

LIFETIMES OF HEAVY-FLAVOUR HADRONS – WHENCE AND WHITHER? ¹

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Abstract

A theoretical treatment for the weak decays of heavy-flavour hadrons has been developed that is genuinely based on QCD. Its methodology as it applies to total lifetimes and the underlying theoretical issues are sketched. Predictions are compared with present data. One discrepancy emerges: the beauty baryon lifetime appears to be significantly shorter than expected. The ramifications of those findings are analyzed in detail.

I. Introduction: The Holy Grail and The Hope

While the fertile minds of theorists have spawned many ideas formulated in the world of quarks and hadrons, they have been conspicuously less productive in translating these ideas into the world of hadrons. Heavy-flavour physics presents us with two new perspectives onto this long-standing embarrassment to the theoretical community.

(i) A detailed study of beauty decays in particular addresses essential elements of the Standard Model (hereafter referred to as *SM*) and thus provides us with fundamental probes of it: what are the values of the KM parameters $V(cb)$, $V(ub)$, $V(td)$ and $V(ts)$; can the *SM* account for $B^0 - \bar{B}^0$ oscillations and rare *B* decays; finally, the ‘Holy Grail’ of beauty physics: CP violation. Hadronization should not be seen as exclusively evil in that context. For without it particle-antiparticle oscillations would not occur and it would become even more difficult for CP violation embedded in the quark Lagrangian to manifest itself in observable transitions.

¹Invited Lecture given at the 6th International Symposium on Heavy Flavour Physics, Pisa, Italy, June 6-10, 1995

(ii) On the quark level there obviously exists a *single* lifetime for a given flavour. Differences in the lifetimes of weakly decaying hadrons carrying the same flavour thus provide us with a yardstick for evaluating the impact of hadronization. From $\tau(K^+)/\tau(K_S) \simeq 140$, $\tau(D^+)/\tau(D^0) \simeq 2.5$ and $0.9 \leq \tau(B^-)/\tau(B_d) \leq 1.2$ one infers that deviations of the lifetime ratios from unity decrease monotonically with the heavy-flavour mass m_Q increasing. This is as expected, and it suggests that hadronization effects can be addressed through an expansion in $1/m_Q$. Heavy flavour decays thus constitute an intriguing lab to study QCD in a novel environment with a new probe, namely m_Q . This give us reason to hope that nonperturbative effects can be brought under theoretical control in beauty decays and maybe even in charm decays.

There exist now four second-generation theoretical technologies providing us with tools to deal with heavy-flavour decays, namely QCD sum rules; Monte Carlo simulations of QCD on the lattice; heavy quark effective theory (HQET) and $1/m_Q$ expansions. These technologies are genuinely based on QCD without a need to invoke a ‘deus ex machina’². Among those only the $1/m_Q$ expansion allows to treat inclusive decays. HQET, for example, deals with formfactors in exclusive semileptonic transitions rather than total widths; those – through the phase space – depend strongly on m_Q , whereas it is the special feature of HQET that m_Q is removed from its Lagrangian. However the $1/m_Q$ expansion benefits from the assistance of the other three techniques.

It is actually somewhat misleading to use the term $1/m_Q$ expansion. For it is primarily the inverse of the *energy release* rather than $1/m_Q$ that provides the expansion parameter. The highest accuracy can thus be expected for $b \rightarrow ul\nu$ transitions, followed by $b \rightarrow cl\nu$ and $b \rightarrow u\bar{d}$ decays; a lower precision holds for $b \rightarrow c\bar{u}d$ and $b \rightarrow u\bar{c}s$; the applicability of these tools to the $b \rightarrow c\bar{c}s$ channels on the other hand is suspect – a concern I will repeatedly return to.

The remainder of my talk will be organized as follows: in Sect.II I will summarize the early phenomenology before introducing the $1/m_Q$ methodology for fully integrated rates in Sect.III; in Sect.IV I describe some applications and compare the resulting predictions with present data before concluding in Sect.V. In presenting the material I adopt three guidelines, namely to stress intuitive arguments over more formal ones (those are given elsewhere); to place the arguments into today’s theoretical landscape and to emphasize the practical relevance for experimental studies. A more detailed discussion can be found in Ref.[1].

II. Early Phenomenology: Myths, Legends and Truths

Having been raised in Bavaria and being now in the service of a catholic university, I appreciate that myths and legends quite often contain more than just a kernel of

²It should be kept in mind, though, that they all require an assumption concerning quark-hadron duality that has not been proven yet; I will briefly comment on that later on.

truth. However it typically is very difficult to ascertain *beforehand* what is truth and what is poetic license (or worse). The same situation applies with respect to phenomenological models.

The starting point is the *spectator* process which contributes *uniformly* to the widths of all hadrons H_Q of a given flavour. It rises so rapidly with m_Q , namely $\Gamma_{spect} \propto G_F^2 m_Q^5$ (for $m_Q < M_W$) that it dominates for large m_Q and thus provides the yardstick by which the non-leading (in m_Q) contributions are evaluated. One mechanism was identified for generating differences in the H_Q lifetimes, namely "Weak Annihilation" (=WA) of Q with the light valence antiquark for mesons or "W Scattering" (=WS) with the valence diquark system for baryons³. Such an analysis had first been undertaken for charm decays. Since WA contributes to Cabibbo allowed decays of D^0 , but not of D^+ mesons (in the valence quark description), it creates a difference in $\tau(D^0)$ vs. $\tau(D^+)$. However the WA rate to lowest order in the strong interactions is doubly suppressed relative to the spectator rate, namely by the helicity factor $(m_q/m_c)^2$ with m_q denoting the largest mass in the final state and by the 'wavefunction overlap' factor $(f_D/m_c)^2$ reflecting the practically zero range of the low-energy weak interactions: $\Gamma_{W-X}^{(0)}(D^0) \propto G_F^2 f_D^2 m_q^2 m_c$. Therefore it had originally been suggested that already charm hadrons should possess approximately equal lifetimes. It then came as quite a surprise when observations showed it to be otherwise – in particular since the first data suggested a considerably larger value for $\tau(D^+)/\tau(D^0)$ than measured today. This caused a re-appraisal of the theoretical situation; its results *at that time* can be summarized in three main points.

(i) There is another significant source for a lifetime difference [2]. Cabibbo-allowed nonleptonic D^+ decays – but not D^0 decays – produce two antiquarks in the final state that carry the same flavour: $D^+ = [c\bar{d}] \rightarrow (s\bar{d}u)\bar{d}$. Thus one has to allow for the *interference* between different quark diagrams in D^+ , yet not in D^0 decays; the \bar{d} valence antiquark in D^+ mesons thus ceases to play the role of an uninvolved bystander and a difference in $\tau(D^+)$ vs. $\tau(D^0)$ will arise. This interference turns out to be destructive, i.e. it prolongs $\tau(D^+)$ over $\tau(D^0)$, but only once the QCD radiative corrections have been included! This effect is usually referred to as 'Pauli Interference' (=PI) although such a name would be misleading if it is interpreted as suggesting that the interference is automatically destructive.

(ii) It was argued [3] that the helicity suppression of the WA contribution to D decays can be vitiated. Evaluating explicitly a W -exchange diagram with gluon bremsstrahlung off the initial antiquark line one finds:

$$\Gamma_{W-X}^{(1)}(D^0) \propto (\alpha_s/\pi) G_F^2 (f_D/\langle E_{\bar{q}} \rangle)^2 m_c^5 \quad (1)$$

³A distinction is often made between W exchange in the s and in the t channel with the former case referred to as 'weak annihilation' and the latter as 'W exchange'. This classification is however artificial since the two operators mix already under one-loop renormalization in QCD.

with $\langle E_{\bar{q}} \rangle$ denoting the average energy of the initial antiquark \bar{q} . Using a non-relativistic wavefunction for the decaying meson one has $\langle E_{\bar{q}} \rangle \simeq m_q$. This contribution, although of higher order in α_s , would dominate over the lowest order term $\Gamma_{W-X}^{(0)}$ since helicity suppression has apparently been vitiated and the decay constant f_D is now calibrated by $\langle E_{\bar{q}} \rangle$ with $f_D/\langle E_{\bar{q}} \rangle \sim \mathcal{O}(1)$ rather than $f_D/m_c \ll 1$. The spectator picture would still apply at asymptotic quark masses, since $\Gamma_{W-X}^{(1)}/\Gamma_{spec} \propto (f_D/\langle E_{\bar{q}} \rangle)^2 \rightarrow 0$ as $m_c \rightarrow \infty$ due to $f_D \propto 1/\sqrt{m_c}$. Eq.(1) – if true – would have a dramatic impact on the theoretical description of weak heavy-flavour decays: WA would be enhanced considerably and be quite significant even in beauty decays. Alternatively it had been suggested [4] that the wavefunction of the D meson contains a $c\bar{q}g$ component where the $c\bar{q}$ pair forms a spin-one configuration with the gluon g balancing the spin of the $c\bar{q}$ pair.

Both effects listed above, namely PI and WA, prolong $\tau(D^+)$ over $\tau(D^0)$.

(iii) A very rich structure emerges in the decays of charm baryons [5]: WS is *not* helicity suppressed already to lowest order in the strong coupling; PI affects the Λ_c , $\Xi_c^{0,+}$ and Ω_c widths in various ways, generating destructive as well as constructive contributions! It is then very hard to make reliable numerical predictions for these baryonic lifetimes beyond the overall qualitative pattern:

$$\tau(\Xi_c^0) < \tau(\Lambda_c) < \tau(\Xi_c^+) \quad (2)$$

Reviews of these phenomenological descriptions can be found in [6].

As it turned out, some of the phenomenological descriptions anticipated the correct results: it is PI that provides the main engine behind the D^+-D^0 lifetime ratio; Λ_c is considerably shorter-lived than D^0 ; the observed charm baryon lifetimes do obey the hierarchy stated in eqs.(2).

Nevertheless the phenomenological treatments had significant shortcomings, both of a theoretical and of a phenomenological nature: (i) No agreement had emerged in the literature about how corrections in particular due to WA and WS scale with the heavy quark mass m_Q . (ii) Accordingly no clear predictions could be made on the lifetime ratios among beauty hadrons, namely whether $\tau(B^+)$ and $\tau(B_d)$ differ by a few to several percent only, or by 20 - 30 %, or by even more! (iii) No unequivocal prediction on $\tau(D_s)$ or $\tau(B_s)$ had appeared. (iv) In the absence of a systematic treatment it is easy to overlook relevant contributions, and that is actually what happened; or the absence of certain corrections had to be postulated in an ad-hoc fashion. Thus there existed both an intellectual and a practical need for a description based on a systematic theoretical framework rather than a set of phenomenological prescriptions; this is provided by the $1/m_Q$ expansion.

III. Methodology of the Heavy Quark Expansion for Total Rates

To begin with, the decay dynamics have to be known on the quark level. The charged current operators for a given combination of quark flavours are naturally defined at scale M_W . The matrix elements for the decay process are evaluated at an ordinary hadronic scale μ_{had} ; therefore one has to evolve these operators from M_W down to μ_{had} , which is done perturbatively. Since $\mu_{had}^2 \ll m_Q^2 \ll M_W^2$ one finds that the perturbative QCD corrections have a very sizeable impact on decay rates. Yet they do not generate any lifetime differences among the hadrons H_Q .

The size of the matrix elements is controlled by nonperturbative dynamics. It is here where the $1/m_Q$ expansion benefits from the results of the other three technologies, since those can determine the size of some of the relevant expectation values.

In analogy to the treatment of $e^+e^- \rightarrow hadrons$ one describes the transition rate into an inclusive final state f through the imaginary part of a forward scattering operator evaluated to second order in the weak interactions [7, 8]:

$$\hat{T}(Q \rightarrow f \rightarrow Q) = i Im \int d^4x \{ \mathcal{L}_W(x) \mathcal{L}_W^\dagger(0) \}_T \quad (3)$$

where $\{.\}_T$ denotes the time ordered product and \mathcal{L}_W the relevant effective weak Lagrangian expressed on the parton level. If the energy release in the decay is sufficiently large one can express the *non-local* operator product in eq.(3) as an infinite sum of *local* operators O_i of increasing dimension with coefficients \tilde{c}_i containing higher and higher inverse powers of m_Q . The width for $H_Q \rightarrow f$ is then obtained by taking the expectation value of \hat{T} between the state H_Q . For semileptonic and nonleptonic decays treated through order $1/m_Q^3$ one arrives at the following generic expression[8]:

$$\begin{aligned} \Gamma(H_Q \rightarrow f) = & \frac{G_F^2 m_Q^5}{192\pi^3} |KM|^2 \left[c_3^f \langle H_Q | \bar{Q}Q | H_Q \rangle + c_5^f \frac{\langle H_Q | \bar{Q}i\sigma \cdot GQ | H_Q \rangle}{m_Q^2} + \right. \\ & \left. + \sum_i c_{6,i}^f \frac{\langle H_Q | (\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q) | H_Q \rangle}{m_Q^3} + \mathcal{O}(1/m_Q^4) \right] \quad (4) \end{aligned}$$

where the dimensionless coefficients c_i^f depend on the parton level characteristics of f (such as the ratios of the final-state quark masses to m_Q); KM denotes the appropriate combination of KM parameters, and $\sigma \cdot G = \sigma_{\mu\nu} G_{\mu\nu}$ with $G_{\mu\nu}$ being the gluonic field strength tensor. The last term in eq.(4) implies also the summation over the four-fermion operators with different light flavours q . It is through the quantities $\langle H_Q | O_i | H_Q \rangle$ that the dependence on the *decaying hadron* H_Q , and on non-perturbative forces in general, enters; they reflect the fact that the weak decay of the heavy quark Q does not proceed in empty space, but within a cloud of light degrees of freedom – (anti)quarks and gluons – with which Q and its decay products can interact strongly. These are matrix elements for on-shell hadrons H_Q ; $\Gamma(H_Q \rightarrow f)$ is thus expanded

into a power series in $\mu_{had}/m_Q < 1$. For $m_Q \rightarrow \infty$ the contribution from the lowest dimensional operator obviously dominates; here it is the dimension-three operator $\bar{Q}Q$.

There are six important qualitative features to be noted about this expansion:

(i) If eq.(1) were indeed correct with the scale for the transition rate set by the low energy quantity $\langle E_{\bar{q}} \rangle$ rather than by m_Q , a $1/m_Q$ expansion would be of dubious, if any, value. Fortunately this contribution turns out to be spurious for *inclusive* transitions: when all terms, in particular also those coming from the interference between the WA and the spectator amplitudes, are summed up, all terms of order $1/\langle E_{\bar{q}}^2 \rangle$ and even $1/\langle E_{\bar{q}} \rangle$ cancel[9]; i.e. an expansion purely in powers of $1/m_Q$ holds for inclusive reactions!

(ii) Since $\langle H_Q | \bar{Q}Q | H_Q \rangle = 1 + \mathcal{O}(1/m_Q^2)$, one reads off from eq.(4) that the leading contribution to the total decay width is *universal* for all hadrons of a given heavy-flavour quantum number; i.e., for $m_Q \rightarrow \infty$ one has derived – from QCD proper – the spectator picture! This is not a surprising result; still it is gratifying.

(iii) Contributions of order $1/m_Q$ would dominate all other effects – if they were present! The heavy quark expansion shows unequivocally that they are absent in total rates due to a subtle intervention of the local colour gauge symmetry. This has many important ramifications [1]: e.g., one can infer that most B_c decays are driven by the decay of the \bar{c} inside the B_c meson and that $\tau(B_c)$ is short, namely well below 1 psec – contrary to some claims in the literature.

(iv) Lifetime differences first arise at order $1/m_Q^2$ and are controlled by the expectation values of dimension-five operators. These terms, which had been overlooked in the original phenomenological analyses, generate a lifetime difference between heavy-flavour *baryons* on one side and *mesons* on the other. Yet apart from small isospin or $SU(3)_{fl}$ breaking they do *not* shift the meson lifetimes relative to each other.

(v) Differences in the meson lifetimes emerge at order $1/m_Q^3$ and are expressed through the expectation values of four-fermion operators; those are proportional to f_M^2 with f_M denoting the decay constant for the meson M . Contributions from what is referred to as WA and PI in the original phenomenological descriptions are systematically and consistently included. Further contributions to the baryon-meson lifetime difference also arise at this level due to WS.

(vi) Since the transitions $b \rightarrow c l \nu$ or $c \rightarrow s l \nu$ are described by an isosinglet operator one can invoke the isospin invariance of the strong interactions to deduce for the semileptonic widths

$$\Gamma_{SL}(B^-) = \Gamma_{SL}(B_d) \quad , \quad \Gamma_{SL}(D^+) = \Gamma_{SL}(D^0) \quad (5)$$

and therefore

$$\frac{\tau(B^-)}{\tau(B_d)} = \frac{BR_{SL}(B^-)}{BR_{SL}(B_d)} \quad , \quad \frac{\tau(D^+)}{\tau(D^0)} = \frac{BR_{SL}(D^+)}{BR_{SL}(D^0)} \quad (6)$$

up to small corrections due to the KM [Cabibbo] suppressed transition $b \rightarrow ul\nu$ [$c \rightarrow dl\nu$] which changes isospin by half a unit. The spectator ansatz goes well beyond eq.(5): it assigns the same semileptonic width to all hadrons of a given heavy flavour. Yet such a property cannot be deduced on *general* grounds: for one had to rely on $SU(3)_{Fl}$ symmetry to relate $\Gamma_{SL}(D_s)$ to $\Gamma_{SL}(D^0)$ or $\Gamma_{SL}(B_s)$ to $\Gamma_{SL}(B_d)$ and no symmetry can be invoked to relate the semileptonic widths of mesons and baryons. There is actually a WA process that generates semileptonic decays on the Cabibbo-allowed level for D_s [and also for B_c], but not for the other heavy-flavour states: the hadrons are produced by gluon emission off the \bar{s} [or the \bar{c}] line. Yet since the relative weight of WA is significantly reduced in meson decays, one does not expect this mechanism to change $\Gamma_{SL}(D_s)$ significantly relative to $\Gamma_{SL}(D^0)$. Contributions to the semileptonic widths arise already in order $1/m_Q^2$. Yet on rather general grounds one predicts the expectation values $\langle P_Q|\bar{Q}Q|P_Q\rangle$ and $\langle P_Q|\bar{Q}i\sigma \cdot GQ|P_Q\rangle$ to be largely independent of the flavour of the light antiquark in the meson and therefore

$$\Gamma_{SL}(D_s) \simeq \Gamma_{SL}(D^0) \quad , \quad \Gamma_{SL}(B_s) \simeq \Gamma_{SL}(B_d) \quad (7)$$

On the other hand, as explained below, the values of the expectation values for these operators are different when taken between baryon states and one expects

$$\Gamma_{SL}(\Lambda_Q) > \Gamma_{SL}(P_Q) \quad (8)$$

Through order $1/m_Q^3$ the non-perturbative corrections in eq.(4) are expressed through the expectation values of three operators. A heavy quark expansion yields

$$\langle H_Q|\bar{Q}Q|H_Q\rangle = 1 - \frac{\langle(\vec{p}_Q)^2\rangle_{H_Q}}{2m_Q^2} + \frac{\langle\mu_G^2\rangle_{H_Q}}{2m_Q^2} + \mathcal{O}(1/m_Q^3) \quad (9)$$

where $\langle(\vec{p}_Q)^2\rangle_{H_Q} \equiv \langle H_Q|\bar{Q}(i\vec{D})^2Q|H_Q\rangle$ denotes the average kinetic energy of the quark Q moving inside the hadron H_Q and $\langle\mu_G^2\rangle_{H_Q} \equiv \langle H_Q|\bar{Q}i\sigma \cdot GQ|H_Q\rangle$.

For the chromomagnetic operator one finds $\langle\mu_G^2\rangle_{P_Q} \simeq \frac{3}{4}(M_{V_Q}^2 - M_{P_Q}^2)$, where P_Q and V_Q denote the pseudoscalar and vector mesons, respectively. Therefore

$$\langle\mu_G^2\rangle_B \simeq 0.37 \text{ GeV} \quad , \quad \langle\mu_G^2\rangle_D \simeq 0.41 \text{ GeV} \quad (10a)$$

For Λ_Q and Ξ_Q baryons one has instead

$$\langle\mu_G^2\rangle_{\Lambda_Q, \Xi_Q} \simeq 0 \quad (10b)$$

since the light diquark system inside Λ_Q and Ξ_Q carries no spin.

For the quantity $\langle(\vec{p}_Q)^2\rangle_{H_Q}$ there exists an estimate from a QCD sum rules analysis[13] yielding $\langle(\vec{p}_b)^2\rangle_B \simeq 0.5 \pm 0.1 \text{ GeV}$ and one can expect one from lattice QCD in the foreseeable future. We do have a model-independent lower bound on it[11], namely

$\langle(\vec{p}_b)^2\rangle_B \geq 0.37 \pm 0.1 \text{ GeV}$. The *difference* in the kinetic energy of Q inside baryons and mesons can be related to the masses of charm and beauty hadrons:

$$\langle(\vec{p}_Q)^2\rangle_{\Lambda_Q} - \langle(\vec{p}_Q)^2\rangle_{P_Q} \simeq \frac{2m_b m_c}{m_b - m_c} \cdot \{[\langle M_B \rangle - M_{\Lambda_b}] - [\langle M_D \rangle - M_{\Lambda_c}]\} \quad (11)$$

where $\langle M_{B,D} \rangle$ denote the ‘spin averaged’ meson masses: $\langle M_B \rangle \equiv \frac{1}{4}(M_B + 3M_{B^*})$ and likewise for $\langle M_D \rangle$ ⁴. Using data one finds: $\langle(\vec{p}_Q)^2\rangle_{\Lambda_Q} - \langle(\vec{p}_Q)^2\rangle_{P_Q} = -(0.07 \pm 0.20)(\text{GeV})^2$; i.e., the present measurement of M_{Λ_b} is not yet sufficiently accurate.

The expectation values for the four-quark operators taken between *meson* states can be expressed in terms of a single quantity, namely the decay constant:

$$\langle H_Q(p) | \bar{Q}_L \gamma_\mu q_L \rangle (\bar{q}_L \gamma_\nu Q_L) | H_Q(p) \rangle \simeq \frac{1}{4} f_{H_Q}^2 p_\mu p_\nu \quad (12)$$

where factorization has been assumed. The theoretical expectations for the decay constants are

$$f_D \simeq 200 \pm 30 \text{ MeV} \quad , \quad f_B \simeq 180 \pm 30 \text{ MeV} \quad (13a)$$

$$f_{D_s}/f_D \simeq 1.15 - 1.2 \quad , \quad f_{B_s}/f_B \simeq 1.15 - 1.2 \quad (13b)$$

The size of the expectation values taken between *baryonic* states are quite uncertain at present. There exists more than one relevant contraction, and for the time being quark model estimates provide us with the only guidance! I will return to this point when discussing predictions of baryon lifetimes.

While there are significant uncertainties and ambiguities in the values of the masses of beauty and charm quarks, their difference which is free of renormalon contributions is tightly constrained:

$$m_b - m_c \simeq \langle M \rangle_B - \langle M \rangle_D + \langle(\vec{p})^2\rangle \cdot \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \simeq 3.46 \pm 0.04 \text{ GeV} . \quad (14)$$

This value agrees very well with the one extracted from an analysis of energy spectra in semileptonic *B* decays [14].

In summary: (i) One expects on rather general grounds that the nonperturbative corrections in *beauty* decays amount typically to no more than a few percent with an expansion parameter $\sqrt{\langle\mu_G^2\rangle_B/m_b^2} \sim 0.13$. (ii) The situation in charm decays on the other hand is unclear since the expansion parameter is considerably larger: $\sqrt{\langle\mu_G^2\rangle_D/m_c^2} \sim 0.46$.

⁴The crucial assumption here is that *c* quarks are sufficiently heavy for a $1/m_c$ expansion to be of practical help.

IV. Applications and Comparisons with the Data

There are three applications I want to discuss here: semileptonic branching ratios, extracting $V(cb)$ from $\Gamma_{SL}(B)$ and the lifetimes of charm and beauty hadrons.

(A) The semileptonic branching ratio of B mesons

The present world average yields[17] $BR_{SL}(B) = 0.1043 \pm 0.0024$. A free parton model treatment leads to $BR_{SL}(b)|_{PM} \simeq 0.15$ which is lowered by perturbative QCD corrections: $BR_{SL}(b)|_{pert.QCD} \simeq 0.125 - 0.135$. The data differ from this expectation by $\sim 15 - 20\%$. A priori one would think that nonperturbative corrections transforming $BR_{SL}(b)$ into $BR_{SL}(B)$ could naturally close the gap since they might be of order $\mu_{had}/m_b \sim 10 - 20\%$ for $\mu_{had} \sim 0.5 - 1$ GeV. Yet, as stated above, the leading nonperturbative contributions arise at order $(\mu_{had}/m_b)^2 \sim 1 - 4\%$. A more detailed analysis shows that $BR_{SL}(B)$ is indeed lowered relative to $BR_{SL}(b)$, but only by $\sim 2\%$ [18]. There exists a loophole, though, in that analysis: the energy release in the channel $b \rightarrow c\bar{c}s$ is not large and terms in the expansion that are formally of higher order in $1/m_b$ might actually be quite significant numerically. There is some theoretical evidence that they would indeed enhance $\Gamma(B \rightarrow [c\bar{c}s])$. If $\Gamma(B \rightarrow [c\bar{c}s\bar{q}]) \sim 2 \cdot \Gamma(b \rightarrow c\bar{c}s)$ were to hold, the non-leptonic B width would be enhanced sufficiently to bring the prediction on $BR_{SL}(B)$ into line with the data. Such a resolution would have another observable consequence: it would raise the charm content N_c of the B decay products quite significantly:

$$N_c|_{expected} \simeq 1.25 - 1.3 \quad \text{if} \quad \Gamma(B \rightarrow [c\bar{c}s\bar{q}]) \simeq 2 \cdot \Gamma(b \rightarrow c\bar{c}s) \quad (15a)$$

$$N_c|_{expected} \simeq 1.15 \quad \text{if} \quad \Gamma(B \rightarrow [c\bar{c}s\bar{q}]) \simeq \Gamma(b \rightarrow c\bar{c}s) \quad (15b)$$

At this meeting we heard about a new preliminary CLEO analysis yielding[19]

$$N_c|_{observed} = 1.17 \pm 0.05 \quad (15c)$$

This is the highest value ever reported for this observable, and I am very much inclined to conclude that for the time being the problem of the ‘baffling’ semileptonic branching ratio has faded away – largely due to a higher than originally observed charm content!

Another less publicized puzzle has also found its resolution [8]: the semileptonic branching ratio of D mesons through order $1/m_c^2$ – i.e., before the $D^+ - D^0$ lifetime difference is generated in order $1/m_c^3$ – is estimated to be around 9% rather than the $\sim 15\%$ expected for c quarks. Thus there is no contradiction with the findings that the lifetime difference is produced mainly by PI rather than by WA.

(B) Extracting $|V(cb)|$ from $\Gamma_{SL}(B)$

Measuring $\Gamma_{SL}(B)$ allows to determine $|V(cb)|$ – if the total semileptonic width can reliably be calculated. Considerable progress has been achieved over the last few years

Observable	QCD ($1/m_c$ expansion)	Data
$\tau(D^+)/\tau(D^0)$	~ 2 [for $f_D \simeq 200$ MeV]	2.547 ± 0.043
$\tau(D_s)/\tau(D^0)$	$1 \pm \text{few } \%$	1.12 ± 0.04
$\tau(\Lambda_c)/\tau(D^0)$	$\sim 0.5^*$	0.51 ± 0.05
$\tau(\Xi_c^+)/\tau(\Lambda_c)$	$\sim 1.3^*$	1.75 ± 0.36
$\tau(\Xi_c^+)/\tau(\Xi_c^0)$	$\sim 2.8^*$	3.57 ± 0.91
$\tau(\Xi_c^+)/\tau(\Omega_c)$	$\sim 4^*$	3.9 ± 1.7

Table 1: QCD Predictions for Charm Lifetimes

in determining the nonperturbative as well as perturbative corrections. It might seem at first that these efforts would go for naught since $\Gamma_{SL}(B)$ depends on the fifth power of the beauty *quark* mass m_b with its intrinsic uncertainties. However it turns out that $\Gamma_{SL}(B)$ depends on $m_b - m_c$ rather than on m_b and m_c separately, and this difference is rather tightly constrained, see eq.(14). With that information one finds[20]

$$|V(cb)|_{incl} \simeq (0.0410 \pm 0.002) \cdot \sqrt{\frac{1.5 \text{ psec}}{\tau_B}} \cdot \sqrt{\frac{BR_{SL}(B)}{0.1043}}$$

It has been claimed that such an extraction is quite unreliable since the perturbative expansion of $\Gamma_{SL}(B)$ is ill-behaved: while the $\mathcal{O}(\alpha_S^2)$ corrections have not been fully determined, an estimate of their weight based on the BLM-prescription seem to indicate that they contain large coefficients of around 10 - 20! Closer scrutiny however shows the following: if the theoretically sound ‘running’ mass evaluated around a scale of 1 GeV is employed, the expansion is well-behaved; i.e., the large corrections get absorbed into the definition of the quark mass[21].

(C) Charm Lifetimes

The expectations [9, 10] for the lifetimes of charm hadrons through order $1/m_c^3$ are juxtaposed to the data in Table 1. The agreement with the data is remarkable considering that the expansion parameter is not much smaller than unity here. A few more detailed comments are in order:

- The D^+-D^0 lifetime difference is driven mainly by PI with WA contributing not more than 10 - 20%. Including renormalization down to μ_{had} is numerically essential. Within the accuracy of the expansion, the data are reproduced.
- The D_s-D^0 lifetime ratio can be treated with better theoretical accuracy, namely of order a few percent. The observed near equality of $\tau(D_s)$ and $\tau(D^0)$ represents rather direct evidence for the *reduced* weight of WA in charm *meson* decays[10].
- The success so far in predicting baryon lifetime ratios is even more remarkable, since the baryonic widths receive contributions of *both signs* and the relevant expectation

Observable	QCD ($1/m_b$ expansion)	Data
$\tau(B^-)/\tau(B_d)$	$1 + 0.05(f_B/200 \text{ MeV})^2[1 \pm \mathcal{O}(10\%)] > 1$ (mainly due to <i>destructive</i> interference)	1.04 ± 0.05
$\bar{\tau}(B_s)/\tau(B_d)$	$1 \pm \mathcal{O}(0.01)$	0.98 ± 0.08
$\tau(\Lambda_b)/\tau(B_d)$	$\sim 0.9^*$	0.76 ± 0.06

Table 2: QCD Predictions for Beauty Lifetimes

values are computed in quark models, as indicated by the asterisk in the Table. ⁵

(D) *Beauty Lifetimes*

Quantitative predictions [16] for the lifetime ratios of beauty hadrons through order $1/m_b^3$ are given in Table 2 together with present data. The predictions follow the same general pattern as in charm decays. Yet the deviations of the lifetime ratios from unity are much smaller since $1/m_b^2 \ll 1/m_c^2$; for the same reason one has more faith in the reliability of the $1/m_Q$ expansion for beauty than for charm decays.

The near-equality with $\tau(B_d)$ refers to the *average* B_s lifetime, $\bar{\tau}(B_s)$. For the difference in the lifetimes of the two B_s mass eigenstates one predicts[15]

$$\frac{\Delta\Gamma(B_s)}{\bar{\Gamma}(B_s)} \equiv \frac{\Gamma(B_{s,short}) - \Gamma(B_{s,long})}{\bar{\Gamma}(B_s)} \simeq 0.18 \cdot \frac{(f_{B_s})^2}{(200 \text{ MeV})^2}, \quad (17)$$

i.e., the largest lifetime difference among B^- , B_d and B_s mesons is expected to be generated by $B_s - \bar{B}_s$ oscillations! One can search for the existence of two different B_s lifetimes by comparing $\tau(B_s)$ as measured in $B_s \rightarrow \psi\eta/\psi\phi$ on one hand and in $B_s \rightarrow l\nu X$ on the other:

$$\tau(B_s \rightarrow l\nu D^{(*)}) - \tau(B_s \rightarrow \psi\eta/\psi\phi) \simeq \frac{1}{2}[\tau(B_{s,long}) - \tau(B_{s,short})] \quad (18)$$

Whether an effect of the size predicted in eq.(17) is large enough to be ever observed in a real experiment, is unclear. Nevertheless one should search for it even if one has sensitivity only for a 50% lifetime difference. For while eq.(17) represents the best presently available estimate, it is not a ‘gold-plated’ prediction. It is conceivable that the underlying computation *underestimates* the actual lifetime difference!

The prediction on $\tau(\Lambda_b)/\tau(B_d)$ appears to be in serious (though not yet conclusive) disagreement with the data. The details of what went into that prediction can be found in ref.[1]; here I want to state only the following conclusion. If $\tau(B_d)$ indeed exceeds $\tau(\Lambda_b)$ by 25 - 30 %, then a ‘theoretical price’ has to be paid: (i) The charm mass represents too low of a scale for allowing to go beyond merely qualitative

⁵In passing one should note that a new element enters in $\Gamma(\Omega_c)$: $\langle\mu_G^2\rangle_{\Omega_c} \neq \langle\mu_G^2\rangle_{\Lambda_c} \simeq 0$ since the light di-quark system inside Ω_c carries spin one.

predictions on charm baryon (or even meson) lifetimes, since it appears that corrections of order $1/m_c^4$ and higher are still important; (ii) the present agreement between theoretical expectations and data on charm baryon lifetimes is largely accidental and most likely would not survive in the face of more precise measurements! At the same time an intriguing puzzle arises: Why are the quark model results for the relevant expectation values so much off the mark for beauty baryons? It is the deviation from unity in the lifetime ratios that is controlled by these matrix elements; finding a 30 % difference rather than the expected 10 % then represents a 300 % error!

(E) The Ratios of Semileptonic Branching Ratios

Both $\Gamma_{SL}(\Lambda_c)$ and $\Gamma_{NL}(\Lambda_c)$ are predicted to differ substantially from the corresponding quantities for D^0 mesons. There is no intrinsic reason why $BR_{SL}(\Lambda_c) \simeq BR_{SL}(D^0) \times \tau(\Lambda_c)/\tau(D^0) \simeq 0.5 BR_{SL}(D^0)$ should hold; the semileptonic Λ_c branching ratio is probably larger than that. Analogous considerations lead to

$$\langle BR_{SL}(beauty) \rangle < \langle BR_{SL}(B) \rangle, \quad (19)$$

where $\langle BR_{SL}(beauty) \rangle$ denotes the average over *all* beauty hadrons and $\langle BR_{SL}(B) \rangle$ that over B mesons.

V. Summary and Outlook

Inclusive heavy-flavour decays can be treated through an expansion in $1/m_Q$ which allows to express the leading nonperturbative corrections through the expectation values of a small number of dimension-five and -six operators. Basically all such matrix elements relevant for *meson* decays can reliably be related to other observables; this allows to extract their size in a model-independent way. For *baryon* decays, however, one has at present to rely on quark model calculations to determine the expectation values of the dimension-six operators relevant for lifetime differences. The numerical results of such computations are of dubious reliability; predictions for lifetime ratios involving heavy-flavour *baryons* therefore suffer from larger uncertainties than those involving only mesons.

In addition to providing us with a more satisfying theoretical framework the $1/m_Q$ expansion yields also practical benefits: it reproduces the charm lifetime ratios within the expected (rather sizeable) uncertainties due to higher order terms; it predicts unequivocally small differences among $\tau(B^-)$, $\tau(B_d)$ and $\bar{\tau}(B_s)$.

At present there exists one glaring phenomenological problem and – not surprisingly, as just indicated – it concerns baryon decays: the observed Λ_b lifetime is shorter than predicted relative to the B_d lifetime. Unless future measurements move it up significantly, one has to pay a theoretical price for that failure. To the degree that the observed value for $\tau(\Lambda_b)/\tau(B_d)$ falls below 0.9 one has to draw the following conclusion: one cannot trust the numerical results of quark model calculations for

baryonic matrix elements – not very surprising by itself; yet furthermore and more seriously it would mean that $1/m_Q^4$ or even higher order contributions are still relevant in charm baryon decays before fading away for beauty decays. Then one had to view the apparently successful predictions on the lifetimes of charm baryons as largely coincidental!

The theoretical analysis of the lifetimes of heavy-flavour hadrons can be improved, refined and extended: (a) *improved* by a better understanding of quark-hadron duality [12, 11, 1]; (b) *refined* by a reliable determination of in particular, but not only, the baryonic expectation values of the relevant dimension-six operators; (c) *extended* by treating Ξ_b decays.

There is a host of future measurements that will probe and advance our understanding of heavy flavour decays:

(i) While there is no theoretical need to measure $\tau(D^+)/\tau(D^0)$ [or $\tau(\Lambda_c)/\tau(D^+)$] more precisely, it is desirable to determine $\tau(D_s)/\tau(D^0)$ with an accuracy of $\sim 1\%$; this will allow us to quantitatively address some aspects of WA that, despite their subtle nature, are important not only for $\Gamma(D_s)$ decays, but also for lepton spectra in D_s and B^- decays [10].

(ii) Measurements of $\Xi^{0,+}$ and Ω_c lifetimes with at least a 10% accuracy are clearly needed. Then one could extract the weight of the various contributions quantitatively and compare it with their theoretical evaluation; this in turn would enable us to isolate discrepancies between data and predictions in charm decays and shed light on problems we encounter when extrapolating to beauty baryon decays.

(iii) Likewise one has to measure $\tau(\Lambda_c)$, $\tau(\Xi_c^-)$ and $\tau(\Xi_c^0)$ *separately*.

(iv) It is mandatory to confirm that $\tau(B_d) \simeq \bar{\tau}(B_s)$ holds within an accuracy of very few percent and to verify that $\tau(B^+)$ *exceeds* $\tau(B_d)$ by a few to several percent. A *future* discrepancy between the predictions on $\tau(B^+)/\tau(B_d)$ or $\tau(B_d)/\bar{\tau}(B_s)$ and the data – in particular an observation that the lifetime for B^+ mesons is *shorter* than for B_d mesons – would have quite fundamental consequences. For the leading deviation of these ratios from unity arises at order $1/m_b^3$ and should provide a good approximation since the expansion parameter is small: $\mu_{had}/m_b \sim 0.13$. The size of this term is given by the expectation value of a four-fermion operator expressed in terms of f_B . A failure in this simple situation would raise very serious doubts about the validity or at least the practical relevance of the $1/m_Q$ expansion for treating fully inclusive nonleptonic transitions; at best this would leave semileptonic transitions in the domain of their applicability. Such a breakdown of quark-hadron duality would *a priori* appear as a quite conceivable and merely disappointing scenario. However such an outcome would have to be seen as quite puzzling *a posteriori*; for in our analysis we have not discerned any sign indicating the existence of such a fundamental problem or a qualitative distinction between nonleptonic and semileptonic decays [23, 11]. Thus

even a failure would teach us a valuable, albeit sad lesson about the intricacies of the strong interactions; for the heavy quark expansion is directly and unequivocally based on QCD with the only additional assumption concerning the workings of quark-hadron duality!

Acknowledgements: I am deeply indebted to my collaborators for their unusual generosity with which they shared their insights with me. It has been a genuine pleasure to attend this inspiring and beautifully organized meeting in Pisa which houses one of the most glorious ensemble of Western architecture. Special treats were provided for all the senses of the participants, in particular also for their ears! This work was supported by the National Science Foundation under grant number PHY 92-13313. I also thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for partial support during the completion of this manuscript.

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