

NSF-ITP-95-64

NUB-TH-3125/95

# Probing Supergravity Grand Unification in the Brookhaven g-2 Experiment

U. Chattopadhyay<sup>(a)</sup> and Pran Nath<sup>(a)1,(b)</sup><sup>(a)</sup>*Department of Physics, Northeastern University, Boston, MA 02115, USA*<sup>(b)</sup>*Institute for Theoretical Physics, University of California**Santa Barbara, CA 93106-4030, USA*

## Abstract

A quantitative analysis of  $a_\mu \equiv \frac{1}{2}(g-2)_\mu$  within the framework of Supergravity Grand Unification and radiative breaking of the electro-weak symmetry is given. It is found that  $a_\mu^{SUSY}$  is dominated by the chiral interference term from the light chargino exchange, and that this term carries a signature which correlates strongly with the sign of  $\mu$ . Thus as a rule  $a_\mu^{SUSY} > 0$  for  $\mu > 0$  and  $a_\mu^{SUSY} < 0$  for  $\mu < 0$  with very few exceptions when  $\tan\beta \sim 1$ . At the quantitative level it is shown that if the E821 BNL experiment can reach the expected sensitivity of  $4 \times 10^{-10}$  and there is a reduction in the hadronic error by a factor of four or more, then the experiment will explore a majority of the parameter space in  $m_0 - m_{\tilde{g}}$  plane in the region  $m_0 \lesssim 400$  GeV,  $m_{\tilde{g}} \lesssim 700$  GeV for  $\tan\beta \gtrsim 10$  assuming the experiment will not discard the

---

<sup>1</sup>Permanent address

Standard Model result within its  $2\sigma$  uncertainty limit. For smaller  $\tan\beta$ , the SUSY reach of E821 will still be considerable. Further, if no effect within  $2\sigma$  limit of the Standard Model value is seen, then large  $\tan\beta$  scenarios will be severely constrained within the current naturalness criterion, ie.,  $m_0, m_{\tilde{g}} \lesssim 1$  TeV.

## 1 Introduction

The high level of experimental accuracy of the measured value of the anomalous magnetic moment of the muon [1] has provided verification to several orders in the perturbation expansion of quantum electrodynamics(QED) [2] as well as put constraints on physics beyond the Standard Model (SM) [3]. Further the E821 experiment currently underway at Brookhaven is expected to improve the accuracy over the previous measurement by a factor of 20 [4]. Simultaneously, it is expected that improved analyses of existing data on ( $e^+e^- \rightarrow hadrons$ ) as well as new data from ongoing experiments in VEPP-2M [5] together with future experiments in DAΦNE [6], BEPC [7] etc. will reduce the uncertainty in the hadronic contributions to a significant level so as to allow for a test for the first time of the electro-weak corrections in the Standard Model. It was pointed out in Refs [8, 9] ( see also Refs [10] for a discussion of previous work and Ref [11] for a discussion of recent work ) that supersymmetric corrections to  $(g - 2)_\mu$  are in general the same size as the electro-weak corrections in supergravity grand unification [12]. Thus improved experiments designed to test the Standard Model electro-weak corrections can also provide a probe of supersymmetric contributions.

Table 1: Contributions to  $a_\mu \times 10^{10}$ 

Nature of Contribution	CASE A	CASE B
Q.E.D. to $O(\alpha/\pi)^5$ Kinoshita <i>et al.</i> [22, 2]	11658470.8(0.5)	11658470.8(0.5)
Hadronic vac. polarization to $O(\alpha/\pi)^2$ Hadronic vac. polarization to $O(\alpha/\pi)^3$ Kinoshita & Marciano [3]	705.2(7.56) Martinovic & Dubnicka [18]	702.35(15.26) Eidelman & Jegerlehner [19, 20]
Light by light hadronic amplitude Bijnens <i>et al.</i> [21]	-9.0(0.5)	-9.0(0.5)
Total hadronic	0.8(0.9)	0.8(0.9)
Electro-weak one-loop Fujikawa <i>et al.</i> [23]	697.0(7.6)	694.2(15.3)
Electro-weak 2-fermion loops Czarnecki <i>et al.</i> [22]	19.5	19.5
Electro-weak 2-boson loops Czarnecki <i>et al.</i> [22]	-2.3(0.3)	-2.3(0.3)
Total electro-weak upto 2-loops	$(-2.0 + 0.045 \times R_b)$ $(15.2(0.3) + 0.045 \times R_b)$	$(-2.0 + 0.045 \times R_b)$ $(15.2(0.3) + 0.045 \times R_b)$
Total (with $R_b = 0$ )	11659183(7.6)	11659180(15.3)

The analyses of Refs [8, 9], however, were done without using the constraints of radiative breaking of the electro-weak symmetry [13] and without the benefit of the recent high precision LEP data [14, 15] to constrain the coupling constants [16]. The purpose of the present work is to include these features in the analysis. Additionally, we investigate the effects on the results due to  $b \rightarrow s\gamma$  constraint [17], and dark matter constraint. We shall find that the expected accuracy of  $a_\mu \equiv \frac{1}{2}(g-2)_\mu$ , in E821 BNL experiment would either allow for supergravity grand unification effects to be visible in the BNL experiment, or if no effect beyond the  $2\sigma$  level is seen, then there will be a significant constraint on the model.

The most recent experimental value of  $a_\mu$  is averaged to be  $a_\mu^{exp} = 1165923(8.4) \times 10^{-9}$ . The quantity within the parenthesis refers to the uncertainty in the last digit. The Standard Model results consist of several parts:

$$a_\mu^{SM} = a_\mu^{qed} + a_\mu^{had} + a_\mu^{E-W} \quad (1)$$

Here  $a_\mu^{qed}$  [2] is computed to  $O(\alpha^5)$  QED corrections.  $a_\mu^{had}$  consists of  $O(\alpha^2)$  [18, 19] and  $O(\alpha^3)$  [3] hadronic vacuum polarizations, and also light-by-light hadronic contributions [21]. We exhibit two different evaluations in Table 1. Case A uses the analysis of Martinovic and Dubnicka [18] who use a detailed structure of pion and kaon form factors in their fits to get the  $O(\alpha^2)$  corrections. Case B uses the  $O(\alpha^2)$  analysis of Eidelman and Jegerlehner [19, 20]. We note that there is almost a factor of two difference between the hadronic errors of case A and case B. For the electro-weak corrections we have used the recent analyses of Czarnecki et al. [22], which include one-loop electro-weak corrections of

the Standard Model [23], 2-loop corrections with fermion loops [22, 24], and partial 2-loop corrections with boson loops [25]. The remaining unknown 2-loop corrections with boson loops are denoted by  $R_b$  following the notation of Ref. [22]. Pending the full 2-loop bosonic contributions in the E-W corrections, the total  $a_\mu^{SM}$  for case A is  $11659180(7.6) \times 10^{-10}$  while for case B one has  $11659200(15.3) \times 10^{-10}$ . We see that the overall error, which is dominated by the hadronic corrections, is about half for case A relative to that for case B.

The new high precision E821 Brookhaven experiment [4] has an anticipated design sensitivity of  $4 \times 10^{-10}$ . This is about 20 times more accurate than that of the CERN measurement conducted earlier. However, as mentioned already one needs in addition an improvement in the computation of the hadronic contribution  $a_\mu^{had}$ , where uncertainties primarily arise due to hadronic vacuum polarization effects. This problem is expected to be overcome soon through further accurate measurements of  $\sigma_{tot}(e^+e^- \rightarrow hadrons)$  for the low energy domain.

## 2 The Minimal Supergravity Model

The framework of our analysis is N=1 Supergravity grand unified theory [12] where the Supergravity interactions spontaneously break supersymmetry at the Planck scale ( $M_{Pl} = 2.4 \times 10^{18}$  GeV) via a hidden sector [12]. Further we assume that the GUT group  $G$  breaks at scale  $Q = M_G$  to the Standard Model gauge group :  $G \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$ . At low energy the symmetry breaking effective potential below the GUT scale  $M_G$  is given by

$$V_{SB} = m_0^2 z_i z_i^\dagger + (A_0 W^{(3)} + B_0 W^{(2)} + h.c.) \quad (2)$$

where  $W^{(2)}, W^{(3)}$  are the quadratic and the cubic parts of the effective superpotential. Here  $W^{(2)} = \mu_0 H_1 H_2$ , with  $H_1, H_2$  being two Higgs doublets, and  $W^{(3)}$  contains terms cubic in fields and involves the interactions of quarks, leptons and Higgs with strength determined by Yukawa couplings. In addition the low energy theory has a universal gaugino mass term  $-m_{\frac{1}{2}} \bar{\lambda}^\alpha \lambda^\alpha$ . The minimal Supergravity model below the GUT scale depends on the following set of parameters.

$$m_0, m_{\frac{1}{2}}, A_0, B_0; \mu_0, \alpha_G, M_G \quad (3)$$

where  $M_G$  and  $\alpha_G$  are the GUT mass and the coupling constant respectively. Among these parameters  $\alpha_G$  and  $M_G$  are determined with the high precision LEP results by using two-loop renormalization group equations  $\alpha_i(M_Z), i = 1, 2, 3$  up to  $M_G$  [26]. Renormalization group analysis is used to break the electro-weak symmetry [13] and radiative breaking allows one to determine  $\mu_0$  by fixing  $M_Z$  and to find  $B_0$  in terms of  $\tan\beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$ . Thus the model is completely parametrized by just four quantities [27]

$$m_0, m_{\tilde{g}}, A_t, \tan\beta \quad (4)$$

Here the universal gaugino mass  $m_{\frac{1}{2}}$  is replaced by the gluino mass  $m_{\tilde{g}}$  and  $A_0$  by  $A_t$ , which is the t-quark A parameter at the electro-weak scale  $M_{EW}$ . In addition to these four parameters and the top quark mass  $m_t$ , one has to specify the sign of  $\mu$ , since the radiative breaking equations determine only  $\mu^2$ . There are 32 new particles in this model (12 squarks, 9 sleptons, 2 charginos, 4 neutralinos, 1 gluino, 2 CP even neutral Higgs, 1

CP odd neutral Higgs and 1 charged Higgs). The masses of these 32 new particles and all their interactions can be determined by the four parameters mentioned above. Thus the theory makes many new predictions [27], and has led to considerable activity [28, 29] to explore the implications of supergravity grand unification and its signals [30]. The allowed parameter space of the model is further constrained by the following i) charge and colour conservation [31], ii) absence of tachyonic particles, iii) a lower bound on SUSY particle masses as indicated by CDF, D0, and LEP data, iv) an upper limit on SUSY masses from the naturalness criterion which is taken as  $m_0, m_{\tilde{g}} < 1$  TeV. Our analysis automatically takes into account important Landau pole effects that arise due the top being heavy and thus in proximity to the Landau pole that lies in the top Yukawa coupling [32]. Additionally, we consider the constraint on  $b \rightarrow s\gamma$  decay rate from the recent CLEO data, and the neutralino relic density constraint.

### 3 Analysis of $(g - 2)_{\mu}^{SUSY}$ and Results

We use Ref. [8] to compute SUSY contributions to  $(g - 2)_{\mu}$ . These contributions arise from Figs. (1a) and (1b). In Fig. (1a) one exchanges 2 charginos  $\tilde{W}_a$ ,  $a=1,2$  with masses  $m_{\tilde{W}_a}$  which are charged spin  $\frac{1}{2}$  Dirac fields and a sneutrino state. In Fig. (1b) one exchanges 4 neutralinos  $\tilde{Z}_{(k)}$ ,  $k=1,2,3,4$  with masses  $m_{\tilde{Z}_{(k)}}$  which are spin  $\frac{1}{2}$  Majorana fields (our labelling of particles satisfy  $\tilde{m}_i < \tilde{m}_j$  for  $i < j$ ), and two scalar smuon mass eigenstates. The mass spectra of the charginos, neutralinos, smuons and of the sneutrino is given in the Appendix for convenience. The one-loop supersymmetric contribution to  $(g - 2)_{\mu}$

then is given by

$$a_\mu^{SU5Y} = \frac{1}{2}(g - 2)_\mu^{SU5Y} = \frac{1}{2}(g^{\tilde{W}} + g^{\tilde{Z}}) \quad (5)$$

For the chargino-sneutrino part referring to [8] we find (with summation over repeated indices implied)

$$g^{\tilde{W}} = \frac{m_\mu^2}{24\pi^2} \frac{A_R^{(a)2}}{m_{\tilde{W}_a}^2} F_1(x_a) + \frac{m_\mu}{4\pi^2} \frac{A_R^{(a)} A_L^{(a)}}{m_{\tilde{W}_a}} F_2(x_a) \quad (6)$$

$$x_a = \left( \frac{m_{\tilde{\nu}}}{m_{\tilde{W}_a}} \right)^2 ; \quad a = 1, 2 \quad (7)$$

The mass of the sneutrino required in Eq. (7) can be determined from Eq. (33). The first term in Eq. (6) contains terms diagonal in chirality whereas the second term has R-L interference terms arising from Yukawa interactions. The functions  $F_1(x)$  and  $F_2(x)$  are defined by

$$\begin{aligned} F_1(x) &= (1 - 5x - 2x^2)(1 - x)^{-3} - 6x^2(1 - x)^{-4} \ln(x) \\ F_2(x) &= (1 - 3x)(1 - x)^{-2} - 2x^2(1 - x)^{-3} \ln(x) \end{aligned} \quad (8)$$

$A_R^{(a)}$  and  $A_L^{(a)}$  of Eq. (6) are given as

$$A_R^{(1)} = -\frac{e}{\sqrt{2} \sin \theta_W} \cos \gamma_1; \quad A_L^{(1)} = (-1)^\theta \frac{e}{\sqrt{2} \sin \theta_W} \frac{m_\mu \cos \gamma_2}{\sqrt{2} M_W \cos \beta} \quad (9)$$

$$A_R^{(2)} = -\frac{e}{\sqrt{2} \sin \theta_W} \sin \gamma_1; \quad A_L^{(2)} = -\frac{e}{\sqrt{2} \sin \theta_W} \frac{m_\mu \sin \gamma_2}{\sqrt{2} M_W \cos \beta} \quad (10)$$

Here the angles  $\gamma_1$  and  $\gamma_2$  are found by using  $\gamma_{1,2} = \tilde{\beta}_2 \mp \tilde{\beta}_1$  where

$$\begin{aligned} \sin 2\tilde{\beta}_{1,2} &= (\mu \pm \tilde{m}_2) [4\nu_{1,2}^2 + (\mu \pm \tilde{m}_2)^2]^{-\frac{1}{2}} \\ \cos 2\tilde{\beta}_{1,2} &= 2\nu_{1,2} [4\nu_{1,2}^2 + (\mu \pm \tilde{m}_2)^2]^{-\frac{1}{2}} \end{aligned} \quad (11)$$



where  $-\pi < 2\tilde{\beta}_{1,2} \leq \pi$ .

The neutralino-smuon loop correction results in

$$\begin{aligned}
g^{\tilde{Z}} &= -\frac{m_\mu^2}{12\pi^2} \frac{1}{m_{\tilde{Z}(k)}^2} \left[ \{s^2(B_k^R)^2 + c^2(B_k^L)^2\} G_1(x_{1k}) + \{c^2(B_k^R)^2 + s^2(B_k^L)^2\} G_1(x_{2k}) \right] \\
&\quad + \frac{m_\mu}{4\pi^2} \frac{sc}{m_{\tilde{Z}(k)}} B_k^R B_k^L [G_2(x_{1k}) - G_2(x_{2k})] \\
&\quad - \frac{m_\mu}{4\pi^2} \frac{C_k}{m_{\tilde{Z}(k)}} \left[ \{c^2 B_k^L - (-1)^{\theta_k} s^2 B_k^R\} G_2(x_{1k}) + \{s^2 B_k^L - (-1)^{\theta_k} c^2 B_k^R\} G_2(x_{2k}) \right] \quad (12)
\end{aligned}$$

Here  $s = \sin \delta$ ,  $c = \cos \delta$ , and

$$x_{rk} = \left( \frac{m_{\tilde{\mu}_r}}{m_{\tilde{Z}(k)}} \right)^2; \quad r = 1, 2; \quad k = 1, 2, 3, 4 \quad (13)$$

The functions  $G_1(x)$  and  $G_2(x)$  are given by

$$\begin{aligned}
G_1(x) &= \frac{1}{2}(2 + 5x - x^2)(1 - x)^{-3} + 3x(1 - x)^{-4} \ln(x) \\
G_2(x) &= (1 + x)(1 - x)^{-2} + 2x(1 - x)^{-3} \ln(x) \quad (14)
\end{aligned}$$

The Yukawa coefficients  $C_k$  are found from

$$C_k = e \frac{m_\mu}{2M_W \cos \beta \sin \theta_W} [\cos \beta O_{3k} + \sin \beta O_{4k}] \quad (15)$$

and  $B_k^R$  and  $B_k^L$  are found by using

$$\begin{aligned}
B_k^R &= e [-O_{1k} + \cot 2\theta_W O_{2k}] \\
B_k^L &= e [-O_{1k} - \tan \theta_W O_{2k}] (-1)^{\theta_k} \quad (16)
\end{aligned}$$

The angle  $\delta$ ,  $\theta_k$  and the quantities  $O_{ij}$  are defined in the Appendix.

Among the two sources of one-loop contributions to  $a_\mu^{SUSY}$  the chargino-sneutrino loop contributes more than the neutralino-smuon loop. This occurs mainly due to the smallness of the mixing angle  $\delta$  (see Eq. (12)) which itself arises from the smallness of the muon to the sparticle mass ratio (see Eq. (31)). Partial cancellations on the right hand side of Eq. (12) are also responsible for a reduction of the neutralino-smuon contribution. For the chargino-sneutrino contribution (see Eq. (6)) one has comparable magnitudes for chirality diagonal and non-diagonal terms for small  $\tan\beta(\sim 1)$ . The non-diagonal terms become more important as  $\tan\beta$  starts to deviate significantly from unity. Indeed, a large value of  $\tan\beta$  results in a large contribution of the chirality non-diagonal term in the chargino-sneutrino part and hence to  $|a_\mu^{SUSY}|$  due to the enhancement arising from  $\frac{1}{\cos\beta} \sim \tan\beta$  in the Yukawa coupling.

There exists a very strong correlation between the sign of  $a_\mu^{SUSY}$  and the sign of  $\mu$  which we now explain. While the chiral interference chargino part of  $a_\mu^{SUSY}$  dominates over the other terms, it is the lighter chargino mass which contributes most dominantly. In fact this part depends on  $A_L^{(1)}A_R^{(1)}$  which from Eq. (9) can be seen to have a front factor of  $(-1)^\theta$ , where  $\theta = 0(1)$  for  $\lambda_1 > 0(< 0)$  where  $\lambda_1$  is the smaller eigenvalue of the chargino mass matrix (see Eq. (26)). For  $\mu > 0$ , one finds  $\lambda_1 < 0$  invariably and for  $\mu < 0$  one has  $\lambda_1 > 0$  for almost all the regions of parameter space of interest. This can be seen by writing  $\lambda_{1,2}$  in the following form

$$\lambda_{1,2} = \frac{1}{2} \left( \left[ 2M_W^2 + (\mu - \tilde{m}_2)^2 - 2M_W^2 \sin 2\beta \right]^{\frac{1}{2}} \mp \left[ 2M_W^2 + (\mu + \tilde{m}_2)^2 + 2M_W^2 \sin 2\beta \right]^{\frac{1}{2}} \right) \quad (17)$$

and noting that the terms containing  $\sin 2\beta$  are only appreciable when  $\tan\beta$  is small ( $\sim 1$ ).

As a result, due to this unique dominance of the chiral interference term involving the lighter chargino mass, one finds as a rule  $a_\mu^{SU\!SY} > 0$  for  $\mu > 0$  and  $a_\mu^{SU\!SY} < 0$  for  $\mu < 0$  with some very few exceptions for the latter case when  $\tan\beta \sim 1$  resulting in a very small  $|a_\mu^{SU\!SY}|$ .

The generic dependence of  $|a_\mu^{SU\!SY}|$  on the remaining parameters  $m_0$ ,  $m_{\tilde{g}}$ , and  $A_t$  is as follows: Regarding  $m_0$ , the dependence results primarily from the chargino-sneutrino sector because the mass spectra depend on  $m_0$ . This results in a decreasing  $|a_\mu^{SU\!SY}|$  with increase in  $m_0$ . Among the other two basic parameters a large gluino mass  $m_{\tilde{g}}$  in general again leads to a smaller  $|a_\mu^{SU\!SY}|$ , due to the resulting larger sparticle masses entering in the loop. The  $A_t$  dependence enters implicitly via the SUSY mass spectra, and also explicitly in the neutralino-smuon exchange diagrams. Figs. (2a) and (2b) show the upper limit of  $|a_\mu^{SU\!SY}|$  in the minimal Supergravity model by mapping the entire parameter space for  $m_0, m_{\tilde{g}} \leq 1$  TeV,  $\tan\beta \leq 30$  and both signs of  $\mu$ . One finds that  $\max |a_\mu^{SU\!SY}|$  increases with increasing  $\tan\beta$  for fixed  $m_{\tilde{g}}$ . As discussed above this happens due to the essentially linear dependence of the chirality non-diagonal term on  $\tan\beta$ . Furthermore, for large  $\tan\beta$  one finds similar magnitudes of  $\max |a_\mu^{SU\!SY}|$  for  $\mu > 0$  and  $\mu < 0$ . This can be understood by noting that for a large  $\tan\beta$  the lighter chargino masses for  $\mu > 0$  and  $\mu < 0$  cases have almost similar magnitudes (see Eq. (17)). This, along with the explanation of the dependence of the sign of  $a_\mu^{SU\!SY}$  on the sign of  $\mu$  accounts for similar  $|a_\mu^{SU\!SY}|$  values for both  $\mu < 0$  and  $\mu > 0$  when  $\tan\beta$  is large. This similarity between the  $\mu > 0$  and  $\mu < 0$  cases is less apparent for small  $\tan\beta$  (ie.  $\tan\beta \lesssim 2$ ) and small  $m_{\tilde{g}}$  values.

Next we include in the analysis  $b \rightarrow s\gamma$  constraint and the dark matter constraint

which have been shown in recent work [33, 34] to generate strong constraints on the parameter space. For the  $b \rightarrow s\gamma$  decay the CLEO Collaboration [17] gives a value of

$$\text{BR}(b \rightarrow s\gamma) = (2.32 \pm 0.5 \pm 0.29 \pm 0.32) \times 10^{-4} \quad (18)$$

Combining the errors in quadrature one has  $\text{BR}(b \rightarrow s\gamma) = (2.32 \pm 0.66) \times 10^{-4}$ . The Standard Model prediction for this branching ratio has an  $O(30\%)$  uncertainty [35] mainly from the currently unknown next-to-leading (NLO) order QCD corrections. Recent Standard Model evaluations give [35]  $\text{BR}(b \rightarrow s\gamma) = (2.9 \pm 0.8) \times 10^{-4}$  at  $m_t \approx 170\text{GeV}$ . The SUSY effects in  $\text{BR}(b \rightarrow s\gamma)$  can be conveniently parametrized by introducing the parameter  $r_{\text{SUSY}}$  which we define by the ratio [36]

$$r_{\text{SUSY}} = \text{BR}(b \rightarrow s\gamma)_{\text{SUSY}} / \text{BR}(b \rightarrow s\gamma)_{\text{SM}} . \quad (19)$$

Several uncertainties that are present in the individual branching ratios cancel out in the ratio  $r_{\text{SUSY}}$ . However, we point out that the NLO corrections would in general be different for the SUSY case than for the SM case due to the presence of different SUSY thresholds [38]. In the present analysis we limit ourselves to the leading order evaluation. In a similar fashion we can define

$$r_{\text{exp}} = \text{BR}(b \rightarrow s\gamma)_{\text{exp}} / \text{BR}(b \rightarrow s\gamma)_{\text{SM}} . \quad (20)$$

Using the experimental result of Eq. (18) and the Standard Model values given above one finds that  $r_{\text{exp}}$  lies in the range

$$r_{\text{exp}} = 0.46 - 2.2 \quad (21)$$

Now normally in SUSY theory one can get rather large deviations from the SM results so that  $r_{SUSY}$  can lie in a rather large range, i.e.,  $\approx (0, 10)$ . Thus the constraint  $r_{SUSY} = r_{exp}$  is an important constraint on the theory. This constraint then excludes a part of the parameter space [34, 36] and reduces the magnitude of the  $max |a_{\mu}^{SUSY}|$  for a given  $m_{\tilde{g}}$ .

The cosmological constraint on the neutralino relic density [39] also plays a very significant role in limiting the parameter space of the model. Theoretically the quantity of interest is  $\Omega_{\tilde{Z}_1} h^2$  where  $\Omega_{\tilde{Z}_1} = \frac{\rho_{\tilde{Z}_1}}{\rho_c}$ . Here  $\rho_{\tilde{Z}_1}$  is the neutralino mass density,  $\rho_c$  is the critical mass density to close the universe and  $h = H/(100km/sMpc)$  where  $H$  is the Hubble constant. Astronomical observations indicate  $h \cong 0.4 - 0.8$  which results in a spread of value for  $\Omega_{\tilde{Z}_1} h^2$ . For our analysis we use a mixture of cold and hot dark matter (CHDM) in the ratio  $\Omega_{CDM} : \Omega_{HDM}=2:1$  consistent with the COBE data. Assuming total  $\Omega = 1$  as is implied by the inflationary scenario and using for the baryonic matter  $\Omega_B=0.1$  we get [36, 39]

$$0.1 \leq \Omega_{\tilde{Z}_1} h^2 \leq 0.4 \quad (22)$$

The combined effects of the  $b \rightarrow s\gamma$  constraint and relic constraint put severe limits on the parameter space [36, 40, 41]. Their effect on  $a_{\mu}^{SUSY}$  is shown in Figs. (2c) and (2d) which are similar to Figs. (2a) and (2b) except for the inclusion of the combined effects of  $b \rightarrow s\gamma$  constraint and relic constraint. Comparison of Figs. (2a) and (2c) and similarly of Figs. (2b) and (2d) show that typically  $max |a_{\mu}^{SUSY}|$  for large  $\tan\beta$  ( i.e.,  $\tan\beta \gtrsim 2$ ) is reduced by about a factor of  $\frac{2}{3}$  for gluino masses below the dip under the combined effects of  $b \rightarrow s\gamma$  and dark matter constraints. However, as is obvious, the most striking effect arises due to the appearance of the dip itself. The existence of such a dip was first noted in

Ref. [36] and is due to the relic density constraint. It is caused by the rapid annihilation of neutralinos near the Z pole which reduces the relic density below the lower limit in Eq. (22) and hence part of the parameter space gets eliminated due to this constraint. In order to get the correct position and depth for this dip one must use the accurate method [39, 42] for calculation of the relic density. The analysis of Figs. (2c) and (2d) show that it would be difficult to discern SUSY effects in the  $(g - 2)_\mu$  experiment for gluino masses around the dip or correspondingly for neutralino masses of  $m_{\tilde{Z}} \sim M_Z/2$ . ( There is a similar dip at  $m_{\tilde{Z}} \sim m_h/2$  which does not appear in the graphs because the remaining parameters have been allowed to range over the full space , but would be manifest once the Higgs mass is fixed.)

Interestingly, even the current limits on  $(g - 2)_\mu$  including the present experimental and theoretical errors place some constraint on the parameter space of supergravity grand unification. Ascribing any new physics to supersymmetry by using  $a_\mu^{SUSY} + a_\mu^{SM} = a_\mu^{exp}$  one may constrain the parameter space of the model in the  $(m_0 - m_{\tilde{g}})$  plane for different  $\tan\beta$  with a consideration of all possible  $A_t$ . We have combined the uncertainty of theoretical estimate and the current experimental uncertainties in quadrature to find that

$$-11.7 \times 10^{-9} < a_\mu^{SUSY} < 22.5 \times 10^{-9} \quad (23)$$

Fig. (2e) exhibits for the case  $\mu < 0$  the excluded regions in the  $(m_0 - m_{\tilde{g}})$  plane for different  $\tan\beta$  values where the excluded domains lie below the curves. We note that the excluded domains depend strongly on the value of  $\tan\beta$  and constraints become more severe as  $\tan\beta$  increases. For  $\tan\beta \lesssim 10$  the constraints on  $(m_0, m_{\tilde{g}})$  are very modest. A

similar analysis holds for  $\mu > 0$  as shown in Fig. (2f), except that here the constraints on  $(m_0, m_{\tilde{g}})$  are generally less severe.

We analyze now the effect of the implications of the predicted experimental accuracy of  $a_\mu$  ( $\sim 4 \times 10^{-10}$ ) to be attained in the Brookhaven experiment for supergravity grand unification. Of course, the present theoretical uncertainty in the hadronic contributions to  $a_\mu$ , mostly arising from the hadronic vacuum polarization effects as discussed earlier, limits the usefulness of such a precise measurement of  $a_\mu$ . The hadronic uncertainty arises from the uncertainty in the computation of [4]

$$R(s) = \frac{\sigma_{tot}(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (24)$$

for the low energy tail of  $(e^+e^- \rightarrow hadrons)$  cross-section. Ongoing measurements in VEPP-2M together with future experiments in DAΦNE, BEPC etc. are expected to reduce this hadronic uncertainty to a significant extent, perhaps by a factor of 4 or more, enhancing correspondingly the usefulness of the precision measurement of  $a_\mu$ .

We present here two analyses. For the first analysis we used the more optimistic estimate of Ref. [18] where the authors made an improved evaluation of  $R(s)$  and  $a_\mu^{had}$  through the use of global analytical models of pion and kaon form factors in addition to the use of a better experimental input of the three-pion  $e^+e^-$  annihilation data in comparison to previous determinations [3]. This corresponds to Case A of Table 1 and gives (setting  $R_b = 0$ ) the result

$$a_\mu^{SM} = 11659183(7.6) \times 10^{-10} \quad (25)$$

For the second analysis we shall make a comparative study over an assumed range of

errors which includes analyses of both cases A and B of Table 1 as well as the possibility of even more constrained errors.

In order to analyze the effect of the predicted accuracy level of the Brookhaven experiment on the models of our discussion we have assumed that the experiment will not discard the Standard Model result within its  $2\sigma$  uncertainty limit. As in the analysis of the current experiment above we ascribe any new physics to supersymmetry by using  $a_\mu^{SUSY} + a_\mu^{SM} = a_\mu^{exp}$  and constrain the parameter space of our model in the  $(m_0 - m_{\tilde{g}})$  plane for different  $\tan\beta$  with a consideration of all possible  $A_t$ . Following the same procedure as before, we combine the uncertainty of theoretical estimate and the expected experimental accuracy level of  $4 \times 10^{-10}$  in quadrature. In our first analysis we use Eq. (25) as the theoretical input and assume that the predicted accuracy in the experimental determination of  $a_\mu$  will be achieved, we then determine the constraints on the SUSY particle spectrum if  $a_\mu^{SUSY}$  lies within the  $2\sigma$  of the combined theoretical and experimental error. In Figs. (3a) to (3f) we exhibit the regions of  $(m_{\tilde{W}_1} - m_{\tilde{\nu}_\mu})$  plane which will be excluded (dark shaded region) if  $a_\mu^{SUSY}$  lies within the  $2\sigma$  limit. We also exhibit the regions which will be partially excluded (light shaded area) because a significant part of the parameter space is eliminated by the constraint, and the allowed region (dotted area). In addition, there is a region (white space) which is inaccessible due to the existence of a lower limit of sneutrino mass ( $m_{\tilde{\nu}_\mu}$ ) [36].

Next we give a comparative analysis of the constraints for cases A and B of Table 1 and also for a case C where the error is reduced by factor of 4 over case B ( see in this context the analysis of Ref. [37] which gives a new evaluation of hadronic contributions



of  $699(4.5) \times 10^{-10}$ ). In Figs. (4a) and (4b), we give a composite display of the excluded regions in the  $(m_0 - m_{\tilde{g}})$  plane for the three cases. We observe that the forbidden region increases in proportion to the decrease in the combined error of theory and experiment. Further, the excluded region increases with increasing value of  $\tan\beta$ . Thus if the combined error decreases by a factor of four (case C) and no effect beyond  $2\sigma$  is seen, then the  $g - 2$  experiment will exclude most of the region in  $(m_0 - m_{\tilde{g}})$  plane for large  $\tan\beta$  as can be seen from Figs. (4e) and (4f). In fact even with the presently large uncertainty in the theoretical values of  $a_{\mu}^{SM}$  one will be able to exclude a significant part of the  $(m_0 - m_{\tilde{g}})$  parameter space if the expected sensitivity of the measurement of  $a_{\mu}$  is reached for large  $\tan\beta$  values (see Figs. (4e) and (4f)). Of course a significant reduction of the uncertainty in  $a_{\mu}^{SM}$  which one expects to be possible in the near future will more stringently constrain the parameter space when combined with the expected sensitivity of the Brookhaven experiment. We have also carried out a similar analysis for the  $a_{\tau}^{SUSY}$  for the tau lepton. This gives  $\max |a_{\tau}^{SUSY}| \sim 1.0 \times 10^{-5}$  for  $\tan\beta = 20$ . Even for this large value of  $\tan\beta$  the predicted value of  $a_{\tau}^{SUSY}$  is too small to observe with present experimental accuracy [43].

## 4 Conclusion

In this paper we have presented an analysis of  $(g - 2)_{\mu}^{SUSY}$  within the framework of supergravity grand unification under the constraint of radiative breaking of the electro-weak symmetry, and the constraint of  $b \rightarrow s\gamma$  and dark matter. One finds that over most of the parameter space the chiral interference light chargino part of  $a_{\mu}^{SUSY}$  dominates and imparts a signature to  $a_{\mu}^{SUSY}$ . Thus as a rule one finds  $a_{\mu}^{SUSY} > 0$  for  $\mu > 0$  and

$a_\mu^{SUSY} < 0$  for  $\mu < 0$  with very few exceptions when  $\tan\beta \sim 1$ . At the quantitative level it is shown that the expected experimental sensitivity of  $(g-2)_\mu$  measurements combined with the expected reduction of error in  $(g-2)_\mu^{SM}$  (by  $\approx O(1/4)$ ) will exclude the  $m_0, m_{\tilde{g}}$  parameters in the domain  $m_0 \lesssim 700$  GeV,  $m_{\tilde{g}} \lesssim 1$  TeV for large  $\tan\beta (\gtrsim 20)$  and stringently constrain the parameter space for lower  $\tan\beta$ . With the same assumptions one will be able to probe the domain  $m_0 \lesssim 400$  GeV,  $m_{\tilde{g}} \lesssim 700$  GeV for  $\tan\beta \gtrsim 10$ . The constraint becomes less severe for smaller values of  $\tan\beta$ . However, even for  $\tan\beta = 5$ , the SUSY reach of the new experiment will be very substantial (see Figs. (4a) and (4b)). Thus one finds that the Brookhaven experiment coupled with the corroborating experiments and analyses designed to reduce the hadronic error will complement SUSY searches at colliders and provide an important probe of the parameter space of supergravity grand unification especially for large  $\tan\beta$ .

We wish to acknowledge useful discussions with R. Arnowitt, W. Marciano and W. Worstell. This research was supported in part by NSF grant number PHY-19306906 and PHY94-07194.

## 5 Appendix

The chargino masses  $m_{\tilde{W}_i} = |\lambda_i|$ ,  $i=1,2$  where  $\lambda_{1,2}$  are the eigenvalues of the chargino mass matrix and are given by

$$\lambda_{1,2} = \frac{1}{2} \left( \left[ 4\nu_2^2 + (\mu - \tilde{m}_2)^2 \right]^{\frac{1}{2}} \mp \left[ 4\nu_1^2 + (\mu + \tilde{m}_2)^2 \right]^{\frac{1}{2}} \right) \quad (26)$$

where

$$\nu_{1,2} = \frac{M_W}{\sqrt{2}}(\sin \beta \mp \cos \beta) \quad (27)$$

and  $\tilde{m}_2$  is obtained from the relation  $\tilde{m}_a = \frac{\alpha_a(M_Z)}{\alpha_3(M_Z)}m_{\tilde{g}}$  where  $\alpha_a(M_Z)$ ,  $a = 1, 2, 3$  are the SU(3), SU(2) and U(1) gauge coupling constants at the Z boson mass.

The neutralino masses  $m_{\tilde{Z}(k)} = |\lambda_k|$  where  $\lambda_k$  are the eigenvalues of the neutralino mass matrix which in the  $(\tilde{W}_3, \tilde{B}, \tilde{H}_1, \tilde{H}_2)$  basis reads [12]

$$M_{\tilde{Z}} = \left( \begin{array}{cc|cc} \tilde{m}_2 & o & a & b \\ o & \tilde{m}_1 & c & d \\ \hline a & c & o & -\mu \\ b & d & -\mu & o \end{array} \right) \quad (28)$$

where  $a = M_Z \cos \theta_W \cos \beta$ ,  $b = -M_Z \cos \theta_W \sin \beta$ ,  $c = -M_Z \sin \theta_W \cos \beta$  and  $d = M_Z \sin \theta_W \sin \beta$  while the quantities  $\theta_k$  that appear in Eq. (16) are defined by  $\theta_k = 0$  (1) for  $\lambda_k > 0$  ( $< 0$ ). The quantities  $O_{ij}$  are the elements of the orthogonal matrix which diagonalizes the neutralino mass matrix.

Smuon masses are given by

$$m_{\tilde{\mu}_{1,2}}^2 = \frac{1}{2} \left( (\tilde{m}_L^2 + \tilde{m}_R^2) \mp \sqrt{(\tilde{m}_L^2 - \tilde{m}_R^2)^2 + 4m_\mu^2(A_t + \mu \cot \beta)^2} \right) \quad (29)$$

Here  $A_t$  is scaled with  $m_0$ , and the L and R parts are given by

$$\begin{aligned} \tilde{m}_L^2 &= m_0^2 + m_\mu^2 + \tilde{\alpha}_G \left[ \frac{3}{2}f_2 + \frac{3}{10}f_1 \right] m_{\frac{1}{2}}^2 + \left( -\frac{1}{2} + \sin^2 \theta_W \right) M_Z^2 \cos 2\beta \\ \tilde{m}_R^2 &= m_0^2 + m_\mu^2 + \tilde{\alpha}_G \left( \frac{6}{5}f_1 \right) m_{\frac{1}{2}}^2 - \sin^2 \theta_W M_Z^2 \cos 2\beta \end{aligned} \quad (30)$$

where  $m_{\frac{1}{2}} = \frac{\alpha_G}{\alpha_3(M_Z)}m_{\tilde{g}}$ ,  $\tilde{\alpha}_G = \alpha_G/4\pi$ ,  $f_k(t) = t(2+\beta_k t)/(1+\beta_k t)^2$  with  $\beta_k = \tilde{\alpha}_G(33/5, 1, -3)$  and  $t = 2\ln(\frac{M_G}{Q})$  at  $Q = M_Z$ .

The mixing angle which describes the left-right mixing for the smuons is determined by the relations

$$\sin 2\delta = \frac{2m_\mu (A_t + \mu \cot \beta)}{\sqrt{(\tilde{m}_L^2 - \tilde{m}_R^2)^2 + 4m_\mu^2 (A_t + \mu \cot \beta)^2}} \quad (31)$$

and

$$\tan 2\delta = \frac{2m_\mu (A_t + \mu \cot \beta)}{(\tilde{m}_L^2 - \tilde{m}_R^2)} \quad (32)$$

A similar result analysis holds for sneutrino masses and one has

$$m_{\tilde{\nu}_\mu}^2 = m_0^2 + \tilde{\alpha}_G \left[ \frac{3}{2}f_2 + \frac{3}{10}f_1 \right] m_{\frac{1}{2}}^2 + \frac{1}{2}M_Z^2 \cos 2\beta \quad (33)$$

## References

- [1] J.Bailey et al., Phys. Lett. **68B**, 191(1977); Nucl. Phys. **B150**, 1 1979; Particle Data Group, Phys. Rev. **D50**, 1171 (1994).
- [2] T. Kinoshita, Phys. Rev. **D47**, 5013 (1993);  
T. Kinoshita, B. Nizic, and Y. Okamoto, Phys. Rev. **D41**, 593 (1990);  
M. Samuel and G. Li, Phys. Rev. **D44**, 3935 (1991); Erratum **D48**, 1879 (1993);  
S. Laporta and E. Remiddi, Phys. Lett. **B301**, 440 (1992);  
S. Laporta, Phys. Lett. **B312**, 495 (1993);  
S. Karshenboim, Yad. Fiz. **56**, 252 (1993).
- [3] T. Kinoshita and W. J. Marciano, in *Quantum Electrodynamics*, edited by T. Kinoshita (World Scientific, Singapore, 1990), pp. 419–478;  
T.Kinoshita, B.Nizic, and Y.Okamoto, Phys. Rev. **D31**, 2108 (1985); T.Kinoshita, Z.Phys. **C56**, S80(1992).

- [4] V. Hughes: In: Proceedings of the Second International Symposium on Particles, Strings and Cosmology, 1991, p. 868 eds. P. Nath, S. Reucroft, and in *Frontiers of High Energy Spin Physics*, edited by T. Hasegawa et al. (Universal Academy Press, Tokyo, 1992), pp. 717–722.
- [5] T. Kinoshita, Talk at the 10th International Symposium on *High Energy Spin Physics*, Nagoya, Nov. 9-14, 1992.
- [6] R. Barbieri and E. Remiddi, in *The DAΦNE Physics Handbook*, ed. L. Maiani et. al., INFN, Frascati (1992) 301.
- [7] *Status report of BEPC and BES*, Z.Zheng and N. Qi (CCAST World lab, Beijing, and Beijing Inst. High Energy Phys.), 1989; In: Tsukuba 1989, Proceedings, e+ e- collision physics, 55-73.
- [8] T. C. Yuan, R. Arnowitt, A. H. Chamseddine and P. Nath, *Z. Phys C* **26** (1984) 407.
- [9] D.A. Kosower, L.M. Krauss, N. Sakai, *Phys. Lett.***133B** (1983) 305.
- [10] P. Fayet: In: *Unification of the fundamental particle interactions*, p. 587 eds. S. Ferrara, J. Ellis, P. van Nieuwenhuizen. New York: Plenum, 1980;  
 J.A. Grifols, A. Mendez, *Phys. Rev.***D26** (1982) 1809;  
 J. Ellis, J.S. Hagelin, D.V. Nanopoulos, *Phys. Lett.***116B** (1982) 283;  
 R. Barbieri, L. Maiani, *Phys. Lett.***117B** (1982) 203.
- [11] J.L. Lopez, D.V. Nanopoulos, X. Wang, *Phys. Rev.***D49** (1994) 366.

- [12] A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett.**29** (1982) 970; P. Nath, R. Arnowitt and A.H. Chamseddine, Applied N=1 Supergravity, ICTP Lecture Series Vol I, 1983 (World Scientific, Singapore 1984);
- R. Arnowitt, and P. Nath, Lectures at VII J.A. Swieca Summer School, Campos de Jordao, Brazil, 1993 (World Scientific, Singapore, 1994); H. Nilles, Phys. Rep.**110** (1984) 1.
- [13] K. Inoue et al., Prog. Theor. Phys. **68** (1982) 927;
- L. Ibañez and G.G. Ross, Phys. Lett.**B110** (1982) 227;
- J. Ellis, J. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett.**125B** (1983) 275;
- L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys.**B250** (1983) 495;
- L.E. Ibañez, C. Lopez and C. Munos, Nucl. Phys.**B256** (1985) 218.
- [14] M. Davier, Proc. Lepton-Photon High Energy Phys. Conference, Geneva, 1991 eds. S. Hegarty et. al. (World Scientific, Singapore, 1991); H. Bethke, Proc. XXVI Conf. on High Energy Physics, Dallas, 1992, ed. J. Sanford, AIP Proc. No. 272(1993).
- [15] J. Erler and P. Langacker, UPR-0632-T; hep-ph/9411203.
- [16] P. Langacker, Proceedings, PASCOS 90 Symposium, Editors, P. Nath and S. Reucroft (World Scientific, Singapore, 1990); J. Ellis, S. Kelly, and D.V. Nanopoulos, Phys. Lett. **249B**, 441 (1990), *ibid*, **260B**, 131(1991); U. Amaldi, W. de Boer, and H. Furstenau, *ibid* **260B**, 447 (1991); F. Anselmo, et al, Nuovo Cim.**104A**,1817(1991).
- [17] R. Ammar *et. al.*(CLEO Collaboration), Phys. Rev. Lett. **71**,674 (1993).
- [18] L. Martinovic and S. Dubnicka Phys. Rev.**D42** (1990) 884.

- [19] S. Eidelman and F. Jegerlehner, PSI-PR-95-1, BUDKERINP-95-5, hep-ph 9502298.
- [20] A similar analysis has also been given by W. A. Worstell and D. H. Brown, who find  $O(\alpha^2)$  hadronic correction to be  $705(15.26) \times 10^{-10}$  ( Boston University Internal Report and private communication with W. Worstell ).
- [21] J. Bijmens, E. Pallante, and J. Prades, preprint NORDITA-95/36 N,P, hep-ph /9505251. A different evaluation of light-by-light hadronic contribution is given by M. Hayakawa, T. Kinoshita, and A. I. Sanda, preprint DPNU-94-21, hep-ph /9503463. They find a contribution of  $-3.6(1.6) \times 10^{-10}$  which is larger and of the opposite sign to that given by Bijmens et al, but still only about one fourth of the size of the Standard Model electro-weak contribution and within the expected error of the Brookhaven experiment.
- [22] A. Czarnecki, B. Krause and W. Marciano, TTP95-19, hep-ph /9506256.
- [23] K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. **D6**, 2923 (1972); R. Jackiw and S. Weinberg, Phys. Rev. **D5**, 2473 (1972); G. Altarelli, N. Cabbibo, and L. Maiani, Phys. Lett. **B40**, 415 (1972); I. Bars and M. Yoshimura, Phys. Rev. **D6**, 374 (1972); W. A. Bardeen, R. Gastmans, and B. E. Lautrup, Nucl. Phys. **B46**, 315 (1972).
- [24] S. Peris, M. Perrottet, and E. de Rafael, preprint CPT-95/P3202, hep-ph/9505405.
- [25] T. V. Kukhto, E. A. Kuraev, A. Schiller, and Z. K. Silagadze, Nucl. Phys. **B371**, 567 (1992).
- [26] M.B. Einhorn and D.R.T. Jones, Nucl. Phys.**B196** (1982) 475.

- [27] R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **69** (1992) 725.
- [28] G. G. Ross and R. G. Roberts, *Nucl. Phys.* **B377**, 971(1992); P. Nath and R. Arnowitt *Phys. Lett.* **B289** (1992) 368. ;M. Drees and M. M. Nojiri, *Nucl. Phys.* **B369**, 54(1992);J. Lopez et al, *Phys.Lett.* **B299**, 262(1993); M.Olechowski and S.Pokorski, *Nucl.Phys.***B404**, 590(1993); G. L. Kane et al, *Phys.Rev.* **D49**, 6173(1994); P. Langacker and N. Polonsky, *Phys. Rev.* **D49**, 1454(1994); D. J. Castano et al, *Phys. Rev.* **D49**, 4882(1994).
- [29] J.Bagger, K.Matchev, D.Pierce, JHU-TIPAC-95001 (1995); P.H.Chankowski, Z.Pluciennick, S.Pokorski, and C.E.Vayonakis, MPI-Pth/95-58.
- [30] H.Baer ,C-H. Chen, R.Munroe, F.Paige and X.Tata, *Phys. Rev.* **D51**, 1046 (1995); V. Barger, M. S. Berger and P. Ohman and R. J. N. Philips, MAD/PH/842(1994); H.Baer, M.Brhlik, R.Munroe and X.Tata, FSU-HEP-950501; UH-511-829-95; W.DeBoer, G.Berkhart, R.Ehret and W.Oberschulte, hep-ph/9507291 and the references quoted therein.
- [31] J.M. Frere, D. Jones, and S. Raby, *Nucl. Phys.* **B222**, 1 (1983); M. Claudson, J. Hall, and I. Hinchliffe, *Nucl. Phys.* **B288**, 501 (1983); M. Drees, M. Gluck, and K. Brassie, *Phys. Lett.* **157B**, 164 (1985).
- [32] P.Nath, J.Wu and R.Arnwitt, NUB-TH-3116/95, CTP-TAMU-07/95 hep-ph /9502388 (to appear in *Phys. Rev. D*).
- [33] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, *Nucl. Phys.* **B353**, 541 (1983).



- [34] J. Wu, R. Arnowitt, and P. Nath, Phys. Rev. **D51**,1371(1995) and the references quoted therein.
- [35] A.J.Buras, M.Misiak, M.Munz and S.Pokorski, Nucl. Phys. **B 424**, 374(1994); M.Ciuchini, E.Franco, G.Martinelli, L.Reina and L.Silvestrini, Phys. Lett. **B 316**, 127 (1993).
- [36] P. Nath, and R. Arnowitt, Phys. Lett. **B336**, 395 (1994); Phys. Rev. Lett. **B289**, 368(1992); R.Arnwitt and P.Nath, CTP-TAMU-14/95-NUB-TH-3121/95.
- [37] A. Z. Dubnickova, S. Dubnicka, P. Strizenec, Dubna Preprint E-2-92-281.
- [38] H. Anlauf, Nucl.Phys.**B430**, 245(1994).
- [39] R. Arnowitt, and P. Nath, Phys. Lett. **299B**, 58 (1993) and Erratum *ibid* **303B**, 403 (1993); P. Nath, and R. Arnowitt, Phys. Rev. Lett. **70** 3696 (1993) ; J. Lopez, D. Nanopoulos, and K. Yuan, Phys. Rev. **D48**,2766(1993).
- [40] F. Borzumati, M. Drees and M. Nojiri, Phys.Rev.**D51**, 341(1995).
- [41] E.Diehl,G.Kane, C.Kolda and J.Wells, UM-TH-94-38(1994).
- [42] K.Griest and D.Seckel, Phys. Rev. **D43**, 3191 (1991); P.Gondolo and Gelmini, Nucl. Phys. **B360**, 145 (1991).
- [43] R. Escribano and E. Masso, Phys. Lett. **B301** (1993) 419; M.A. Samuel and G. Li, Phys. Rev. Lett.**67** (1991) 668.

## Figure Captions

**Fig. 1:**

Fig. (1a) Chargino-sneutrino one-loop exchange diagram which contributes to  $a_\mu^{SUSY}$ .

Fig. (1b) Neutralino-smuons one-loop exchange diagram which contributes to  $a_\mu^{SUSY}$ .

**Fig. 2:**

Fig. (2a) The upper limit of  $|a_\mu^{SUSY}|$  vs.  $m_{\tilde{g}}$  for the case  $\mu < 0$  in the Minimal Supergravity model for different values of  $\tan\beta$  in the region  $\tan\beta \leq 30$ , when one allows the remaining parameters  $A_t$  and  $m_0$  to vary over the parameter space subject to the naturalness criterion of  $m_0 \leq 1$  TeV.

The dashed horizontal line is the current  $2\sigma$  limit as given by Eq. (23)

Fig. (2b): Same as Fig. (2a) except that  $\mu > 0$ .

Fig. (2c): Same as Fig. (2a) except that the constraint from  $b \rightarrow s\gamma$  and dark matter as discussed in the text are included.

Fig. (2d): Same as Fig. (2c) except that  $\mu > 0$ .

Fig. (2e): Excluded regions (regions enclosed by the curves towards the origin) in the  $(m_0 - m_{\tilde{g}})$  plane under the current theoretical and experimental limits of  $a_\mu$  for various values of  $\tan\beta$ , for the case  $\mu < 0$ .

Fig. (2f): Same as Fig. (2e) except that  $\mu > 0$ .

**Fig. 3:**

Fig. (3a) Display in the  $(m_{\tilde{W}_1} - m_{\tilde{\nu}_\mu})$  plane of the allowed (shown in dots), disallowed

(shown in squares) and partially allowed (shown in cross) regions corresponding to the  $2\sigma$  limit of Case A of the theoretical evaluation of  $a_\mu$  and the predicted level of accuracy of the Brookhaven experiment, for the case  $\tan\beta = 5$  and  $\mu < 0$ . The white area between the excluded region and  $m_{\tilde{W}_1}$  axis remains inaccessible due to the existence of a lower limit of sneutrino mass  $m_{\tilde{\nu}_\mu}$  [36].

Fig. (3b): Same as Fig. (3a) except that  $\mu > 0$ .

Fig. (3c): Same as Fig. (3a) except that  $\tan\beta = 10$ .

Fig. (3d): Same as Fig. (3c) except that  $\mu > 0$ .

Fig. (3e): Same as Fig. (3c) except that  $\tan\beta = 30$ .

Fig. (3f): Same as Fig. (3e) except that  $\mu > 0$ .

**Fig. 4:**

Fig. (4a): Excluded regions (regions enclosed by the curves towards the origin) for  $\tan\beta = 5$  and  $\mu < 0$  in the  $2\sigma$  limit, after combining in quadrature the predicted Brookhaven experimental uncertainty and different levels of uncertainty of theoretical estimates corresponding to Cases A, B, and C as discussed in the text.

Fig. (4b): Same as Fig. (4a) except that  $\mu > 0$ .

Fig. (4c): Same as Fig. (4a) except that  $\tan\beta=10$ .

Fig. (4d): Same as Fig. (4c) except that  $\mu > 0$ .

Fig. (4e): Same as Fig. (4a) except that  $\tan\beta=30$ .

Fig. (4f): Same as Fig. (4e) except that  $\mu > 0$ .