IFUP – TH-42-95
hep-ph/9507379

July 1995

Lepton Flavour Violations in SO(10) with large $\tan\beta$

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Abstract

In supersymmetric SO(10) with large $\tan\beta$ and Yukawa coupling unification $\lambda_t = \lambda_b = \lambda_\tau$, we compute the rates for lepton flavour violating processes. Experiments in progress or foreseen for $\mu \to e\gamma$, $\mu \to e$ conversion (and the dipole moment of the electron) are shown to provide very significant tests of the theory for all slepton masses up to about 1 TeV.

1 Introduction

The possibility to understand the rationale of the quantum numbers of quarks and leptons strongly supports the view of a local symmetry larger than the standard model one, relevant at, or close to, the Planck scale. In this context, a phenomenologically very relevant question arises: is this larger symmetry already present in the field theory which emerges below the Planck scale? Indeed, the measured values of the standard model gauge couplings seem to favour the possibility of an intermediate stage of gauge-unification, around $M_{\rm G} \approx 2\cdot 10^{16}$ GeV. The exploration of all possible signals of such unification acquires therefore a great significance. Among these signals, one which is especially relevant in the case of supersymmetric unification, is the violation of lepton flavour [1].

Soon after the first formulation of phenomenologically viable supersymmetric models, it was realized that, in absence of lepton flavour conservation, a generic slepton mass matrix would give rise to uncontrollably large LFV [2]. Without a theoretical guideline, it is however virtually impossible to make a reliable prediction for the corresponding rates. On one side the mixing angles in the leptonic sector are unknown; on the other side, and even more importantly, there is in general no control of the amount of non-degeneracy in the slepton masses, which is essential to undo a GIM-like cancellation in the relevant amplitudes. For lepton-slepton mixing angles of the order of the CKM ones, the $\tilde{e}$ and $\tilde{\mu}$ sleptons must be extremely degenerate [2, 3] in order not to exceed the experimental upper bounds on LFV processes, while a $\mathcal{O}(1)$ splitting between the $\tilde{\tau}$ and the other sleptons may be acceptable. To avoid a dangerous non degenerate sfermion spectrum, a flavour universality hypothesis was made on the soft SUSY breaking terms. Moreover, this hypothesis has been made plausible in some models, for example when SUSY breaking is communicated to observable fields by gravity [4]. In this case the universality holds at the Planck scale.

Remarkably enough, a controllable source for flavour non-universality is present in minimal supersymmetric unified models, as pointed out by Barbieri and Hall [1]. When the SUSY breaking parameters evolve to low energy, their universality is lost if non flavour-universal couplings are present. A significant splitting between the $\tilde{\tau}$ mass and the $\tilde{e}$ and $\tilde{\mu}$ ones is induced by the top Yukawa coupling (and, possibly, the other third generation couplings). As a consequence, the rates for LFV processes can be predicted in the usual parameter space of the MSSM. As found in [5], these rates are of key interest as tests of supersymmetric unification.

The hierarchy between the third generation fermion masses can be interpreted as a hierarchy between their Yukawa couplings. In this case λ_τ cannot influence significantly the sleptons masses and the resulting LFV rates were computed in [5]. It is the purpose of the present work to extend this analysis to theories where the $t/b, \tau$

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splitting is instead attributed to a hierarchy between the vacuum expectation values of the two Higgs doublets, or, in the usual language, to a large $\tan\beta \equiv v_{\rm u}/v_{\rm d} \sim m_t/m_b$. This possibility is particularly interesting since it allows for a SO(10) GUT where not only the gauge couplings but also the third generation Yukawa couplings unify at $M_{\rm G}$ [6, 7]. In this paper we compute, in the large $\tan\beta$ region, the minimal amount of flavour violations that in a wide class of supersymmetric unified models we expect to be transferred from the hadronic to the leptonic sector by the unified third generation Yukawa coupling.

In section 2 we present and motivate the model in which we calculate the lepton flavour violations (LFV). We explain in which sense this model gives the minimal amount of lepton flavour violating effects we expect to be present in a general class of unified models. In section 3 we show how the well known problems of the large $\tan\beta$ region are resolved by an unification scale D-term contribution to the soft SUSY breaking masses, which also fixes the main features of the spectrum. In section 4 we show our predictions for the lepton flavour violating processes, and in section 5 we discuss how they are expected to be modified in more general models. In appendices A and B we give and solve the renormalization group equations (RGEs) above and below the unification scale.

2 The model

We restrict our attention to unified theories with SO(10) gauge group, because they furnish the main (unique?) motivation for considering a large $\tan\beta \sim m_t/m_b$ value as an interesting one, notwithstanding the difficulties in obtaining it in a radiative electroweak symmetry breaking scenario.

To exploit the full predictability of SO(10) gauge unified theories (GUT), we only consider softly broken supersymmetric field theory models, assumed to be valid up to the Planck scale, in which

1. the two MSSM Higgs doublets, $h^{\rm u}$ which gives mass to up-quarks and $h^{\rm d}$ which gives mass to down-quarks and to leptons, lie in a single 10 (‘Φ’) vector representation of SO(10);

2. the third generation particles lie in a single 16 (‘Ψ_3’) spinorial representation of SO(10), together with a right handed neutrino;

3. the only relevant couplings for lepton flavour violations are the unified gauge coupling and the unified third generation Yukawa coupling;

4. the light generation Yukawa couplings are supposed to arise from non renormalizable operators, allowing in this way for non trivial physical flavour-mixing angles
$$f = \lambda \Psi_3 \Psi_3 \Phi + \text{n.r.o};$$

5. the soft SUSY-breaking scalar masses are supposed to be *universal at the Planck scale*[4]. In view of the different origin of the various Yukawa couplings, it would not be satisfactory to make a similar hypothesis for the trilinear A-terms — and we will not do it;

6. the unification gauge group SO(10) directly breaks to the standard model gauge group at the unification scale. The D-term contribution to the soft SUSY breaking scalar masses corresponding to the reduction of the rank of the gauge group should be non zero[5] in order to minimize the fine-tuning necessary for a correct electroweak symmetry breaking. The unified gauge β-function coefficient is not an important parameter [5].

Before being able to compute the LFV effects in this model we should solve two new kind of problems — one of technical and one of fundamental nature — both peculiar of the large $\tan\beta$ region. To be able to calculate the rates for the LFV processes, one has to control both the $m_{\tilde\tau}/m_{\tilde\mu}, m_{\tilde e}$ splitting and the lepton-slepton mixing angles. In previous calculations of LFV effects for moderate values of $\tan\beta$, two different and independent sources produced these two effects: the mass splitting was induced by the (diagonalizable) up quark Yukawa couplings matrix above the unification scale, while the mixing angles resulted from the ones in the lepton Yukawa matrix, linked by unification physics to the Cabibbo-Kobayashi-Maskawa angles.

[4]This assumption is done in order to reduce the number of free parameters and is perfectly consistent with our intention of compute the minimal amount of LFV. In fact, apart from accidental cancellations, possible flavour violations already present in the soft masses at the Planck scale would simply add to the renormalization induced ones, while non universal but flavour-symmetric soft breaking masses would not substantially alter the LFV effects.

[5]This can be achieved without introducing new sources of LFV.

The technical problem is that now, below the unification scale, also the $\tan\beta$ enhanced τ Yukawa coupling contributes to the intergenerational slepton mass splitting — this is discussed in appendix B. This coupling also gives some reduction of the lepton-slepton mixing angles, both in the left and in the right sector. Appendix B.3 is devoted to this computation.

The fundamental problem is that now, above the unification scale, all the Yukawa couplings necessary to give mass to the light fermions, may also induce lepton-slepton mixing angles. We cannot say anything about them, without understanding the dynamical mechanism that generates the light generation Yukawa couplings. However, if we insist on computing only the 'minimal' effects, we may (and we shall) assume that the main consequence of renormalization effects above the unification scale consists in making the third generation sfermions in the Ψ_3 lighter than the other ones, *without* generating non-diagonal entries in the slepton mass matrices.

In this case the lepton-slepton mixing angles only come from the lepton Yukawa coupling matrix, and we have to connect them with the CKM angles. These angles, which measure the misalignment between the up and the down quark angles in the left sector, are the only experimentally accessible angles below the Fermi scale. We will again assume, as in the previous analyses, that the down-quark and lepton Yukawa matrices are equal at the unification scale. However the mixing angles in the down-quark Yukawa matrix may now only be a partial contribution the the measured CKM ones, with the other contribution coming from a non diagonal up-quark Yukawa matrix. The unknown up-quark mixing angles, if comparable in size to the CKM ones, could generate an electric dipole for the u quark giving in this way a contribution of the same order of the d quark one to the neutron electric dipole moment. Due to the larger mass hierarchy among the up-quarks, it looks however more likely that the mixing angles in the up sector be smaller than the ones in the down sector. We stick to this simplifying hypothesis in the following, postponing to section 5 a discussion of how we may expect the mixing angles to be distributed between the up, the down and lepton Yukawa coupling matrices.

We can now summarize the additional hypotheses that complete the description of the model. At the unification scale, in the supersymmetric basis (in which there are no flavour violations at the gaugino vertices)

7. the sfermions mass matrices are flavour-diagonal, that is

$$\boldsymbol{m}^2_\Psi = \mathrm{diag}(m^2_{\Psi_1}, m^2_{\Psi_1}, m^2_{\Psi_3}). \tag{2.1}$$

   The expressions for these masses in term of the universal soft breaking parameters at the Planck scale are given in Appendix A;

8. the down and lepton Yukawa coupling matrices $\boldsymbol{\lambda}^{\rm d}$ and $\boldsymbol{\lambda}^{\rm e}$, *equal* and *symmetric*, are the only non flavour-diagonal matrices present in the theory.

With these assumptions we are now ready to start the actual calculations. We will however first recall the constraints on $\lambda_{\rm G}$, the common value of the third generation Yukawa couplings λ_t, λ_b and λ_τ at the unification scale. This parameter plays a crucial role in the determination of the LFV rates. We may fix the $\tan\beta$ value as a function of $\lambda_{\rm G}$ by requiring the τ mass m_τ to have its measured value

$$\tan\beta \simeq \frac{v\lambda_\tau(M_Z)}{m_\tau/\eta_\tau} = 97\lambda_\tau(M_Z)$$

where $v = 174\,\mathrm{GeV}$ and $\eta_\tau = 0.986$ is the QED renormalization for the τ mass between m_τ and the Z-pole. It is interesting to compute the reduction in the $\alpha_3(M_Z)$ value in the $\overline{\rm MS}$ scheme, as predicted from gauge coupling unification, due to the Yukawa terms in the two loop RGEs between the Fermi and the unification scale. This is shown in fig 1, where the $\alpha_3(M_Z)$ reference value in the case of zero Yukawa couplings is the one obtained with all the unknown threshold and gravitational corrections [8] set to zero and with the input values of the electroweak couplings used in [9]. We see that, for $\mathcal{O}(1)$ values of the top Yukawa coupling at the unification scale, in the large $\tan\beta$ region this effect gives a reduction in the predicted $\alpha_3(M_Z)$ of order of its present 1σ experimental error; this reduction becomes three times smaller for moderate values of $\tan\beta$.

The third generation unified Yukawa coupling $\lambda_{\rm G}$ is a very important parameter for lepton flavour violations too. There exist two almost equivalent upper bounds on it. The maximum value that $\lambda_{\rm G}$ can assume without developing a Landau pole below the Planck scale at $2.4\cdot 10^{18}\,\mathrm{GeV}$ is around $\lambda_{\rm G} \lesssim 1.4$. The second upper bound arise from the correct electroweak symmetry breaking requirements, that we will discuss in the next section. There are also two lower bounds on $\lambda_{\rm G}$. To obtain a top quark mass in the CDF range [10], $\lambda_{\rm G} \gtrsim 0.5$ is needed [6, 7]. In this case all the third generation Yukawa couplings at the Z-pole are greater than about $3/4$ of their maximum possible values ("*InfraRed fixed point*"), which are, for $\alpha_3(M_Z) = 0.121$,

$$\lambda_t^{\max}(M_Z) = 1.06, \qquad \lambda_b^{\max}(M_Z) = 0.99, \qquad \lambda_\tau^{\max}(M_Z) = 0.64. \tag{2.2}$$

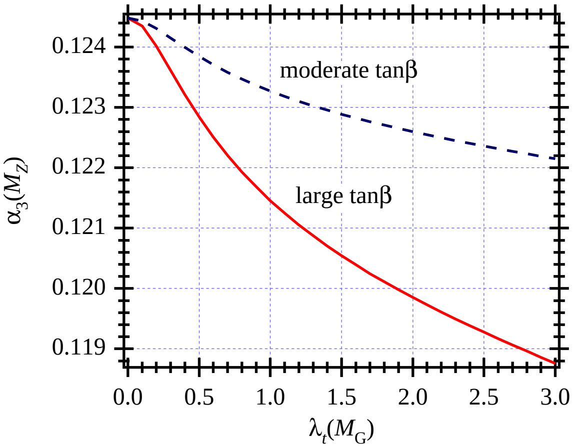

Figure 1: Two loop prediction for $\alpha_3(M_Z)$ as function of the top quark Yukawa coupling at the unification scale, $\lambda_{t\mathrm{G}}$, in the cases of moderate $\tan\beta$, $\lambda_{t\mathrm{G}} \gg \lambda_{b\mathrm{G}}, \lambda_{\tau\mathrm{G}}$ (dashed line), and large $\tan\beta$, $\lambda_{t\mathrm{G}} = \lambda_{b\mathrm{G}} = \lambda_{\tau\mathrm{G}}$ (solid line), with all threshold and gravitational effects set to zero.

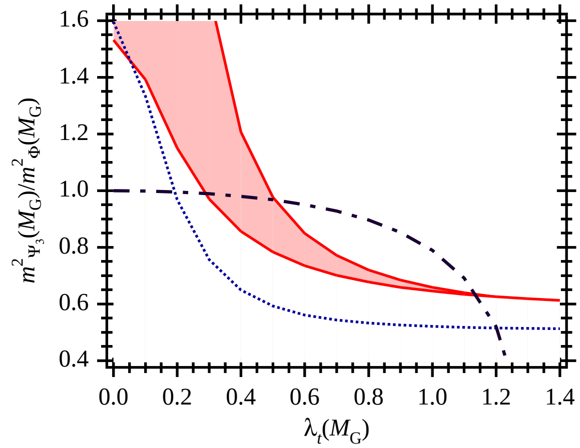

Figure 2: The allowed interval for $m^2_{\Psi_3\mathrm{G}}/m^2_{\Phi\mathrm{G}}$ (gray area) and the prediction for its value from Planck scale universality (dot-dashed line) as a function of λ_{G} in the low fine-tuning region of the soft SUSY breaking parameters. Also shown is the subdominant limit from the $\tilde{L}_3$ mass (dotted line).

This gives a $\tan\beta$ value in the range $45 \div 60$.

To predict the correct value of the b/τ mass ratio even higher values of λ_{G} seem indeed to be necessary. There may be a conflict between this requirement and the upper bounds previously mentioned. One should not forget, however, the various uncertainties that can affect the b/τ mass prediction:

- sizeable dependence on the value of $\alpha_3(M_Z)$ in the range $0.110 \div 0.125$;
- $\tan\beta$ enhanced one loop quantum correction to the bottom mass [7, 11];
- second-third generation mixing contribution to the b and/or τ masses [12];
- mixing between third generation particles and other heavy ones, induced by SU(5)-breaking vacuum expectation values [13];
- a possible right-handed tau neutrino Yukawa coupling effect in the RGEs, if its mass is lower than the unification mass as cosmological and phenomenological considerations may indicate [12, 14].

For these reasons, in the following, we will avoid to impose a correct b/τ mass ratio, and neglect all the problems related to it.

3 Structure of the allowed parameter space

In this section we will discuss for which values of the SUSY breaking parameters it is possible to obtain a phenomenologically acceptable theory. The reader may want to skip this section and jump directly to the following one, where we present the predictions on the $\mu \to e\gamma$ branching ratio, that constitute the principal aim of this paper.

It is well known that the price we have to pay in order to have Yukawa unification, is the need to fine tune the parameters which determine $\tan\beta$ and M_Z through the minimization conditions of the MSSM potential

$$\frac{2\mu B}{\mu_{\mathrm{u}}^2 + \mu_{\mathrm{d}}^2} = \sin 2\beta \qquad \text{and} \qquad \frac{\mu_{\mathrm{u}}^2 \tan^2\beta - 1\mu_{\mathrm{d}}^2}{\tan^2\beta - 1} = -\frac{M_Z^2}{2}. \tag{3.1}$$

In the large $\tan\beta$ region these equations become

$$\frac{\mu B}{\mu_{\mathrm{u}}^2 + \mu_{\mathrm{d}}^2} \approx \frac{1}{\tan\beta} \qquad \text{and} \qquad \mu_{\mathrm{u}}^2 \approx -\frac{M_Z^2}{2}. \tag{3.2}$$

The first minimization condition requires μ and/or B to be much smaller than the pseudoscalar Higgs mass $m_A^2 = \mu_{\rm u}^2 + \mu_{\rm d}^2$. Since B gets sizeable renormalization corrections from the trilinear A-terms and from the gaugino masses (see appendix B), and m_A^2 is generated from the SUSY breaking scalar masses, the best way to satisfy the large $\tan\beta$ condition in (3.2) with the minimal amount of fine tuning is to restrict our analysis to the case where the scalar masses are the dominant soft SUSY-breaking parameters [7, 14].

The problem is that, since the RGEs for $\mu_{\rm u}^2$ and $\mu_{\rm d}^2$ are similar and the two Higgs doublets come from the same SO(10) multiplet, it is difficult to reconcile the two requirements that

- $\mu_{\rm u}^2$ must be negative to break the electro-weak gauge symmetry as in (3.2), while
- $\mu_{\rm d}^2$ must be positive enough so that the squared pseudoscalar Higgs mass is positive.

This conciliation is impossible if we assume that the only relevant soft SUSY breaking term is an universal scalar mass m_0^2, either at the unification scale [7], or at the Planck scale. In these cases the renormalization effects may indeed induce a small difference between the two Higgs squared masses, but only in the bad direction $\mu_{\rm u}^2 \geq \mu_{\rm d}^2$. To reconcile the two requirements it is possible to move to the 'hard fine tuning' region where the GUT scale gaugino mass is larger than the scalar masses. Other possible solutions consist in relying on positive *ad hoc* large $\mathcal{O}(20\% - 30\%)$ GUT threshold corrections to the ratios of SO(10)-linked quantities, like λ_t^2/λ_b^2, or $m^2_{\tilde{u}_R}/m^2_{\tilde{d}_R}$, or directly to $\mu_{\rm d}^2/\mu_{\rm u}^2$ [15]. All these corrections would however introduce new uncontrolled uncertainties.

The best solution is given by the well known fact that, at the scale where the rank of the gauge group is reduced by spontaneous breaking, possible additional contributions to the soft SUSY breaking scalar masses arise from the D-terms associated with the broken diagonal (Cartan) generators. These contributions are generated whenever the soft SUSY-breaking masses of the fields whose vacuum expectation values reduce the rank of the group are different.

In the $\mathrm{SO}(10) \to \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ case, even with all the SUSY breaking scalar masses degenerate at the Planck scale, an amount of non-degeneracy may be produced by different interactions of the 16_H and $\overline{16}_H$ fields whose vacuum expectation values reduce the rank of the group. In this way a non vanishing SU(5)-invariant correction to the soft SUSY breaking scalar masses arises from the D-term associated with the broken $\mathrm{U}(1)_X$ generator. Decomposing the SO(10) fields in SU(5) multiplets

$$\Phi = \bar{5} \oplus 5, \qquad \Psi_i = 10_i \oplus \bar{5}_i \oplus 1_i$$

these corrections modify the matching conditions at the unification scale in the following, very precise, way:

$$\begin{array}{ll} m_{\bar{5}}^2 = m_\Phi^2 + 2m_X^2 (= m^2_{h^{\rm d}}) & m_5^2 = m_\Phi^2 - 2m_X^2 (= m^2_{h^{\rm u}}) \\ m_{\bar{5}_i}^2 = m_{\Psi_i}^2 - 3m_X^2 (= m^2_{\tilde{L}_i} = m^2_{\tilde{d}_{Ri}}) & m_{10_i}^2 = m_{\Psi_i}^2 + m_X^2 (= m^2_{\tilde{Q}_i} = m^2_{\tilde{e}_{Ri}} = m^2_{\tilde{u}_{Ri}}) \end{array} \tag{3.3}$$

where m_X^2 is the only additional parameter. These corrections enter the RGE equations from the unification scale to the Fermi one in a very peculiar way (see appendix B):

- these new m_X^2 contributions and the old usual SUSY breaking parameters evolve in an independent way;
- in a very good approximation the $\mathrm{U}(1)_X$ D-term corrections to the masses are not renormalized.

We now explain how all the main features of the low energy spectrum emerge in a very clear way from the analytical approximation presented in appendix B.2. The full numerical calculation is only necessary to get the details.

The low energy Higgs mass parameters may be written as

$$\mu_{\rm u}^2 \equiv m^2_{h^{\rm u}} + \mu^2 \quad \approx \quad m^2_{\Phi \rm G} + \mu^2 + x_2^h M_{5\rm G}^2 - \tfrac{3}{7} 3 I_t - 2m_X^2 \tag{3.4a}$$

$$\mu_{\rm d}^2 \equiv m^2_{h^{\rm d}} + \mu^2 \quad \approx \quad m^2_{\Phi \rm G} + \mu^2 + x_2^h M_{5\rm G}^2 - \tfrac{3}{7} 3 I_b + 2m_X^2 \tag{3.4b}$$

where $x_2^h \approx 0.53$ and $I_t \approx I_b$ is the Yukawa coupling induced renormalization group correction, approximately equal for the two Higgs doublets (see appendix B.2 for an analytical expression). We may neglect this small difference, because we rely on a positive $m_X^2 \approx (m_A^2 + M_Z^2)/4$ to obtain the desired splitting between the two Higgs mass parameters.

Is there any limitation to this way of obtaining the desired symmetry breaking pattern? We see from eq. (3.3) that a positive m_X^2 term also has the dangerous effect of decreasing the squared masses of the sparticles in the three $\bar{5}_i$ of SU(5). Above some value of m_X^2, one of them will become negative. The dominant upper bound on m_X^2 will be given by the third generation masses, because they are further reduced by Yukawa effects in the renormalization. The $m^2_{\tilde{b}_R} > 0$ constraint will dominate at low values of the gaugino masses, because the squarks get larger Yukawa couplings than the sleptons. Above some value of M_2 (around $m_{\tilde{e}_R}/5$) the dominant constraint will become $m^2_{\tilde{L}_3} > 0$. This is because the squarks get positive corrections to their masses from the gluinos, corrections that are bigger than those that the other gauginos give to the slepton masses.

Let us first understand these bounds in the limit where the soft scalar masses, parametrized by $m^2_{\Phi\mathrm{G}}$ and $m^2_{\Psi_i\mathrm{G}}$, much larger than M_Z^2, are the only non zero SUSY breaking parameters. The dominant constraints $m_A^2 > 0$ and $m^2_{\tilde{b}_R} > 0$ will restrict the ratio $m^2_{\Psi_3\mathrm{G}}/m^2_{\Phi\mathrm{G}}$ in a λ_G-dependent way. From our analytical approximation we get

$$\frac{21-5\rho_t}{2(7+5\rho_t)} \lesssim \frac{m^2_{\Psi_3\mathrm{G}}}{m^2_{\Phi\mathrm{G}}} \lesssim \frac{7}{6\rho_t} - \frac{1}{2}, \qquad \text{where} \quad \rho_t \approx \frac{\lambda_t^2(M_Z)}{(1.05)^2}.$$

This agrees very well with the full numerical calculation, shown in fig. 2, from which we can see that, in this limit, the allowed area closes at $\lambda_\mathrm{G} \simeq 1.2$. In fig. 2 we have also shown the predicted value for the mass ratio assuming universality at M_Pl

$$\frac{m^2_{\Psi_3\mathrm{G}}}{m^2_{\Phi\mathrm{G}}} = \frac{1-\frac{15}{14}\rho_\mathrm{G}}{1-\frac{12}{14}\rho_\mathrm{G}}, \qquad \text{where} \quad \rho_\mathrm{G} = \frac{\lambda^2_\mathrm{G}}{\lambda^2_\mathrm{max}(M_\mathrm{Pl})}.$$

It is now easy to understand what happens when we turn on small non zero values for the other SUSY-breaking parameters. This is the case we are most interested in. We fix the universal scalar mass at some high value, for example imposing $m^2_{\tilde{e}_R} = 1\,\mathrm{TeV}$ as in the LFV plots of the next section, and we examine the spectrum as a function of the remaining parameters. Its main characteristic are the following.

> For a fixed $\mathcal{O}(1)$ value of λ_G and any value of the (almost irrelevant) A-parameters the allowed region in the $\{M_2, \mu\}$ plane is the strip
>
> $$\mu^2 \approx (5.6\rho_t - 2.2\rho_t^2 - 0.5)M^2_{5\mathrm{G}} + \Delta^2 \tag{3.5}$$
>
> which becomes narrower for higher values of λ_G. Along its 'lower' boundary, given by the $m_A^2 > 0$ condition, the pseudoscalar and charged Higgs are light. Along its 'upper' boundary, given by the $m^2_{\tilde{L}_3} > 0$ ($m^2_{\tilde{b}_R} > 0$) constraint for values of M_2 greater (smaller) than about $m_{\tilde{e}_R}/5$, a light $\tilde{\tau}_L$ ($\tilde{b}_R$) is present. In the intermediate allowed region the masses of these particles do not exceed at the same time $1/3$ of $m_{\tilde{e}_R}$.

This behaviour may be seen from the LFV plots in fig. 3, in which the full numerical solutions have been employed. Its features can also be understood from the analytic approximation. The dashed area corresponds to the excluded region. The lower $m_A^2 > 0$ limit of (3.5) is easily obtained from eq. (3.4) using the analytic approximation for I_t, eq. (B.4). The fact that the upper $m^2_{\tilde{L}_3} > 0$ border is almost parallel to it is not an accident and, again, is easily understood from the analytic approximation. The reason is that the dependence on the gaugino mass and on the μ term of the doublet slepton mass is dominated by the $-3m_X^2$ term, which contain a $\mathcal{O}(M_3^2)$ term from I_t. The other renormalization effects are negligible in comparison to it.

The origin of the Δ term in eq. (3.5) is also easily understood from fig. 2. For values of λ_G that give the correct value for the $m^2_{\Psi_3\mathrm{G}}/m^2_{\Phi\mathrm{G}}$ ratio, the strip starts from zero values of M_2 and μ. In the most interesting region, $\lambda_\mathrm{G} \in 0.6 \div 1.1$, the predicted ratio is somewhat high, and the strip starts from a non-zero value of $|\mu|$ (typically $|\mu| > \Delta \sim m_{\tilde{e}_R}/5$); in the remaining case $\lambda_\mathrm{G} \gtrsim 1.1$ a non-zero value of M_2 will be instead necessary.

4 Leptonic flavour violations

Let us briefly recall from [5] the main structure of the lepton flavour violations.

The most significant observables are the $\mu \to e\gamma$ decay, the $\mu \to e$ conversion in atoms and the electric dipole moment d_e (included in the list together with the genuine lepton flavour violations because generated by the same mechanism [16]).

In SO(10)-like theories, where LFV are mediated by both the left and the right handed sleptons, a unique loop integral F, with dimensions mass^{-2}, gives the dominant contribution to all these processes

$$\mathrm{B.R.}(\mu\to e\gamma) = 5.0\cdot 10^{-12} \times \frac{|V^{\mathrm{e}_R}_{e\tilde{\tau}} V^{\mathrm{e}_L}_{\mu\tilde{\tau}}|^2 + |V^{\mathrm{e}_R}_{\mu\tilde{\tau}} V^{\mathrm{e}_L}_{e\tilde{\tau}}|^2}{2\cdot(0.01\cdot 0.04)^2} \frac{|F|^2}{(1\,\mathrm{TeV})^{-4}} \tag{4.1a}$$

$$\mathrm{C.R.}(\mu\to e \text{ in Ti}) = 2.5\cdot 10^{-14} \times \frac{|V^{\mathrm{e}_R}_{e\tilde{\tau}} V^{\mathrm{e}_L}_{\mu\tilde{\tau}}|^2 + |V^{\mathrm{e}_R}_{\mu\tilde{\tau}} V^{\mathrm{e}_L}_{e\tilde{\tau}}|^2}{2\cdot(0.01\cdot 0.04)^2} \frac{|F|^2}{(1\,\mathrm{TeV})^{-4}} \tag{4.1b}$$

$$d_e = 2.9\cdot 10^{-27}\, e\cdot\mathrm{cm} \times \frac{|V^{\mathrm{e}_R}_{e\tilde{\tau}} V^{\mathrm{e}_L}_{e\tilde{\tau}}|}{(0.01)^2} \frac{|F|}{(1\,\mathrm{TeV})^{-2}} \sin\varphi \tag{4.1c}$$

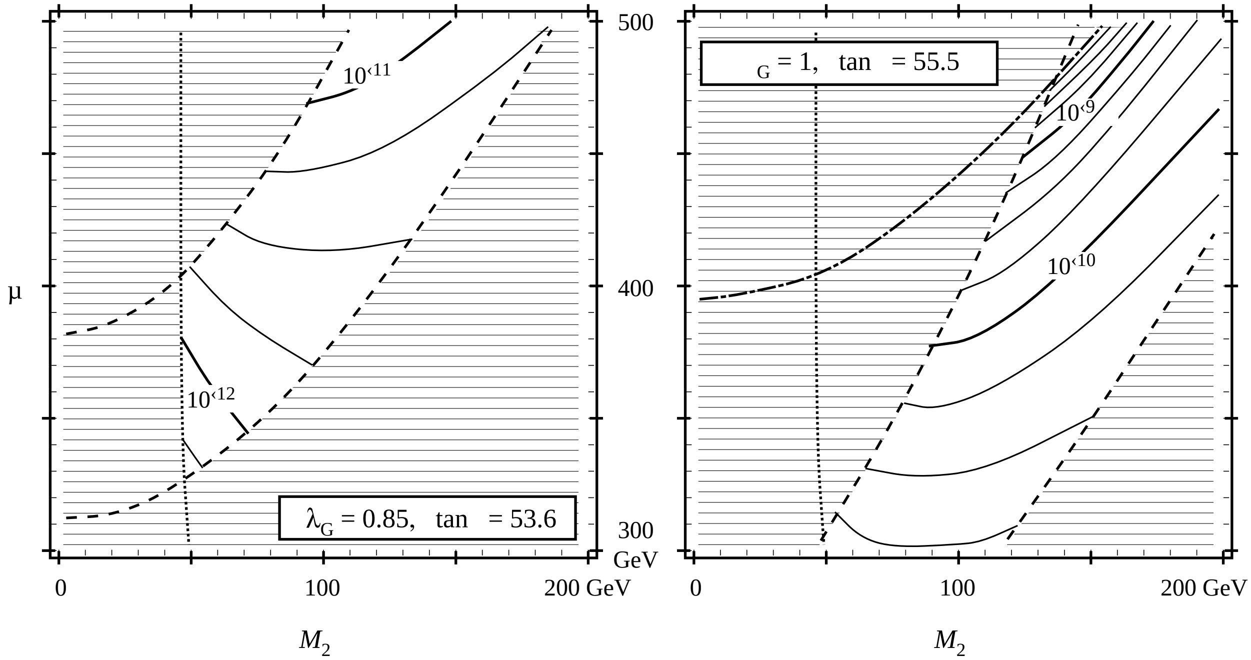

Figure 3: Contour-plots for B.R.$(\mu \to e\gamma)$ in the plane $\{M_2, \mu\}$ for $m_{\tilde{e}_R} = 1\,\mathrm{TeV}$ and $\lambda_{\rm G} = \{0.85, 0.1\}$.

where $\sin\varphi$ is a CP violating phase and V^{e_R} (V^{e_L}) are the left (right) handed lepton-slepton mixing angles which appear at the gaugino vertices in the mass eigenstate basis for leptons and sleptons. With our assumptions the lepton-slepton mixing angles are linked to the CKM ones in the following way

$$|V^{\mathrm{e}_{L,R}}_{e_i\tilde{\tau}}(M_Z)| = y_t y_b y_\tau |V_{td_i}(M_Z)| \times \frac{m^2_{\tilde{e}_{L,R}}(M_{\rm G}) - m^2_{\tilde{\tau}_{L,R}}(M_{\rm G})}{m^2_{\tilde{e}_{L,R}}(M_Z) - m^2_{\tilde{\tau}_{L,R}}(M_Z)}, \qquad i = 1, 2. \tag{4.2}$$

The dominant constraint is today given by the $\mu \to e\gamma$ decay, for which, at present, B.R.$(\mu \to e\gamma) < 4.9 \cdot 10^{-11}$ [17]. The experimental study of $\mu \to e$ conversion, currently limited by C.R.$(\mu \to e \text{ in Ti}) < 10^{-12}$ [18], may undergo a very significant progress in the near future [19]. For large values of the CP violating phase φ, the bound on the electric dipole moment of the electron, $|d_e| < 4.3 \cdot 10^{-27}\, e \cdot \mathrm{cm}$ [20] gives the same restriction in parameter space as $\mu \to e\gamma$.

The dominant amplitude F, given by Feynman graphs containg both left-handed and right-handed sleptons[6], is proportional to the left-right slepton mixing term in the Lagrangian, which gets contributions proportional either to the A-terms or to $\mu\tan\beta$. In view of $\tan\beta = 45 \div 60$, we assume that $\mu\tan\beta \gg A$ and we consequently neglect the A-terms contributions[7]. We are also neglecting LFV mediating diagrams which employ only right-handed (or left-handed) sleptons because of a (m_μ/m_τ) suppression factor [5]. One may wonder whether this is plausible since, due to the $\mathrm{U}(1)_X$ D-term, the $\tilde{\tau}_L$ may be significantly lighter than the $\tilde{\tau}_R$. But these graphs are indeed negligible because the graphs with left-right slepton mixing have an additional enhancement factor $\tan\beta$. It is therefore possible to approximate the amplitude with[8]

$$F = \mu\tan\beta[G_2(m^2_{\tilde{\tau}_L}, m^2_{\tilde{\tau}_R}) - G_2(m^2_{\tilde{e}_L}, m^2_{\tilde{\tau}_R}) - G_2(m^2_{\tilde{\tau}_L}, m^2_{\tilde{e}_R}) + G_2(m^2_{\tilde{e}_L}, m^2_{\tilde{e}_R})], \tag{4.3}$$

where

$$\begin{aligned} G_2(m^2) &= \sum_{n=1}^{4} \frac{H_{n\tilde{B}}}{M_{N_n}} (H_{n\tilde{B}} + \cot\theta_{\rm W} H_{n\tilde{W}_3}) \cdot g_2(\frac{m^2}{M^2_{N_n}}), \\ G_2(m_1^2, m_2^2) &= \frac{G_2(m_1^2) - G_2(m_2^2)}{m_1^2 - m_2^2}, \qquad g_2(r) = \frac{1}{2(r-1)^3}[r^2 - 1 - 2r\ln r]. \end{aligned}$$

[6] With 'left-handed slepton' we mean the supersymmetric partner of corresponding left-handed lepton.

[7] The variations of the A terms in their allowed range has actually little influence on the determination of allowed region of the $\{M_2, \mu\}$ plane and on the $\mu \to e\gamma$ amplitude.

[8] The contribution from the $m_\tau \mu \tan\beta$ LR-mixing term to the $\tilde{\tau}$ masses is negligible if the sleptons are heavy enough so that the experimental upper bounds on the LFV effects are not exceeded.

In this equation N_n, $n = 1, \ldots, 4$ are the four neutralino mass eigenstates, of mass M_{N_n}, related to the bino and the neutral wino by

$$\begin{aligned} \tilde{B} &= \textstyle\sum_{n=1}^{4} N_n H_{n\tilde{B}}, \\ \tilde{W}_3 &= \textstyle\sum_{n=1}^{4} N_n H_{n\tilde{W}_3}. \end{aligned} \tag{4.4}$$

So, the only low energy parameters on which F, and consequently the LFV processes, depend are

- the left (right) handed lepton-slepton mixing angles;
- the μ parameter and the neutralino masses. Using the GUT relation $M_1 = M_2 \cdot \alpha_1(M_Z)/\alpha_2(M_Z)$, all the neutralino masses may be computed in terms of M_2 and μ, that we take as free parameters; the sign of μ turns out to be an irrelevant parameter in almost all of the parameter space.
- the slepton masses: for a given value of the right-handed selectron mass the other slepton masses may be computed as function of $m_{\tilde{e}_R}$, M_2, μ, and of the top Yukawa coupling via the Planck-scale universality hypothesis. We forget the weak dependence on the τ A-term.

We present now the contour-plots of the B.R.$(\mu \to e\gamma)$ in the plane $\{M_2, \mu\}$ for $|V_{ts}(M_Z)| = 0.04$, $|V_{td}(M_Z)| = 0.01$ and at fixed values of the right-handed selectron mass $m_{\tilde{e}_R} = 1\,\mathrm{TeV}$, and of the top quark Yukawa coupling at the unification scale, λ_{G}. We have explained in the previous section why we choose to restrict the plane to low M_2 and μ values, and why the allowed range is a strip. The A-terms and the sign of μ are additional but irrelevant parameters.

In fig 3 we show our predictions for B.R.$(\mu \to e\gamma)$ in the allowed area of the $\{M_2, \mu\}$ plane, for $m_{\tilde{e}_R} = 1\,\mathrm{TeV}$ and $\lambda_{\mathrm{G}} = \{0.85, 0.1\}$. The allowed area is limited from below by the $m_A^2 > 0$ condition (dashed line) and from above by $m^2_{\tilde{b}_R} > 0$ (dashed line) and $m^2_{\tilde{L}_3} > 0$ (dot-dashed line). The bound at small M_2 (dotted line) is obtained by requiring the charginos to be heavier than $45\,\mathrm{GeV}$. The plane is also restricted to values of M_2 and μ lower than $m_{\tilde{e}_R}$ for which the necessary fine-tuning is less severe. As can be seen from the plots the $m^2_{\tilde{L}_3} > 0$ condition is more relevant for higher values of the unified third generation coupling: for this reason the dependence on λ_{G} is stronger than the naive expectation $F \propto \lambda_{\mathrm{G}}^2$.

The rates for the other interesting LFV processes may also be easily deduced from the graphs, since they are connected to $\mu \to e\gamma$ in the following way:

$$\frac{\mathrm{B.R.}(\mu \to e\gamma)}{5 \cdot 10^{-11}} \approx 4 \frac{\mathrm{C.R.}(\mu \to e \text{ in Ti})}{10^{-12}} \approx \left(\frac{|d_e|/\sin\varphi}{10^{-26}\, e \cdot \mathrm{cm}} \right)^2 \tag{4.5}$$

For other values of the right-handed selectron mass than the one considered, the factor F scales as $(1\,\mathrm{TeV}/m_{\tilde{e}_R})^2$.

The main result is that, in the large $\tan\beta$ region, the present experimental limits on LFV already constrain in a significant way the parameter space. More importantly, the experiments in progress and/or foreseen will be able to probe the theory up to $\tilde{e}$ and $\tilde{\mu}$ masses of about $1\,\mathrm{TeV}$ in all of the significant region of the remaining parameter space.

5 More general and realistic models

In this section we discuss the problem of how we may expect the mixing angles in the lepton Yukawa coupling matrix to be linked to the measured CKM angles by unification physics.

We assume, as explained before, that at the unification scale all the significant flavour violating terms are contained in the three Yukawa coupling matrices $\boldsymbol{\lambda}^{\mathrm{a}}$, where $\mathrm{a} = \{\mathrm{u}, \mathrm{d}, \mathrm{e}\}$, defined by

$$f_{\mathrm{MSSM}} = h^{\mathrm{u}} u_R \boldsymbol{\lambda}^{\mathrm{u}} Q + h^{\mathrm{d}} d_R \boldsymbol{\lambda}^{\mathrm{d}} Q + h^{\mathrm{d}} L \boldsymbol{\lambda}^{\mathrm{e}} e_R + \mu h^{\mathrm{u}} h^{\mathrm{d}}.$$

In order to find the physical flavour-violating angles, we parametrize the Yukawa coupling matrices in the usual way

$$\boldsymbol{\lambda}^{\mathrm{a}} = \boldsymbol{U}^{\mathrm{a}\dagger} \cdot \mathrm{diag}(\lambda_{\mathrm{a}_1}, \lambda_{\mathrm{a}_2}, \lambda_{\mathrm{a}_3}) \cdot \boldsymbol{V}^{\mathrm{a}}$$

and rotate the fermion fields to their mass eigenbasis. Other than the quark-squark mixing angles and the CKM angles, given by $\boldsymbol{V} = \boldsymbol{V}^{\mathrm{u}} \cdot \boldsymbol{V}^{\mathrm{d}\dagger}$, we are left with the lepton-slepton mixing matrices at the gaugino vertices

$$\mathcal{L} \supset (\bar{\lambda} \bar{e}_R \boldsymbol{V}^{\mathrm{e}_R} \tilde{e}_R + \tilde{e}_R^* \boldsymbol{V}^{\mathrm{e}_R\dagger} e_R \lambda) + (\lambda L \boldsymbol{V}^{\mathrm{e}_L} \tilde{L}^* + \tilde{L} \boldsymbol{V}^{\mathrm{e}_L\dagger} \bar{L} \bar{\lambda})$$

with, in the left sector $\boldsymbol{V}^{\mathrm{e}_L} = \boldsymbol{U}^{\mathrm{e}}$ and, in the right sector $\boldsymbol{V}^{\mathrm{e}_R} = \boldsymbol{V}^{\mathrm{e}}$. The symbol λ indicates the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauginos.

In models where the up-quark Yukawa couplings are not unified with those of the down-quarks and of the leptons (because either the fermions or the two Higgs doublets are not contained in a single representation of the unification gauge group) the only problem is to find the link between $\boldsymbol{\lambda}^{\mathrm{e}}$ and $\boldsymbol{\lambda}^{\mathrm{d}}$. In fact, due to the sufficiently large flavour symmetry, the up-quark Yukawa matrix may be diagonalized at the Planck scale, so that the CKM mixing angles are entirely contained in $\boldsymbol{\lambda}^{\mathrm{d}}$. In this case the only problem is to connect the lepton mixing angles with the down-quark ones.

In our SO(10) model, where also the up-quark Yukawa matrix is unified with the down-quark and lepton ones, the up-quark mixing angles can not be freely rotated away. So, the full unification of the Yukawa couplings poses the further problem of separating the contributions to CKM mixing angles from the up and the down quark mixing matrices. However the full unification also allows for more predictions in the Yukawa sector. This issue has already been studied. In particular, some work has been done in trying to discover phenomenologically acceptable textures of the Yukawa coupling matrices which can be justified by the physics at the unification scale. The aim of these works [21, 22] consists in studying whether unified theories may be predictive in the flavour sector. For this reason, textures with a minimal number of non zero entries were constructed, which can accommodate the observed spectrum of lepton and quark masses and CKM mixing angles in terms of a reduced set of free parameters. Since we are assuming that the sfermion mass matrices at the unification scale are diagonal but not flavour degenerate, these textures give us also a prediction of how the CKM angles are connected with the angles in the leptonic sector which give rise to the LFV processes.

Of some interest are textures of the form [22]

$$\boldsymbol{\lambda}^{\mathrm{u}} = \lambda_{\mathrm{G}} \begin{pmatrix} 0 & C_u & 0 \\ C_u & 0E & c^L_{\mathrm{u}}B \\ 0 & c^R_{\mathrm{u}}B & 1 \end{pmatrix}, \qquad \boldsymbol{\lambda}^{\mathrm{d}} = \lambda_{\mathrm{G}} \begin{pmatrix} 0 & 1C & 0 \\ 1C & 1E & c^L_{\mathrm{d}}B \\ 0 & c^R_{\mathrm{d}}B & 1 \end{pmatrix}, \qquad \boldsymbol{\lambda}^{\mathrm{e}} = \lambda_{\mathrm{G}} \begin{pmatrix} 0 & 1C & 0 \\ 1C & 3E & c^R_{\mathrm{e}}B \\ 1 & c^L_{\mathrm{e}}B & 0 \end{pmatrix}$$

where λ_{G}, B, C and E are free parameters (E may be chosen as the only complex one; C_u, not relevant for our considerations, may be given by a different operator than C, or it may be linked to it by $C_u = -\frac{1}{27}C$ [22]), all the predicted Clebsh factors has been explicitly written down, and, in order to obtain acceptable predictions for m_c/m_t and V_{cb}, only nine distinct possibilities exist for the Clebsh coefficients c^L_{a} and c^R_{a} of the '23' operator.

The Cabibbo angle is mainly given by $\boldsymbol{\lambda}^{\mathrm{d}}$ rather than by $\boldsymbol{\lambda}^{\mathrm{u}}$, and the the corresponding angle in the leptonic sector is $1/3$ of it. So, defining $\chi_{L,R} \equiv c^{L,R}_{\mathrm{e}}/(c^L_{\mathrm{d}} - c^L_{\mathrm{u}})$, The Clebsh factors which multiply the rates for the LFV processes are

$$(\frac{1}{3}\chi_L\chi_R)^2 \times \left\{ \begin{array}{l} \text{B.R.}(\mu \to e\gamma) \\ \text{C.R.}(\mu \to e) \end{array} \right. , \qquad \text{and} \qquad (\frac{1}{9}\chi_L\chi_R) \times d_e \tag{5.1}$$

In the nine possible cases for the '23' operator the values of the Clebsh correction factors for the $\mu \to e\gamma$ and $\mu \to e$ rates range around one in the interval $10^{-1} \div 10$ with the exception of two models in which they are around $10^{\pm 2}$. The constant suppression factor, 1/9, due to the Georgi-Jarlskog factor [21] is generally compensated by the other factors. For the electric dipole moment of the electron the Clebsh correction factors range from $10^{-1} \div 1$.

At least based on the examples considered, we conclude that neglecting all Clebsh factors (assumption 8. of this paper) gives, with an uncertainty of one order of magnitude, a correct estimate of their effects in the LFV rates, and probably a slight over-estimate of the electric dipole moment of the electron by a factor 3.

6 Conclusions

In supersymmetric theories with no conservation of lepton flavour, processes like $\mu \to e\gamma$, $\mu \to e$ conversion or similar may play a very important role. We have computed the rates for lepton flavour violating processes in a supersymmetric SO(10) model with large $\tan\beta$ and Yukawa coupling unification $\lambda_t = \lambda_b = \lambda_\tau$. These rates are the minimal ones we expect to be present in more general gauge and Yukawa unified models. The main result is that, in the large $\tan\beta$ region, the present experimental limits on LFV already constrain in a significant way the parameter space. More importantly, the LFV experiments in progress and/or foreseen will be able to probe the theory up to $\tilde{e}$ and $\tilde{\mu}$ masses of about 1 TeV in all of the significant region of the remaining parameter space.

Acknowledgements
The authors are extremely grateful to Riccardo Barbieri for many discussions and for a careful reading of the manuscript.

i	b_i	c_i^Q	c_i^u	c_i^d	c_i^L	c_i^e	c_i^{u}	c_i^{d}	c_i^{e}
1	$\frac{33}{5}$	$\frac{1}{30}$	$\frac{8}{15}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{6}{5}$	$\frac{13}{15}$	$\frac{7}{15}$	$\frac{9}{5}$
2	1	$\frac{3}{2}$	0	0	$\frac{3}{2}$	0	3	3	3
3	-3	$\frac{8}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	0	0	$\frac{16}{3}$	$\frac{16}{3}$	0

Table 1: Values of the RGE coefficients in the MSSM.

A Renormalization from M_{Pl} to M_{G}

Neglecting all couplings except the gauge and the third generation unified Yukawa one, the solutions to all the one loop RGEs between $E_{\max} = M_{\mathrm{Pl}}$ and $E_{\min} = M_{\mathrm{G}}$ can be given analytically.

All the equations and their solutions may easily be adapted from the ones of the model defined in eq. (22) and discussed in appendix A of [5], provided that the 10-plet 'Φ_{d}', its couplings and its A-terms are erased and that the coupling named 'λ_t' is identified with the unified third generation coupling λ. The transcription is made easier by the fact that the same notations and the same values of the high energy parameters have been used in this article.

B Renormalization from M_{G} to M_Z

Neglecting all couplings except the gauge g_i ($i = \{1,2,3\}$) and the third generation Yukawa ones $\lambda_{\mathrm{a}_3} = \lambda_a$ ($\mathrm{a} = \{\mathrm{u},\mathrm{d},\mathrm{e}\}$ and $a \equiv \mathrm{a}_3 = \{t,b,\tau\}$), the one loop RGEs between M_{G} and M_Z are

$$\frac{d}{dt}\frac{1}{g_i^2} = b_i, \qquad \frac{d}{dt}\frac{M_i}{g_i^2} = 0 \tag{B.1a}$$

$$\frac{d}{dt}\lambda_{\mathrm{a}_g}^2 = \lambda_{\mathrm{a}_g}^2 (c_i^{\mathrm{a}} g_i^2 - S_{\mathrm{a}_g b}\lambda_b^2) \tag{B.1b}$$

$$\frac{d}{dt}A_{\mathrm{a}_g} = c_i^{\mathrm{a}} g_i^2 M_i - S_{\mathrm{a}_g b}\lambda_b^2 A_b \tag{B.1c}$$

$$\frac{d}{dt}\mu = \tfrac{1}{2}(2c_i^h g_i^2 - S_b\lambda_b^2)\mu \tag{B.1d}$$

$$\frac{d}{dt}B = 2c_i^h g_i^2 M_i - S_b\lambda_b^2 A_b \tag{B.1e}$$

$$\frac{d}{dt}m_{R_g}^2 = 2c_i^R g_i^2 M_i^2 - \delta_{g3} Z_{Rb}\lambda_b^2 (X_b + A_b^2) - \tfrac{3}{5} Y_R g_1^2 X_Y \tag{B.1f}$$

where $g = \{1,2,3\}$ runs over the generation number, the values of the numerical coefficients b_i and c_i are given in table 1 while those of S and Z are obtained in the following way: due to the non renormalization theorem, the running of the Yukawa couplings is only induced by wave function renormalizations. So let Z_{Ra} be proportional to the contribution of the Yukawa coupling λ_a^2 to the wave-function renormalization of the field R — it is different from zero only for the third generation particles and the for the two Higgs doublets. Its values are

$$Z^T = \begin{array}{c} \\ t \\ b \\ \tau \end{array} \begin{array}{c} \begin{array}{ccccccc} h^{\mathrm{d}} & h^{\mathrm{u}} & \tilde{Q}_3 & \tilde{t}_R & \tilde{b}_R & \tilde{\tau}_R & \tilde{L}_3 \end{array} \\ \begin{pmatrix} 0 & 3 & 1 & 2 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 & 1 \end{pmatrix} \end{array}$$

Then $S_b = \sum_{R=h^{\mathrm{u}},h^{\mathrm{u}}} Z_{Rb} = (3,3,1)$ and $S_{\mathrm{a}_g b} = \sum_{R(\mathrm{a}_g)} Z_{Rb}$ where the sum is extended over the fields involved in the Yukawa coupling λ_{a_g}, that is

$$S_{ab} = \begin{pmatrix} 6 & 1 & 0 \\ 1 & 6 & 1 \\ 0 & 3 & 4 \end{pmatrix} \qquad \text{and} \qquad S_{\mathrm{a}_1 b} = S_{\mathrm{a}_2 b} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 3 & 1 \end{pmatrix}.$$

The X's are linear combination of sparticle masses defined by $X_a = \sum_{R(a)} m_R^2$ and $X_Y = \mathrm{Tr}\, Y_R m_R^2$, or, more explicitely, by

$$X_t = m_{h^{\mathrm{u}}}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{t}_R}^2, \qquad X_b = m_{h^{\mathrm{d}}}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{b}_R}^2, \qquad X_\tau = m_{h^{\mathrm{d}}}^2 + m_{\tilde{L}_3}^2 + m_{\tilde{\tau}_R}^2,$$

$$X_Y = (m^2_{h^{\rm u}} - m^2_{h^{\rm d}}) + \sum_g (m^2_{\tilde{Q}_g} - m^2_{\tilde{L}_g} - 2m^2_{\tilde{u}_{Rg}} + m^2_{\tilde{d}_{Rg}} + m^2_{\tilde{e}_{Rg}}).$$

Parametrizing the solutions for these combinations in terms of I_a and I_Y

$$\begin{array}{lcl} \frac{d}{dt} X_a = 2c_i^{\rm a} g_i^2 M_i^2 - S_{ab}\lambda_b^2(X_b + A_b^2) & \Rightarrow & X_a = X_{a{\rm G}} + x_2^a M_{5{\rm G}}^2 - 3S_{ab} I_b \\ \frac{d}{dt} X_Y = b_1 g_1^2 X_Y & \Rightarrow & X_Y = X_{Y{\rm G}}(1 - f_1^{-1}) \equiv X_{Y{\rm G}} - I_Y \end{array}$$

we may then write the following formal solutions for all the equations

$$\begin{aligned} g_i(M_Z) &= g_{5{\rm G}}/f_i^{1/2} \qquad M_i(M_Z) = M_{5{\rm G}} \cdot f_i^{-1}(E) && \text{(B.2a)}\\ \lambda_{{\rm a}_g}(M_Z) &= \lambda_{{\rm a}_g}(M_{\rm G}) E_{\rm a}^{1/2} \cdot \textstyle\prod_b y_b^{S_{{\rm a}_g b}} && \text{(B.2b)}\\ A_{{\rm a}_g}(M_Z) &= A_{{\rm a}_g}(M_{\rm G}) + x_1^{\rm a} M_{5{\rm G}}^2 - S_{{\rm a}_g b} I'_b && \text{(B.2c)}\\ \mu(M_Z) &= \mu(M_{\rm G}) \cdot E_h \cdot \textstyle\prod_b y_b^{S_b} && \text{(B.2d)}\\ B(M_Z) &= B(M_{\rm G}) + 2x_1^h M_{5{\rm G}} - S_b I'_b && \text{(B.2e)}\\ m^2_{R_g}(M_Z) &= m^2_{R_g}(M_{\rm G}) + x_2^R M_{5{\rm G}}^2 - \delta_{g3} Z_{Rb} 3 I_b + \tfrac{3}{5}\tfrac{Y_R}{b_1} I_Y && \text{(B.2f)} \end{aligned}$$

where $f_i(E) \equiv 1 + b_i g_{\rm G}^2 t(E)$,

$$E_\alpha \equiv \prod_{i=1}^3 f_i^{c_i^\alpha/b_i} \qquad \text{and} \qquad x_n^R \equiv \sum_{i=1}^3 \frac{c_i^R}{b_i}[1 - f_i^{-n}(M_Z)].$$

The parameters

$$y_a(\lambda_{\rm G}) \equiv \exp\left[-\frac{1}{2}\int \lambda_a^2\, dt\right],$$

I_a and I'_a contain *all* the the Yukawa couplings induced corrections.

The CKM matrix elements between a light and the heavy generation evolve as $V_{i3}(M_Z) = V_{i3}(M_{\rm G})/y_t y_b$, while the elements between the light generations are not renormalized.

The problem of solving the full set of equations is now reduced to the one of finding solutions for the parameters y_a, I_a and I'_a. This can be done both numerically and analytically.

B.1 Numerical exact solution

In our SO(10) model the boundary conditions at the unification scale for the X's are particularly simple: $X_{X{\rm G}} = 4m_X^2$ gets contributions only from the U(1)$_X$ D-terms, while the three $X_{a{\rm G}}$ are all equal and not affected by them: $X_{\rm G} = 2m^2_{\Psi_3} + m^2_\Phi$. For this reason the form of I_a ed I'_a for the most general SO(10) symmetric boundary conditions may be given in terms of few function of the unified Yukawa coupling

$$\begin{aligned} 3S_{ab} I_b &= a_a(\lambda_{\rm G}) X_{\rm G} + b_a(\lambda_{\rm G}) M_{5{\rm G}}^2 + c_a(\lambda_{\rm G}) A_{3{\rm G}} M_{\rm G} + d_a(\lambda_{\rm G}) A_{3{\rm G}}^2 && \text{(B.3a)}\\ S_{ab} I'_b &= a'_a(\lambda_{\rm G}) A_{3{\rm G}} + b'_a(\lambda_{\rm G}) M_{5{\rm G}} && \text{(B.3b)} \end{aligned}$$

It is easy to see that $a_a = a'_a$. For a given value of $\lambda_{\rm G}$ a very efficient numerical procedure for obtaining the values of these coefficients consists in solving numerically the RGEs four times for four particular choices of $X_{\rm G}$, $M_{\rm G}$ and $A_{\rm G}$. Another possibility consists in obtaining the RGEs for these coefficients and solving them.

In conclusion we have written the most general solution for all the RGE in terms of 18 numerical functions. We show them once for all in fig. 4. The solution for the most general SO(10)-invariant boundary conditions are obtainable by inserting their values in equations (B.2) and (B.3).

B.2 Analytical approximate solution

Analytical approximate solutions are obtained setting $S_{ab} \approx 7\,{\rm diag}(1,1,1)$. Then

$$y_t \approx y_b \approx [1 + 12\lambda_{\rm G}^2]^{-1/14}, \qquad y_\tau \approx [1 + 4.4\lambda_{\rm G}^2]^{-1/14}.$$

Defining $\rho_a \equiv 1 - y_a^{14}$, the approximate values of the Yukawa couplings at the Z pole are $\lambda_t(M_Z) \approx \lambda_b(M_Z) \approx 1.05 \cdot \rho_t^{1/2}$ and $\lambda_\tau(M_Z) \approx 0.70 \cdot \rho_\tau^{1/2}$. Approximate expressions for the dimensionful parameters are

$$\begin{aligned} 3I_t \approx 3I_b &\approx \rho_t[X_{\rm G} + (13 - 5\rho_t) M_{5{\rm G}}^2 + (1 - \rho_t) A_{\rm G}^2 + 4.5(1 - \rho_t) A_{\rm G} M_{5{\rm G}}] && \text{(B.4a)}\\ 3I_\tau &\approx \rho_\tau[X_{\rm G} + (8 - 5\rho_\tau) M_{5{\rm G}}^2 + (1 - \rho_t)\mathcal{O}(A_{\rm G} M_{5{\rm G}}, A_{\rm G}^2)] && \text{(B.4b)}\\ I'_t \approx I'_b &\approx \rho_t[A_{\rm G}^2 + 2.2 M_{5{\rm G}}] && \text{(B.4c)}\\ I'_\tau &\approx \rho_\tau[A_{\rm G}^2 + 1.4 M_{5{\rm G}}] && \text{(B.4d)} \end{aligned}$$

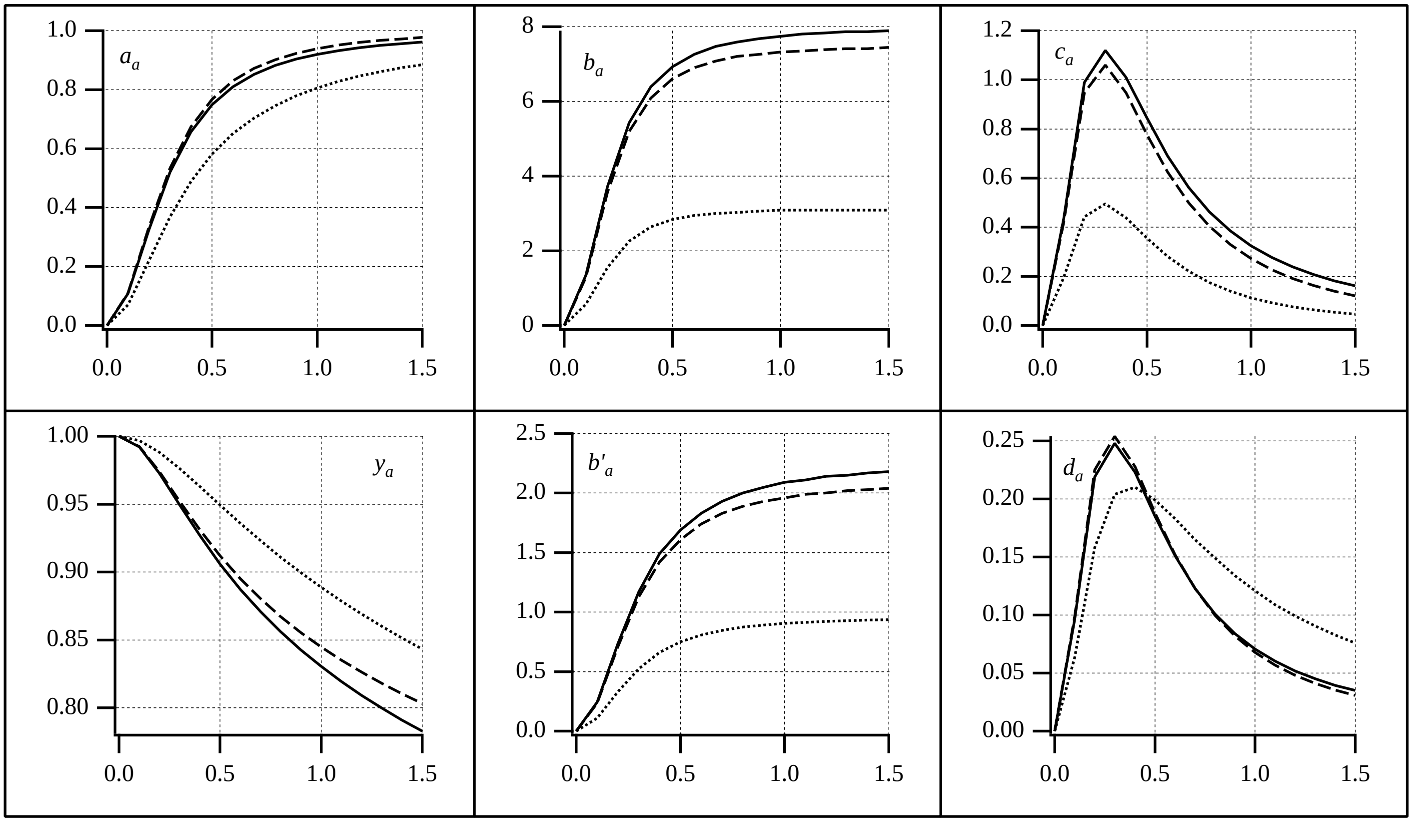

Figure 4: Values of a_a, b_a, c_a, d_a, b'_a and y_a ($a = t, b, \tau$) as function of $\lambda_{\rm G}$. The index a runs over t (solid line), b (dashed line) and τ (dotted line).

B.3 Evolution of the lepton-slepton mixing angles below the unification scale

The lepton-slepton mixing may be described by two matrices of physical mixing angles, $(\boldsymbol{V}^L)_{e_i\tilde{e}_j}$ and $(\boldsymbol{V}^R)_{e_i\tilde{e}_j}$, that, in a supersymmetric basis (where the lepton-slepton-gaugino interactions are flavour blind) of sleptons mass eigenstates measure, respectively, the different orientation in flavour space between the left (right) slepton mass matrix and the left (right) part of the lepton Yukawa coupling matrix $\boldsymbol{\lambda}^{\rm e}$. In the calculations we have employed a non supersymmetric physical basis of mass eigenstates for both the leptons and the sleptons. In this case the LFV mixing matrices appear at the lepton-slepton-gaugino vertex.

To consider their evolution it is better to employ a supersymmetric basis in which the lepton Yukawa coupling matrix is diagonal. Then it will remain diagonal throughout renormalization. This is due to the fact that if there are no LFV at the unification scale, the large λ_τ Yukawa coupling may render the $\tilde{\tau}$'s lighter than the other sleptons, but does not generate LFV. The evolution equations for all the small non-diagonal elements δm^2 of both the left and the right slepton mass matrix between the third generation and a light one are

$$\frac{d}{dt}\delta m^2 = -2\lambda_\tau^2 \delta m^2 \qquad \Rightarrow \qquad \delta m^2(M_Z) = \delta m^2(M_{\rm G}) \cdot y_\tau$$

Then the mixing angles at the gaugino vertices in the lepton and slepton physical mass basis,

$$V^{{\rm e}_L}_{e_i\tilde{\tau}} = \frac{(\delta m^2)^L_{\tilde{e}_i\tilde{\tau}}}{m^2_{\tilde{e}_L} - m^2_{\tilde{\tau}_L}} \qquad \text{and} \qquad V^{{\rm e}_R}_{e_i\tilde{\tau}} = \frac{(\delta m^2)^R_{\tilde{e}_i\tilde{\tau}}}{m^2_{\tilde{e}_R} - m^2_{\tilde{\tau}_R}},$$

suffer also a reduction due to the increase in mass difference between the $\tilde{\tau}$s and the other sleptons. However this is compensated in physical effects by the reduced mass of the stau leptons.

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