical boundstates the binding energy is much bigger than μ_R . The typical NR parameter V_{int}/μ_R , an estimation of the characteristic relative velocity, is no longer small and a full relativistic approach has to be adopted instead. Moreover the heavy part of the spectrum has to contain necessarily multi- ϕ states, not only the two particle sector. Pair excitations are now energetically available since typical momenta flowing through internal gluon lines are much bigger than the pair energy threshold. This means that our NR counting of glueball states is clearly underestimating

the real number of them contributing to the total pressure. A careful counting of states is necessary in order to determine an accurate description of the glueball gas pressure near the critical temperature T_c .

Like in QCD_2 with fermions, one can use light cone quantization and obtain a full relativistic Bethe-Salpeter equation for multi- ϕ bound states valid for any constituent mass. This study has been already carried out quantitatively by *Demeterfi et al.* [9]. In the description of the highest spectrum the constituent mass can be neglected respect to the boundstate mass ($m \gg \mu_R$). Thus we can study the growth of the density of the most massive states by analyzing their model in the limit in which the constituent mass equals zero. Their results in this limit are very interesting since they show that the density of states exhibits the Hagedorn behavior [10],

$$\rho_H(m) \approx g_2^{b-1} a m^{-b} e^{m/T_0} \quad [a] = 1 \quad (36)$$

Although only an estimation of the Hagedorn temperature $T_0(\epsilon) = (1.4-1.5)\sqrt{N_c/\pi}(g_3^2/\epsilon)$ is given, the previous result is enough to obtain the relevant contribution to the glueball gas pressure near T_c . The ultrarelativistic boundstates are much more abundant than the NR ones (compare the exponential behavior in Eq.(36) to the NR density). This results in a stronger raise of the glueball pressure with T and therefore in a decrease of the critical temperature (see Fig.2a).

In principle, the constants P_ϕ^{vac} and P_{GB}^{vac} in Eqs. (33) and (34) may be different. The former corresponds to the pressure of the non-confining vacuum (free gluons), whereas the latter corresponds to the confining vacuum generating the Fock space of singlet color states. The difference between these two pressures is given by the difference between the vacuum thermodynamical potentials of the two phases ($\Omega = -P$)

$$P_\phi^{vac} - P_{GB}^{vac} = \Omega_{\text{non-conf}}^{vac} - \Omega_{\text{conf}}^{vac} = -\mathcal{E}_{\text{conf}}^{vac} \quad (37)$$

where $\mathcal{E}_{\text{conf}}^{vac}$ is the vacuum energy of the confining phase.

The vacuum structure of pure gauge QCD_2 is trivial for it is a free massless theory, as it is apparent in a physical axial gauge (e.g. $a_1 = 0$). Obviously it is manifestly invariant under

dilatation transformations and thus its vacuum energy is zero after zero point renormalization ($\mathcal{E}_{vac} \sim \langle T_{\alpha\alpha} \rangle \sim \langle \partial_\alpha j_\alpha^{\text{dil}} \rangle = 0$).

Pure gauge QCD_3 on a narrow strip is a confining theory. However the vacuum structure derived from the strip action (12) is the same as in pure gauge QCD_2 . After integrating the transverse fluctuation ϕ we obtain an effective action for the 2D gluon field whose first term is identical to that of pure gauge QCD_2 . This term renormalizes the coupling constant but in a *momentum independent* way, $g_2^R = Z(\mu)g_2$. Thus no breaking of dilatation symmetry is encountered and we meet the same result as before [11]

$$\mathcal{E}_{\text{conf}}^{vac} \sim \langle T_{\alpha\alpha}^{\text{strip}} \rangle \sim \langle \partial_\alpha (j_\alpha^{\text{dil}})^{\text{strip}} \rangle = 0 \quad (38)$$

The previous result can be likewise related to the absence of a gluon condensate ($\langle f^2 \rangle = 0$) in the theory represented by the strip action (12). As we mentioned before the long distance effective action is proportional to that of pure gauge QCD_2 , which being a free massless theory cannot generate a non-zero gluon condensate.

According to Gibbs criterion, at the critical temperature it must be true, (see Fig.2)

$$P_{GB}(T_c; \epsilon) = P_{\text{plasma}}(T_c; \epsilon) \quad (39)$$

Once we know the two constants appearing in the expressions for the gluon and glueball gas pressure are the same, the equalization equation yielding T_c is completely defined. The two constants cancel in Eq.(39) just rendering an equation for the T-dependent parts of Eqs. (33) and (34). We can now find the evolution of T_c with the width valid for narrow strips. Keeping only the leading dependent terms in ϵ one obtains the following expression for the critical temperature evolution,

$$T_c(\epsilon) \stackrel{\epsilon \ll 1}{\approx} T_c^0(\epsilon) + B \left(\frac{T_0(\epsilon)}{\mu(\epsilon)} \right) \ln \left(\frac{\mu(\epsilon)}{T_0(\epsilon)} \right) + C \left(\frac{T_0(\epsilon)}{\mu(\epsilon)} \right) \quad (40)$$

From the 3D point of view the two dimensional coupling constant is a width dependent object, $g_2 = g_3^2/\epsilon$. If we wished to compare to numerical simulations in 3D, the natural variables to work with would be (g_3, L) or (g_3, ϵ) . The 3D coupling constant g_3 defines completely the color flux generated by the static quark-antiquark pair in the infinite volume

limit. The string tension, for example, can be expressed in terms of it since g_3^2 carries dimensions of mass. If we put the system on a strip (an asymmetric box, in the case of a lattice simulation), the natural question to answer is what happens to the infinite volume values of relevant observables such as the string tension, the glueball masses or the critical temperature under the new different boundary conditions. This is equivalent to fix the value of g_3 (maintain the infinite volume quark-antiquark configuration) and tune only the strip width L .

In Fig.3 we give an approximate representation of the critical temperature as a function of ϵ . It is obtained by solving numerically the equalization equation (39) for different values of ϵ . However it is worth observing that the critical temperature does not exist in the exact line limit of the strip. This is not because the effective 2D coupling constant blows up in that limit, which pushes T_c to infinity. In the strict $\epsilon \rightarrow 0$ limit both the plasma and the glueball gas pressure are exactly zero independently of temperature. The equalization equation has no solution for $\epsilon = 0$. The previous feature is not related to the dimensional running of the 2D coupling constant, but to the effective decoupling of the transverse gluon $\mu_R \xrightarrow{\epsilon \rightarrow 0} \infty$ in that limit. Therefore we recover in our strip approach a previously known result, pure gauge QCD_2 is always in the same (confining) phase.

From the previous analysis one concludes that the existence of the two phase structure is a non-trivial quantum effect produced by the interaction between longitudinal and transverse degrees of freedom. Simpler models, as the non-compact 3D abelian gauge theory, lacks such property. As a consequence, they do not show the rich phase structure of the non-abelian theory [5].

6 Conclusions

Our study establishes the preeminent role of transverse dynamics in the generation of a deconfining phase. Unlike their longitudinal counterparts, transverse gluons are dynamical and their dynamics is responsible for the formation of a screening mass for the 2D gluon. The loss of its massless character entails a radical change in a confinement order parameter such as the v.e.v. of the Polyakov loop. It becomes non-zero thus indicating the presence

of a deconfining phase transition. It is highly remarkable that even a very small amount of transverse physics is enough to produce such a drastic change in the thermodynamics of pure gauge QCD_2 .

Although the theory cannot be simple even for very small values of the strip width -it has to account for a transition phase-, an easier approach may be taken in that case. The Fock space of the theory changes radically depending on what side of the critical interphase we are describing. The partition function and all thermodynamic functions behave very differently below and above T_c because of that circumstance. The advantage of the small ϵ approach is it allows us to simplify considerably the dynamics of the physical states in both phases, but, at the same time, without spoiling the structure of their particular Fock spaces. In another words, in the small ϵ regime we always find only color singlet states (glueballs) in the confining phase, and free gluon particles above T_c . However both glueballs and gluons interact weakly among them in their respective phases.

In this way we observe that the thermodynamics of the deconfining phase can be accurately described by a free gas of transverse and longitudinal gluons. Unquestionably we cannot predict the appearance of typical non-analyticities at the transition point with such a simple model. However the approach is as good as one wishes (just by manipulating ϵ) provided we exclude the singular point. A clear signal that the non-analytical behavior is really included within the formalism is encountered in the effective action for the gauge field modes Eq.(31). The non-zero modes are massless. If we integrated them in order to get the effective potential for the gauge field zero mode (an order parameter of confinement), we would face strong IR divergences already at one loop level. In finite temperature field theory this is a clear indication of a non-analytical behavior in T. This non-analyticity will show up in the form of infrared divergences in the perturbative expansion (even off mass-shell). Typical perturbative high T calculations break down, and one has to resort to complicated infinite resummations to find the non-analyticity behavior and predict T_c [12]. Fortunately in our case, with the exception of the critical point, all these effects are suppressed above T_c when ϵ is very small.

The confining phase consists of weakly interacting gluon boundstates, which at low T behaves as a mix-up of inert glueball gasses. This fact is directly associated to the NR

character of the low-lying spectrum. However the spectrum also contains fully relativistic states since there is no upper bound for the binding energy. These states form the high part of the spectrum. The high glueball spectrum is extremely populated because it is made out of many multi-gluon states whose energy levels are very close to each other. The number of such states grows fastly with the mass, as a relativistic treatment points out [9]. They exhibit the Hagedorn behavior.

After heating the system sufficiently we can get to excite the Hagedorn spectrum. However this supply of heat does not turn immediately into an increasing of the gas temperature. Most of the energy is used to excite the great amount of massive states accessible to the system instead of increasing the thermal motion of the less energetical states. The energy necessary to produce a small change in the temperature becomes increasingly higher. The gas pressure blows up at a limiting temperature $T_0(\epsilon)$ due to the exponential raise of the density of states. There exists then a temperature for which the pressure of the glueball gas becomes higher than that of the free gluon gas. Thermodynamically the latter is favored and the system evolves to this phase until the thermodynamical equilibrium is restored.

From the microscopical point of view we can understand the phase transition as produced by a certain instability arising as a result of increasing fluctuations. We have seen that multi-gluon states are allowed by gauge dynamics on the strip. They are extremely heavy and thus the probability of a multi-gluon fluctuation is very small at low T . However we know that the pressure of the glueball gas grows rapidly near the Hagedorn temperature. The reason for such behavior is that an increasing amount of energy is needed to excite the large quantity of multi-gluon states with similar masses, formed by an increasing number of constituent gluons (this number grows with the glueball mass [9]). Thus the probability of generating such multi-gluon fluctuations with a huge number of constituent gluons becomes bigger and bigger as we approach T_0 . It is natural then to think of T_c as the temperature for which the multi-gluon fluctuation involves an infinite number of gluons. The glueball gas turns unstable and the gluon plasma is formed.

As it happened with the glueball spectrum of the model, the results here presented show an appealing agreement with our knowledge of the thermodynamics of higher dimensional gauge theories. It is interesting to remark why one expects these results to describe, at least

qualitatively, the features of the 3D theory. Our approach consists in defining the whole 3D theory under special boundary conditions (the strip compactification). The analysis developed here and in our previous work [1] accounts for the behavior of pure gauge QCD_3 in the regime of very small widths. The dynamics of the 2D effective action Eq.(12), obtained to describe the long distance longitudinal physics of pure gauge QCD_3 , is defined completely by the 3D parameters (g_3, L) . The 2D effective coupling constant and the 2D scalar mass Eq.(13) are fixed by the 3D inputs. In this sense one deals with the real theory and not with a 2D model of it. In the latter we would take g_2 and μ as free parameters to tune after comparing to 3D data.

Certainly our analysis is restricted to a validity region ($\epsilon \ll 1$) which lies far from the realistic domain, the infinite width limit. However our unreachable non-perturbative *transverse* regime ($\epsilon \approx 1$) is the natural scenario for lattice simulations. The study of asymmetric lattices (one lattice dimension substantially bigger than the others) complements perfectly our results. The small ϵ regime is precisely the forbidden territory for the lattice simulation, which is limited by the finite lattice spacing constraint. When both analysis merge, one discovers the two following relevant features [13]:

1. One can extrapolate the small ϵ results (i.e., in Eq.(23)) to the $\epsilon \geq 1$ regime in a continuous way. This gives rise to a non-zero value for the QCD_3 glueball masses and the string tension in the $\epsilon \rightarrow \infty$ limit.
2. There exists a stabilization of the ϵ flow at a critical ϵ^* (corresponding to the critical width L^*). The string tension and the glueball spectrum tend quickly to a constant, their QCD_3 value, above the critical point ϵ^* .

The previous results strongly suggest that the description of the low energy longitudinal dynamics of pure gauge QCD_3 can be performed using a generalization of the 2D effective action Eq.(12). That is, through an effective action defined at the critical ϵ^* ,

$$\mathcal{L}_{strip}(\epsilon^*) = \text{Tr} \left[\frac{1}{2} f_{\alpha\beta}^{*2} + (D_\alpha^* \phi)^2 + \mu^{*2} \phi^2 + U(\phi; g_2^*, \epsilon^*) \right] \quad (41)$$

The low energy effective action contains only the terms with less number of covariant derivatives in the zero mode fields a_α and ϕ . All the parameters defining the 2D effective

action get frozen at the critical width. In particular, the 2D coupling constant and the scalar mass acquire their infinite width value already at ϵ^* , ($g_2 \rightarrow g_2^*$, $\mu \rightarrow \mu^*$). The same happens to the effective potential U .

The previous consideration allows us to understand why it is possible to get a sensible model of pure gauge QCD_3 using 2D adjoint matter gauge theories of the type (41) [2, 6, 9]. In this approach, the coupling constant g_2 and the scalar mass μ are no longer fixed by the 3D theory. They are just input parameters. We cannot know them until we solve the underlying theory in the critical region. However it is reasonable to expect that there exist a set of 2D parameters which may describe accurately the low energy dynamics of the 3D theory.

The qualitative features of the string tension and the glueball spectrum are kept along their trajectories in ϵ space. They get stabilized at the critical value of the strip width L^* , which is identified as the flux tube width. Therefore if the zero temperature properties of the model do not seem to disappear in the infinite volume limit, we have good reasons to believe that the qualitative picture here developed for the deconfining phase transition can be also preserved. With maybe only one exception, the gluon condensate. Our analysis does not predict any difference between its values in both phases.

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Figure Captions

Fig.1: Diagrams contributing to the a_n self-energy at high T. Dashed lines correspond to ϕ modes, wavy lines correspond to gluon modes.

Fig.2 : Pressure versus temperature in the relativistic confining (dashed lines), NR confining (dot-dashed lines) and deconfining (solid lines) phase ($g_3 = 1, \epsilon = 0.01$). The vacuum pressure is taken to be zero in both phases (see text): (a) The critical crossing for a relativistic high energy spectrum; (b) The critical crossing for the NR spectrum.

Fig.3: Estimated evolution of the critical temperature with the strip width ($L = \epsilon, g_3 = 1$).