Reheating after Supercooling

in the Chiral Phase Transition

Stanisław Mrówczyński*

High-Energy Department, Sołtan Institute for Nuclear Studies,

ul. Hoża 69, PL - 00-681 Warsaw, Poland

Berndt Müller†

Department of Physics, Duke University, Durham, N.C. 27708-0305, USA

(July 1995)

Abstract

The chirally symmetric quark-gluon plasma produced in energetic heavy-ion collisions is predicted to supercool at the late stages of its evolution. The thermal energy is then transformed into the potential energy associated with an energetically unfavorable field configuration. Since the system is in an unstable state it eventually rolls down to the true minimum of the effective chiral potential. When this motion is described in terms of the sigma-model, we find that the energy of the coherent $\sigma-$field is very efficiently converted into pionic excitations due to anharmonic oscillations around this minimum. The system is expected to partially thermalize before its disintegration.

12.38.Mh, 12.39.Fe, 25.75.+r

Typeset using REVTEX

*E-mail: MROW@FUW.EDU.PL

†E-mail: MULLER@PHYSICS.PHY.DUKE.EDU

The process of hadronization being of essentially nonperturbative character is not well understood at present. This leads to large theoretical uncertainties in describing the late stages of temporal evolution of the quark-gluon plasma predicted to be produced in very energetic heavy ion collisions. Recently, the hadronization of an extended quark-gluon plasma has been studied within the nucleation model [1,2] by considering the growth of hadronic bubbles in the quark-gluon plasma undergoing a first-order phase transition. Thermal equilibrium between both phases was assumed. The authors of refs [1] found that the conversion of the quark-gluon plasma into hadrons is *slow* with a characteristic time of order 50-100 fm/c, resulting in significant supercooling before the plasma hadronizes. Since the microscopic mechanism which actually converts quarks and gluons into hadrons remains unknown, it is unclear to what extent the assumption of thermal equilibrium in both phases holds.

Inspired by the very recent work by Kofman, Linde, and Starobinsky [3], who studied the reheating after inflation in the early Universe, we here discuss a nonequilibrium scenario, where the hadronization coincides with the chiral phase transition. When the quark-gluon plasma is supercooled, the system is in a moderately excited state over a chirally symmetric false vacuum. In this state, the original thermal energy of the plasma is partially transformed into vacuum energy. Once the instability is triggered, the system rolls down to the true chirally asymmetric minimum of the effective potential. Since the oscillations around this minimum are anharmonic, the coherent field is converted into pionic excitations. As we argue below, the conversion mechanism is fast and strongly couples the long wavelength modes to those of short wavelength. Thus, the pions are expected to partially thermalize before freeze-out. Our picture is closely related to that of Rajagopal and Wilczek [4] who studied the excitation of long-wavelength modes during the decay of a supercooled, false chiral vacuum state. However, we are here concerned not with the formation of coherent field domains, which has been numerically studied extensively [4,5], but rather with the excitation of quasi-thermal modes of the pion field.

In the following, we analyse this hadronization mechanism within the linear sigma model defined by the effective Lagrangian

$$\mathcal{L}(t,x) = \frac{1}{2}\partial^\mu\phi_a\partial_\mu\phi_a - \frac{\lambda}{4}(\phi_a\phi_a - v^2)^2 + m_\pi^2 v\chi \tag{1}$$

where $\phi_a \equiv (\chi, \vec{\pi})$ and the parameters are $\lambda \approx 20$, $v \approx 90$ MeV, $m_\pi \approx 140$ MeV. It is important to stress that in the deconfined, chirally symmetric phase the fields ϕ_a are treated as an effective description of quarks and gluons. The ϕ_a-field is thought to dominate the low-energy dynamics of quarks and gluons near the transition, see e.g. [4]. Below the critical temperature the chiral symmetry is broken and one deals with the *physical* fields $\sigma = v_0 - \chi$ and $\vec{\pi}$, which correspond to the sigma mesons and pions, respectively. Here $v_0 \approx v$ is the minimum of the potential in (1).

When the system is significantly supercooled in the symmetric chiral phase, where $\sigma \approx v$ and $\vec{\pi} \approx 0$, the energy density is dominated by the potential energy

$$\varepsilon_{\rm pot} = \frac{\lambda}{4} v^4 \ .$$

If this energy is converted into thermal energy of the pion gas, i.e.

$$\varepsilon_{\rm pot} = \varepsilon_{\rm th} = \frac{\pi^2}{10} T^4 \ ,$$

where the pion mass is neglected[1], one finds the temperature $T \approx 135$ MeV, which is close to the freeze-out temperature of pions observed in relativistic nucleus-nucleus collisions [6]. Thus, even the complete supercooling of the system with subsequent reheating is not in conflict with the data. Let us then discuss the scenario in more detail.

The sigma and pion fields satisfy the following equations of motion:

$$\left[\partial^2 + 2\lambda v^2 - 3\lambda v\sigma + \lambda\sigma^2 + \lambda\vec{\pi}^2\right]\sigma = \lambda v \vec{\pi}^2 \ , \tag{2}$$

$$\left[\partial^2 + m_\pi^2 - 2\lambda v\sigma + \lambda\sigma^2 - \lambda\vec{\pi}^2\right]\vec{\pi} = 0 \ . \tag{3}$$

---

[1]Since the average thermal energy of a massless pion is $3T$, the zero mass or ultrarelativistic approximation is not bad even for temperatures of the order of the pion mass.

We use these equations to discuss the temporal evolution of the system which initially is in the chirally symmetric phase. The fields are treated as classical. Since the system is assumed to be significantly supercooled, the initial values of the fields are taken as $\sigma \approx v$ and $\vec{\pi} \approx 0$. Then, the system appears on the top of a "Mexican hat" potential. When the $\sigma-$field is rolling down to the potential minimum at $\sigma = 0$, its amplitude decreases. Therefore one expects that $v^2\sigma > v\sigma^2 > \sigma^3$, and as a zeroth approximation, we neglect in eqs. (2, 3) the terms which are quadratic and cubic in the fields. In this way we get the equations

$$\left[\partial^2 + m_\sigma^2\right]\sigma^{(0)} = 0\ , \qquad \left[\partial^2 + m_\pi^2\right]\vec{\pi}^{(0)} = 0\ , \tag{4}$$

with $m_\sigma \equiv \sqrt{2\lambda}v = 600$ MeV, which have plane wave solutions. Keeping in mind the initial conditions, we choose the solutions of eqs. (4) as

$$\sigma^{(0)} = \sigma_0 \cos(m_\sigma t + \varphi)\ , \qquad \vec{\pi}^{(0)} = 0\ , \tag{5}$$

where the field $\sigma^{(0)}$ is assumed to be homogeneous. The solution (5) describes undamped collective oscillations of the $\sigma-$field around its minimum. Now we substitute the solutions (5) into eqs. (2, 3) and keep the terms no more than quadratic in the $\sigma-$field and linear in the $\vec{\pi}-$field. Changing the time variable ($2z \equiv m_\sigma t + \varphi$) and writing down the equations for modes labeled by the momentum $\mathbf{k}$ we get

$$\left[\frac{d^2}{dz^2} + A_\sigma - 2q_\sigma \cos(2z)\right]\sigma_{\mathbf{k}}^{(1)}(z) = 0\ , \tag{6}$$

$$\left[\frac{d^2}{dz^2} + A_\pi - 2q_\pi \cos(2z)\right]\vec{\pi}_{\mathbf{k}}^{(1)}(z) = 0\ , \tag{7}$$

where

$$A_\sigma \equiv 4\frac{m_\sigma^2 + \mathbf{k}^2}{m_\sigma^2}\ , \qquad q_\sigma \equiv 3\frac{\sigma_0}{v}\ , \tag{8}$$

and

$$A_\pi \equiv 4\frac{m_\pi^2 + \mathbf{k}^2}{m_\sigma^2}\ , \qquad q_\pi \equiv 2\frac{\sigma_0}{v}\ . \tag{9}$$

Eqs. (6,7) are written down in the form of the well known Mathieu equation [7], which corresponds to the wave equation in elliptic coordinates. As discussed above $\sigma_0 < v$, thus $0 < q_\sigma < 3$ and $0 < q_\pi < 2$, while $A_\sigma \geq 4$ and $A_\pi \geq 0.2$.

The solutions of Mathieu's equation are of the form

$$F_\nu(z) = e^{i\nu z} P(z) ,$$

where the constant ν, which is called the characteristic exponent, depends on A and q and $P(z)$ is a periodic function with the period π. It is easy to see that for an infinitesimally small q there are narrow regions of resonant solutions of the equations (6,7) for $A \approx N^2$ where N is an arbitrary integer number. Then, the characteristic frequency of the wave equation is a multiple of the frequency of the stimulating force. Let us consider for illustration the first resonant region around $A = 1$. The parameters A, q and ν are then connected by the equation [7]

$$A = \nu^2 + \frac{q^2}{2(\nu^2 - 1)} , \quad \text{for } \nu \neq 1 .$$

For $A = 1$ one immediately finds

$$\nu^2 = 1 \pm \frac{i}{\sqrt{2}} q .$$

Decomposing ν into the real and imaginary parts with $|\mathrm{Re}\,\nu| \gg |\mathrm{Im}\,\nu|$, we get

$$\mathrm{Im}\,\nu \approx \pm \frac{i}{2\sqrt{2}} q .$$

As expected, we have found an unstable mode which generally grows exponentially in time.

When the amplitude of the stimulating force represented by q increases, the narrow resonances change into wide resonant bands in the $A - q$ plane. The imaginary part of the characteristic exponent ν is negative there and the solutions of Mathieu's equation are unstable. For example, when $q = 1$ the resonant regions extend for A from the intervals (0, 1.9), (3.9, 4.4), (9.06, 9.08) etc. As seen, the first two resonances are rather broad while the third and higher ones remain narrow for $q = 1$.

Although our zeroth order solution (5) is homogeneous, i.e. $\mathbf{k} = 0$, it effectively couples (via eqs. (6, 7)) not only to the long wavelength pion modes but also to those with $|\mathbf{k}| \approx m_\sigma$.

Let us consider how fast these modes grow. For a given value of A and q, the characteristic exponents can be read from the charts presented in [7]. For $q_\sigma = 3$ and $q_\pi = 2$, which are the maximal values, one finds for $A_\sigma \approx A_\pi \approx 4$ the imaginary characteristic exponents equal 0.35 and 0.17, respectively. Then, the particle numbers proportional to $|\sigma_{\mathbf{k}}^{(1)}|^2$ and $|\vec{\pi}_{\mathbf{k}}^{(1)}|^2$ grow in time as $e^{t/\tau}$ with $\tau_\sigma = 0.9$ fm/c and $\tau_\pi = 1.9$ fm/c, where $\tau^{-1} = m_\sigma \mathrm{Im}\ \nu$. The characteristic time τ_π of growth of the occupation of single-particle pion modes appears much smaller than the nucleation time found in [1]. Therefore, the mechanism described here can indeed be responsible for hadronization of the quark-gluon plasma.

The hadronization which proceeds simultaneously with the transition from the chirally symmetric to asymmetric state has been discussed in [9]. The hadronization time has been identified there with the characteristic time of rolling down from the top of the "Mexican hat" to the potential minimum, which is of order 0.5 fm/c. In our opinion, we deal with "physical" pions only when the coherence of the σ and $\vec{\pi}$ fields breaks down due to the anharmonic oscillations around the true vacuum state. Therefore, we identify the hadronization time with τ_π found above which equals about 2 fm/c.

An important feature of our hadronization scenario is the strong coupling of the soft coherent modes to those with $|\mathbf{k}| \approx m_\sigma$. This implies that the spectrum of produced pions is rather broad, similar to a thermal spectrum at $T \approx m_\pi$. Therefore, we expect at least partial thermalization of the system. The reheating or even overheating due to the rapid release of the latent heat has also been advocated [10] within the nucleation model [1]. The thermal spectrum of hadrons observed in relativistic heavy-ion collisions supports such an expectation. It might be of critical importance for the so-called Disoriented Chiral Condensates actively discussed recently [4,5,8].

It has been suggested that coherent domains of the pion field can appear in the nonequilibrium chiral phase transition of the quark-gluon system created in high energy collisions. The phenomenon, analogous to the formation of misaligned domains in a ferromagnet, would be observed by the coherent pion emission when the domains relax to the ground state with vanishing pion field. Specifically, one expects significant fluctuations in the number of neutral and charged pions. The soft pions are expected to be particularly sensitive to the domain formation. Numerical simulations of the linear sigma model represented by the Lagrangian (1) have shown [4,5] that the disoriented condensates can indeed appear under favorable conditions.

As the reheating proceeds in the hadronizing system, it will generate additional background on which it is more difficult to observe the pions from coherent domains. We note that the numerical simulations [4] do not fully take into account the short wavelength modes excited by eqs. (6,7) due to the finite lattice spacing, typically of 1 fm. Therefore, the coupling of the coherent soft modes to the harder ones could easily be underestimated.

In summary, if the quark-gluon plasma produced in relativistic heavy-ion collisions is significantly supercooled at the late stage of its evolution, its thermal energy is converted into potential energy of the $\sigma-$field. The system then rolls down from the chirally symmetric to the asymmetric state and the hadronization coincides with the chiral phase transition. The physical pions emerge when the system anharmonically oscillates around the true vacuum. Due to the efficient resonant coupling between the soft and hard modes, the system is expected to thermalize again, at least partially, on a short time scale.

Acknowledgements:

We thank the European Centre for Theoretical Studies in Nuclear Physics and Related Areas in Trento (Italy) for warm hospitality. This work was supported, in part, by the U.S. Department of Energy and the Polish Committee of Scientific Research under grants DE-FG05-90ER40592 and 2-P03B-195-09, respectively.

REFERENCES

[1] L. P. Csernai and J.I. Kapusta, *Phys. Rev.* **D46** (1992) 1379; L. P. Csernai and J.I. Kapusta, Phys. *Rev. Lett.* **69** (1992) 737; L. P. Csernai, J.I. Kapusta, Gy. Kluge, and E. E. Zabrodin, *Z. Phys.* **C58** (1993) 453.

[2] G. Lana and B. Svetitsky, *Phys. Lett.* **B285** (1992) 251.

[3] L. Kofman, A. Linde, and A.A. Starobinsky, *Phys. Rev. Lett.* **73** (1994) 3195.

[4] K. Rajagopal and F. Wilczek, *Nucl. Phys.* **B404** (1993) 577.

[5] S. Gavin, A. Gocksch, and R.D. Pisarski, *Phys. Rev. Lett.* **72** (1994) 2143; M. Asakawa, Z. Huang, and X.-N. Wang, *Phys. Rev. Lett.* **74** (1995) 3126.

[6] J. Baechler et al., *Phys. Rev. Lett.* **72** (1994) 1419; J. Letessier, J. Rafelski, and A. Tounsi, *Phys. Lett.* **B328** (1994) 499; P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, *Phys. Lett.* **B344** (1995) 43.

[7] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover Publications, New York, 1965).

[8] A. Anselm and M. Ryskin, *Phys. Lett.* **B226** (1991) 482; J.-P. Blaizot and A. Krzywicki, *Phys. Rev.* **46** (1992) 246; J.D. Bjorken, *Acta Phys. Pol.* **B23** (1992) 561; S. Gavin and B. Müller, *Phys. Lett.* **B329** (1994) 486.

[9] L. P. Csernai and I.N. Mishustin, *Phys. Rev. Lett.* **74** (1995) 5005.

[10] T. Csörgö and L. P. Csernai, *Phys. Lett.* **B333** (1994) 494.