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BRST Cohomology and Vacuum Structure of Two-Dimensional Chromodynamics

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Abstract

Using a formulation of QCD_2 as a perturbed conformally invariant theory involving fermions, ghosts, as well as positive and negative level Wess-Zumino-Witten fields, we show that the BRST conditions become restrictions on the conformally invariant sector, as described by a G/G topological theory. By solving the corresponding cohomology problem we are led to a finite set of vacua. For $G = SU(2)$ these vacua are two-fold degenerate.

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Quantum Chromodynamics in 1+1 dimensions (QCD_2) has been subject of numerous investigations in the past twenty five years [1], [2]. However, unlike its abelian counterpart, the exactly soluble Schwinger model [9], one had up to recently no hint at its exact integrability. Moreover, traditional topological arguments based on instantons suggest that the vacuum of QCD_2 is unique, unlike the case of the Schwinger model, where this vacuum is known to be infinitely degenerate. However, arguments have been presented [3] in favor of the existence of a discrete, but finite set of QCD_2 vacua in higher representations of the fermions.

The recent formulation [4] of QCD_2 as a perturbed conformally invariant Wess-Zumino-Witten-type theory [5] turns out to provide an appropriate starting point for a dynamical investigation of the physical Hilbert-space structure of QCD_2. The fundamental framework is provided by the BRST analysis of QCD_2 in this formulation, as carried out in ref. [7]. In particular it will be our aim to investigate the possible existence of degeneracy of the QCD_2 vacuum.In this respect it will be useful to point out the parallelisms with the Schwinger model in the decoupled formulation.We shall show that the conformally invariant sector of QCD_2 is described by a level-one G/G topological field theory, thus allowing for a complete classification of the ground states. By explicitly solving the cohomology problem for G=SU(2), we find that the vacuum is two-fold degenerate in the left and right moving sector, respectively.

In order to provide the necessary framework, we briefly review the essential results of refs. [4],[7].

In the light-cone gauge $A_+ = 0$, the QCD_2-partition function reads[2]

$$Z = \int \mathcal{D}A_- \int \mathcal{D}\chi_1^{(0)} \mathcal{D}\chi_1^{\dagger(0)} \int \mathcal{D}\psi_2 \mathcal{D}\psi_2^\dagger \int \mathcal{D}b_- \mathcal{D}c_- e^{iS_{GF}} \tag{1}$$

with the corresponding gauge-fixed Lagrangian [7]

$$\mathcal{L}_{GF} = tr\frac{1}{8}(\partial_+ A_-)^2 + \chi_1^\dagger i\partial_+ \chi_1 + \psi_2^\dagger iD_- \psi_2 + tr(b_- i\partial_+ c_-) \tag{2}$$

where b_-, c_- are Grassman-valued ghost fields arising from the gauge-fixing condition.

[2]Our conventions are: $\chi = \binom{\chi_1}{\chi_2}$, $\gamma_+ = 2\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \gamma_- = 2\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, A_\pm = A_0 \pm A_1$, $\partial_\pm = \partial_0 \pm \partial_1$, $A_\mu = A_\mu^a t^a$, etc., with $[t^a, t^b] = ifabct^c$ and $tr(t^a t^b) = \delta^{ab}, fabcfabd = \frac{1}{2}C_V \delta cd.$

Local decoupled formulation of QCD_2

Making the change of variable

$$\begin{aligned} A_- &= \frac{i}{e} V\partial_- V^{-1} \\ \psi_2 &= V\chi_2 \end{aligned} \tag{3}$$

one is led to the factorized partition function [4, 7]

$$Z = Z_F^{(0)} Z_{gh}^{(0)} Z_V \tag{4}$$

where

$$Z_V = \int \mathcal{D}V e^{-i(c_V+1)\Gamma[V] - \frac{i}{8e^2}\int d^2x\ tr[\partial_+(V\partial_- V^{-1})]^2} \tag{5}$$

and

$$\begin{aligned} Z_F^{(0)} &= \int \mathcal{D}\bar{\chi}\mathcal{D}\chi e^{i\int d^2x \bar{\chi} i\partial\!\!\!/ \chi} \\ Z_{gh}^{(0)} &= \int \mathcal{D}b_\pm \mathcal{D}c_\pm e^{i\int d^2x \mathrm{tr}[\mathrm{b_+ i\partial_- c_+ + b_- i\partial_+ c_-}]} \end{aligned} \tag{6}$$

with $\Gamma[g]$ the WZW functional

$$\Gamma[g] = \frac{1}{8\pi}\int d^2x\ tr\partial^\mu g^{-1}\partial_\mu g + \frac{1}{4\pi}\ tr\int_0^1 dr \int d^2x \varepsilon^{\mu\nu}\tilde{g}^{-1}\dot{\tilde{g}}\tilde{g}^{-1}\partial_\mu\tilde{g}\tilde{g}^{-1}\partial_\nu\tilde{g} \tag{7}$$

where $\tilde{g}(1,x) = g(x), \tilde{g}(0,x) = \mathbb{1}$. The partition function exhibits a BRST symmetry in the left- and right-moving sector, implying the existence of conserved right- and left-moving BRST currents [7]

$$\begin{aligned} J_\mp^{(B)} &= \mathrm{tr}\left[c_\mp\Omega_\mp - \frac{1}{2}b_\mp\{c_\mp, c_\mp\}\right] \\ &\partial_\pm J_\mp^{(B)} = 0 \end{aligned} \tag{8}$$

where

$$\begin{aligned} \Omega_- \equiv -\frac{1}{4e^2}\mathcal{D}_-(V)\partial_+(Vi\partial_- V^{-1}) &- \left(\frac{1+C_V}{4\pi}\right) Vi\partial_- V^{-1} \\ &+ \chi_1\chi_1^\dagger + b_-\{c_-, c_-\} \approx 0 \end{aligned} \tag{9}$$

and

$$\Omega_+ \equiv \frac{1}{4e}V^{-1}[\partial_+^2(Vi\partial_- V^{-1})]V - \frac{1+C_V}{4\pi}V^{-1}i\partial_+ V + \chi_2\chi_2^\dagger + b_+\{c_+, c_+\} \approx 0 \tag{10}$$

are first-class constraints, with $\Omega_- \approx 0$ playing the role of the Gauss law. It is interesting to note that

$$\partial_-\Omega_+ = -V^{-1}\partial_+\Omega_- V \tag{11}$$

implying

$$\mathcal{D}_-\partial_+ A_- + (1+C_V)\frac{e^2}{\pi}A_- = 0. \tag{12}$$

Note that the term proportional to C_V has been ignored in the literature.

Non-local decoupled formulation of QCD_2

The partition function (5) involves 4th order derivatives. In order to reduce this order to 2nd order, we introduce an auxiliary field E via the identity

$$e^{\frac{i}{4e^2}\int \frac{1}{2}\mathrm{tr}[\partial_+(Vi\partial_- V^{-1}]^2} = \int \mathcal{D}E e^{-i\int \frac{1}{2}\mathrm{tr}\left[\frac{e^2}{\pi}E^2 + \frac{E}{\sqrt{\pi}}\partial_+(Vi\partial_- V^{-1})\right]}. \tag{13}$$

Making the change of variable [4],[7]

$$E = \sqrt{\pi}\left(\frac{1+C_V}{2\pi}\right)\frac{1}{\partial_+}(\beta^{-1}i\partial_+\beta) \tag{14}$$

and making use of the Polyakov-Wiegmann identity [6]

$$\Gamma[gh] = \Gamma[g] + \Gamma[h] + \frac{1}{4\pi}\int d^2x \ \mathrm{tr}\left(g^{-1}\partial_+ g h\partial_- h^{-1}\right) \tag{15}$$

one arrives at the alternative representation [4],[7]

$$Z = Z_F^{(0)} Z_{gh}^{(0)} Z_{\tilde{V}} Z_\beta \tag{16}$$

with

$$Z_{\tilde{V}} = \int \mathcal{D}\tilde{V} \exp\{-i(1+C_V)\Gamma[\tilde{V}]\} \tag{17}$$

where $\tilde{V} = \beta V$, and

$$Z_\beta = \int \mathcal{D}\beta \exp\left\{ i\Gamma[\beta] + i\left(\frac{1+C_V}{2\pi}\right)^2 e^2 \int \frac{1}{2}\mathrm{tr}\left[\partial_+^{-1}(\beta^{-1}\partial_+\beta)\right]^2 \right\}, \qquad (18)$$

Note that the WZW action enters with negative level $-(1+C_V)$. This will be very important in the following section.

There exist [7] two BRST currents associated with the partition function (16):

$$\tilde{J}_\pm^{(B)} = \mathrm{tr}\left[c_\pm \tilde{\Omega}_\pm - \frac{1}{2} b_\pm \{c_\pm, c_\pm\}\right] \qquad (19)$$

where

$$\tilde{\Omega}_- \equiv \chi_1 \chi_1^\dagger + \{b_-^{(0)}, c_-^{(0)}\} - \frac{1+C_V}{4\pi} \tilde{V} i\partial_- \tilde{V}^{-1} \approx 0 \qquad (20)$$

$$\tilde{\Omega}_+ \equiv \chi_2 \chi_2^\dagger + \{b_+^{(0)}, c_+^{(0)}\} - \frac{1+C_V}{4\pi} \tilde{V}^{-1} i\partial_+ \tilde{V} \approx 0 \qquad (21)$$

represent first class constraints. These shall play a central role in the characterization of the QCD_2 vacuum.

Finally let us rewrite Z_β in (18) in terms of an auxiliary field C_- as follows:

$$Z_\beta = \int \mathcal{D}\beta \int \mathcal{D}C_- e^{iS'} \qquad (22)$$

where

$$S' = \Gamma[\beta] + \int \left[\frac{1}{2}\mathrm{tr}(\partial_+ C_-)^2 + \left(\frac{1+C_V}{2\pi}\right) e\,\mathrm{tr}(C_- \beta^{-1} i\partial_+ \beta)\right]. \qquad (23)$$

By gauging the $gh - \tilde{V} - \beta$ sector following method of ref. [8], one discovers one further constraint

$$\left(\frac{1+C_V}{2\pi}\right) e\beta C_- \beta^{-1} \left(\frac{1+C_V}{4\pi}\right) \tilde{V} i\partial_- \tilde{V}^{-1} - \frac{1}{4\pi}\beta i\partial_- \beta^{-1} + \{b_-, c_-\} \approx 0. \quad (24)$$

As emphasized in [4], this constraint is 2nd class with respect to the constraints (20, 21), and serves to determine the auxiliary field C_-.

The Schwinger model revisited

In order to gain some feeling for the constraints (2.9), (2.10), (2.21) and (2.22) it is useful to see what these constraints correspond to in the $U(1)$ case.

In the $U(1)$ case, $C_V = 0$ (corresponding to decoupled Faddeev-Popov ghost from the outset). Parametrizing V in (3) by

$$V = e^{i2\sqrt{\pi}\phi}, \tag{25}$$

the WZW functional $\Gamma[V]$ and Maxwell term in (2.5) reduce to

$$\Gamma[V] = \int \frac{1}{2}(\partial_\mu \phi)^2, \quad S_{Max} = \left(\frac{2\sqrt{\pi}}{e}\right)^2 \int (\Box\, \phi)^2 \tag{26}$$

respectively, so that the partition function (4) reads

$$Z = Z_F^{(0)} Z_{gh}^{(0)} \int \mathcal{D}\phi e^{i\int d^2x\{-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{\pi}{2e^2}(\Box\phi)^2\}} \tag{27}$$

Notice that ϕ is a negative metric field, corresponding to the fact that the WZW action $\Gamma[V]$ enters in (5) with negative level.

From (26) follows the equation of motion

$$\Box\,(\Box + \frac{e^2}{\pi})\phi = 0 \tag{28}$$

the constraints $\Omega_\pm$ (9) and (10) take the form

$$\Omega_\pm = \frac{\sqrt{\pi}}{2e}\partial_\mu(\Box + \frac{e^2}{\pi})\phi - e\bar{\chi}\gamma_\mu\chi \approx 0 \tag{29}$$

In the spirit of [8] these constraints are obtained by the gauging of the effective action in (3.5) as follows:

$$\begin{aligned}
&\bar{\chi} i\partial\!\!\!/ \chi \to \bar{\chi}(i\partial\!\!\!/ + W\!\!\!\!/\,)\chi \\
&-\int \frac{1}{2}(\partial_\mu\phi)^2 \to -\frac{1}{2}\int (\partial_\mu\phi + \frac{1}{2\sqrt{\pi}}W_\mu)^2 \\
&\frac{\pi}{2e^2}\int (\Box\,\phi)^2 \to \frac{\pi}{2e^2}\int \left[\partial^\mu(\partial_\mu\phi + \frac{1}{2\sqrt{\pi}}W_\mu)\right]^2
\end{aligned} \tag{30}$$

where W_μ is an external field. Parametrizing this field as

$$W_\mu = \epsilon_{\mu\nu}\partial^\nu \psi + \partial_\mu \zeta \tag{31}$$

one finds

$$\begin{aligned} -\frac{1}{2}\int(\partial_\mu\phi)^2 &\to -\frac{1}{2}\int(\partial_\mu\tilde{\phi})^2 + \frac{1}{8\pi}\int(\partial_\mu\psi)^2 \\ \frac{\pi}{2e^2}\int(\Box\,\phi)^2 &\to \frac{\pi}{2e^2}\int(\Box\,\tilde{\phi})^2 \end{aligned} \tag{32}$$

where $\tilde{\phi} = \phi + \frac{1}{2\sqrt{\pi}}\zeta$. The ψ-dependent term cancels against the anomaly arising from the fermionic integration, so that the gauged partition function coincides with (27). Following [8], the variation of the partition function with respect to W_μ then leads to the constraints (29). Notice that the Klein-Gordon operator $(\Box + \frac{e^2}{\pi})$ projects out the massive mode of ϕ satisfying (28), leaving one only with the massless mode. Hence, (29) corresponds to constraints on the massless (conformally invariant) sector of the theory. Indeed, both the curl and divergence of Ω_μ vanishes.

In the Schwinger model it is clear how to separate the massless (negative metric) field from the massive (physical) excitations. The procedure corresponds to the transition from the local to the non-local formulation of section 2, and consists in introducing the auxiliary field E via (13) with the parametrization (26). This leads to the new effective Lagrangian

$$\tilde{\mathcal{L}} = \bar{\chi} i \partial\!\!\!/ \chi + b_+ i\partial_- c_+ + b_- i\partial_+ c_- - \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial^\mu E\partial_\mu E - \frac{e^2}{2\pi}E^2 \tag{33}$$

where

$$\eta = \phi - E \tag{34}$$

The Lagrangian $\tilde{\mathcal{L}}$ plays the role of the "non-local" QCD_2 Lagrangian of section 2.

We see that η is the negative metric, zero mass field in the usual parametrization of the Schwinger model. In the non-abelian formulation, the fields $\beta, \tilde{V}$ and V take the role of E, η and ϕ, respectively, the correspondences being given by $\beta = \exp(-2i\sqrt{\pi}E), \tilde{V} = \exp(2i\sqrt{\pi}\eta)$ as well as (25).

According to our discussion in section 2, we expect two first-class constraints, and one second-class constraint. The first two one obtains by gauging the fermion-eta sector in a way analogous to the first two equations in

(30). This leads to the first-class constraints

$$\tilde{\Omega}_\mu := \bar{\chi}\gamma_\mu\chi - \frac{1}{2\sqrt{\pi}}\partial_\mu\eta \approx 0 \tag{35}$$

which replace the constraints (20) and (21) in the abelian case. Condition (35), when implemented on the physical states, is just the familiar requirement [10]

$$\partial_\mu(\varphi + \eta)|phys\rangle = 0 \tag{36}$$

where φ is the "potential" of the free fermionic current

$$\bar{\chi}\gamma_\mu\chi = \frac{-1}{2\sqrt{\pi}}\partial_\mu\varphi. \tag{37}$$

Condition (36) characterizes the physical Hilbert space of the Schwinger model, and in particular its ground state structure, which turns out to be infinitely degenerate. In the non-abelian case, this role is taken up by the constraints (20) and (21).

In the notation of this section, the partition function (22) takes the form

$$Z_\beta = \int \mathcal{D}\beta\mathcal{D}C_- e^{i\int\mathcal{L}'} \tag{38}$$

with

$$\mathcal{L}' = -\frac{1}{2}(\partial_\mu\eta)^2 + \frac{1}{2}(\partial_\mu E)^2 + C_-\partial_+E + \frac{1}{2}(\partial_\mu C_-)^2 \tag{39}$$

The gauging of $\mathcal{L}'$, following the procedure of ref. [8] corresponds to the replacement of $\mathcal{L}'$ by

$$\mathcal{L}_W = \mathcal{L}' + W_+(\partial_-\eta + \partial_-E - \frac{e}{\sqrt{\pi}}C_-) \tag{40}$$

with $W_+ = \partial_+\phi$. Expression (40) can be written as

$$\mathcal{L}'' = -\frac{1}{2}(\partial_\mu\eta')^2 + \frac{1}{2}(\partial_\mu E')^2 + C_-\partial_+E' + \frac{1}{2}(\partial_\mu C_-)^2 \tag{41}$$

with $\eta' = \eta + \phi,\ E' = E + \phi$. Hence the partition functions associated with $\mathcal{L}''$ and $\mathcal{L}'$ coincide, implying the constraint

$$-\frac{e}{\sqrt{\pi}}C_- - \partial_-\eta + \partial_-E = 0 \tag{42}$$

which is just the abelian version of (24).

The physical Hilbert space of the Schwinger model factorizes into a massive Fock space and a massless one. The condition (36) only implies a restriction on the massless (conformally invariant) sector which describes the ground state of the theory. This restriction is equivalent to the corresponding BRST condition. Indeed, the action associated with the Lagrangian (33) is invariant under the BRST transformation

$$\begin{aligned}
&2\sqrt{\pi}\delta\eta = -i\epsilon c_- \\
&\delta\chi_1 = \epsilon c_-\chi_1,\ \delta\chi_2 = 0 \\
&\delta c_- = \delta c_+ = 0 \\
&\delta b_- = -\epsilon\chi_1^\dagger\chi_1 - \frac{\epsilon}{2\sqrt{\pi}}\partial_-\eta,\ \delta b_+ = 0
\end{aligned} \tag{43}$$

and a similar transformation obtained by the substitutions $\chi_1 \leftrightarrow \chi_2,\ c_\pm \leftrightarrow c_\mp$ and $b_\pm \leftrightarrow b_\mp$.

These symmetries imply the conservation of the BRST currents,

$$\tilde{J}_\pm^{(B)} = c_\pm\tilde{\Omega}_\pm, \qquad \partial_\mp\tilde{J}_\pm^{(B)} = 0 \tag{44}$$

with $\tilde{\Omega}_\pm$ given by (35). The condition (36) is seen to be equivalent to the BRST condition

$$\tilde{Q}_\pm|\Psi_0\rangle = 0 \tag{45}$$

where $\tilde{Q}_\pm$ is the (nilpotent) charge associated with the currents (44), and $|\Psi_0\rangle$ labels the ground states.

The matter and negative metric part of $\tilde{\Omega}_\pm$ separately satisfy a Kac-Moody algebra with level $k = 1$ and $k = -1$, respectively.

As is well known, there exists an infinite set of solutions of (45). This is quite unlike the case of QCD_2, where the vacuum degeneracy is finite, as we now demonstrate.

The QCD_2 vacuum

From our discussion in section 2 we conclude that the physical states of QCD_2 are obtained by solving the BRST conditions

$$Q_\pm^{(B)}|\Psi\rangle = 0, \qquad |\Psi\rangle \in \mathcal{H}_{phys} \tag{46}$$

with identification of states differing by BRST exact states. In (46), $Q_{\pm}^{(B)}$ are the charges associated with the BRST currents (8) or, equivalently, (19). We shall preferably work with the currents (19) of the non-local formulation, where the BRST condition (46) becomes a restriction on the conformally invariant sector of $\mathcal{H}_{phys}$ describing the ground states $|\Psi_0\rangle$ of QCD_2:

$$Q_{\pm}^{(B)}|\Psi_0\rangle = 0 \tag{47}$$

The crucial observation now is that the solution of (47) in the conformally invariant $f{-}gh{-}\tilde{V}$ sector of (16) is identical to the solution of the cohomology problem of a level one G/G coset WZW model, which corresponds to a topological field theory. In the following we shall restrict ourselves to $G = SU(2)$.

The constraints $\Omega_{\pm}$ involve the matter currents

$$J_{\pm}^{a} = \frac{1}{2}\bar{\chi}t^{a}\gamma_{\pm}\chi \tag{48}$$

the negative metric-field currents

$$\tilde{J}_{-}^{a} = -\frac{1+C_V}{4\pi}tr\left(t^a\tilde{V}i\partial_{-}\tilde{V}^{-1}\right),\ \tilde{J}_{+}^{a} = -\frac{1+C_V}{4\pi}tr\left(t^a\tilde{V}^{-1}i\partial_{+}\tilde{V}\right) \tag{49}$$

and the ghost currents

$$J_{\pm}^{a(gh)} = f_{abc}b_{\pm}^{b}c_{\pm}^{c} \tag{50}$$

Since the two light-cone components can be treated independently, we shall omit the subscript $\pm$ from here on.

The solution of the BRST condition (47) is constructed in terms of the highest weight eigenstates $|J,\tilde{J}\rangle$ of the charges associated with isospin-3 component of the currents (48) and (49) (zero modes in the corresponding Laurent expansion)

$$J_0^3|J,\tilde{J}\rangle = J|J,\tilde{J}\rangle,\ \tilde{J}_0^3|J,\tilde{J}\rangle = \tilde{J}|J,\tilde{J}\rangle \tag{51}$$

with the highest weight condition

$$J_0^{1+i2}|J,\tilde{J}\rangle = 0,\ \tilde{J}_0^{1+i2}|J,\tilde{J}\rangle = 0. \tag{52}$$

We define our Fock space be requiring that the state $|J,\tilde{J}\rangle$ be annihilated by the "positive" frequency parts of the currents (47)-(49):

$$J_{n>0}^{a}|J,\tilde{J}\rangle = 0, \quad \tilde{J}_{n>0}^{a}|J,\tilde{J}\rangle = 0 \tag{53}$$

$$c^a_{n>0}|J,\tilde{J}\rangle = 0, \quad b^a_{n>0}|J,\tilde{J}\rangle = 0 \tag{54}$$

In (53) and (54) the subscript n labels the modes in the Laurent expansion of the corresponding operators. Since b^a and c^a are canonically conjugate fields, we further require

$$b^a_0|J,\tilde{J}\rangle = 0 \tag{55}$$

In order to obtain the states satisfying the BRST condition (47), in terms of the states $|J,\tilde{J}\rangle$ defined above, we follow closely the work of ref. [12] and make use of the Wakimoto realization [11] of the (level one) Kac-Moody currents J^a and $\tilde{J}^a$:

$$\begin{aligned} J^+_n &= a_n \\ J^3_n &= \sqrt{\frac{3}{2}}\phi_n + \sum_m a_m d_{n-m} \\ J^-_n &= n d_n - \sqrt{6}\sum_{n,m} \phi_m d_{n-m} \\ &- \sum_{m,n,k} d_m d_k a_{n-m-k}, \end{aligned} \tag{56}$$

where a_n and d_m are canonically conjugate pairs, $[d_n, a_m] = \delta_{m+n}$ and $[\phi_m, \phi_n] = m\delta_{m+n}$. In the case of $U(1)$ (Schwinger model), $a_m = d_m = 0$, and $\sqrt{\frac{3}{2}}\phi_n \to \frac{1}{2\sqrt{\pi}}\partial\varphi$, where the field φ is defined in (37). An analogous decomposition in terms of $\tilde{a}_n, \tilde{d}_n$ and $\tilde{\phi}_n$ is made[3] for $\tilde{J}^{\pm}_n, \tilde{J}^3_n$

Expressing the BRST charge in terms of the Wakimoto variables, it can be decomposed into terms of given degrees by attributing to the fields $c, d, \tilde{d}$ and $\phi^+(h, a, \tilde{a}, \phi^-)$ the degree $+1(-1)$, where $\phi^{\pm}$ are defined by $\phi^{\pm} = \frac{1}{\sqrt{2}}(\phi \pm i\tilde{\phi})$. It turns out that it suffices to study the states which are annihilated by the operator $Q^{(0)}$ of lowest degree, which is nilpotent, as well as quadratic in the fundamental excitations:

$$Q^{(0)}|\Psi_0\rangle = 0 \tag{57}$$

$$Q^{(0)} = \sum_n c^-_{-n} a_n + 2\sqrt{3}\sum_n c^3_{-n}\phi^-_n + c^+_{-n}\tilde{a}_n \tag{58}$$

The total current, as well as the zero'th mode of the energy momentum tensor (Virasoro operator L_0) correspondingly take the form,

$$\Omega^3_0 = J + \tilde{J} + \sum_n \left[: a_{-n} d_n : - : \tilde{a}_{-n}\tilde{d}_n : - f_{3cb} : c^c_n b^b_{-n} : \right] \tag{59}$$

[3] Actually there are some technical subleties, for which we refer the reader to ref. [12].

$$L_0 = \frac{1}{3}[J(J+1) - \tilde{J}(\tilde{J}+1)] + \sum_{n\neq 0}\Big\{n : a_{-n}d_n$$
$$+\tilde{a}_{-n}\tilde{d}_n + g_{bc}b^b_{-n}c^c_n] : +\phi^+_{-n}\phi^-_n\Big\} \quad (60)$$

As shown in ref. [12], the physical states must also be annihilated by these operators:

$$\Omega^3_0|\Psi_0\rangle = 0, \quad L_0|\Psi_0\rangle = 0 \quad (61)$$

since Ω^3_0 and L_0 turn out to be BRST exact. This implies $\tilde{J} = -J - 1$, as well as the absence of non-zero- mode excitation, and allows one to write the physical states as linear combinations of

$$|n_d, n_{\tilde{a}}, n_+, n_-\rangle = (d_0)^{n_d}(\tilde{a}_0)^{n_{\tilde{a}}}(c^+_0)^{n_+}(c^-_0)^{n_-}|J, -J-1\rangle \quad (62)$$

Implementation of condition (57) is then found to imply [12]

$$|\Psi_0\rangle = c^+_0|J, -(J+1)\rangle \quad (63)$$

In order to completely classify the vacua, we still need to know the values which J can take. From the representation theory of Kac-Moody algebras with central charge k [13] one learns that the allowed values of J are finite and parametrized by

$$2J+1 = r - (s-1)(k+2)$$
$$2J+1 = -r + s(k+2) \quad (64)$$

where $s = 1, ..., q$ and $r = 1, ..., p-1$, where p and q are coprime and defined by $k+2 = p/q$. For our case $k = 1$. We therefore conclude that $J = 0, \frac{1}{2}$, that is, we have a two-fold degeneracy of the ground state.

Summarizing, we have shown, the BRST conditions implied restrictions on the conformally invariant (vacuum) sector of QCD_2. By systematically exploring these conditions, we have shown that for a given chirality the vacuum of $SU(2) - QCD_2$ is two-fold degenerate.

The same conclusion is suggested by other, quite different considerations based on the equivalence of the level $k, G/G$ model on the space Σ to the so-called BF theories, which in turn are equivalent to a Chern-Simons theory on $\Sigma \times S^1$ [14]. The result obtained in [14] for $G = SU(2)$ in particular can be interpreted as the existence of $k+1$ states. This agrees with the result (64) obtained from representation theory, which for k=integer($s = 1$) reduces to just one condition, $2J + 1 = r, r = 1, ..., k + 1$. In our case, $k = 1$.

In the case $G = SU(N)$ we also expect a discrete, growing number of vacua, in accordance with the results of ref. [14]. In the case of the Schwinger model [9] and its generalization to the Cartan subalgebra of $U(N)$ [15], the vacuum is infinitely degenerate. In the $U(1)$ case this is known since the work of Lowenstein and Swieca [10]; in the framework of section 4, these vacua are given by $|\Psi_0\rangle = |J, -J\rangle$, and the infinite degeneracy is a consequence of J taking all integer values.

In the non-local formulation of QCD_2 we have seen that the massive sector (described by Z_β) completely separates from the conformally invariant one (describing the vacuum sector). In [16] the S-matrix has been computed up to pole factors describing the bound states. These factors may differ according to the choice of vacuum.

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