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The theoretical calculation of the Rossby number and the ‘non-local’ convective overturn time for pre-main sequence and early post-main sequence stars

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ABSTRACT

This paper provides estimates of convective turnover time scales for Sun-like stars in the pre-main sequence and early post-main sequence phases of evolution, based on up-to-date physical input for the stellar models. In this first study, all models have solar abundances, which is typical of the stars in the Galactic disk where most of the available data have been collected. A new feature of these models is the inclusion of rotation in the evolutionary sequences, thus making it possible to derive theoretically the Rossby number for each star along its evolutionary track, based on its calculated rotation rate and its local convective turnover time near the base of the convection zone. Global turnover times are also calculated for the complete convection zone. This information should make possible a new class of observational tests of stellar theory which were previously impossible with semi-empirical models, particularly in the study of stellar activity and in research related to angular momentum transfer in stellar interiors during the course of stellar evolution.

Subject headings: general – stars: interiors – stars: evolution

1. Introduction

There is a rapidly growing body of observations relating to the study of solar-stellar phenomena which will lead to a better understanding of the internal dynamics of the pre-main sequence and early post-main sequence evolutionary phases of stars like the Sun. But before this wealth of data can be fully understood, the role of the convection zone structure, its depth and overturn time scale, and its interaction with rotation need to be clarified. This is of importance for understanding not only the mechanism of angular momentum transfer in stars, but also the evolution of rotating stars, both in the pre main-sequence and post main-sequence phases. The interaction of rotation with convection is now widely believed to be responsible for the generation of stellar dynamos and the observed stellar magnetic activity and activity cycles.

Durney and Latour (1978) (see also Durney, Mihalas and Robinson 1981, and Durney and Robinson 1982) made an important step forward in relating the principles of mean-field dynamo theory to the observations. They showed that if a stellar dynamo is responsible for the observed stellar activity, there should be a relation between magnetic activity and the characteristics of rotating stellar convection zones. With the help of dimensional arguments, they pointed out the significance of the Rossby number (proportional to the ratio of the rotation period to the convective turnover time) in the dynamo mechanism. Since in the dynamo model, the dynamo action is believed to take place at the base of the convection zone, anchored in the radiative layers just below the convective interface, the convective turnover time of the deepest part of the convection zone is the most relevant in the evaluation of the Rossby number. Soon afterwards, the availability of precise rotation periods for magnetically active stars (Baliunas et al. 1983) made it possible to test this hypothesis in a semi-empirical way by combining convective turnover times derived from model convection zones, such as the models of Gilman (1980), with the observed rotation periods (Noyes 1983). This was done by a number of researchers (Mangeney and Praderie 1984; Hartmann et al. 1984; Noyes et al. 1984) for stars near the main sequence and for pre-main sequence stars (Simon et al. 1985). Since then a large number of studies have been performed correlating different types of magnetic activity indices to such semi-empirically derived Rossby numbers and to other parameters such as the stellar rotation rate (Basri 1987; Shrijver and Rutten 1987; Simon and Fekel 1987; Dobson and Radick 1989).

The pattern of convective velocities as a function of effective temperature and age derived from stellar models, which serve as input in Rossby number calculations, depend sensitively on the particulars of the stellar interior models, either input parameters such as chemical composition, or mass, or physics input such as opacities or the equation of state used. In addition, because of the well-known non-linearity of the equation of stellar structure, it is unadvisable to construct stellar envelope models by simple inward integration without applying the interior boundary conditions, as the early calculations frequently did; fully consistent interior models are needed. An important step in relating activity observations to self consistent stellar evolutionary tracks and the predicted evolutionary changes in convective overturn times was made by Gilliland (1985, 1986). Other calculations of Rossby numbers, based on complete main sequence stellar models, have also been

published by Rucinski and Vandenberg (1986, 1990). Since then, rapid progress has been made in our knowledge of stellar opacities and equation of state (see e.g. Rogers and Iglesias 1994), and much improved models of the Sun and Sun-like stars can be constructed (Guenther et al 1992; Chaboyer et al. 1995; Guenther, Kim and Demarque 1995).

The theory of rotating stellar evolution has also advanced. The work of Endal and Sofia (1978, 1981), which included the spin-down due to a stellar wind, and introduced into stellar evolution the effects of various rotationally induced mixing processes acting on different time scales, thus relating angular momentum transfer to internal mixing, opened up new ways of confronting theory and observation (Pinsonneault et al. 1989, 1990; Chaboyer et al. 1995).

The purpose of this paper is to provide estimates of turnover time scales for Sun-like stars in the pre-main-sequence and early post main-sequence phases of evolution, based on up-to-date physical input for the stellar models. In this first study, all models have solar abundances, which are typical stars in the Galactic disk. Another new feature of these models is the inclusion of rotation. Because the evolution of internal rotation has been included in the models, it is possible to derive theoretically the Rossby number for each star along its evolutionary track based on the theoretical estimates for both the convective turnover time and the rotation rate of the convection zone. These internally self consistent models should make possible a new class of observational tests of stellar theory which were impossible with semi-empirical models.

We describe convection by the mixing-length formalism in the usual way. While it is known that the convective velocities near the stellar surface are not well described by the mixing-length approximation (Kim et al. 1995a,b), the convective turnover time scales calculated here are dominated by the conditions near the base of the convection zone, where the temperature gradient is for all practical purposes adiabatic, and the mixing-length approximation is known to provide an adequate description of convection, at least in an average sense (Chan and Sofia 1989; Lydon et al. 1992). For this reason, the Rossby number estimates should be little affected (subject to a constant scale factor) by improvements in our understanding of convection. We emphasize, however, that for many other purposes, such as describing the outer layers where radiation plays a dominant role, the mixing length approximation is inadequate, and more refined convection models that take into consideration the interplay between convection and radiation, are needed (Kim et al. 1995a,b). This conclusion applies in particular for understanding the structure of the transition superadiabatic layer at the top of the convection zone. It is also likely to apply for understanding the behavior of magnetic fields, the details of the generation of acoustic noise in stellar chromospheres, and the driving of p-modes in Sun-like stars.

Section 2 describes the series of stellar models with masses ranging from $0.5M_{\odot}$ to $1.2M_{\odot}$ which were evolved from the fully convective pre-main-sequence Hayashi phase to the sub-giant phase. The calculation of the convective turnover time and of the Rossby number and their evolution as a function of time are considered in Sections 3 and 4, respectively. Finally, we briefly discuss the results in section 5.

2. Calculations

2.1. Stellar models

A series of stellar models with masses ranging from 0.5 to $1.2M_{\odot}$ (in $0.1M_{\odot}$ increments), have been evolved from a fully convective pre-main sequence model to the sub-giant phase. The OPAL opacities tables (Iglesias and Rogers 1991), constructed for the solar mixture of Anders and Grevesse(1989) were used, together with the Kurucz (1991) low temperature opacities. The Kurucz (1992) model atmospheres served as surface boundary conditions. The numerical tolerances and input physics were identical for all evolutionary runs, and similar to those adopted by Chaboyer et al. (1995). All models used the parameters derived for the standard solar model, where the initial X , Z , and the mixing length ratio α are varied until a solar model at the solar age of 4.55Gyr (Guenther 1989) has the observed solar values of luminosity, radius, and Z/X . In addition, the solar surface rotational velocity and 7Li depletion were used to calibrate the rotation and diffusion parameters of all evolutionary sequences, as described in sections 2.2 and 2.3 below. The solar model in this calibration matches the solar radius and luminosity to within 0.01%, while the surface Z/X matches the observed value to within 1.0%. The model also reproduced the observed solar rotation rate and Li depletion to within 1.5%. Table 1 summarizes the characteristics of the models and their input parameters.

Figure 1 shows the evolutionary tracks in the H-R diagram. For the internal rotation rates considered here, rotation has a negligible effect on both the rate of evolution and the path of the evolutionary track in the H-R diagram (Pinsonneault et al. 1989; Deliyannis et al. 1989).

2.2. Rotation

All models used in this paper have been constructed using a version of the Yale Rotating Stellar Evolution Code (Prather 1976, Pinsonneault 1988). Recently, the YREC has been improved in the microscopic diffusion and its interaction with rotational mixing (Chaboyer et al. 1995). The calculation has been carried out using this improved version.

The evolutionary sequences were started from fully convective pre-main sequence models in the Hayashi phase. At first, the whole star rotates as a rigid body, and spins up as it contracts. The torque due to the stellar wind then takes over and spins the star down (see below), and in the process of transferring out internal angular momentum, progressively depletes 7Li in the star. The evolution of the internal angular momentum distribution follows the approach of Pinsonneault et al (1989), which includes the effects of rotationally induced instabilities in the radiative layers. This results in a state of differential rotation. The Sun seems to be overdepleted in 7Li compared with other stars of its age and spectral class. Since the amount of mixing in a star increases as its initial rotational velocity increases, it is likely that the Sun was initially a rapid rotator. Therefore, even though observations show that the majority of T-Tauri stars have rotation velocities around

10km/s , we have chosen an initial rotation velocity of 30km/s for the solar model. Observations of T-Tauri stars also indicate that there is no large difference in surface rotation rates between high and low mass stars over the mass range we study, i.e. $0.5 \sim 1.2M_{\odot}$ (Bouvier 1991; Bouvier et al. 1993), and for this reason, we have applied the same rotation parameters to all masses. Figure 2 shows the evolution of the rotation period of our theoretical models. It is important to note that changing the initial rotation velocity affects the time-scale for early spin-down, but does not change appreciably the final configuration. This is due to the fact that our calibration requires the solar model to rotate at the present rotation rate of the Sun (the adopted initial rotational velocity affects primarily only the present ${}^7\text{Li}$ abundance, which is not particularly relevant to the convective turnover calculations of this paper). Note in Figure 2 that the rotation period depends sensitively on mass for a given age.

When calculating the evolution of the Rossby number, the adopted wind law is the most important input of our rotating models, as internal structural effects of rotation are minimal in these models which rotate relatively slowly. A modified version of Kawaler’s parameterization for the loss of angular momentum due to magnetic stellar wind (Kawaler 1988) has been used (Chaboyer et al. 1995). It is given by:

$$\begin{aligned} \frac{dJ}{dt} &= f_k K_w R^{2-N} M^{-N/3} \dot{M}^{1-2N/3} \omega^{1+4N/3} & (\omega < \omega_{crit}), \\ \frac{dJ}{dt} &= f_k K_w R^{2-N} M^{-N/3} \dot{M}^{1-2N/3} \omega \omega_{crit}^{4N/3} & (\omega \geq \omega_{crit}), \end{aligned}$$

where R is the radius in units of the solar radius (R_{\odot}), M is the mass in units of the solar mass (M_{\odot}), ω_{crit} introduces a saturation level into the angular momentum loss law (set to $1.5 \times 10^{-5} \text{ s}^{-1}$), $K_w = 2.036 \times 10^{33} (1.452 \times 10^9)^N$ in cgs units, and \dot{M} is the mass-loss rate in unit of $10^{-14} M_{\odot} \text{ yr}^{-1}$ (set to 2.0). Here, it is primarily the exponent N in the wind model (a measure of the magnetic field geometry), which determines the rate of angular momentum loss with time. We have adopted the value of 1.5 for N , which reduces to the empirical Skumanich (1972) law near the main sequence i.e.,

$$v_{rot} \propto \tau^{-.51},$$

where v_{rot} is equatorial rotation velocity, and τ is the age in Gyr. The constant factor in the wind model, f_k , determines the total amount of the angular momentum loss. We adjust f_k for a given N to give the solar surface rotation velocity at the solar age. We use the observed value at about 30 degrees from the equator, 1.86km/s , since this value is close to the mean value which from the seismology data, the interior of the Sun appears to approach (Libbrecht and Morrow 1991)

For the sake of completeness, the transport of angular momentum and chemical elements have also been taken into account in the models. Two types of rotation-induced mixing – the dynamical shear instabilities and the Solberg-Hoiland instability – and three type of secular instabilities – meridional circulation, the Goldreich-Schubert-Fricke instability, and the secular shear instability – were included in the calculations (Chaboyer et al. 1995, Pinsonneault et al. 1989).

In this study, the uncertain effects of other secular instabilities are treated as free parameter, and the diffusion coefficients are set to be the same as that of meridional circulation, f_c . The value of f_c is fixed in the solar model by requiring the model lithium depletion to match the solar value at the solar age. The depletion of Li is inferred by comparing the cosmic abundance with the photospheric abundance. A comparison of the meteoric abundance with the photospheric abundance shows the depletion to be a factor of 140_{-30}^{+40} (Anders and Grevesse 1989). We have therefore set the solar 7Li depletion factor equal to 140.

2.3. Diffusion

The microscopic diffusion coefficients of Michaud and Proffitt (1993) have been used. They have the advantage of being valid not only for 4He and 1H , but also for 3He , 6Li , 7Li , and 9Be . Comparison with the diffusion coefficients of Thoul et al. (1994) indicates that the Michaud-Proffitt coefficients are good to within 15%. Finally we note that when diffusion is taken into account, the surface Z/X is not a constant during a stellar evolution calculation. As 4He diffuse with respect to hydrogen with the relatively short time scale, the model structures are affected. Measurements of the solar photosphere do not actually determine Z – they measure Z/X (i.e. $[Fe/H]$). The Anders and Grevesse (1989) photospheric mixture with meteoritic Fe gives $Z/X = 0.0267 \pm 0.001$. Thus, our model of the present Sun was constrained to match this number.

3. Global turnover time and Rossby number

The close connection between stellar rotation and its chromospheric emission can be described in terms of general stellar dynamo models. In the mean-field dynamo theory, a dimensionless parameter, the dynamo number, characterizes the model behavior. The dynamo number is essentially proportional to the inverse square of the Rossby number, N_R , which is the ratio of the stellar rotation period to the local convective turnover time (Durney and Latour 1978; Noyes et al 1984). Thus, in principle, one could draw a theoretical Rossby number vs. magnetic activity diagram.

In practice, however, our knowledge on stellar convection is too limited to calculate ‘correct’ convective turnover times. The characteristic length scales as well as the velocities are not well known. Even when one decides to resort to the mixing length approximation, there are still uncertainties: the mixing length ratio α is assumed to be the same for all stars with different masses and/or at different evolutionary stage, which is probably not be quite correct. In addition, some assumption must be made as to where in the convection zone the dynamo process is operating, since the convective overturn time is strongly depth dependent. For example, Gilman (1980) set the characteristic convective overturn time equal to the convective overturn time one scale height above the bottom of the convective zone. On the other hand, Gilliland (1986)

determined the turnover time at a distance of half of the mixing length above the base of the convection zone.

3.1. Global convective turnover time

To depict the characteristics of convection at each stellar evolution stage, two parameters have been calculated; the ‘global’ convective turnover time, and the Rossby number. For the characteristic time scale of convective overturn, the ‘non-local’ (or ‘global’) convective turnover time has been calculated at each time step. It is:

$$\tau_c = \int_{R_b}^{R_*} \frac{dr}{v}$$

where R_b is the location of the bottom of the surface convection zone, which is defined where the $\nabla - \nabla_{ad} = 0$, R_* is the total radius of the stellar model, and v is the local convective velocity. Figure 3 shows the evolution with time of the convective turnover time. Note that in the pre-main sequence phase, τ_c varies rapidly with time. Near the main sequence, τ_c remains nearly constant and is primarily a function of mass.

3.2. Rossby number

For the Rossby number calculation, the characteristic convective overturn time was set equal to the ‘local’ convective overturn time at a distance of half of the mixing length $\frac{\alpha H_P}{2}$ above the base of the convection zone. The ratio of the rotation period to this characteristic convective overturn time is used to characterize the Rossby number in the deep convection zone, where dynamo generation of magnetic fields is thought to occur.

$$N_R = 2\pi v / \alpha \Omega H_P$$

where v is the characteristic convective velocity, α is the mixing length ratio, Ω is rotational velocity, and H_P is the local pressure scale height. It turns out that **the evolution of the ‘local’ turnover time is the same as the ‘non-local’ one, except for a scaling factor.** The evolution of the inverse squared Rossby number is illustrated in Figure 4. This quantity, sometimes called the ‘dynamo number’, is believed to be proportional to the strength of magnetic activity. Note that the dynamo number depends on both the age and the mass of the star.

4. Isochrones

Theoretical isochrones offer the opportunity to test stellar evolution theory in star clusters where stars are coeval and formed from a gas cloud of uniform composition. Conversely, when properly calibrated, isochrones can become a powerful tool to study the properties of field stars.

Isochrones were constructed using the evolutionary tracks for the ages of 0.2, 0.5, 0.7, 1.0, 4.55 (the solar age), 10, and 15 Gyr. Their characteristics are listed in Table 2. Figure 5 shows a plot of isochrones of the non-local turnover time vs. $\log T_{eff}$ (the solid lines). For comparison, a few isochrones of the local turnover time are shown (the dotted lines), in the same figure. Isochrones of the non-local turnover time vs. rotation period are given in Figure 6. In Figure 7, rotation period vs. $\log T_{eff}$ is shown, where the increase of $\log T_{eff}$ can be understood as the increase of the stellar mass, because of the proximity of the main sequence. The right most point of each line is for $0.5 M_{\odot}$. Assuming our treatment of stellar rotation is correct, then one can use Figure 7 to uniquely determine stellar mass and age from the effective temperature and the rotation period. Figure 8 shows the the inverse square of the Rossby number, N_R^{-2} vs. $\log T_{eff}$. Once empirical relations between N_R^{-2} and magnetic activity indices are determined, one can use Figure 8 for determination of the age and the mass of a star by observing its effective temperature and an activity index. Figure 9 is the plot of N_R^{-2} vs. rotation period, where for an isochrone, the right most point of the line represent the lowest mass. We see that, given the assumptions implicit in our discussion, our grid of theoretical evolutionary tracks provide the means to determine the age and the mass of a star from a measurement of its rotation period and an activity index.

5. Discussion

Estimates of turnover time scales and the Rossby number are provided, for Sun-like stars in the pre-main sequence and early post-main sequence phases of evolution, based on up-to-date physical input for the stellar models, and including rotation.

We expect the results in this paper to be robust, since the convective turnover timescale is weighted toward the deepest part of the convection zone, where the shortcomings of the mixing length approximation are least important. This is the reason why our ‘global’ and ‘local’ convective time scale give the same result except for a scaling factor (e.g. Figure 5). This is consistent with recent numerical simulations of convection (e.g. Chan and Sofia 1989; Kim et al. 1995b) which confirm the validity of the mixing length approximation in the limit of deep and efficient convection.

In this study, all models have solar abundances; they will therefore find applications in the interpretation of the rotational history and magnetic activity indices for stars in young star clusters and Sun-like field stars, which are the most common stars in our part of the Galactic disk. Caution must be exercised, however, with stars with chemical composition that differ appreciably from solar. Both the depth of the convection zone and the convective velocities are known to depend on opacities and equation of state. As more detailed observations about the rotational properties and magnetic activity of very metal-poor and very metal rich stars become available, the sensitivity of convective turnover timescales to chemical composition parameters will need to be explored in some detail.

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Figure Captions

Figure 1: The evolutionary tracks in the theoretical HR-diagram. Several isochrones have also been drawn for 1 Gyr to 15 Gyr. The lower age isochrones are indistinguishable from the 1 Gyr isochrone on the scale of this diagram.

Figure 2: The rotation period as a function of age and mass. The slope near the main sequence of each mass curve corresponds to the Skumanich law with a proportionality factor which depends on mass.

Figure 3: The non-local convective turnover time as a function of age and mass. Near the main sequence, τ_c remains nearly constant with time for a given mass.

Figure 4: The dynamo number ($\propto N_R^{-2}$), often used as a measure of magnetic activity strength, as a function of age and mass. This plot (together with Figures 8 and 9) provides the means of calibrating magnetic activity indices along the main sequence of a star cluster.

Figure 5: The global and local convective turnover times as a function of effective temperature and age.

Figure 6: Global convective turnover time as a function of rotation period and age.

Figure 7: Rotation period as a function of effective temperature and age.

Figure 8: The dynamo number as a function of effective temperature and age.

Figure 9: The dynamo number as a function of rotation period and age.

Table 1. Input physics

Parameter	Input
Mass	$0.5 \sim 1.2 M_{\odot}$
Mixing length ratio	1.86315
Weight fraction of hydrogen, X	0.70952
Weight fraction of all heavy elements, Z	0.01926
Mixture of heavy elements	Anders-Grevesse(1989)
The exponent in the wind model, N	1.5
The constant factor in the wind model, f_k	17.4837
The diffusion coefficient, f_c	0.05575
Initial rotation velocity	30 km s^{-1}
Opacity tables	OPAL with Kurucz opacity tables for low temperature
Atmosphere	Kurucz model atmosphere
Equation of state	The standard implementation with Debye-Hückel correction
The microscopic diffusion	Michaud and Proffitt (1993)

Table 2. Isochrones

M/M_{\odot}	$\log T_{eff}$	$(B - V)^a$	$\log L/L_{\odot}$	τ_c^b (day)	$N_R^{-2}^c$	Rotation Period (day)
0.20 Gyr						
0.5	3.54	1.55	-1.50	125.10	6.18	23.41
0.6	3.59	1.37	-1.19	104.06	27.58	9.18
0.7	3.64	1.16	-0.88	83.19	28.40	7.23
0.8	3.68	0.99	-0.61	66.50	24.13	6.31
0.9	3.72	0.83	-0.36	53.15	18.90	5.69
1.0	3.75	0.69	-0.14	40.27	13.30	5.19
1.1	3.78	0.57	0.06	27.49	7.73	4.67
1.2	3.80	0.49	0.24	13.95	2.48	4.13
0.50 Gyr						
0.5	3.53	1.60	-1.53	137.40	11.13	19.19
0.6	3.59	1.37	-1.18	106.56	13.57	13.44
0.7	3.64	1.16	-0.87	84.84	11.77	11.46
0.8	3.68	0.99	-0.60	67.18	9.31	10.25
0.9	3.72	0.83	-0.35	53.07	7.01	9.33
1.0	3.75	0.69	-0.13	39.97	4.82	8.57
1.1	3.78	0.57	0.07	26.97	2.56	7.94
1.2	3.80	0.49	0.26	13.14	0.69	7.42
0.70 Gyr						
0.5	3.53	1.60	-1.53	140.32	9.26	21.32
0.6	3.59	1.37	-1.18	107.55	10.14	15.69
0.7	3.64	1.16	-0.87	84.88	8.42	13.59
0.8	3.68	0.99	-0.60	67.40	6.57	12.22
0.9	3.72	0.83	-0.35	52.96	4.92	11.12
1.0	3.75	0.69	-0.13	40.00	3.37	10.21
1.1	3.78	0.57	0.08	26.76	1.77	9.44
1.2	3.80	0.49	0.27	12.87	0.46	8.81

Table 2—Continued

M/M_{\odot}	$\log T_{eff}$	$(B - V)^a$	$\log L/L_{\odot}$	τ_c^b (day)	$N_R^{-2}{}^c$	Rotation Period (day)
1.00 Gyr						
0.5	3.53	1.60	-1.53	141.14	7.15	24.45
0.6	3.59	1.37	-1.18	108.04	7.30	18.61
0.7	3.64	1.16	-0.87	85.43	5.90	16.29
0.8	3.69	0.95	-0.59	67.49	4.55	14.68
0.9	3.72	0.83	-0.34	52.74	3.40	13.37
1.0	3.75	0.69	-0.12	39.86	2.31	12.28
1.1	3.78	0.57	0.09	26.32	1.19	11.34
1.2	3.80	0.49	0.29	12.40	0.30	10.56
2.00 Gyr						
0.5	3.53	1.60	-1.53	142.19	4.11	32.61
0.6	3.59	1.37	-1.18	108.88	3.73	26.23
0.7	3.64	1.16	-0.86	85.26	2.91	23.24
0.8	3.69	0.95	-0.58	67.28	2.22	20.98
0.9	3.73	0.78	-0.32	52.62	1.63	19.19
1.0	3.76	0.65	-0.09	39.25	1.07	17.72
1.1	3.78	0.57	0.14	25.00	0.53	16.23
1.2	3.80	0.49	0.34	11.44	0.13	15.07
4.55 Gyr						
0.5	3.53	1.60	-1.52	144.87	1.80	49.92
0.6	3.59	1.37	-1.16	110.05	1.54	40.97
0.7	3.64	1.16	-0.84	85.10	1.19	36.26
0.8	3.69	0.95	-0.54	66.70	0.89	32.77
0.9	3.73	0.78	-0.26	51.21	0.65	29.83
1.0	3.76	0.65	0.00	37.49	0.41	27.66
1.1	3.78	0.57	0.25	24.82	0.22	25.14
1.2	3.78	0.57	0.44	26.42	0.28	23.40

Table 2—Continued

M/M_{\odot}	$\log T_{eff}$	$(B - V)^a$	$\log L/L_{\odot}$	τ_c^b (day)	$N_R^{-2}{}^c$	Rotation Period (day)
10.00 Gyr						
0.5	3.53	1.60	-1.50	145.61	0.73	78.38
0.6	3.59	1.37	-1.13	109.32	0.62	64.26
0.7	3.65	1.11	-0.78	83.95	0.47	56.79
0.8	3.70	0.91	-0.45	65.26	0.33	52.86
0.9	3.74	0.74	-0.11	50.58	0.24	48.69
1.0	3.75	0.69	0.28	50.94	0.32	42.01
15.00 Gyr						
0.5	3.54	1.55	-1.48	144.95	0.39	107.04
0.6	3.60	1.33	-1.10	108.32	0.33	87.34
0.7	3.66	1.07	-0.73	83.08	0.24	78.18
0.8	3.71	0.87	-0.34	64.97	0.18	70.55
0.9	3.74	0.74	0.13	68.27	0.34	53.99

^a Revised Yale Isochrones and Luminosity Functions (Green et al. 1987)

^b $\tau_c = \int_{R_b}^{R_*} \frac{dr}{v(r)}$, where R_b is the location of the bottom of the surface convection zone, R_* is the total radius of the stellar model, and $v(r)$ is the convective velocity as a function of radius.

^c $N_R = 2\pi v / \alpha \Omega H_P$ where v is the local convective velocity at a distance of the half of the mixing length $\frac{\alpha H_P}{2}$ above the base of the convection zone, α is the mixing length ratio, Ω is rotational velocity, and H_P is the local pressure scale height.